

Modelling Haircuts: Evidence from NYSE Stocks

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Abstract

This study aims to model lenders' haircut decision specifically for stocks. The mathematical model showed that lenders face a trade-off between profit and risk exposure in a secured loan; consequently, haircuts are determined in the solvency as a stochastic variable. It was assumed coherently to industry practice that lenders use parametric VaR for collateral valuation. In this model, lenders' probability selection in the VaR approach indicates their risk tolerance, which was captured through to asset liquidity and market volatility expectations. The model implementation on NYSE domestic stocks showed that stocks' haircuts have similar classification pattern with asset liquidity.

JEL classification numbers: G21

Keywords: Asset haircuts, VaR, collateral valuation, collateral constraint

1. Introduction

The aftermath of the 2008 financial crisis inclined academia, finance industry, and financial regulatory institutions to place more importance on leverage and asset liquidity². Different than other financial crisis we have seen in last couple decades, the 2008 financial crisis had started in the debt market and infected to the equity

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² See for example (Acharya & Viswanathan, 2010), (Acharya, Gale, & Yorulmazer, 2011), (Adrian & Boyarchenko, 2012), (Adrian & Shin, 2010), (Bruce & Kenneth, 2013), (Brunnermeier & Pedersen, 2009), (Cao, 2008), (Cipriani, Fostel, & Houser, 2013), (Cornett, McNutt, Strahan, & Tehranian, 2011), (Fostel & Geanakoplos, 2008), (Fostel & Geanakoplos, 2012), (Fostel & Geanakoplos, 2014), (Garleanu & Pedersen, 2011), (Geanakoplos, 2010), (Kahraman & Tookes, 2015), (Longworth, 2010), (Schularic & Taylor, 2012), (Simsek, 2013), (Thornton, 2009), (Tobias, Moench, & Shin, 2013) among many others.

market almost a year later (Krishnamurthy, 2010). While the sharp decrease on asset prices was reducing borrowers' equity, dampen lenders' risk appetite by high volatility and low liquidity resulted significant increases on asset haircuts (collateral constraint). Borrowers were forced to pay the debt more than to roll it. Financial institutions with excessive leveraged portfolios had funding liquidity problem and forced to fire-sell, which decreased market liquidity and accelerated the decrease on asset price. The record high levels of borrowing costs and asset haircuts caused a complete malfunction in some funding markets (Tanju, 2014). Because of that, buying toxic assets and accepting them as collateral for lending were other important policies of the FED besides the lowering interest rates (Adam, Nicolae, & Lasse, 2010).

As it was mentioned above, the significant increases on the asset haircuts was one of the most important triggers of the crisis. That impact was seen immediately on the Repurchase Agreements (Repo) market, which is the biggest short-term debt market for investors³. While haircuts had been around zero for fixed income instruments before the crisis, they gradually increased to 20% for non-subprime related fixed income assets and 100 % for subprime related fixed assets after the crisis (Gorton & Metrick, 2010).

Despite the fact leverage and collateral constraints have been becoming popular subjects in finance literature, the lack of haircut data is an obstacle for empirical studies. We aimed to analyze and model lenders' behavior for asset haircuts specifically for stocks in this study. In that aspect, the study will facilitate empirical studies, which requires haircut data for the stocks. Beside the academic works, it will be useful for risk management practice as well.

Asset haircuts are one of the main components of secure loans. In general, a lender is exposed to two major risks for a loan. The first risk is credit risk, which is a risk not to collect the principle end of the borrowing term and the second risk factor is interest risk, which is a risk not to collect the interest payment on determined periods. The basic idea behind secured loan is holding some asset as collateral to avoid those risks. Since all risky assets are subject to uncertainty, lenders determine a risky assets' collateral value lower than their market price, which is expressed as haircut.

$$\tau_i^t = \frac{P_i^t - P_{i,c}^t}{P_i^t} \quad (1)$$

$P_i, P_{i,c}$ are asset i 's market and collateral value, respectively and $P_i^t > P_{i,c}^t$ for risky assets. τ_i^t is asset i 's haircut.

³ Its estimated daily volume is \$5-\$10 trillion dollars between 2007 and 2009 (Baklanova, Copeland, & McCaughrin, 2015)

From the definition above, the higher collateral value assets have lower haircuts, which allow asset holders to use higher leverage. In other words, haircuts create limits for leverage in secured loans. The leverage factor, which measures the maximum level leverage for an asset in a secured loan, is determined by the haircut.

$$L_i^t = \frac{1 - \tau_i^t}{\tau_i^t} \quad (2)$$

L_i^t is the leverage factor for asset i .

If the haircut is 0.20 for an asset, the asset's leverage factor will be 4. That haircut (or leverage factor) enables an investor to invest up to \$1 on the same asset by using \$0.20 his/her own money and borrowing \$0.80, or the investor can use \$1 the asset as a collateral and can borrow up to \$4 cash.

In the mathematical model, the lender maximizes his/her profit from a secured loan by avoiding credit and interest risk within a probability. Haircuts were used as a decision parameters in the maximization process by holding the assumption that borrowers and lenders are price taker for interest rates. Collateral valuation, which is lender's assessment of the asset has a key role in the process. It is important to note that the asset has different function for the lender and borrower. While, the borrower holds the asset as an investment in a specific period, the lender holds the asset as a collateral against to credit and interest risk. Because of that functional difference, there are three important dissimilarities between the lender and borrower's asset valuations. The first one is the period. The lender just considers the loan period. Financial secured loans like repo contracts are mostly short term financial transactions like daily, weekly or monthly. However, borrowers can invest the asset longer than lenders by rolling the short term loan in the financial credit markets, like repo market. The second dissimilarity comes up with parameters in their decision functions. Borrower considers both expected return and risk of the asset, but lender just the asset's risk. In other words, lender just tries to predict lowest price level for the asset during the borrowing period with a probability. In this parallel, we used parametric Value at Risk (VaR) approach to model lender's assessment. The VaR approach measures the worst expected loss of the asset, under normal market conditions, given the time interval and probability. The VaR approach is a common method in the practice to decide initial margin and haircuts (Longworth, 2010). The intuition to use VaR is clear; if the sum of the credit and interest risk of the loan are lower or equal to asset worst case value, then the lender secures the loan. The third dissimilarity in the valuations is related with asset liquidity. In case borrower fails on the loan requirements, lender liquidates the asset rather than hold it for an investment purpose. Assets with lower liquidity create risk to accelerate decreasing on the asset price. The short liquidation period inhibits the scheduling to be able to reduce asset liquidity impact. On the other hand, because of borrower's investment period relatively longer, asset liquidity is less considerable factor for them. There is a consensus that asset liquidity has a significant impact on leverage constraints in the

finance literature. In general, regulations restrict illiquid assets more than liquid assets for leverage. Similarly, investors have higher risk tolerance for liquid assets.

In this study, we mathematically modeled asset haircuts as a dynamic, asset based, asset liquidity linked collateral constraints. A haircut of an asset (collateral constraint) determines the asset's debt capacity. Asset's debt capacity has two essential functions for an investor. The first function is using the asset's collateral value to buy the asset itself. In that sense, the asset is used as a collateral to finance itself. The second function is using the assets collateral value to have liquidity for funding needs other than buying the asset itself. In practice, the REPO market is a very common application for secured loans for financial assets. Hence, the mathematical model for asset haircuts was designed compatible with Repo market contracts.

2. Literature Review

Black (1972) pioneered borrowing constraints in modern finance theory. His study showed the market equilibrium for asset pricing in the absence of risk-free rate and under the borrowing constraints. Black demonstrated that two-fund separation theorem is still valid even if investors are constrained for borrowing, as long as short-sales are possible. Bernanke and Gertler (1989) studied leverage on a macro level in a neoclassical business cycle model. Geanakoplos (2003)- (2010) showed shocks cause a significant decrease in asset prices and leverage together, which reveals leverage cycles. Hindy and Huang (1995); Geanakoplos (1997)- (2003)- (2010); Gârleanu and Pedersen (2011); Tobias, Moench, and Shin (2013); Cipriani, Fostel, and Houser (2013) studied for leverage and asset pricing in a general equilibrium model.

