Journal of Statistical and Econometric Methods, vol.2, no.1, 2013, 61-79
ISSN: 1792-6602 (print), 1792-6939 (online)
Scienpress Ltd, 2013

On Bayesian prediction of future median generalized order statistics using doubly censored data from type-I generalized logistic model

Tahani A. Abushal[1]

Abstract

This paper is concerned with the problem of deriving expressions for the Bayesian predictive survival functions for the median of future sample of generalized order statistics having odd and even sizes. Both of the informative and future samples are drawn from a population whose distribution is truncated type-I generalized logistic distribution TTIGL (β, α, τ). Doubly type II censored data and two sample technique have been used here. Bayesian prediction intervals using two independent samples, based on informative prior is obtained. Bayesian prediction intervals for: upper order statistics and upper records are considered as special cases. Numerical computations based on simulation study are given to illustrate the performance of the procedures.

Keywords: Bayesian prediction; Double censoring; Logistic distribution; Two-sample prediction; Order statistics; Records; Generalized order statistics

[1] Department of Mathematics, Umm AL-Qura University, Saudi Arabia
e-mail:taabushal@uqu.edu.sa

Article Info: *Received* : December 2, 2012. *Revised* : January 11, 2013
Published online : March 1, 2013

1 Introduction

In Kamps (1995) generalized order statistics (GOS) have been introduced as a unified approach to several models of ordered random variables. Such models are ordinary order statistics (OOS) [David and Nagaraja (2003), Arnold, Balakrishnan and Nagaraja (1992)], records [Ahsanullah (1994) and Arnold, Balakrishnan and Nagaraja (1998)], sequential order statistics [Cramer and Kamps (1996)] and ordering via truncated distributions and censoring schemes [Kamps (1995)]. Kamps's book (1995) gave several applications such as a variety of disciplines, recurrence relation for moments of order statistics and characterizations. Several authors utilized the GOS in their work, such authors, among others, are Ahsanullah (2000), Habibullah and Ahsanullah (2000), Kamps and Gather (1997), Keseling (1999), Cramer and Kamps (2000), Pawlas and Szynal (2001), Ahmad and Fawzy (2003), AL-Hussaini and Ahmad (2003a,b), AL-Hussaini (2004),Ahmad (2007, 2010) and Jaheen (2002, 2005). Several authors have predicted future order statistics and records from homogeneous and heterogeneous populations that can be represented by single or finite mixtures of distributions. For a good survey see, AL-Hussaini and Jaheen (1995, 1996, 1999), AL-Hussaini and Ahmad (2003b), Ali Mousa (2003) and AL-Hussaini (1999, 2001). The logistic distribution is one of the oldest growth models. The truncated logistic distribution plays a role in a variety of applications, also, the type I generalized logistic distribution has applications in the theoretical and practical fields. For more details on the logistic and half-logistic distributions, see Balakrishnan (1985, 1992), Balakrishnan and Wang (1991) and Balakrishnan and Chan (1992). AL-Angary (1997) introduced the truncated type I generalized logistic distribution which was denoted by $TTIGL(\beta, \gamma, \alpha)$ and described some of its properties. Some studies have discussed the truncated type I generalized logistic distributions such as, AL-Hussaini and Ateya (2003, 2005) and AL-Hussaini et al.(2006). A random variable X is said to have a truncated type I generalized logistic distribution with vector of parameters $\Theta = (\beta, \alpha, \tau)$ if it's probability density function (PDF) is given by

$$f(x;\theta) = \frac{\alpha}{\tau(1-2^{-\alpha})} \exp[-(x-\beta)/\tau][1+\exp[-(x-\beta)/\tau]]^{-(\alpha+1)}, \qquad x \geq \beta, (\beta \geq 0, \tau > 0, \alpha > 0). \tag{1}$$

