Theoretical Mathematics & Applications, vol.3, no.1, 2013, 145-159
ISSN: 1792- 9687 (print), 1792-9709 (online)
Scienpress Ltd, 2013

The Effect of Heat Absorption on a Variable Viscosity Reactive Couete Flow under Arrhenius Kinetics

Anthony R. Hassan[1] and Jacob A. Gbadeyan[2]

Abstract

This paper investigates the effect of heat generation on a variable viscosity reactive hydrodynamic Couette flow through a channel where viscosity is temperature dependent. The non – linear equations of momentum and energy governing the flow system are solved using Adomian Decomposition Method. Graphical results for velocity and temperature profiles are presented and discussed.

Mathematics Subject Classification: 80A, 80A32

Keywords: Arrhenius kinetics; Couette flow; heat generating; hydrodynamic and variable viscosity

---

[1] Department of Mathematics, Tai Solarin University of Education, Ijagun, Ogun State, Nigeria, e-mail: anthonyhassan72@yahoo.co.uk
[2] Department of Mathematics, University of Ilorin, Ilorin, Nigeria.

Article Info: *Received* : November 30, 2012. *Revised* : January 19, 2013
Published online : April 15, 2013

1 Introduction

The Couette flow of a reactive fluid has been of interest to many researchers of fluid mechanics due to its numerous engineering and industrial applications, for example, in lubrication coal slurries, polymer solutions or melts, hydrocarbon oils, grease etc. as discussed by N.S. Kobo and O.D. Makinde (2010).

In this respect, various aspects of Couette flow have been studied, for example, J. Lopez – Lemus and R. M. Velasco (1999) studied the characteristics of the Couette flow in terms of temperature jump in the wall and the slip velocity. Also, G. Bodosa and A. K. Borkakati (2003) investigated an unsteady flow of an incompressible and electrically conducting fluid between two horizontal parallel plates, one of which is at rest, other moving in its own plane with a velocity in the presence of a uniform transverse magnetic field. Furthermore, Hazem Ali Attia (2005) examined the effect of hall term on the unsteady Couette flow of a visco elastic fluid under the influence of an applied uniform magnetic field whereas Isom Herron and Fritzner Soliman (2006) in their study showed that without the axial magnetic field, with only the torodial field due to an axial current, the Couette flow is strictly stable to linear axis symmetric disturbances.

However, T. Hayat and A. H. Kara (2006) determined the exact solutions to the partial differential equation that arose in the modeling of a Couette flow of a third – grade fluid with variable magnetic field subject to certain initial and boundary conditions. Also, T. Hayat, M. Sajid and M. Ayub (2007) considered a series solution for generalised Couette flow using homotopy analysis method and its convergence. More so, S. Abelman, E. Momoniat and T. Hayat (2009) presented a numerical solution for the study of Couette of a thermodynamic compatible third grade fluid filling the porous space in a rotating frame where partial slip effects were taken into account.

Considering the importance of reactive fluids because of the physical nature, especially in lubricants which are usually subjected to the differential heat exchange processes like in internal combustion engines. T. Chinyoka (2011) modelled and numerically solved the shear flow of chemically reactive Oldroyd – B liquids subjected to thermal convections. Already, N.S. Kobo et al (2010) investigated the inherent irreversibility associated with the Couette flow of reacting variable viscosity combustible materials under Arrhenius kinetics and evaluated the entropy production. In addition, T. Chinyoka and O. D. Makinde (2011) developed an unconditionally stable and convergent semi – implicit finite difference scheme and utilise it to computationally investigate the transient heat transfer in the generalised Couette flow of a reactive variable viscosity third – grade liquid with asymmetric convective cooling and their result showed that there was a transient increase in both then fluid velocity and temperature with an increase in the reaction strength, viscous heating and fluid viscosity parameter.