Due to lack of obtainable data for haircuts, there are limited number empirical studies in the literature. Garleanu & Pedersen (2011) used Credit Default Swaps (CDS) premiums as a proxy. However, CDS premium are available for just limited asset classes. In contrast this study, leverage or borrowing constraints are studied in investor based like, (Adrian, Etula, & Muir, 2014); (Nuño & Thomas, 2013); (Adrian & Shin, 2010); (Adrian, Moench, & Shin, 2010); (Brunnermeier & Yuliy, 2016); (Adrian & Shin, 2014); (Acharya & Viswanathan, 2010); (Gromb & Vayanos, 2002). Investor based studies focus on a leverage ratio from investor's balance sheet⁴.

We defined asset haircuts as a function of asset liquidity and market volatility expectation besides the asset's risk itself in a mathematical model. As we mentioned before, the parametric VaR was used to capture lenders collateral valuation. Investors' risk tolerance, which is reflected as a probability in the parametric VaR

⁴ Empirical studies use aggregate leverage ratio from intermediary's balance-sheets like (Adrian, Etula, & Muir, 2014)

changes depending on market expectations and asset liquidity. Similar to this study, (Adrian & Shin, 2010); (Adrian & Boyarchenko, 2012), (Brunnermeier & Pedersen, 2009); (Zigrand, Shin, & Daniels, 2009) used VaR in their study.

Modelling asset haircuts mathematically as a stochastic variable has not been studied in the literature. This study's one of the most important contributions to literature is that suggesting a mathematical model to predict asset haircuts in a dynamic structure.

3. Methodology

We started the study with a mathematical model to define lenders behavior for haircut in secured loans. The secured loans in financial markets have two fundamental functions. The first function is using the asset's collateral value to buy the asset itself. In that sense, the asset is used as a collateral to finance itself. The second function is using the assets collateral value to have liquidity for funding needs other than buying the asset itself. In practice, the REPO market is a very common application for secured loans for financial assets. Hence, the mathematical is also compatible with Repo market contracts. The study continued with collateral valuation, where parametric VaR was used and a field study to capture lenders' risk appetite change depending on asset liquidity and market volatility. In the last part of the study, the NYSE stocks haircuts were predicted and analyzed through to the model for the period 01/1993-12/2014.

4. The Mathematical Model

In this model, investors use secure loans like repo contracts for borrowing. In a typical secured loan, the lender is exposed to two major risks. The first risk is credit risk, which is a risk not to collect the principle end of the borrowing term. Credit risk is the most important risk factor for a lender. The second risk factor is interest risk, which is a risk not to collect the interest payment on determined periods. In most cases, interest risk is relatively lower than credit risk. The basic idea behind secure loans is holding some asset as collateral to avoid those risks.

As compatible with the industry practice, it was assumed that borrowers and lenders funding costs (interest rates) are determined in a competitive market. Both lender and borrower are price takers. In other words, except the haircut and asset collateral value, all other parameters are given. As will be explained later with details, it is assumed that the lender has a function to determine asset's collateral value. By holding those assumptions, lender's profit function can be written as below for 1\$ asset i ;

$$\begin{aligned}\pi_L &= R_L - C_L = P_i^{t-1}(1 - \tau_i^t)r_b - P_i^{t-1}(1 - \tau_i^t)r_L \\ &= P_i^{t-1}(1 - \tau_i^t)(r_b - r_L)\end{aligned}\quad (3)$$

π_L is the lender's profit function,

R_L and C_L are the lender's revenue and cost function respectively,

P_i is asset i's price,

r_L and r_b are lender and borrowers funding cost respectively,

τ_i asset i's haircut.

In order to avoid credit and interest risks, lender requires the collateral ($P_{i,c}$) from the borrower. Hence, the lender's risk exposure in this secured loan can be expressed as a ratio. Hence, the lender's risk exposure in this secured loan can be expressed as a ratio;

$$\gamma_i^{t-1} = \frac{P_i^{t-1}(1 - \tau_i^t)(1 + r_b)}{P_{i,c}^{t-1}} \quad (4)$$

γ_i is risk exposure ratio for asset i,

$P_{i,c}$ is asset i's collateral value.

If the risk exposure ratio is 1, there is a solvency for the lender. In other words, the lender holds same \$ amount value of the asset as a collateral against the risks of the loan. If the ratio is higher than 1, the lender cautiously holds the asset as a collateral more than total \$ value of risks. Similarly, the ratio values lower than 1 indicate that the lender is taking some degree of those risks by holding less collateral than the total risk of the loan.

Since, it is assumed that all parameters are given but haircuts, the haircuts relation with the profit and risk functions are determinative for lender's behavior. As seen from derivatives below in profit and risk exposure functions respect to haircuts, haircut has reverse relationship with profit and risk exposure. Higher haircuts decrease both profit and risk exposure, while lower haircuts increase them.

$$\frac{d\pi_L}{d\tau_i} = -P_i^{t-1}(r_b - r_{b,c}) < 0 \quad (5)$$

$$\frac{d\gamma_i}{d\tau_i} = -\frac{P_i^{t-1}(1 + r_b)}{P_{i,c}^{t-1}} < 0 \quad (6)$$

Summing up, the lender determines profit and risk exposure through haircut. However, derivatives respect to haircut in profit and risk exposure function show a trade-off between increasing the profit and reducing the risk exposure for the lender. Therefore, lender's profit maximization problem can be written as follows;

$$\begin{aligned} & \max P_i^{t-1}(1 - \tau_i^t)(r_b - r_{b,c}) \\ \text{Subject to } & P_{i,c}^{t-1} \geq \left[\left(P_i^{t-1}(1 - \tau_i^t) \right) (1 + r_b) \right] \\ & \text{where } \tau_i^t \in R \end{aligned} \quad (7)$$

and haircut level, which maximizes lenders profit in **solvency**;

$$\tau_i^t = 1 - \frac{P_{i,c}^{t-1}}{P_i^{t-1}(1 + r_b)} \quad (8)$$

The trade-off between profit and risk exposure requires to determine haircuts on the solvency level. That mathematical approach strongly relies on two assumptions. The first one is, both lender and borrower are price taker for the interest rates. It is very reasonable and realistic assumption due the short-term financial lending market structure. In those markets (typically repo market), parties decide for large volume transactions in a short time. Because of that deals are done primarily based on interest rate, the collateral requirement are shaped mostly by lender. Also, Central Banks control the overnight borrowing rate by being an active player in the market. In any case, investors (except the money authority) are price takers. As a result, interest rate is determined independently from collateral values. Therefore, asset haircuts are directly determined by assets' collateral values, which requires **the second assumption** that the lenders have a function to determine assets' collateral value. The study will continue with analyzes on the second assumption.

4.1. Determining Assets' Collateral Value

Because all risky assets' values change over time, they are subject to uncertainty. On the other hand, the lender's risk, which is sum of the credit and the interest risks is certain from the beginning of the loan process. As shown in Equation (4), in case the collateral value lower than total risk, the lender is exposure some degree of risk. In order to avoid such a situation, the lender values the asset by considering the highest lost on its value with a probability during the borrowing period. The price difference between market and collateral value (haircut) reflects the lender's perceived risk of loss for per unit of the asset in a liquidation at any point of time during the borrowing period.

