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A Study of Budget Deficit Impact on Household Consumption in Morocco : A Copulas Approach

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Abstract

In this paper, we examine the validity of one of the most controversial issues in the economic research called the *Ricardian equivalence hypothesis*. It stipulates that there is no effect of budget deficit (BD) on household consumption (HC) [3]. Our approach is based on a *Copulas model* in the objective to select the best dependence structure between the two variables BD and HC in Moroccan economy during the 1980-2011 period where other econometric methods, like the vector autoregressive analysis model, do not give any answer. Especially, we use the *Farlie-Gumbel-Morgenstern (FGM) family of copulas* and we show that there is a significant non-linear cause-effect relationship between the BD and HC variables. We determine various conditional probabilities of the household consumption once the budget deficit is fixed. We found that a conditional distribution of the household consumption varies significantly for each fixed level of the budget deficit, and so an expansionist fiscal policy can improve the household consumption unlike to the restrictive one. This result permits us to reject the Ricardian equivalence hypothesis for the Moroccan economy.

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1 Introduction

The impact of the fiscal stimulus on the economy has been the subject of long debate between different currents of the economic thought. [3] studied the impact of fiscal stimulus, financed by the government debt, on the household consumption. He argued that, under certain restrictive assumptions such us the rational expectations of consumers, the infinite time horizon of private agents, intergenerational altruism and the flat-rate tax, a fiscal stimulus has no effect on household consumption. This result is known as the Ricardian Equivalence Hypothesis (REH).

Several empirical studies have been carried out to validate this hypothesis. However, they have given mixed results and so have not reached to a consensus on the REH. [1] highlighted some empirical studies that have attempted to validate this hypothesis on the basis of an aggregate consumption. These studies have led to divergent conclusions by authors who have either signed acceptance of the REH [18, 26, 24, 8, 19, 9, 2] or rejected it [5, 11, 23, 21, 4, 12, 14].

The methodology used by these authors is mainly based on linear regression models. Nevertheless, the lack of such models is related to: (i) some limitations in the case of non-linear links between the explanatory variables and the variables to explain, (ii) the margins variables are assumed to be *a priori* Gaussian.

To close this gap, we introduce *copulas method* which is especially advantageous when the dependence structure between variables is not linear and non-Gaussian. This method has been applied on many fields. For instance, the Copulas method studies the extreme values problem; it is a useful tool to estimate the maximal peak flows values of the tributaries of a river [10]. It is also used to measure the Value-at-Risk (VaR) of portfolios in management risk [6, 7] and in other areas like environmental sciences [15] and bioinformatics [17].

In this article, we suggest the use of the copula approach as a new method to treat the area of public finance. We model the *dependence structure* between macroeconomic and fiscal policy variables. Especially, we state the impact of budget deficit[3] on household consumption in Moroccan economy.

The article is organized as follows. In section 2, we present briefly the copula theory by focusing on the Farlie-Gumbel-Morgenstern (FGM) family. In section 3,

[3] The budget deficit is defined conventionally as the difference between total government revenue and their expenditures.

we model the dependence structure between the BD and HC variables through observed datasets and a bivariate distribution using FGM copulas within Moroccan economy during the 1980-2011 period. Section 4 concludes.

2 Methodology

2.1 Bivariate Copulas theory

The dependence between two or more variables is usually represented by linear models. This dependence is often measured by the Pearson correlation coefficient, which estimates the linear dependence between variables. However, in the case of a nonlinear dependence, alternative measures are proposed in the literature, in particular, those which are based on ranks such as Kendall's tau (τ) and Spearman's rho (ρ). Note that the Kendall's τ is defined by:
$\tau_{XY} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0]$ and Spearman's ρ is defined by :
$\rho_{XY} = 3P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0]$
where (X_1, Y_1) and (X_2, Y_2) are random pairs which are identically distributed. These measures can be explicitly expressed via copulas. To briefly outline the copulas notion, we define a bivariate copula by a distribution function C, with uniform marginal distributions on [0, 1], C: $[0, 1]^2 \rightarrow [0, 1]$. This is justified by a fundamental result known as Sklar's theorem.