All the above investigations are restricted to heat transfer or heat and mass transfer problems. However, of late, the study of heat generation or absorption effects is important in view of several physical problems, such fluids as described in Ahmed M. Salem and Mohamed Abd El – Aziz (2008) undergone exothermic or endothermic chemical reactions in which they considered the problem of heat and mass transfer by steady flow of an electrically conducting fluid over a continuously stretching sheet in the presence of heat generation or absorption effects and mass diffusion of chemical species which is subjected to a strong external magnetic field. In a similar situation, Basant K. Jha and Abiodun O. Ajibade (2009) investigated the natural convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input and concluded that the influence of suction/injection is suppressed by large value of heat sink. Most recently, S. Das, S. K. Guchhait and R. N. Jana (2012) investigated the radiation effects on unsteady MHD free convective Couette flow of heat generation/absorbing fluid and concluded that the interaction between the radiation, MHD effects, buoyancy forces and the heat generation induced by a vertical motion of the plate can affect the configuration of the flow field significantly.

In spite of all these studies, the heat generating fluid with radiation absorption has received little attention; hence the main purpose of this present investigation is to extend the work of N.S. Kobo et al (2010) and to study the effect of internal heat generating on a reactive hydrodynamic fluid through a channel where viscosity is temperature dependent. In the rest of the paper; in section 2, the problem is formulated, non – dimensionalised and is solved in section 3 using Adomian decomposition method. Presentations of analytical results of the problem are shown graphically in section 4 and section 5 gives the concluding remarks.

2 Formulation of the Problem

Consider the steady flow of a reactive, viscous and incompressible fluid flowing in x – direction between two parallel plates of which the upper plate is moving with velocity u and the lower plate is kept stationary with width (H) and length (L). The heat absorption terms in this problem is assumed to be the type used in Basant K. Jha et al (2009)

$$Q = Q_0(T_0 - \overline{T}) \tag{1}$$

Also, the temperature – dependent viscosity $\overline{\mu}$ and chemical reaction kinetic (G) can be expressed following N.S. Kobo et al (2010) as:

$$\overline{\mu} = \mu_0 e^{\frac{E}{R\overline{T}}} \quad \text{and} \quad G = QC_0 A e^{\frac{E}{R\overline{T}}} \tag{2}$$

With these conditions, the continuity, momentum and energy equations governing the fluid flow in dimensionless form is given thus:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\varepsilon^2 \operatorname{Re}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + 2\varepsilon^2 \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \varepsilon^2\frac{\partial v}{\partial x}\right)\right] \tag{4}$$

$$\varepsilon^4 \operatorname{Re}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+2\varepsilon^2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\varepsilon^2\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\varepsilon^2\frac{\partial v}{\partial x}\right)\right] \tag{5}$$

$$\varepsilon^2 Pe\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\varepsilon^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\lambda e^{\frac{T}{1+\beta T}}+\mu\phi-\delta T \tag{6}$$

where $$\phi = Br\left[2\varepsilon^2\left(\frac{\partial u}{\partial x}\right)^2+2\varepsilon^2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\varepsilon^2\frac{\partial v}{\partial x}\right)^2\right].$$

The following non dimensionalise quantities in (1) and (6)

$$y=\frac{\bar{y}}{\varepsilon L},\quad x=\frac{\bar{x}}{L},\quad u=\frac{\bar{u}}{U},\quad v=\frac{\bar{v}}{\varepsilon U},\quad \varepsilon=\frac{H}{L},\quad T=\frac{E(\bar{T}-T_0)}{RT_0^2},\quad p=\frac{\varepsilon^2 L\bar{p}}{\mu U},$$

$$Br=\frac{E\mu_0 U^2}{kRT_0^2}e^{-\frac{E}{RT_0}},\quad \lambda=\frac{QEAa^2C_0}{kRT_0^2}e^{-\frac{E}{RT_0}},\quad \operatorname{Re}=\frac{\rho UL}{\mu_0},\quad \beta=\frac{RT_0}{E},$$

$$Pe=\frac{\rho C_p UL}{k},\quad \mu=\frac{\bar{\mu}}{\mu_0}e^{-\frac{E}{RT_0}}\ \text{and}\ \delta=\frac{Q_0H^2}{k} \tag{7}$$

Invoking the channel aspect ratio $0<\varepsilon<<1$, and then axial pressure gradient $\left(\frac{\partial p}{\partial x}=0\right)$ is zero for Couette flow, then equations (3) to (6) become