The parametric VaR approach, which is one of the common methods to determine asset collateral value in industry practice was used to model lenders' assessment for collateral valuation. The VaR approach is statistical form of the question that "What could be the lowest price of the asset for given probability and time window by assuming the normal market condition of asset returns and market to market pricing. Hence, the asset collateral value can be defined by subtracting VaR of the asset from its current price.

$$P_{i,c}^t = P_i^t - VaR_\alpha \quad (9)$$

It is assumed that asset returns follow geometric random walk process, and asset returns' distribution is log-normal.

$$p^t = p^{t-1} e^{\mu + \sigma \varepsilon^t} \quad (10)$$

$$r = \ln\left(\frac{p^t}{p^{t-1}}\right) \sim IID N \quad (11)$$

P is asset price,

r is asset's logarithmic return,

μ asset's expected return,

σ asset return's standard deviation,

ε is a random variable, ($\varepsilon \sim IID N(0,1)$)

Additionally, it is assumed that the mean of the returns (μ) are zero, for simplicity. It is also not far away from the reality, assets return are close to 0 in most repo contracts (like daily, weekly). Hence, the asset pricing process will be as follows;

$$p^t = p^{t-1} e^{\sigma \varepsilon^t \sqrt{T}} \quad (12)$$

It can be written to express gain or loss as follows,

$$P_i^t - P_i^{t-1} = P_i^{t-1} [e^{\sigma \varepsilon^t \sqrt{T}} - 1] \quad (13)$$

$$G_i^t = -[P_i^t - P_i^{t-1}] \quad (14)$$

If, $G_i^t < 0$ is Gain and $G_i^t > 0$ is loss, then VaR can be defined as follows;

$$1 - \alpha = \mathbb{P}(G_i^t \geq VaR) = \mathbb{P}(-[P_i^t - P_i^{t-1}] \geq VaR)$$

$$p = \int_{-\infty}^{\alpha} f(r) dr \quad (15)$$

$$1 - \alpha = \mathbb{P}([P_i^t - P_i^{t-1}] \leq -VaR) \quad (16)$$

$$1 - \alpha = \mathbb{P}\left(P^{t-1}[e^{\sigma\varepsilon^t\sqrt{T}} - 1] \leq -VaR\right) \quad (17)$$

$$1 - \alpha = \mathbb{P}\left([e^{\sigma\varepsilon^t\sqrt{T}}] \leq 1 - \left(\frac{VaR}{P^{t-1}}\right)\right) \quad (18)$$

$$1 - \alpha = \mathbb{P}\left(\sigma\varepsilon^t\sqrt{T} \leq \log\left[1 - \left(\frac{VaR}{P^{t-1}}\right)\right]\right) \quad (19)$$

$$1 - \alpha = \mathbb{P}\left(\varepsilon^t \leq \log\left[1 - \left(\frac{VaR}{P^{t-1}}\right)\right]/\sigma\sqrt{T}\right) \quad (20)$$

$$Z_\alpha = \log\left[1 - \left(\frac{VaR}{P^{t-1}}\right)\right]/\sigma\sqrt{T} \quad (21)$$

$$VaR_\alpha = P^{t-1}(1 - e^{-Z_\alpha\sigma\sqrt{T}}) \quad (22)$$

Therefore, the asset's collateral value can be expressed as follows;

$$VaR_\alpha = P^{t-1}(1 - e^{-Z_\alpha\sigma\sqrt{T}}) \quad (23)$$

The asset collateral value, defined in Equation.9 will be as follow, after the substitution,

$$P_{i,c}^t = P_i^t - VaR_\alpha = P^{t-1} \left[1 - \left(1 - e^{-Z_\alpha\sigma\sqrt{T}}\right)\right] = P^{t-1} e^{-Z_\alpha\sigma\sqrt{T}} \quad (24)$$

Finally, the asset haircut can be written as follow, by using Equation.24

$$\tau_i^t = 1 - \frac{P_{i,c}^{t-1}}{P_i^{t-1}(1 + r_b)} = 1 - \frac{P^{t-1} e^{-Z_\alpha\sigma\sqrt{T}}}{P_i^{t-1}(1 + r_b)} = 1 - \frac{e^{-Z_\alpha\sigma\sqrt{T}}}{1 + r_b} \quad (25)$$

As seen from the Equation. 25, the confidence level has a very special role in the model. Other than confidence level, all parameters are given. Hence, the lender's probability (confidence level) preference is the unique parameter to reflect his/her risk appetite. When the lender wants to take a higher risk, s/he picks higher confidence level, which corresponds lower probability value inside the formula. That confidence level selection's corresponding probability value turns out lower haircut. Lower haircut provides using higher leverage.

As can be noticed in the Equation. 25, the confidence level stands separately from the asset return's risk. Basically, the confidence level decision, which reflects lender's risk appetite is shaped by other factors. We claimed that lenders pick the probability (risk tolerance) depending on **asset liquidity, asset liquidity risk, and market volatility expectation**. Different than our approach, (Acharya & Viswanathan, 2010); (Brunnermeier & Pedersen, 2009); (Garleanu & Pedersen, 2011) focused **market liquidity and leverage** relationship. In those studies, intermediaries face funding liquidity problems as a consequence of contraction on their borrowing capacities due to market frictions.

$$\alpha_i^{t-1} = f(c_i, \sigma_{c_i}, \varphi^t) \quad (26)$$

where c_i, σ_{c_i} expected asset liquidity and asset liquidity risk respectively and φ is market volatility.

Asset liquidity is a term to refer degree of ease and certainty of value, which a security can be converted into cash. Other than cash, every asset has a degree of liquidity that is determined by: How quickly it can be converted to cash and how much the price of the asset must be reduced to do so. It is common view that lenders have low risk tolerance for illiquid assets than liquid assets. That axiom can be explained from the definition above with two aspects of liquidity. In the first aspect, lenders intuitively expect higher uncertainty on illiquid assets values than liquid assets. On the other hand, cash conversion capacity as a second aspect shows that in case lender has to liquidate the collateral, the decrease on the asset price will be higher on illiquid assets than liquid assets.

Hicks addressed, liquidity is spectral (Hicks, 1962). Cash is an extreme case of the spectrum. Therefore, **the fully marketable asset can be even illiquid based on movements of the spectrum of the asset liquidity** (Wood, 1989). Depending on cash conversion capacity aspect, the asset liquidity risk becomes another factor for lenders' risk tolerance. This claim has not been studied nor examined directly in the literature. The idea behind the claim is simple; even if the asset has higher liquidity (higher expected asset liquidity) it could have a higher asset liquidity risk (higher asset liquidity standard deviation), which causes risk not to have enough liquidity during the liquidation.

Market volatility is another factor that investors consider to decide asset haircut (collateral constraint). It is a simple and not arguable claim. Lenders have a low-risk appetite, if uncertainty (volatility) is high in the market. As a result of low-risk appetite, investors will require higher haircut in borrowing to minimize credit and interest rate risk. Similarly, borrowers risk appetite is lower to avoid a margin call or solvency risk. In parallel to this study, market volatility appears directly inside most of the models (Bernanke & Gertler, 1989); (Tobias , Moench, & Shin, 2013); (Adrian & Boyarchenko , 2012); (Fostel & Geanakoplos, 2008).

4.2. Predicting Assets' Haircuts

Since there is no available data for asset haircuts, they were predicted through the mathematical model. In the model, lenders determine the haircuts by selecting a probability, which reflects their risk appetite (or tolerance). As we claimed that asset liquidity first and second moments and market volatility are the parameters, lenders consider to select the probability. A field study survey was used to generate sample population to predict lenders that behavior. The basic idea behind using the field study survey is that having sample probability selection of participants by given information to predict (approximate) them. As will be explained in further detail in the following titles, the survey participants were requested to select a probability on a normal distribution of specific asset's return chart by assessing a given asset price quotation table and chart. The input parameters, which were the quotation tables and asset liquidity charts were randomly generated for a specific expected asset liquidity, asset liquidity deviation and with a distribution. The probability selections reveals the participant's maximum price fall expectation on the asset price for expected asset liquidity and liquidity risk pairs.