Theorem (Sklar, 1959). *Let F_{XY} (.) be a joint distribution function with margins F_X (.) and F_Y (.). Then there exists a copula C(.) such that for all x, y in $\mathbb{R}^2$,*

$$F_{XY}(x, y; \theta_x, \theta_y, \theta_C) = C(F_X\ (x; \theta_x), F_Y\ (y; \theta_y), \theta_C) \qquad (1)$$

If F_X (.) and F_Y (.) are continuous, then C(.) is unique ; otherwise, C(.) is uniquely determined on Range F_X x Range F_Y . Conversely, if C(.) is a copula and F_X (.) and F_Y (.) are the marginal cumulative distribution functions, then the function F_{XY} (.) defined by (1) is a joint cumulative distribution function with margins F_X (.) and F_Y (.).

From Sklar's theorem [25], a bivariate distribution can be expressed into its marginal distributions and a copula function. Thus, copulas allow to model the dependence structure of bivariate random variables by using only its margins.

Several copulas can be used to represent the dependence structure (Table 1). The parameter θ_C indicates the strength of the dependence. Once the coefficient is greater, the dependence is stronger. In fact, the positive value of θ_C indicates a positive dependence while the negative value indicates a negative dependence.

Table 1: Examples of copulas and their properties

Copulas	Parameter θ_c	bivariate Copula $C(u,v)$	Kendall's Tau (τ)
independent		$u_1 u_2$	$\tau = 0$
Clayton	$\theta > 0$	$(u^{-\theta} + v^{-\theta} - 1)^{\frac{-1}{\theta}}$	$\tau = \frac{\theta}{\theta+2}$
Gumbel	$\theta \in [1, +\infty]$	$exp - [(-log(u))^{\theta} + (-log(v))^{\theta}]^{\frac{1}{\theta}}$	$\tau = 1 - \frac{1}{\theta}$
Farlie-Gumbel-Morgenstern	$\theta \in [-1, 1]$	$uv + \theta uv(1-u)(1-v)$	$\tau = \frac{2}{9}\theta$

In our case, the dependence structure between fiscal deficit and household consumption has not *a priori* any sign. It can be positive, negative or null depending on the underlying theoretical framework. Thus, we use Farlie-Gumbel-Morgenstern copula because it offer a great flexibility to model dependence structure without any restriction on parameter sign. In contrast, for example Gumbel and Clayton copulas provide only positive dependence.

2.2 Parameter estimation

In order to estimate equation 1 in Sklar's theorem, we use the maximum likelihood estimation method. Firstly, we differentiate both sides of equation 1 and we get:

$$f_{XY}(x,y;\theta_x,\theta_y,\theta c) = f_X(x;\theta x)\, f_Y(y;\theta y)\mathrm{C}(\mathrm{u},\mathrm{v},\ \theta x,\theta y,\theta c) \tag{2}$$

Where$\mathrm{f_{XY}}$, $\mathrm{f_X}$, $\mathrm{f_Y}$ and $\mathrm{c(u,v)}$ are density functions given by :
$f_{XY} = \frac{\partial^2 F_{XY}}{\partial x \partial y}, f_X = \frac{\partial F_X}{\partial x}, f_Y = \frac{\partial F_Y}{\partial y}$ and $c(u,v) = \frac{\partial^2 C(F_X(u,v),F_Y(u,v))}{\partial u \partial v}$ with :
$u = F_X(x)$ and $v = F_Y(y)$

Secondly, we introduce the logs in equation 2, and we get:

$$L_{XY}\left(\theta_x,\theta_y,\theta_c\right) = L_X\left(x;\theta_x\right) + L_Y\left(y;\theta_y\right) + L_{\mathrm{C}}(\mathrm{u},\mathrm{v},\ \theta_x,\theta_y,\theta_c) \tag{3}$$

where $L_{XY} = \log(f_{XY})$, $L_X = \log(f_X)$, $L_Y = \log(f_Y)$, $L_C = \log(c(\mathrm{u},\mathrm{v}))$

To estimate the parameters (θ_x, θ_y, θ_c), we use usually two methods: *one-step inference* and *two-step inference* (or Inference Function for Margins (IFM)). The first method is based on full maximum likelihood (ML) estimator, which is obtained by maximizing $\mathrm{L_{XY}}$. Under standard regularity conditions, the ML estimator is consistent, asymptotically efficient, and asymptotically normal. However, it is judged more difficult to implement in practice.

To make the inference on the parameters in an easy way, [15] proposed the alternative approach of IFM, which is based on two-step. In the first step, we estimate the marginal parameters θ_x and θ_y by maximizing L_X and L_Y respectively.