The reliability and hazard rate functions are given by

$$R(x) = \frac{1-\omega^{-\alpha}}{1-2^{-\alpha}}, \tag{2}$$

$$r_\theta(x) = \frac{\alpha(\omega-1)\omega^{-\alpha-1}}{\tau(1-\omega^{-\alpha})}, \tag{3}$$

where $\omega = 1 + e^{-(x-\beta)/\tau}, x \geq \beta$. In our study we will take $\beta = 0$, then the vector of parameters will be $\theta = (\alpha, \tau)$. For a value x_i of the random variable X, let

$$\left.\begin{aligned} \omega_i &= 1 + \exp^{-\frac{x_i}{\tau}}, \\ \varepsilon_i(\alpha,\tau) &= 1 - {\omega_i}^{-\alpha}, \\ \eta_i(\alpha,\tau) &= \frac{(\omega_i - 1){\omega_i}^{-\alpha-1}}{\varepsilon_i(\alpha,\tau)}. \end{aligned}\right\} \tag{4}$$

So, (1), (2) and (3) can be written in the following forms (with $\beta = 0$) as

$$f(x_i;\alpha,\tau) = \frac{\alpha}{\tau}(1-2^{-\alpha})^{-1}\varepsilon_i(\alpha,\tau)\eta_i(\alpha,\tau), x_i > 0, (\tau > 0, \alpha > 0). \tag{5}$$

The reliability and hazard rate functions are given, respectively, by (6) and (7)

$$R(x_i) = (1-2^{-\alpha})^{-1}\varepsilon_i(\alpha,\tau), \tag{6}$$

and

$$r_\theta(x_i) = \frac{\alpha}{\tau}\eta_i(\alpha,\tau). \tag{7}$$

We will write ε_i, η_i instead of $\varepsilon_i(\alpha,\tau), \eta_i(\alpha,\tau)$. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample (rs) of size n drawn from a population whose *cdf* is $F(x)$ and *pdf* is $f(x)$. Let $X_{1;n,m,k}, \ldots, X_{n;n,m,k}$ be the corresponding GOS, where $m \geq 1$, $k \geq 1$, see Kamps (1995). It was shown by Ahmad and Abu-Shal (2008) that the joint PDF of the GOS $X_{r_1;n,m,k}, \ldots, X_{r_\ell;n,m,k}$, can be written, for $0 < r_1 < \cdots < r_\ell < 1, r_0 = 0, r_{\ell+1} = n+1, x_0 = -\infty, x_{r_\ell+1} = \infty$, as

$$f_{r_1,\ldots,r_\ell}(x_{r_1},\ldots,x_{r_\ell}) = C(i;r_\ell)[h_m(F_\theta(x_{r_1})) - h_m(0)]^{r_1-1}[\overline{F}_\theta(x_{r_\ell})]^{\gamma_{r_\ell}-1}$$

$$\times[\prod_{i=1}^{\ell-1}[\overline{F}_\theta(x_{r_i})]^m f_\theta(x_{r_i})][h_m(F_\theta(x_{r_{i+1}})) - h_m(F_\theta(x_{r_i}))]^{r_{i+1}-r_i-1}]f_\theta(x_{r_\ell}), \tag{8}$$

for $F_\theta^{-1}(0+) < x_{r_1} \le \cdots \le x_{r_\ell} < F_\theta^{-1}(1)$, where

$$\left.\begin{aligned} C(i;r_\ell) &= C_{r_\ell-1}/\prod_{i=0}^{\ell-1}(r_{i+1}-r_i-1)! \\ C_{r_\ell-1} &= \prod_{i=1}^{r_\ell}\gamma_i, \quad \gamma_i = k+(n-i)(m+1) \\ h_m(z) &= -(1-z)^{m+1}/(m+1), \quad m\neq -1 \\ &\quad -\ln(1-z), \quad m=-1 \end{aligned}\right\}. \tag{9}$$