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right)=0 \tag{8}$$

$$\frac{d^2T}{dy^2}+\lambda e^{\frac{T}{1+\beta T}}+\mu Br\left(\frac{du}{dy}\right)^2-\delta T=0 \tag{9}$$

where $$\mu=e^{-\frac{T}{1+\beta T}} \tag{10}$$

with the following boundary conditions

$$u=0\quad T=0\qquad on\quad y=0\ \text{and}$$

$$u=1\quad T=0\qquad on\quad y=1 \tag{11}$$

For many cases of interest, we take $\beta=0$. A good example of this approximation is the Bratu – type equation for combustion as in Y.A.S. Aregbesola (2003).

With (10) and $\beta=0$, (8) and (9) reduce to

$$\frac{d}{dy}\left(e^{-T}\frac{du}{dy}\right)=0 \tag{12}$$

and

$$\frac{d^2T}{dy^2}+\lambda e^T+e^{-T}Br\left(\frac{du}{dy}\right)^2-\delta T=0 \tag{13}$$

Integrating (12), we obtain

$$\frac{du}{dy}=me^T \tag{14}$$

where m is a constant of integration to be evaluated using the boundary conditions stated in (11).

Substituting (14) into (13) to obtain

$$\frac{d^2T}{dy^2}+\lambda e^T+m^2Br\,e^T-\delta T=0 \tag{15}$$

and more conveniently, (15) can be written as

$$\frac{d^2T}{dy^2}+\gamma e^T-\delta T=0 \tag{16}$$

where $\gamma=\lambda+m^2\,Br$ and m is to be determined by using the boundary conditions.

3 Method of Solution

For convenience, we take the approximation of the exponential function and show that the result converges for small parameters of δ and γ as:

$$e^T=1+T+\frac{T^2}{2}+O(T^3) \tag{17}$$

Substituting (17) into (16), we obtain:

$$\frac{d^2T}{dy^2}+\gamma(1+T+\frac{T^2}{2})-\delta T=0 \tag{18}$$

Integrating (18), we obtain the integral equation,

$$\frac{dT}{dy} = A + \int_0^y \left[\delta T - \gamma(1 + T + \frac{T^2}{2}) \right] dy \tag{19}$$

Where $T'(0) = A$ is a constant to be determined by using the boundary condition.

Integrating (19) again, we have

$$T = Ay + \int_0^y \int_0^y \left[\delta T - \gamma(1 + T + \frac{T^2}{2}) \right] dy\, dy \tag{20a}$$

In a better form, we obtain the following

$$T = Ay + \delta \int_0^y \int_0^y (T) dy\, dy - \gamma \int_0^y \int_0^y (1) dy\, dy - \gamma \int_0^y \int_0^y (T) dy\, dy - \frac{\gamma}{2} \int_0^y \int_0^y (T^2) dy\, dy \tag{20b}$$

We define a series solution of the form

$$T(y) = \sum_{n=0}^{\infty} T_n(y) \tag{21}$$

Substituting (21) into (20b), we get

$$\begin{aligned} T(y) = \sum_{n=0}^{\infty} T_n(y) = Ay + \delta \int_0^y \int_0^y (\sum_{n=0}^{\infty} T_n(y)) dydy - \gamma \int_0^y \int_0^y (1) dydy - \gamma \int_0^y \int_0^y \sum_{n=0}^{\infty} T_n(y) dydy \\ - \frac{\gamma}{2} \int_0^y \int_0^y \left[\sum_{n=0}^{\infty} T_n(y) \right]^2 dydy \end{aligned} \tag{22}$$

We let the nonlinear term be represented as

$$\sum_{n=0}^{\infty} B_n = \left[\sum_{n=0}^{\infty} T_n(y) \right]^2 \tag{23}$$

Taking the Taylor's expansion, we obtained few terms for B_n as follows

$$\begin{aligned} B_0 &= (T_0(y))^2 \\ B_1 &= 2T_0(y)T_1(y) \\ B_2 &= 2T_2(y)T_0(y) + (T_1(y))^2 \\ B_3 &= 2T_3(y)T_0(y) + 2(T_1(y)T_2(y)) \\ &\ldots \end{aligned} \tag{24}$$