In the second step, we estimate the copula parameters θ_c by maximizing L_C, given the estimated parameters for the marginal models. For this second approach, the estimator is consistent and asymptotically normal when the regularity conditions hold as shown in [16]. Moreover, this procedure is highly efficient, easy to implement, and convenient when there are many parameters to be estimated. Thus, we adopt this second method in our application.

2.3 Generating samples from FGM copula

[13] proposed a general algorithm to simulate a copula. They introduced the idea of simulating the joint distribution of (X, Y) by simulating the conditional distribution of X given Y in a recursive manner. The proposed algorithm is summarized by [20] as follows:

1. Generate U and V independent $U(0, 1)$ random numbers.
2. Set $X = F_X^{-1}(U_1)$ and $c_0 = 0$.
3. Calculate recursively Y as the solution of :

$$V = F_Y(Y / x) = \frac{\varphi^{-1}[c_1 + \varphi[F_Y(y)]}{\varphi^{-1}(c_1)} \tag{4}$$

Where $c_1 = \varphi[F_Y(y)]$.

3 Case study: budget deficit and household consumption in Morocco

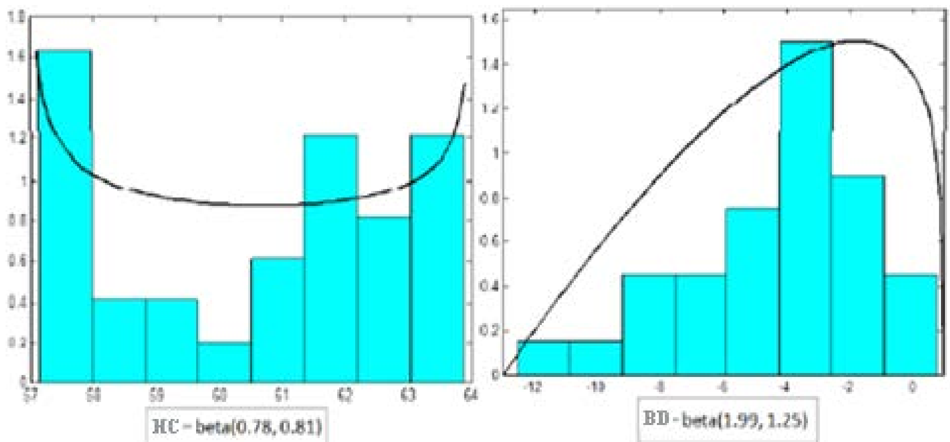
3.1Application the two-step approach

The dependence between random variables is completely described by their joint distribution that can be described by a copula method, using only the margins and copulas functions. Firstly, we adjust the HC and BD variables by an adequate distribution and we pointed out that the variation supports of these variables are respectively represented by the bounded intervals [0, 1] and [-1, 1]. From the boudedness of the intervals, it appears that the best way to fit the HC and BD series data is to use the beta distribution, providing a greater flexibility. We recall that a beta distribution is defined by the following density function:

$$f(x, p, q) = \frac{1}{B(p,q)(b-a)^{p+q-1}}(x-a)^{p-1}(b-x)^{q-1}1_{[a,b]}(x) \tag{5}$$

The parameters p and q have been estimated by the *maximum likelihood estimation* (MLE) method. The estimated parameters for the household consumption are $\hat{p} = 0.78$, $\hat{q} = 0.81$ and for the budget deficit are $\hat{p} = 1.99$, $\hat{q} = 1.25$.

To validate our result, we experiment the test of χ^2 that provide an acceptable compliance between the observed data and the theoretical density (figure 1). We calculate the related *p-value* to the hypotheses $\mathrm{HC} \sim \mathrm{beta}(0.78, 0.81)$ and $\mathrm{BD} \sim \mathrm{beta}(1.99, 1.25)$, obtaining respectively 0.6 and 0.15. This result indicates clearly that the adjustments are adequate[4].



(a) **Histogram of HC with beta density** (b) **Histogram of BD with beta density**

FIGURE 1: Ajustement for household consumption and budget deficit

Secondly, we need to estimate the copula parameter θ_c. However, there are different varieties of copulas. Thus, to make a choice, we calculate the Pearson correlation coefficient between the household consumption and the budget deficit and we find b=-0.41. According to [27], when the Pearson correlation lies between -0.5 and 0.43, we opt for the FGM copula, which is often used in the case of a weak dependence.

To estimate the parameter θ_c via FGM copula, we use the method of moments and we obtain $\hat{\theta}_c = \frac{9}{2}\hat{\tau} = -1.035$[5]. The negative sign of this coefficient indicates that an expansionist fiscal policy (BD<0) can improve household consumption but a restrictive one (BD>0) can not. This result seems to be consistent with the Keynesian theory.