The joint PDF of the first r GOS $X_{1;n,m,k},\ldots,X_{r;n,m,k}$, for $1\le r\le n$ can be obtained if we choose $r_1=1, r_2=2,\ldots,r_\ell=r$ in (8), see Kamps (1995). If we choose, in (8) and (9), $r_1=s, r_2=s+1,\ldots,r_\ell=s+\ell-1\equiv r$, we can easily show that

$$f_{s,\ldots,r}(x_s,\ldots,x_r) = \frac{C_{r-1}}{(s-1)!}[h_m(F_\theta(x_s))-h_m(0)]^{s-1}[\prod_{i=s}^{r-1}[\overline{F}_\theta(x_i)]^m f_\theta(x_i)][\overline{F}_\theta(x_r)]^{\gamma_r-1}f_\theta(x_r),$$

$$= \frac{(-1)^{s-1}C_{r-1}}{(m+1)^{s-1}(s-1)!}[\prod_{i=s}^{r-1}[\overline{F}_\theta(x_i)]^m f_\theta(x_i)]f_\theta(x_r)[\overline{F}_\theta(x_r)]^{\gamma_r-1}$$

$$\times\sum_{\ell=0}^{s-1}\omega_\ell^{(s)}[\overline{F}_\theta(x_s)]^{(s-\ell-1)(m+1)}, \quad m\neq -1,$$

$$\frac{(-1)^{s-1}k^r}{(s-1)!}[\ln(\overline{F}_\theta(x_s))]^{s-1}[\overline{F}_\theta(x_r)]^{k-1}\times f_\theta(x_r)\prod_{i=s}^{r-1}r_\theta(x_i), \quad m=-1, \tag{10}$$

where $\omega_\ell^{(s)} = (-1)^\ell\binom{s-1}{\ell}$ and $\gamma_r = k+(m+1)(n-r)$.

Suppose that n items are simultaneously put on a life test and that for some reasons, the first $s-1$ failure times were not observed. The observed failure times are start only from the sth to the rth failure time, $1<s<r<n$. These ordered observations are referred to as a doubly type II censored data. Type II censoring is obtained when $s=1$ and the complete sample is obtained when $s=1$ and $r=n$. For more about doubly censored sample, reader is referred to Khen et al. (2011, 2010), among others. Suppose that $X_s < X_{s+1} < \cdots < X_r$,where $1<s<r<n$, where $X_i\equiv X_{i;n,m,k}, i=1,2,...,n$ be a doubly type-II censored random sample (informative). Let $Y_1<Y_2<\cdots<Y_N$,where $Y_i\equiv Y_{i;N,M,K}, i=1,2,\ldots,N,\ M>0,\ K>0$ be a second independent generalized ordered random sample (of size N) of future observations drawn from the same distribution. Based on such a doubly type II censored observations, we want to predict the future median (unobserved) $Y_N \equiv Y_N^\star$. Then for ξ being a positive integer ≥ 1 we have:

$$Y_N^\star = \begin{cases} Y_{\xi;N,M,K}, & N = 2\xi - 1 \\ \frac{Y_{\xi;N,M,K}+Y_{\xi+1;N,M,K}}{2}, & N = 2\xi. \end{cases} \tag{11}$$

2 Density functions of future median

2.1 The case of odd N

In the case of the odd future sample size where $N = 2\xi - 1, \xi = 1, 2, 3, ..., N$, let $Y_N^\star$ donete the median of future generalized order statistics. For a given θ, the PDF of $Y_N^\star$ is given, see Kamps (1995), by

$$f_{Y_N^\star}(y|\theta) = \frac{C_{\xi-1}^\star}{(\xi-1)!(M+1)!}[\overline{F}_\theta(y)]^{\gamma_\xi^\star - 1} f_\theta(y) g_M^{\xi-1}[F_\theta(y)], \tag{12}$$

where

$$\left.\begin{array}{l} C_{\xi-1}^\star = \prod_{j=1}^{\xi} \gamma_j^\star, \quad \gamma_j^\star = K + (N-j)(M+1) \\ g_M(y) = h_M(y) - h_M(0), \quad y \in (0,1) \\ h_M(y) = -(1-y)^{M+1}/(M+1), \qquad M \neq -1 \\ \qquad\quad - \ln(1-y), \qquad\qquad M = -1 \end{array}\right\}. \tag{13}$$