4.3. The Field Study Survey

The field study survey was completed in three steps. In the first step, the survey's inputs were generated with a distribution assumption. Because we claimed that lender's risk appetite is shaped by asset liquidity and market volatility, it was aimed to provide representative levels of those parameters to survey participants. In the second step, the survey was implemented. The survey was designed as a web-based implementation for practicality purpose. Social media was used to reach and invite relevant people. In the last step, survey participants' responses (probability selections) was regressed against to survey inputs (asset liquidity, asset liquidity risk).

4.3.1. The Survey Inputs

Because asset liquidity cannot be observed directly, the asset bid-ask prices spreads were used as a proxy. The bid-ask spreads were calculated relative to midpoint price as follows;

$$s_i^t = \frac{2(P_{i,a}^t - P_{i,b}^t)}{P_{i,a}^t + P_{i,b}^t} \quad (27)$$

s_i is asset i 's bid-ask spread,

$P_{i,a}$ is asset i 's ask price,

$P_{i,b}$ is asset i 's bid price.

Bid-ask price spreads are easy to analyze and obtain compared to other asset liquidity proxies like trading volume, trading frequency, quote size, trade size, price impact coefficient etc. Besides that, bid-ask price spreads reflect all motions of asset liquidity concept at some level.

In order to capture normal, high and low asset liquidity levels in the stock market; First, NYSE domestic stocks⁵, which have trading information from 01/1993 to 12/2014 in Center for Research in Security Prices (CRSP) data system are sorted based on their daily bid-ask spreads. Secondly, three randomly-selected⁶ representative portfolios were generated. Those portfolios are composed of five similar stocks regarding their bid-ask spread averages. As seen in Table.1 Portfolio (1), Portfolio (2), and Portfolio (3) represent high liquid, normal liquid and illiquid assets, respectively.

Table 1: Representative Portfolios' Expected Spreads

	Portfolio (1)	Portfolio (2)	Portfolio (3)
Expected Spreads($E(s)$)	0.000684	0.003118	0.011352

In the next step, the bid-ask spread's distribution was analyzed in those asset groups. Despite the fact asset's bid-ask spreads do not have a common pattern for distribution, the log normal distribution was the most pervasive. Hence, it was concluded that assuming log-normal bid-ask spread assumption for survey inputs is a proper approach.

For market volatility expectations, the VIX index was used as a proxy. Because of the fact that VIX is based on option contracts volatility, it is a very suitable

⁵ We excluded stocks, which their price is higher than \$1000 or lower than \$1 in the period, Certificates and closed-end funds, ADR's, SBI's, ATC's and REIT's.

⁶ From most liquid to most illiquid, every 30th stock was selected.

representative to measure market expectations. Nevertheless, VIX is the most common proxy for this purpose in the literature and industry practice. The first three quantiles of monthly VIX index values, which are averaged from daily values between 01/1993 and 12/2014 were used for low, normal and high market volatility expectations respectively.

Table 2: VIX index values for low, normal and high volatility.

	VIX _L	VIX _N	VIX _H
VIX	13.67	18.49	28.50

One of the essential requirement of field study surveys is simplicity. Modeling human behavior within a complex system may cause serious problems and unrealistic results. However, using three parameters (expected bid-ask spreads, expected bid-ask spreads' standard deviation and market volatility expectation) for input makes the survey complex for participants. In order to sustain that required simplicity and practicality, we decided to feed survey visual and numerical inputs with just expected bid-ask spreads and expected bid-ask spreads' deviation.

On the other hand, omitting the market volatility expectation is an essential problem. There is a common acceptance that market volatility expectations have clear impact on haircuts. Therefore, instead of omitting market volatility expectations in the field study survey for the simplicity and practicality, it was decided to measure their impact indirectly by using functional relationship among to input parameters. In order to do that, market volatility expectation is incorporated in the model through to bid-ask spread's standard deviation via the balanced panel data regression below. The stocks, which were filtered above used in the following econometric model⁷ for this purpose.

$$\ln(\widehat{\sigma}_{sp}^{it}) = \gamma_0 + \gamma_1 \ln(\mu_{sp}^{it}) + \gamma_2 \ln(\text{VIX}^t) + \varepsilon^{it} \quad (28)$$

σ_{sp}^{it} is asset i's bid-ask spreads deviation at time t,

μ_{sp}^{it} is asset i's bid-ask spreads mean at time t,

VIX^t is VIX index value at time t.

⁷ Quarterly daily averaged bid-ask spreads and VIX index data values are used. Because of normality problem in the linear model, Box-Cox transformation procedure was followed, and the suggested form log-log was used.

Table 3: Expected bid-ask spread's standard deviation

	γ_0	$\ln(\mu_{sp})$	$\ln(VIX)$
Coefficients	-2.079	0.908	0.266
t	(0.0427)	(0.0027)	(0.0127)
P	0	0	0
$R^2 = 0.7647$			

Through the regression result above, by using pairs of bid-ask spread means and VIX index values (Table.1-2) the bid-ask spread standard deviations were obtained. In order to represent extreme conditions, the regression results increased with 20% confidence level (upper and lower bounds predictions). As shown on the Appendix. 1, the process above generated 27 bid-ask spread means and (expected) standard deviation pairs. With those pairs of bid-ask spread means and standard deviations 27 different lognormal distributed series were generated in the Matlab random number generator function. In the last step, those 27 different series were separately presented in a chart and 12 days closing price quotation table.

4.3.2. Implementation of the Survey

An online field study survey was prepared by using 27 sample series, which had been prepared above. On the survey, each series was presented in a chart and a 12 days closing price quotation table as inputs. Survey participants were asked to select a confidence level in a given normal distribution chart, by considering that bid-ask spread's chart and quotation table in each case. Because the expected asset return and standard deviation were same in the each case, participants confidence level selection changed just based on given asset liquidity charts and quotation tables. Additionally, the market bid-ask spread, which is average of three the sample portfolios' spreads mean was added on each chart. The objective in adding the market bid-ask information is to provide a benchmark for the survey participants to make comparative analysis among the cases. Survey participants were requested to pick an alpha (confidence level) level on a given normal distribution chart by a using a cursor on a scale, which was divided evenly from 1 to 10. Every confidence level selection on the chart also shows the corresponding price with the given standard deviation for a day later and a scale level, which is from 1 to 10^8 . Also, the normal distribution curve was colored red for the α and green for the $1-\alpha$ area. According to selection; when scale is close 1 (for lower alpha values) the $1-\alpha$ area is getting darker green, and the α area is getting darker red, or when the scale is close to 10, the $1-\alpha$ area is getting lighter green, and the α area is getting lighter red. A sample case is presented in Appendix. 2 .

⁸ Where 10 is the lowest risk tolerance and 1 is the highest risk tolerance on the scale.

Even the survey features provide options and simplicities to reveal participants decisions (like alpha level, corresponding price level, scale and color change on the normal distribution chart), yet still its input and decision evaluation require some degree technical qualification. In order to minimize having an irrelevant survey responses, we invited people who are in the industry or academia (in the field) and provided them with a short informative video link from youtube.com about bid-ask spreads. Social media like LinkedIn, Yahoo groups were used to invite the participants. The survey's web link was sent to mail groups, which their members are risk analysts, investment banking professionals, and academic people. The list for those social media groups and survey responses are exhibited in Appendix. 3-4, respectively. The survey was posted on the web address www.mehmetbenturk.net between 03/01/2016 and 04/15/2016. The average time to take the survey was approximately 15-20 minutes. We had 44 participations. The survey responses are presented in Appendix. 5.