Then the zeroth component of (22) can be written following the modification of A.M. Wazwaz and El-Sayed (2001)

$$T_0[y]=-\gamma\int_0^y\int_0^y(1)dy\,dy \tag{25}$$

$$T_1[y]=Ay+\delta\int_0^y\int_0^y(T_0)dy\,dy-\gamma\int_0^y\int_0^y(T_0)dy\,dy-\frac{\gamma}{2}\int_0^y\int_0^y(B_0)dy\,dy \tag{26}$$

$$T_{n+1}[y]=\delta\int_0^y\int_0^y(T_n)dy\,dy-\gamma\int_0^y\int_0^y(T_n)dy\,dy-\frac{\gamma}{2}\int_0^y\int_0^y(B_n)dy\,dy \tag{27}$$

Obtaining few terms of the series leads to

$$T_0[y]=-\frac{\gamma}{2}y^2 \tag{28}$$

$$T_1[y]=Ay-\frac{\gamma\delta}{24}y^4+\frac{\gamma^2}{24}y^4-\frac{\gamma^3}{240}y^6 \tag{29}$$

$$\begin{aligned}T_2[y]=&-\frac{A\gamma}{6}y^3+\frac{A\delta}{6}y^3+\frac{A\gamma^2}{40}y^5-\frac{\gamma^3}{720}y^6+\frac{\gamma^2\delta}{360}y^6-\frac{\gamma\delta^2}{720}y^6+\frac{\gamma^4}{2240}y^8\\&-\frac{\gamma^3\delta}{2240}y^8+\frac{\gamma^5}{43200}y^{10}\end{aligned} \tag{30}$$

$$\begin{aligned}T_3[y]=&-\frac{A^2\gamma}{24}y^4+\frac{A\gamma^2}{120}y^5-\frac{A\gamma\delta}{60}y^5+\frac{A\delta^2}{120}y^5-\frac{A\gamma^3}{280}y^7+\frac{A\gamma^2\delta}{280}y^7\\&+\frac{\gamma^4}{40320}y^8-\frac{\gamma\delta^3}{40320}y^8-\frac{\gamma^3\delta}{13440}y^8+\frac{\gamma^2\delta^2}{13440}y^8+\frac{A\gamma^4}{4320}y^9-\frac{\gamma^5}{44800}y^{10}\\&+\frac{\gamma^4\delta}{22400}y^{10}-\frac{\gamma^3\delta^2}{44800}y^{10}+\frac{127\gamma^6}{39916800}y^{12}-\frac{127\gamma^5\delta}{39916800}y^{12}-\frac{\gamma^7}{8985600}y^{14}\end{aligned} \tag{31}$$

Taking the partial sum of the series, we have

$$T(y)=\sum_{n=0}^{3}T_n(y)$$

It is important to note that the accuracy of the series can be drastically improved by computing more terms of the series.

$$\begin{aligned}T[y]=&Ay-\frac{\gamma}{2}y^2-\frac{A\gamma}{6}y^3+\frac{A\delta}{6}y^3-\frac{A^2\gamma}{24}y^4+\frac{\gamma^2}{24}y^4-\frac{\gamma\delta}{24}y^4+\frac{A\gamma^2}{30}y^5\\&-\frac{A\gamma\delta}{60}y^5+\frac{A\delta^2}{120}y^5+\frac{\gamma^2\delta}{360}y^6-\frac{\gamma\delta^2}{720}y^6-\frac{\gamma^3}{180}y^6-\frac{A\gamma^3}{280}y^7+\frac{A\gamma^2\delta}{280}y^7\\&-\frac{\gamma^3\delta}{1920}y^8+\frac{19\gamma^4}{40320}y^8-\frac{\gamma\delta^3}{40320}y^8+\frac{\gamma^2\delta^2}{13440}y^8+\frac{A\gamma^4}{4320}y^9-\frac{11\gamma^5}{241920}y^{10}\\&+\frac{\gamma^4\delta}{22400}y^{10}-\frac{\gamma^3\delta^2}{44800}y^{10}-\frac{127\gamma^5\delta}{39916800}y^{12}+\frac{127\gamma^6}{39916800}y^{12}-\frac{\gamma^7}{8985600}y^{14}\end{aligned} \tag{32}$$