By substituting Eqn.(13) in Eq.(12), yields

$$f_{Y_N^\star}(y|\theta) = \frac{C_{\xi-1}^\star}{(\xi-1)!(M+1)!}[\overline{F}_\theta(y)]^{\gamma_\xi^\star - 1} f_\theta(y)[h_M(F_\theta(y)) - h_M(F_\theta(0))]^{\xi-1}. \tag{14}$$

By expanding $[h_M(F_\theta(y)) - h_M(F_\theta(0))]^{\xi-1}$ binomially, $f_{Y_N^\star}(y|\theta)$ can be written as

$$f_{Y_N^\star}(y|\theta) \propto [\overline{F}_\theta(y)]^{\gamma_\xi^\star - 1} f_\theta(y) \Sigma_{j=0}^{\xi-1} w_\xi^{(j)} [h_M(F_\theta(y))]^j, \tag{15}$$

By making use of Eq.(13) and (15), we obtain

$$f_{Y_N^\star}(y|\theta) \propto \begin{cases} [\overline{F}_\theta(y)]^{\gamma_\xi^\star - 1} f_\theta(y) \Sigma_{j=0}^{\xi-1} \omega_\xi^{(j)} [\overline{F}_\theta(y)]^{j(M+1)}, & M \neq -1 \\ [\overline{F}_\theta(y)]^{K-1} f_\theta(y) [\ln(\overline{F}_\theta(y))]^{\xi-1}, & M = -1. \end{cases} \quad (16)$$

By substituting Eqn.(5),(6) and (7) in Eq.(16), we obtain.

$$f_{Y_N^\star(y|\theta)} \propto \begin{cases} \frac{\alpha}{\tau}\eta_y[(1-2^{-\alpha})^{-1}\varepsilon_y]^{\gamma_\xi^\star} \times \sum_{j=0}^{\xi-1} \omega_\xi^{(j)} [(1-2^{-\alpha})^{-1}\varepsilon_y]^{j(M+1)}, & M \neq -1 \\ \frac{\alpha}{\tau}\eta_y[(1-2^{-\alpha})^{-1}\varepsilon_y]^{K} & \\ \times [ln[(1-2^{-\alpha})^{-1}\varepsilon_y]]^{\xi-1}, & M = -1. \end{cases} \quad (17)$$

where $\gamma_\xi^\star = K + (N-\xi)(M+1)$ and $\omega_\xi^{(j)} = (-1)^j \binom{\xi-1}{j}$.

2.2 The case of even N

It can be shown when N is even, that the PDF of the median $Y_N^\star$ for a given θ, is given by

$$f_{Y_N^\star}(y|\theta) \propto \begin{cases} \sum_{j=0}^{\xi} \omega_j(\xi)\psi_j(y \mid \theta), & M \neq -1, \\ \varpi(y \mid \theta), & M = -1, \end{cases} \quad (18)$$

where

$$\left.\begin{array}{l} \psi_j(y|\theta) = \int_0^y [\overline{F}_\theta(z)]^{j(M+1)+M}[1-F_\theta(2y-z)]^{\gamma_{\xi+1}^\star - 1} \\ \times f_\theta(2y-z) f_\theta(Z)dz, \quad M \neq -1, \\ \varpi(y|\theta) = \int_0^y H_\theta(z)H_\theta(2y-z)[S(z)]^{\xi-1} \\ \times [1-F_\theta(2y-z)]^K dz, \quad M = -1. \end{array}\right\}. \quad (19)$$