The survey's statistical summary is presented in Appendix 4. The most salient statistical result on the summary table is the significant differences between the minimum and maximum values. Since the survey aims to approximate investors' behavior in a controlled condition, that results are not a surprise. In reality, investors may behave significantly different from each other. Besides that, even survey participants are professionals (specifically from risk management society) or academia in the finance field, some survey participants may have evaluated the survey's input wrong. Therefore, the average-median differences and standard deviations could be more meaningful to evaluate the survey results. The difference between averages and median values are not in a wide range, which indicate the survey is successful in term of capturing investors' behavior. Similarly, standard deviations are also in a narrow range, which confirms the results in general. The highest standard deviations are observed in extreme cases like; high bid-ask spread mean and low bid-ask spread volatility or low bid-ask spread mean and high bid-ask spread volatility.

4.3.3. Regressing the Survey Results

In this step, we used the survey results in a regression analysis to model investors' confidence level selection behavior. The survey responses, which are in a finite set between 0 and 1 caused classification problem in the linear model. Because there is no 0 or 1 in the responses (depended variable), the problem was solved by using log-odds transformation (Papke & Wooldridge, 1996).

$$y_n = \ln\left(\frac{\alpha_n}{1 - \alpha_n}\right) \quad (29)$$

Due to normality problem in linear form, Box-Cox transformation procedure was followed and the form in Equation.30 was used.

$$\left(\frac{1}{y_n}\right) = \vartheta_0 + \vartheta_1\mu_{sp} + \vartheta_2\widehat{\sigma}_{sp} + \varepsilon \quad (30)$$

α_i^n is n. survey participant's alpha selection on case number i.

$\mu_{sp,i}$, $\widehat{\sigma}_{sp,i}$ are the bid-ask spread mean and standard deviation, respectively, on case number i.

Table 4: Expected confidence level

	γ_0	μ_{sp}	$\widehat{\sigma}_{sp}$
Coefficients	-0.522593	12.13654	6.297317
T	(-216.99)	(12.15)	(26.30)
P	0	0	0
$R^2 = 0.6425$			

All the coefficients of the regression are statistically significant. The R^2 of the regression is 64%, which is substantial. Despite the fact, bid-ask spreads' mean and standard deviation have inverse impact on the lenders probability selection, lenders consideration for expected bid-ask spreads' mean roughly two times more than bid-ask spreads' standard deviation.

5. The Empirical Study

Since we have asset liquidity and market volatility proxies' observations for all stocks, the regression model enables to estimate haircuts for the market general, besides to any stock haircut. In order to evaluate the model results, it was implemented to stocks in NYSE. More specifically, we used all stocks were quoted in NYSE between 01/02/1993-12/31/2014, except:

- Companies their headquarters are not in the US.
- Stocks, their price is higher than \$1000.
- Certificates, ADRs, SBIs
- Americus Trust Components (Primes and Scores).
- Closed-end funds, REIT's

The implementation was done by with monthly data set in fourth steps. In the first step, monthly stock returns, stock return's standard deviations, and bid-ask spreads were calculated for each stock. As a second step, the monthly expected bid-ask spreads' deviation for each stock was calculated through the regression in Table.3 (Equation.31). As explained previously, the market volatility expectation (VIX) incorporates to the model in this step.

$$\ln(\widehat{\sigma}_{sp}) = -2.079 + 0.908\ln\mu_{sp} + 0.26\ln_{VIX} + \varepsilon \quad (31)$$

In the third step, probability selection (confidence level) for each stock was estimated through the regression model in Table.4 (Equation.31), where bid-ask spreads' mean and expected bid-ask spreads' standard deviations from the second step were independent variables.

$$\begin{aligned} & \frac{1}{\ln\left(\frac{\alpha_n}{1-\alpha_n}\right)} \\ & = -0.522593 + 12.13654(\mu_{sp}) + 6.297317(\widehat{\sigma}_{sp}) \\ & + \varepsilon \end{aligned} \quad (32)$$

In the final step, the probability selections from prior step placed in the Equation.25 to get monthly haircuts for each stock. Because the computations are monthly, 22 days was used for the period (T). The 10-years US-Treasury bonds' monthly returns were used as a proxy for borrowing cost (r_b). In addition to those, a calibration factor was added to the mathematical model to correctly capture financial crises' impact. The calibration factor requires that the haircuts are increased as a safety factor;

20% if the VIX is higher than 20 and lower than 25,

30% if the VIX is higher than 25 and lower than 30

50% for when the VIX is higher than 30.

5.1. Test Portfolios and Market Portfolio

With the data defined above, 3 asset liquidity based test portfolios and a market portfolio, which is a simple average of all stocks were created.

Most liquid portfolio: It contains the most liquid 20 stocks of each month, between 01/1993-12/2014.

Liquid Portfolio: It contains the first 20 assets from the median, which were sorted based on asset liquidity for each month, between 01/1993-12/2014.

Illiquid asset: It contains the most illiquid 20 assets of each month, between 01/1993-12/2014.

Using asset liquidity based test portfolios enables to test and analyze the model's sensitivity for the liquidity. Since, haircut is modeled directly linked with the asset liquidity, the portfolios' haircuts must reflect that classification property. In other words, liquid portfolio must have the lowest haircut, while the most illiquid the highest. Similarly, market portfolio's haircut must lower than the most illiquid portfolio's haircut and higher than most liquid portfolio's haircut.

Table 5: Portfolios' Haircuts and Leverage Factors

	Average	Standard Deviation	Maximum	Minimum
Most Liquid Portfolio's Haircut	0.15	0.04	0.23	0.09
Most Liquid Portfolio's Leverage Factor	6.13	1.71	10.18	3.34
Liquid Portfolio's Haircut	0.20	0.03	0.31	0.14
Liquid Portfolio's Leverage Factor	4.02	0.79	6.38	2.19
Most Illiquid Portfolio's Haircut	0.30	0.07	0.47	0.18
Most Illiquid Portfolio's Leverage Factor	2.55	0.84	4.60	1.11
Market Portfolio's Haircut	0.23	0.04	0.32	0.18
Market Portfolio's Leverage Factor	3.43	0.64	4.45	2.16