Using (32), we obtain *u(y)* by integrating (14) and we get:

$$
\begin{aligned}
u[y] = m[1 &+ \frac{A}{2}y + (\frac{A^2}{6} - \frac{\gamma}{6})y^2 + (\frac{A\delta}{24} - \frac{A\gamma}{6})y^3 + (\frac{\gamma^2}{30} - \frac{A^2\gamma}{24} + \frac{A^2\delta}{30} - \frac{\gamma\delta}{120})y^4 \\
&+ (\frac{19A\gamma^2}{720} - \frac{A^2\gamma}{144} - \frac{17A\gamma\delta}{720} + \frac{A\delta^2}{720})y^5 \\
&+ (\frac{7A^2\gamma^2}{720} - \frac{19\gamma^3}{5040} - \frac{2A^2\gamma\delta}{315} + \frac{17\gamma^2\delta}{5040} + \frac{A^2\delta^2}{315} - \frac{\gamma\delta^2}{5040})y^6 \\
&+ (\frac{A^3\gamma^2}{1152} - \frac{11A\gamma^3}{2688} - \frac{A^3\gamma\delta}{1152} + \frac{A\gamma^2\delta}{280} - \frac{A\gamma\delta^2}{640})y^7 \\
&+ (\frac{A^4\gamma^2\delta}{10368} - \frac{73A^2\gamma^3}{60480} + \frac{83\gamma^4}{181440} + \frac{55A^2\gamma^2\delta}{36288} - \frac{7\gamma^3\delta}{17280} - \frac{A^2\gamma\delta^2}{2160})y^8 \\
&+ (\frac{11\gamma^2\delta^2}{60480} + \frac{A^2\delta^3}{6480} - \frac{\gamma\delta^3}{362880})y^8 + O(y^9)]
\end{aligned} \tag{33}
$$

where m is a constant to be determined using the boundary condition $u(1) = 1$.

4 Discussion of Results

In this paper, we present the effects of internal heat generation on the variable viscosity reactive Couette fluid flow under Arrhenius kinetics through a channel with isothermal plates. Analytical result for the nonlinear ordinary differential equation for the temperature field is presented by Adomian Decomposition method. Effects of internal heat generation parameters (δ) are presented and discussed in this section.

Table 1 shows the computation of rapid convergence of Adomian Decomposition Method. The use of Adomian decomposition method has been applied to a wide class of problems in the sciences; the method has shown reliable results in supplying analytical approximations that converge very rapidly.

Figure 1 shows the blow up of solution γ. This shows that there exist upper and lower solutions with a critical turning point such that $0 \le \gamma < \gamma_c$ and the effect of internal heat generation is significantly observed as γ tends to 2.8078627802 compared to 3.9528312148 in N.S. Kobo et al (2010). The reduction in the value is due to the presence of internal heat caused within the flow system.

The fluid temperature profiles in the normal direction are shown in Figures 2 and 3. It is clearly noted that as heat generation parameter (δ) increases, the temperature reduces in the sense that there is absorption of heat in the process of flow as shown in Figure 2. Also, the contrast is observed in Figure 3 as the fluid temperature increases with increasing value of γ. This is attributed to an increase in heat generation within the flow system due to exothermic reaction of the reactants within the fluid. For both figures, the minimum and maximum temperatures are observed to be at lower and upper plate surfaces respectively.