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4 Our calculations are implemented in MATLAB via “chi2gof " function.

5 We note that in the case of the smaller sample, the maximum likelihood procedures and the method of moments lead to the same estimation [22].

3.2 Simulation of the joint distribution via Farlie-Gumbel-Morgenstern copula

To simulate the joint distribution, we use the FGM copula and we give the following function of the bivariate distribution (HC, BD):

$$f_{HC,BD}(x,y)=f_{HC}(x)\,f_{BD}(y)-1.035\,f_{CM}(x)\,f_{BD}(y)\big(1-2F_{HC}(x)\big)\big(1-2F_{BD}(x)\big) \quad (6)$$

where f_{HC} and f_{BD} represent density functions of HC and BD respectively while F_{HC} and F_{BD} their cumulative distribution functions. The graphic representation of density distribution is given in Figure 2:

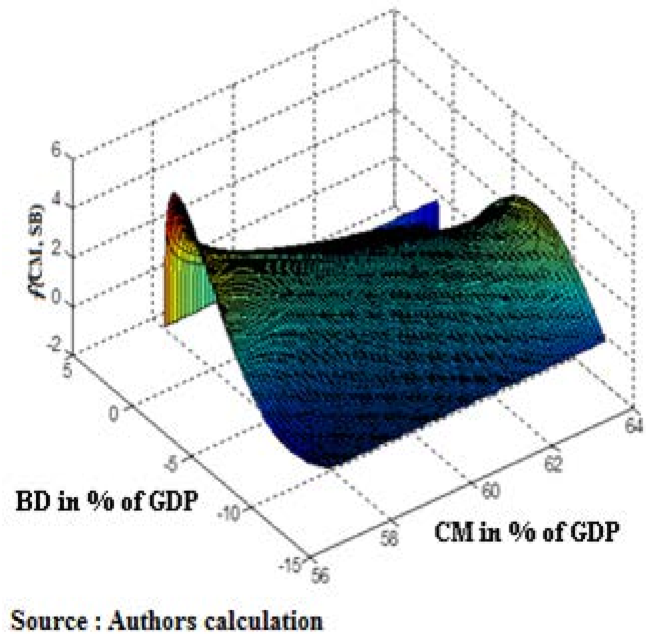
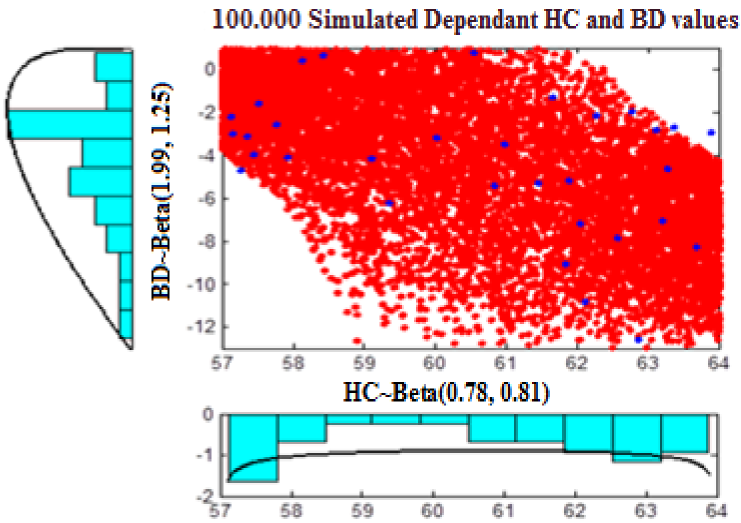


Figure 2: The joint distribution of deficit budget and household consumption

Using the algorithme presented in section 2.3, 100.000 has been simulated from the bivariate distribution (HC, BD).



Source : Authors calculation

Notes: The simulated data points are marked in red color. The observed data of (HC, BD) are marked in blue color.