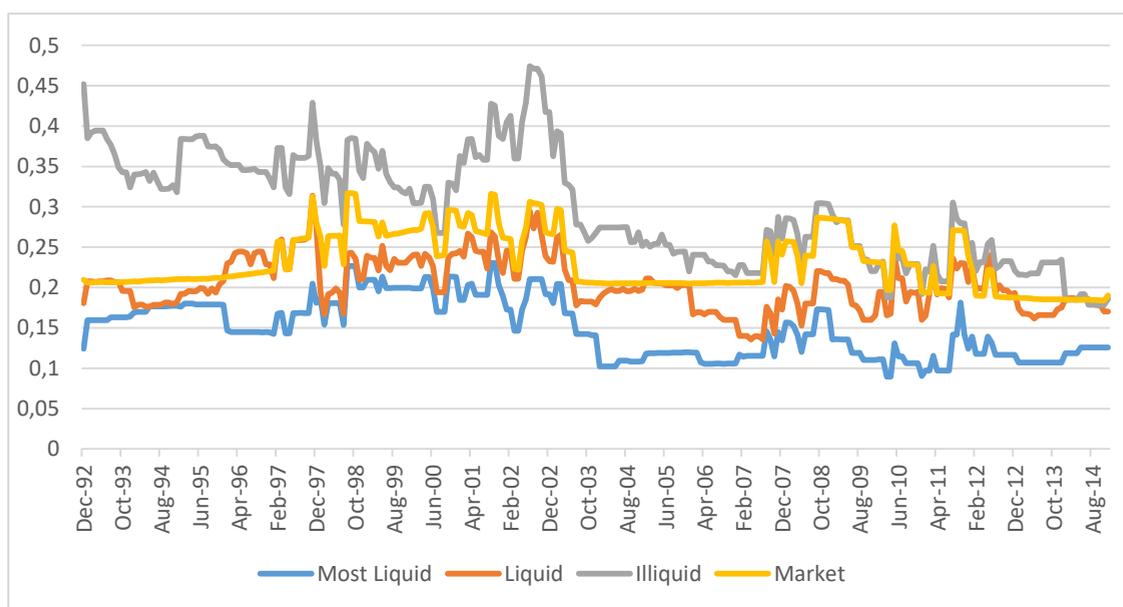


Figure 1: Portfolio Haircuts

Even though there is no obtainable data for the stocks' haircuts, there is an intuitive range to evaluate the test and market portfolios' haircuts predictions' plausibility. As seen on the Figure.1, all test and market portfolios' haircuts are in the acceptable ranges. Additionally, one of the limited data for stocks' haircuts is a survey, which was conducted by Committee on the Global Financial System (Longworth, 2010). According to that survey, prime stocks' haircuts are in 10-15 % ranges. Since, the most liquid portfolio contains prime stocks, it is proper to compare with that. The most liquid portfolio's haircut average is 15%, which is in that range. In the same perspective, liquid and market portfolios represent approximately same class assets with the none-prime stocks. Liquid and market portfolios' average haircut (20% and 23%, respectively) are close to none-prime stock's haircut range (12-20%).

Gorton and Metrick (2010) compiled asset haircuts based on subprime relationship. In that study, assets' haircut difference is 20% for before the 2008 financial crisis. The average haircut difference between the most liquid portfolio and most illiquid portfolio is 15%, which is slightly less than Gorton and Metrick (2010)'s findings. However, it should be noticed that test portfolios are based on stocks, there is no representative portfolio for bonds. Since, bond based portfolios haircut are lower than stocks, it can be said that the mathematical model results are also coherent with Gorton and Metrick (2010)'s results.

As seen from Figure.1, stocks' haircut have been following slightly a lower trend after 2000. That pattern is closely related with increasing asset liquidity, which is result of globalization, algorithmic trading and low interest rates policies. In that perspective the asset liquidity-leverage spiral can be seen on the difference between

the most liquid stocks and other stocks' haircut. The boom in leveraged transactions after 2000 due to algorithmic trading and lower borrowing cost have increased most liquid stocks' liquidity more than others. The higher liquidity has provided higher leverage, which also increases asset liquidity.

Haircuts reflect liquidity and market volatility properly. While the most liquid portfolio has the lowest haircuts and the most illiquid portfolio has the highest haircuts. Similarly, all stocks' haircuts increase significantly in financial crises. Since the model includes asset liquidity risk besides the asset's risk itself, illiquid stocks' haircuts have much higher volatility than liquid assets. On the other hand, market portfolio' haircut has very similar statistical properties with liquid portfolio.

6. Conclusions

This study suggests an asset based, stochastic haircut model specifically for stocks. While the asset collateral valuation was incorporated into the model with parametric VaR, the risk appetite of lenders was captured through to asset liquidity (first and second moments) and market volatility expectations by indirectly a survey based prediction. The survey's statistical significance confirms that lenders consider asset liquidity and market volatility expectations besides the asset risk itself for haircut decision. The concordance between the empirical studies' results and the limited data for stocks haircuts justifies the methodology of the study. Hence, the study's findings allows to examine many recent theoretical claims about asset collaterals.

Not surprisingly, as asset illiquidity and market volatility increase, haircuts increase. However, very illiquid stocks' haircut deviation is significantly (about 2 times) more than liquid stocks. Because of that, the haircut gap between very illiquid and liquid stocks increase much more in financial crisis than other times.

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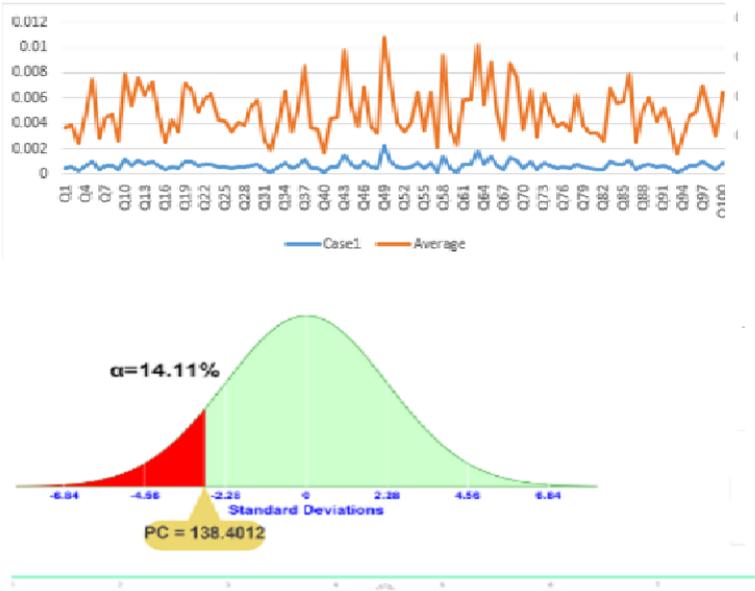
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Appendix 1: Bid-Ask Spreads and Expected Bid-Ask Spreads' Standard Deviations

μ_{sp}	$\widehat{\sigma}_{sp}$
0.000684	0.000335
0.000684	0.000363
0.000684	0.000408
0.003118	0.001329
0.003118	0.001441
0.003118	0.001616
0.011352	0.004298
0.011352	0.004657
0.011352	0.005225
0.000684	0.0001195
0.000684	0.0001239
0.000684	0.0001344
0.003118	0.0004813
0.003118	0.0004824
0.003118	0.0005516
0.011352	0.0014502
0.011352	0.0015924
0.011352	0.0018044
0.000684	0.0010341
0.000684	0.0010556
0.000684	0.0012163
0.003118	0.0038123
0.003118	0.0041335
0.003118	0.004699
0.011352	0.0121389
0.011352	0.0126564
0.011352	0.0147683

Appendix. 2



Bid	Ask	Close
151.02	151.14	151.14
149.62	149.71	149.62
149.91	150.01	150.01
150.09	150.16	150.09
150.37	150.39	150.39
149.22	149.28	149.22
140.55	140.64	140.64
140.92	141.01	140.92
143.95	144.09	144.09
144.68	144.78	144.68
143.61	143.66	143.66
141.88	142.00	141.88

Appendix 3: The list of social media used in the survey.

Group Name	Number of Member	Region	Web address
Risk Managers	3033	Global	http://finance.groups.yahoo.com/group/FinEngineer
Smart quant	927	Global	www.smartquant.com
Risk Management In Turkey	510	Turkey	Risk Management in Turkey (Yahoo Groups)
Indian Stock Market	15480	India	http://www.crnindia.com
Risk, Regulation & Reporting	170781	Global	Risk, Regulation & Reporting(LinkedIn)
Risk Managers	106985	Global	Risk Managers (LinkedIn)
Capital Markets, Private Equity & Global Finance Group	93440	Global	Capital Markets, Private Equity & Global Finance Group(LinkedIn)
Financial Analyst Club Worldwide	145731	Global	Financial Analyst Club Worldwide(LinkedIn)
RIMS, the risk management society	54404	Global	RIMS, the risk management society (LinkedIn)
Financial Risk Management Network	64791	Global	Financial Risk Management Network (LinkedIn)
Financial Analysts	29147	Global	Financial Analysts (LinkedIn)

Appendix 4: Survey results statistical summary.