Table 1: Computation showing rapid convergence of Adomian Decomposition Method

[$y = 0.1$, $\delta = 0.1$and $\gamma = 0.1$]

n	T_n	$\sum_{n=0}^{k} T_n$
0	-0.0005	-0.0005
1	0.00500021	0.00450021
2	1.25005×10^{-10}	0.00450021
3	-1.04175×10^{-9}	0.00450021

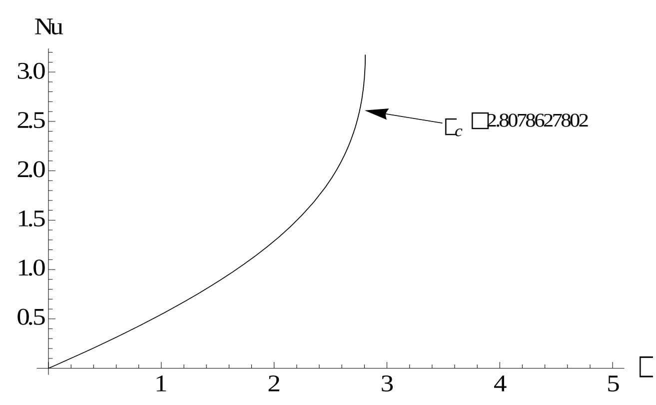

Figure 1: Graph showing the blow – up of γ

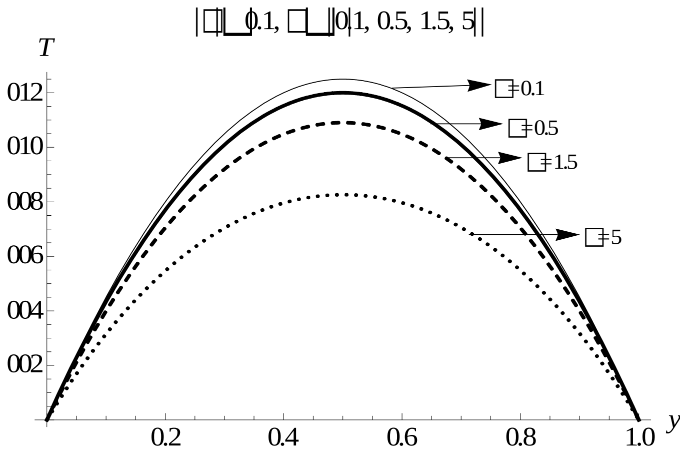

Figure 2: Temperature Profiles for several values of δ

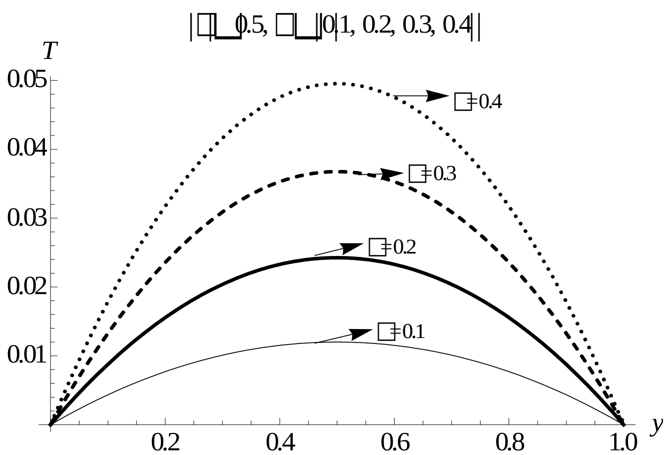

Figure 3: Temperature Profiles for several values of γ

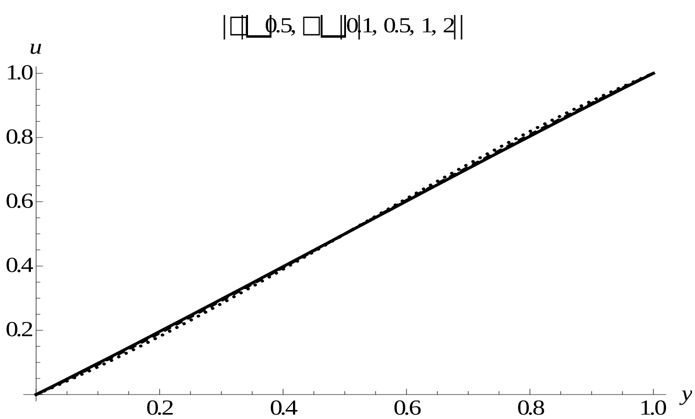

Figure 4: Velocity profiles for several values of γ

Figures 4 and 5 show the velocity profiles for increasing values of δ and γ. It is observed that the fluid velocity is zero at the lower stationary plate and increases gradually towards the upper plate that is moving. As δ and γ increases, the Arrhenius kinetic increases making the profile to increase nonlinearly across the channel to the upper moving plate where y = 1.