By substituting Eqn.(4),(5) and (6) in Eq.(19), we obtain

$$\left.\begin{array}{l} \psi_j(y \mid \theta) = \int_0^y \frac{\alpha^2}{\tau^2}[(1-2^{-\alpha})^{-1}\varepsilon_z]^{(M+1)(j+1)} \\ \times [(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^{\gamma_{\xi+1}^\star}\eta_z\eta_{2y-z}dz, \quad M \neq -1, \\ \varpi(y \mid \theta) = \int_0^y \frac{\alpha^2}{\tau^2}[(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^K \\ \times [\ln[(1-2^{-\alpha})^{-1}\varepsilon_z(\alpha,\tau)]]^{\xi-1}\eta_z\eta_{2y-z}dz, \quad M = -1. \end{array}\right\}. \quad (20)$$

Based on the GOS $X_{s;n,m,k}, X_{s+1;n,m,k}, \ldots, X_{r;n,m,k}$, for $0 \leq s \leq \cdots \leq r \leq n$, the likelihood function can be written, see Ahmad and Abu-Shal (2008), as

$$L(\theta \mid \underline{x}) \propto [h_m(F_\theta(x_s)) - h_m(0)]^{s-1}[\prod_{i=s}^{r-1}[\overline{F}_\theta(x_i)]^m f_\theta(x_i)][\overline{F}_\theta(x_r)]^{\gamma_r-1} f_\theta(x_r)$$

$$= \begin{cases} [\prod_{i=s}^{r-1}[\overline{F}_\theta(x_i)]^m f_\theta(x_i)][\overline{F}_\theta(x_r)]^{\gamma_r-1} f_\theta(x_r) & \\ \times \sum_{\ell=0}^{s-1} \omega_\ell^{(s)} [\overline{F}_\theta(x_s)]^{(s-\ell-1)(m+1)}, & m \neq -1, \\ [\ln(\overline{F}_\theta(x_s))]^{s-1}[\overline{F}_\theta(x_r)]^{k-1} & \\ \times f_\theta(x_r) \prod_{i=s}^{r-1} r_\theta(x_i), & m = -1, \end{cases} \tag{21}$$

where $\Theta = (\alpha, \tau)$ and $\underline{x} = (x_{s;n,m,k}, x_{s+1;n,m,k}, \ldots, x_{r;n,m,k}) = (x_s, \ldots, x_r)$. Using Eqs.(4),(5),(6) and (7) in Eq.(21), we get

$$L(\alpha, \tau \mid \underline{x}) \propto \begin{cases} \frac{(\frac{\alpha}{\tau})^{r-s+1}}{(1-2^{-\alpha})^{\gamma_1}} [\varepsilon_r]^{\gamma_r+1} [\prod_{i=s}^{r} \varepsilon_i^{m+1} \eta_i] \sum_{\ell=0}^{s-1} \omega_\ell^{(s)} & \\ \times (1-2^{-\alpha})^{\ell(m+1)} \varepsilon_s^{(m+1)(s-\ell-1)}, & m \neq -1, \\ \frac{(\frac{\alpha}{\tau})^{r-s+1}}{(1-2^{-\alpha})^{k}} [ln \frac{\varepsilon_s}{1-2^{-\alpha}}]^{s-1} \varepsilon_r^k [\prod_{i=s}^{r} \eta_i], & m = -1. \end{cases} \tag{22}$$

Suppose that the conjugate prior density,which is measured by a function $\pi(\alpha, \tau)$ given by

$$\pi(\alpha, \tau) = \pi_1(\tau|\alpha)\pi_2(\alpha). \tag{23}$$

Suppose that $\pi_1(\tau|\alpha)$ is Gamma (c_1, α),$\pi_2(\alpha)$ is Gamma (c_2, c_3) with respective densities

$$\pi_1(\tau \mid \alpha) \propto \alpha^{c_1} \tau^{c_1-1} \exp(-\tau\alpha), \alpha, \tau > 0, (c_1 > 0), \tag{24}$$

$$\pi_2(\alpha) \propto \alpha^{c_2-1} \exp(-c_3\alpha), \alpha > 0, (c_2, c_3 > 0). \tag{25}$$

It then follows, by substituting (24) and (25) in (23), that the prior PDF of α and τ is given by

$$\pi(\alpha, \tau) \propto \alpha^{c_1+c_2-1} \tau^{c_1-1} \exp[-\alpha(\tau + c_3)], \alpha, \tau > 0, (c_1, c_2, c_3 > 0), \tag{26}$$

where c_1, c_2 and c_3 are the prior parameters (or hyper-parameters).