SURVEY RESULTS STATISTICAL SUMMARY							
	Case1	Case2	Case3	Case4	Case5	Case6	Case7
Average	14.17	14.51	13.51	13.36	13.15	12.44	11.98
Maximum	15.71	15.83	15.36	14.58	14.20	13.74	13.74
Minimum	9.09	7.25	5.39	8.69	10.05	5.34	5.75
Median	14.33	14.68	14.30	13.62	13.21	12.83	12.41
Standard Deviation	0.99	1.39	2.13	1.08	0.74	1.37	1.57
	Case8	Case9	Case10	Case11	Case12	Case13	Case14
Average	12.06	11.58	11.70	11.54	10.23	10.14	9.71
Maximum	13.41	13.28	13.21	14.12	11.94	11.90	11.83
Minimum	4.43	3.99	3.99	2.06	4.28	3.42	1.73
Median	12.44	12.05	12.28	11.87	10.51	10.50	10.33
Standard Deviation	1.60	1.93	1.95	1.88	1.53	1.59	2.10
	Case15	Case16	Case17	Case18	Case19	Case20	Case21
Average	8.98	8.82	8.77	8.99	7.56	6.54	6.58
Maximum	11.47	11.78	11.37	12.88	9.97	9.45	14.12
Minimum	3.74	0.20	0.20	3.69	0.28	0.14	0.29
Median	9.27	9.21	9.23	9.23	7.89	6.75	6.53
Standard Deviation	1.69	2.20	2.10	1.87	2.04	2.05	2.41
	Case22	Case23	Case25	Case24	Case26	Case27	
Average	6.58	5.42	5.61	4.79	3.88	3.19	
Maximum	12.66	8.78	14.12	8.20	8.80	7.12	
Minimum	0.34	0.64	1.70	0.33	0.10	0.04	
Median	6.68	5.41	5.41	4.61	3.82	3.18	
Standard Deviation	2.28	2.19	2.23	2.15	2.46	2.29	

Survey Date	Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8	Case9	Case10	Case11	Case12	Case13	Case14	Case15	Case16	Case17	Case18	Case19	Case20	Case21	Case22	Case23	Case25	Case24	Case26	Case27
02/26/2016 18:20	12.94	13.55	9.45	12.08	12.42	10.13	9.24	10.64	9.78	9.78	9.62	7.87	7.57	7.28	7.13	7.10	7.29	8.39	5.88	5.23	6.64	8.05	5.81	7.19	7.24	7.24	6.08
03/01/2016 17:40	12.89	12.88	8.83	10.61	13.22	9.32	8.88	10.48	7.62	7.62	8.28	6.34	6.52	4.08	6.04	3.51	5.01	5.95	3.87	3.13	2.71	4.51	3.12	4.27	4.72	3.64	3.76
03/02/2016 20:19	14.31	14.49	14.58	13.78	13.61	12.90	12.90	12.73	12.56	12.56	12.90	11.94	11.85	11.67	10.79	10.70	10.78	10.78	9.19	8.05	8.04	8.03	6.89	6.56	5.86	4.68	4.23
03/03/2016 08:58	14.39	14.39	14.39	13.99	13.99	13.58	13.33	13.11	12.89	12.86	12.84	11.55	11.53	11.53	11.12	11.11	11.01	10.95	8.33	6.84	6.88	6.77	5.30	5.48	3.95	2.33	0.72
03/06/2016 19:46	14.13	14.48	14.66	13.33	12.98	11.84	11.74	11.74	11.74	11.74	11.74	10.52	10.74	9.90	8.49	8.43	8.43	8.43	6.94	5.61	5.56	5.50	4.23	4.13	3.12	1.76	0.77
03/06/2016 20:09	14.43	14.37																									
03/07/2016 18:53	14.29	14.64	14.81	13.62	13.29	12.25	12.03	12.02	11.65	10.94	10.90	10.14	9.61	9.26	7.89	7.36	7.73	7.75	6.38	5.54	5.46	5.41	4.62	4.43	4.53	3.46	2.38
03/07/2016 19:46	14.28	14.47	14.55	13.76	13.34	12.35	12.08	12.12	11.72	11.66	11.66	10.19	9.94	9.50	8.09	7.96	7.76	7.66	6.15	5.10	4.92	4.78	3.76	4.71	3.38	1.66	1.26
03/08/2016 09:43	14.15	14.13	14.14	13.72	13.73	13.32	13.01	12.85	12.69	12.69	12.63	11.83	11.81	11.70	11.42	11.42	11.37	11.35	8.31	6.59	6.58	6.58	4.86	4.84	3.16	1.42	0.91
03/08/2016 13:24	14.49	14.84	15.02	13.69	13.69	12.90	12.54	12.54	12.55	12.54	12.41	10.43	10.67	10.42	9.63	9.63	9.63	9.63	8.84	8.03	8.05	8.05	7.26	7.23	6.48	5.69	4.90
03/09/2016 16:17	14.30	14.66	14.84	13.65	13.59	12.88	12.47	12.39	12.25	12.15	12.18	10.45	10.37	10.32	9.25	9.52	9.48	9.46	8.69	8.00	7.97	7.96	7.18	7.37	6.70	5.95	5.20
03/09/2016 18:22	14.68	15.14	15.35	13.73	13.37	12.05	11.73	11.59	11.25	11.10	11.13	8.78	8.33	7.90	6.77	6.56	6.11	6.11	4.12	3.10	3.11	2.96	1.97	2.56	0.77	0.43	0.30
03/10/2016 06:53	14.35	14.35	14.35	13.67	13.05	11.72	11.21	11.19	12.43	12.43	12.40	11.19	11.18	11.16	10.46	10.44	10.44	10.43	9.73	7.28	6.39	5.50	5.17	4.54	4.14	3.44	2.74
03/10/2016 09:03	14.09	14.08	14.09	13.13	13.12	12.14	11.59	11.63	11.67	11.45	11.45	10.69	10.58	10.58	9.50	9.57	9.58	9.59	8.61	7.75	7.72	7.70	6.85	7.00	6.16	5.22	4.28
03/10/2016 15:28	14.66	14.71	14.74	14.26	14.20	13.74	13.14	12.98	12.77	12.77	12.73	11.26	11.24	11.19	10.67	10.65	10.63	10.62	9.92	9.34	9.33	9.33	8.75	8.08	7.92	7.43	7.12
03/11/2016 10:32	14.47	14.83	15.00	13.67	13.33	12.18	12.18	12.19	12.19	12.17	12.11	11.31	11.12	10.77	9.27	9.18	9.18	9.18	7.68	6.37	6.28	6.20	4.99	5.54	4.13	2.51	0.88
03/11/2016 10:36	11.57	7.25	7.01	8.68	10.04	10	5.7	4.43	3.99	3.99	2.05	4.27	8.75	2.34	5.68	3.2	6.83	3.69	3.94	4.82	2.51	7.38	3.68	6.14	5.23	5.75	5.91
03/11/2016 15:47	14.33	14.33	14.33	13.35	13.18	12.03	11.93	11.93	11.93	11.77	11.73	10.44	10.27	10.03	8.77	8.61	8.75	8.75	7.43	6.27	6.25	6.24	5.20	5.33	4.16	2.94	1.70
03/12/2016 09:45	14.40	14.45	14.49	13.75	13.75	13.08	12.28	12.06	11.85	11.85	11.85	10.63	10.64	10.64	9.97	9.96	9.87	9.81	9.15	8.48	8.45	8.45	7.00	7.66	7.14	6.43	5.73
03/13/2016 12:18	14.43	14.78	14.96	13.90	13.55	12.67	12.45	12.46	12.11	12.05	12.03	11.27	11.06	10.71	9.57	9.40	9.29	9.25	8.02	6.99	6.90	6.85	5.88	5.23	5.26	4.01	2.70
03/17/2016 17:14	12.45	15.53	7.15	9.03	14.11	5.33	7.15	12.42	5.31	5.31	14.11	5.31	3.41	1.72	3.73	0.19	0.19	5.23	0.2	0.13	0.2	0.33	0.63	1.6	4.66	0.96	1.23
03/18/2016 08:41	14.12	14.30	14.37	13.32	12.89	11.65	11.34	11.26	10.74	10.73	10.69	9.89	9.66	9.22	7.55	7.47	7.30	7.24	5.57	4.18	4.02	3.93	2.57	2.01	1.70	0.77	0.61
03/18/2016 08:51	14.32	14.32	14.32	13.66	13.64	12.96	12.60	12.60	12.59	12.49	12.49	10.51	10.41	10.43	9.73	9.71	9.70	9.70	9.00	8.36	8.33	8.31	7.69	7.20	7.13	6.14	5.75
03/21/2016 10:10	14.05	14.41	14.57	13.64	13.56	13.07	12.66	12.64	12.53	12.39	12.37	10.70	10.99	10.50	9.76	9.81	9.88	9.88	9.30	8.83	8.81	8.81	8.34	7.61	8.00	7.51	7.01
03/21/2016 14:22	14.33	14.69	14.87	13.55	13.02	11.69	11.23	11.28	10.79	10.79	10.78	9.57	9.30	8.77	6.92	6.79	6.77	6.75	6.55	4.67	4.61	4.55	3.02	3.45	2.88	0.20	0.04
03/21/2016 15:12	14.34	14.79	15.01	13.68	13.55	12.77	12.26	12.10	11.82	11.61	11.56	10.80	10.63	10.51	9.60	9.54	9.45	9.42	8.51	7.78	7.70	7.66	6.91	7.30	6.49	5.61	4.73
03/21/2016 16:33	14.11	14.12	14.13	13.12	12.80	11.46	10.89	10.93	10.54	10.59	10.51	9.09	8.92	8.56	6.89	6.80	6.63	6.56	4.99	3.56	3.23	3.10	1.79	2.52	0.88	0.55	0.39
03/21/2016 18:39	14.14	14.23	14.28	13.48	13.04	11.93	11.92	11.85	11.33	11.32	11.26	10.47	10.23	9.78	8.24	8.01	7.86	7.73	6.19	4.87	4.69	4.54	3.38	4.39	2.90	0.95	0.24
03/22/2016 11:10	14.50	15.03	15.29	13.97	13.95	13.41	13.15	13.15	13.15	13.19	12.09	11.05	11.03	11.04	10.48	10.43	10.48	10.48	9.92	8.51	8.27	8.19	6.80	6.17	5.58	3.87	2.15
03/22/2016 13:23																											
03/22/2016 14:23	14.53	14.65	14.72	14.13	14.04	13.55	13.45	13.35	13.21	13.08	12.16	10.71	10.59	10.44	9.92	9.77	9.78	9.74	9.16	8.70	8.62	8.60	8.16	7.19	6.12	5.30	4.46
03/22/2016 15:50	14.27	14.48	12.47	13.70	13.43	12.58	12.42	12.34	12.11	11.95	12.01	10.87	10.65	10.39	9.28	9.11	9.15	9.12	7.46	6.23	6.16	6.10	5.00	5.13	4.20	2.78	1.72
03/22/2016 16:18	14.06	14.09	14.09	13.12	12.79	11.46	10.92	10.91	10.56	10.52	10.52	8.52	8.24	7.90	6.20	6.10	6.01	6.13	4.34	3.01	3.55	2.96	1.97	2.00	0.33	0.11	0.07
03/23/2016 01:14	14.67	15.14	15.36	14.22	14.20	13.74	13.34	13.14	13.15	13.18	12.84	11.90	11.84	11.56	11.42	11.62	11.35	11.35	9.90	9.34	9.23	9.33	8.71	7.08	7.20	7.61	7.02
03/23/2016 10:12	14.41	15.82	14.38	13.83	13.68	12.95	13.73	12.59	12.44	12.40	11.92	10.60	10.48	10.34	9.47	9.37	9.39	9.37	8.18	7.16	7.08	7.04	6.06	5.71	4.93	3.23	3.08
03/23/2016 12:33	14.05	15.79	12.45	13.11	12.73	13.74	10.91	13.39	10.56	13.14	10.56	8.65	8.25	7.98	6.21	6.02	6.66	6.33	4.78	3.44	3.06	2.96	1.47	2.09	0.77	0.10	0.09