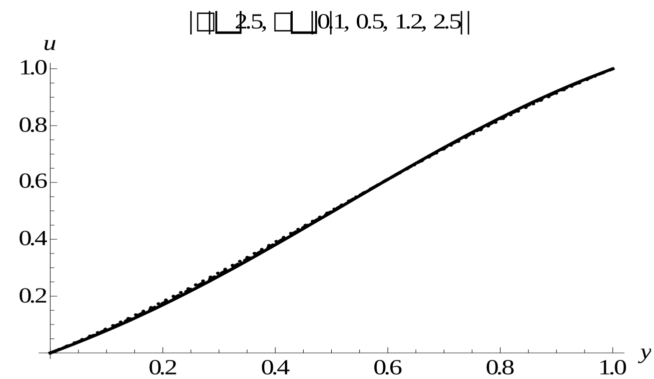

Figure 5: Velocity profiles for several values of δ

5 Conclusion

We have investigated the effect of heat absorption on a variable viscosity reactive hydrodynamic Couette flow through a channel where viscosity is temperature dependent. The non – linear equations of momentum and energy governing the flow system are solved using Adomian decomposition method. Comparing the result to N.S. Kobo et al (2010), there is great influence of heat generation parameter on the flow system and that it is significant in the reduction of the fluid temperature.

Acknowledgement The authors will like to appreciate Dr. Samuel Olumide Adesanya (of Redeemer's University, Redemption City, Nigeria) for his valuable contributions.

List of Constants

$$A = \frac{1}{720720\gamma}\big(8648640 - 1441440\gamma + 288288\gamma^2 - 30888\gamma^3 + 2002\gamma^4 + 1441440\delta - 144144\gamma\delta + 30888\gamma^2\delta + 72072\delta^2 - \sqrt{286}\sqrt{(261534873600 - 87178291200\gamma + 2905943040\gamma^2 - 2957834880\gamma^3 + 480864384\gamma^4 - 61909848\gamma^5 + 5389956\gamma^6 - 293748\gamma^7 + 9163\gamma^8 + 87178291200\delta - 23247544320\gamma\delta + 4410806400\gamma^2\delta - 792215424\gamma^3\delta + 90882792\gamma^4\delta - 6743880\gamma^5\delta + 293748\gamma^6\delta + 11623772160\delta^2 - 2179457280\gamma\delta^2 + 468756288\gamma^2\delta^2 - 43459416\gamma^3\delta^2 + 3371940\gamma^4\delta^2 + 726485760\delta^3 - 72648576\gamma\delta^3 + 14486472\gamma^2\delta^3 + 18162144\delta^4)}\big)$$

$$m = 3628800/(3628800 + 1814400A + 604800A^2 - 604800\gamma - 604800A\gamma - 151200A^2\gamma - 25200A^3\gamma + 120960\gamma^2 + 95760A\gamma^2 + 35280A^2\gamma^2 + 3150A^3\gamma^2 + 350A^4\gamma^2 - 13680\gamma^3 - 14850A\gamma^3 - 4380A^2\gamma^3 - 504A^3\gamma^3 + 1660\gamma^4 + 1743A\gamma^4 + 151200A\delta + 120960A^2\delta - 30240\gamma\delta - 85680A\gamma\delta - 23040A^2\gamma\delta - 3150A^3\gamma\delta + 12240\gamma^2\delta + 12960A\gamma^2\delta + 5500A^2\gamma^2\delta + 252A^3\gamma^2\delta - 1470\gamma^3\delta - 2097A\gamma^3\delta + 5040A\delta^2 + 11520A^2\delta^2 - 720\gamma\delta^2 - 5670A\gamma\delta^2 - 1680A^2\gamma\delta^2 - 126A^3\gamma\delta^2 + 660\gamma^2\delta^2 + 657A\gamma^2\delta^2 + 560A^2\delta^3 - 10\gamma\delta^3 - 219A\gamma\delta^3 + \cdots)$$