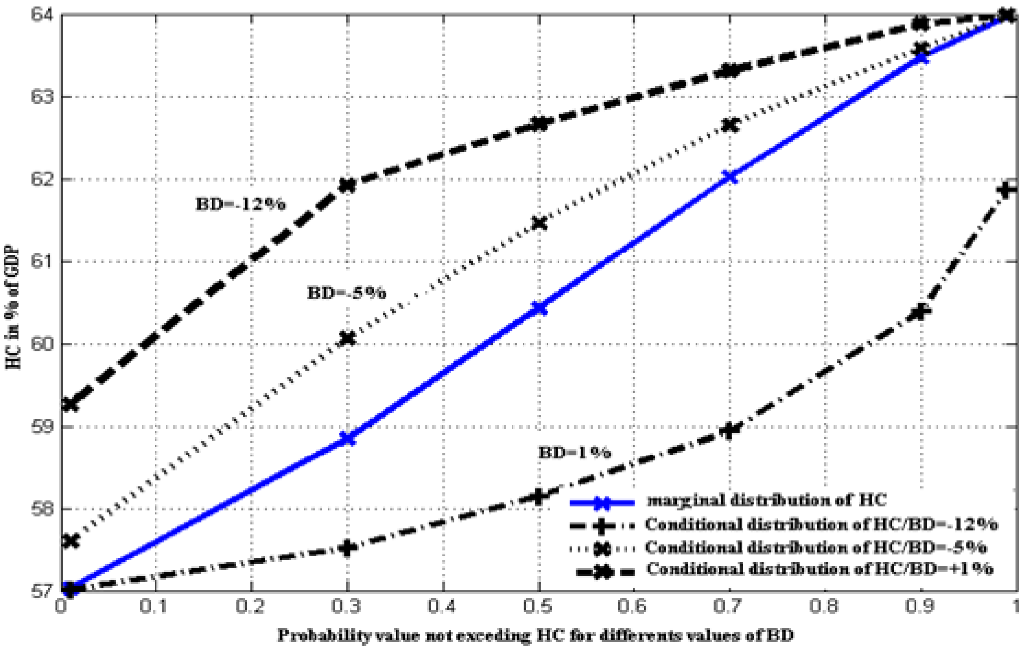
Figure 3: Scatter plot of 100.000 points generated by the dependence structure of FGM copula with beta margins.

Now, we calculate the conditional probability of HC (given BD), using the Monte Carlo simulations. Our results are reported in Table 2 that present the conditional probability of household consumption without exceeding a value *x* for different budget deficit levels greater than a value *y* i.e., P(HC<x/BD>y). Through these conditional probability, the conditional quantiles for any values of budget deficit can be estimated.

Table 2: The conditional probability of the nonexceedence HC for a given values of BD

	-4.03%	**-1.17%**	**-0.21%**	**0.31%**	**0.67%**	**0.81%**	**0.97%**	**P (BD>y)**
60.42%	0,76	0,36	0,19	0,09	0,04	0,014	0,0014	0,5
62.77%	0,86	0,43	0,23	0,11	0,04	0,017	0,0017	0,2
63.47%	0,88	0,45	0,24	0,12	0,05	0,018	0,0021	0,1
63.77%	0,89	0,45	0,24	0,13	0,06	0,018	0,0021	0,05
63.92%	0,90	0,47	0,25	0,15	0,06	0,020	0,0022	0,02
63.96%	0,92	0,48	0,28	0,16	0,08	0,022	0,0023	0,01
63.99%	0,93	0,54	0,33	0,21	0,09	0,031	0,0025	0,001
P (HC<x)	0,5	0,2	0,1	0,05	0,02	0,01	0,001	

To illutrate clearly Table 2, we provide in figure 4 bellow conditional quantiles of the HC variable for some levels of BD for -12%, -5% and 1%.



Source : Authors calculation

Figure 4: Quantiles of the conditional distribution of the consumption for particular levels of budget deficit (-12%, -5% and +1%)

Normally, if the BD and HC variables are statistically independent, the marginal distribution of HC and its conditional distribution (given BD) will be equal. Neverthless, as shown in Figure 4, the conditional distribution of HC is significatively different from the marginal distribution of HC. Thus, we conclude that the BD and HC variables are statically dependent. Moreover, for any given probability, the household consumption is greater, for an expansionist fiscal policy, than a restrictive one, e.g. with a probability of 90%, the household consumption is approximately equal to 64% (in GDP) for a budget deficit of -12% while it is equal to 63.5% for a budget deficit of -5% and becames only 60.2% for a budget surplus of 1%.

4 Conclusion

In this paper, we have studied the validity of REH hypothesis in Morrocan economy during the period 1980-2011. We have used FGM-Copula as an alternative aproach. This method we allowed to inspect the dependence structure between the HC and BD variables . We have foud that this dependence relation is negative and hence the REH hypothesis does not hold for Moroccan economy. Thus, the fiscal policy can improve the household consumption.

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