03/24/2016 13:52	14.07	15.81	12.42	13.16	12.76	13.72	13.74	13.41	13.24	13.21	12.82	11.80	11.76	11.83	11.47	11.77	11.35	11.31	9.89	9.45	9.36	9.33	8.35	8.08	8.20	7.61	6.92
03/24/2016 15:28	14.17	15.80	13.07	13.36	13.08	13.48	12.79	13.12	12.08	12.93	11.77	10.32	10.29	10.04	9.01	8.88	8.97	8.96	7.52	6.52	6.47	6.44	5.46	5.12	4.56	3.89	3.38
03/24/2016 19:12	9.08	10.57	5.38	12.23	11.23	10.51	8.59	6.71	5.86	5.86	5.63	7.83	5.68	5.35	8.13	8.96	4.77	12.87	9.54	7.55	14.11	12.66	5.63	14.11	6.86	8.79	4.49
03/24/2016 23:25	14.04	15.83	12.43	14.25	12.78	13.74	13.73	13.37	13.23	12.08	10.52	8.56	8.28	7.85	6.46	6.00	6.15	6.45	4.57	3.07	3.01	2.96	1.50	2.71	0.45	0.13	0.05
03/25/2016 10:32	15.70	15.54	12.39	13.11	12.75	11.46	10.90	10.92	10.57	13.19	12.88	11.80	11.83	11.31	10.97	11.78	11.34	11.65	9.80	9.01	9.40	9.33	8.40	8.00	8.01	7.56	6.88
03/25/2016 11:19	14.45	15.31	12.76	13.59	12.93	13.05	12.70	12.63	12.21	12.53	11.90	10.26	10.13	9.95	8.94	8.79	8.82	8.80	7.36	6.35	6.30	6.27	5.29	5.16	4.47	3.80	3.35
03/25/2016 13:06	14.23	15.29	13.45	13.49	13.26	13.10	12.55	12.70	12.02	12.41	11.78	10.36	10.25	10.00	9.02	8.90	8.94	8.92	7.40	6.68	6.34	6.31	5.33	5.18	4.45	3.72	3.19
03/25/2016 16:40	15.71	15.84	13.41	14.24	13.25	13.74	13.72	13.39	13.28	13.18	12.82	11.83	11.90	11.82	11.02	11.56	11.33	11.30	9.97	9.01	9.31	9.33	8.78	8.09	8.20	7.61	6.98
03/25/2016 17:27	14.05	15.29	12.41	13.14	12.71	11.46	10.88	12.38	11.94	12.10	11.69	10.37	10.25	9.98	9.05	8.93	8.78	8.93	7.56	6.50	6.41	6.44	5.50	5.23	4.52	3.49	3.18
03/26/2016 09:06	14.58	15.52	12.93	13.58	13.01	12.89	12.44	12.61	12.15	12.58	11.88	10.82	10.73	10.61	9.79	9.75	9.76	9.70	8.29	7.41	7.38	7.36	6.50	6.01	5.74	5.07	4.53
03/26/2016 14:26	14.38	15.69	12.70	13.41	12.91	13.21	12.40	12.76	11.92	12.78	11.71	10.34	10.16	9.58	8.97	8.85	8.89	8.87	7.41	6.42	6.38	6.35	5.36	5.20	4.51	3.84	3.33
03/26/2016 15:14	14.25	15.34	13.73	13.49	13.24	13.13	12.52	12.68	11.99	12.51	11.70	10.50	10.52	10.26	9.35	9.25	9.27	9.21	7.77	6.83	6.80	6.77	5.85	5.63	5.07	4.46	3.91