Nomenclature

L	Channel characteristic length	μ	Viscosity
E	Activation energy	Re	Reynolds number
G	Reaction Kinetic	U	Velocity scale (ms^{-1})
R	Universal gas constant	Pe	Peclet Number
ρ	Fluid density	k	Thermal conductivity coefficient ($Wm^{-1}k^{-1}$)
ε	Channel aspect ratio		
λ	Frank – Kamenettski parameter	Q	Heat generation term (W)
β	Activation energy parameter	Br	Brinkman number
δ	Heat generation parameter	Q_o	Dimensional heat generation coefficient
T	Fluid temperature (k)		
T_o	plate surface temperature (k)	A	Reaction rate constant
C_o	Reactant species initial concentration	p	Pressure
		u	Axial velocity (ms^{-1})

References

[1] Ahmed M. Salem and Mohamed Abd El – Aziz, Effect of Hall currents and chemical reaction on hydromagnetic flow of a Stretching vertical surface with Internal heat generation/absorption, *J. of Applied Mathematical Modelling,* **32,** (2008), 1236 – 1254.

[2] A.M.Wazwaz and El-Sayed, A new modification of the Adomian decomposition Method for linear and nonlinear operators, *J. of Applied Maths Computation*, **122**, (2001), 393 – 405.

[3] Basant K. Jha and Abiodun O. Ajibade, Free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input, *J. of International Communications in Heat and Mass Transfer*, **36**, (2009), 624 – 631

[4] G. Bodosa and A. K. Borkakati: MHD Couette flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field, *Theoret. Appl. Mech*., **30** (1), (2003), 1 – 9.

[5] Hazem Ali Attia: Unsteady Couette flow with heat transfer of a viscoelastic fluid considering the hall effect, *Kragujevac J. Sci*., **27**, (2005), 5 – 22.

[6] Isom Herron and Fritzner Soliman: The stability of Couette flow in a torodal magnetic field, *J. of Applied Mathematics Letters*, **19**, (2006), 1113 – 1117.

[7] J. Lopez – Lemus and R. M. Velasco: Slip boundary conditions in Couette flow, *J. of Physica A,* **274**, (1999), 454 – 465.

[8] N. S. Kobo and O. D. Makinde: Second Law Analysis for a Variable Viscosity Reactive Couette Flow under Arrhenius Kinetics, *Hindawi Publishing Corporation Mathematical Problems in Engineering*, **2010**, (2010), 1 – 15.

[9] S. Abelman, E. Momoniat and T. Hayat: Couette flow of a third grade fluid with rotating frame and slip condition, *J of Nonlinear Analysis: Real world Applications*, **10,** (2009), 3329 – 3334.

[10] S. Das, S. K. Guchhait and R. N. Jana: Radiation effects on unsteady MHD

free convective Couette flow of heat generation/absorbing fluid; *Int. J. of Computer Applications*, **39**(3), 2012, 0975 – 0987.

[11] T. Chinyoka: Two – dimensional flow of chemically reactive viscoelastic fluids with or without the influence of thermal convection. *J. of Commun Nonlinear Sci Numer Simulat* **16,** (2011), 1387 – 1395.

[12] T. Chinyoka and O. D. Makinde: Analysis of transient generalized Couette flow of a reactive variable viscosity third – grade liquid with asymmetric convective cooling, *J. of Mathematical and Computer Modeling,* **54**, (2011), 160 – 174.

[13] T. Hayat and A. H. Kara: Couette flow of a third – grade fluid with variable magnetic field, *J. of Mathematical and Computer Modeling*, **43**, (2006), 132 – 137.

[14] T. Hayat, M. Sajid and M. Ayub; A note on series solution for generalized Couette flow, *J. of Communications in Nonlinear Science and Numerical Simulation,* **12**, (2007),1481 – 1487.

[15] Y.A.S. Aregbesola: Numerical solution of Bratu problems using the method of weighted residuals, *Electric Journal of Southern African Math. Sci Association (SAMSA),* **3**, (2003), 1 – 7.