Using the likelihood function (22) and the prior (26) the posterior probability density function of α and τ for given informative data, say, $\pi^\star(\alpha, \tau \mid \underline{x})$ is given by

$$\pi^\star(\alpha, \tau \mid \underline{x}) \propto L(\alpha, \tau; \underline{x})\pi(\alpha, \tau). \tag{27}$$

Using the likelihood function (22) and the prior (26) the posterior pdf of α and τ can be written using (27) as

$$\pi^{\star}(\alpha,\tau \mid \underline{\mathrm{x}}) \propto \begin{cases} \alpha^{\Im}\tau^{\wp}[\varepsilon_r]^{\gamma_{r+1}}(1-2^{-\alpha})^{-\gamma_1}\exp[-\alpha(\tau+c_3)][\prod_{i=s}^{r}\varepsilon_i^{m+1}\eta_i] & \\ \times\sum_{\ell=0}^{s-1}\omega_\ell^{(s)}(1-2^{-\alpha})^{\ell(m+1)}\varepsilon_s^{(m+1)(s-\ell-1)}, & m\neq -1 \\ \alpha^{\Im}\tau^{\wp}(1-2^{-\alpha})^{-k}\exp[-\alpha(\tau+c_3)][\ln\frac{\varepsilon_s}{1-2^{-\alpha}}]^{s-1} & \\ \times\varepsilon_r^{k}[\prod_{i=s}^{r}\eta_i], & m=-1, \end{cases} \tag{28}$$

where $\Im = c_1+c_2+r-s$ and $\wp = c_1-r+s-2$.

3 Bayesian prediction

3.1 The case of odd future sample size

By making use of Eqn.(28) and (17), yields the Bayes predictive density function of the future median $Y_N^{\star}, N=2\xi-1, \xi=1,2,3,...,N,$ given the $(r-s+1)$ gos's, denoted by $h_{Y_N^{\star}}(y|\theta)$ as

$$h_{Y_N^{\star}}(y \mid \underline{\mathrm{x}}) = \int_{\Theta} f_{Y_N^{\star}}(y|\theta)\pi^{\star}(\theta \mid \underline{\mathrm{x}})d\theta, \quad y > x_r. \tag{29}$$

By making use of Eqs.(28) and (17)in (29), we obtain

$$f_{Y_N^{\star}}(y|\theta)\pi^{\star}(\theta \mid \underline{\mathrm{x}}) \propto \begin{cases} \alpha^{\Im+1}\tau^{\wp-1}\varepsilon_r^{\gamma_{r+1}}\eta_y\exp[-\alpha(\tau+c_3)][\Pi_{i=s}^{r}\varepsilon_i^{m+1}\eta_i][(1-2^{-\alpha})^{-1}\varepsilon_y]^{\gamma_{\xi^{\star}}} \\ \times\sum_{j=0}^{\xi-1}\sum_{\ell=0}^{s-1}\zeta_{\ell,j}^{(\xi,s)}(1-2^{-\alpha})^{-\gamma_\ell+1}[(1-2^{-\alpha})^{-1}\varepsilon_y]^{j(M+1)} \\ \times\varepsilon_s^{(m+1)(s-\ell-1)}, \quad m\neq -1, M\neq -1, \\ \alpha^{\Im+1}\tau^{\wp-1}\eta_y(1-2^{-\alpha})^{-k}\exp[-\alpha(\tau+c_3)][(1-2^{-\alpha})^{-1}\varepsilon_y]^{K} \\ \times[ln\frac{\varepsilon_s}{(1-2^{-\alpha})}]^{s-1}\varepsilon_r^{k}[\Pi_{i=s}^{r}\eta_i][ln[(1-2^{-\alpha})^{-1}\varepsilon_y]]^{\xi-1}, \quad m=-1, M=-1, \end{cases} \tag{30}$$

where

$$\zeta_{\ell,j}^{(\xi,s)} = (-1)^{\ell+j}\binom{s-1}{\ell}\binom{\xi-1}{j}. \tag{31}$$

To obtain $(1-\tau)100$ Bayesian prediction interval BPI for a future generalized order statistic $Y_N^{\star}$, say (L,U), we solve simultaneously the following two nonlinear equations, numerically,

$$P[Y_N^{\star} > L|\underline{\mathrm{x}}] = \int_L^{\infty} h_{Y_N^{\star}}(y|\theta)dy = 1-\frac{\tau}{2}, \tag{32}$$

$$P[Y_N^\star > U|\underline{\mathrm{x}}] = \int_U^\infty h_{Y_N^\star}(y \mid \underline{\mathrm{x}})dy = \frac{\tau}{2}. \tag{33}$$

Equation (32) and (33), can be solved by using Newton-Raphson iteration form as follows:

$$L_{j+1} = L_j - \frac{\int_{L_j}^\infty h_{Y_N^\star}(y \mid \underline{\mathrm{x}})dy - (1-\frac{\tau}{2})}{-h_{Y_N^\star}(L_j \mid \underline{\mathrm{x}})}, \tag{34}$$

$$U_{j+1} = U_j - \frac{\int_{U_j}^\infty h_{Y_N^\star}(y \mid \underline{\mathrm{x}})dy - \frac{\tau}{2}}{-h_{Y_N^\star}(U_j \mid \underline{\mathrm{x}})}, \tag{35}$$

where the initial values L_o,U_o can be taken equal to x_r. The integrals in (34) and (35) can be obtained using the routine QDAGI in IMSL.

3.1.1 Order statistics case

The Bayes prediction density function of the future median $Y_N^\star, N = 2\xi - 1, \xi = 1, 2, 3, ..., N$, given the informative sample $x_s, \ldots, x_r$, can be written from (29) and (30), when $m = 0$, $k = 1$,$M = 0$ and $K = 1$ for as $Y_N^\star$ as

$$h_{Y_N^\star(y|\theta)} = A_1 \int_0^\infty \int_0^\infty \alpha^{\Im+1}\tau^{\wp-1}\varepsilon_r^{n-r}\eta_y \exp^{-\alpha(\tau+c_3)}[\Pi_{i=s}^r \varepsilon_i\eta_i]$$
$$\times[(1-2^{-\alpha})^{-1}\varepsilon_y]^{N-\xi+1}\sum_{j=0}^{\xi-1}\sum_{\ell=0}^{s-1}\zeta_{j,\ell}^{(\xi,s)}(1-2^{-\alpha})^{-(n-\ell)}$$
$$\times[(1-2^{-\alpha})^{-1}\varepsilon_y]^j\varepsilon_s^{s-\ell-1}d\alpha d\tau, \tag{36}$$

where A_1 is a normalizing constant. Using the iteration forms (34) and (35), the routine QDAGI in IMSL to compute the double and triple integrals we obtain the prediction bounds of $Y_N^\star$.

3.1.2 Record values case

Making use of Eq.(30), yields the Bayes predictive density function of the future median $Y_N^\star, N = 2\xi - 1, \xi = 1, 2, 3, ..., N$ when $m = -1$, $k = 1$,$M = -1$ and $K = 1$ for $Y_N^\star$ as

$$h_{Y_N^\star}(y \mid \underline{\mathrm{x}}) = A_1 \int_0^\infty \int_0^\infty \alpha^{\Im+1}\tau^{\wp-1}\eta_y\varepsilon_y\varepsilon_r(1-2^{-\alpha})^{-2}\exp[-\alpha(\tau+c_3)]$$
$$\times[\Pi_{i=s}^r\eta_i][\ln[(1-2^{-\alpha})^{-1}\varepsilon_y]]^{\xi-1}[\ln\frac{\varepsilon_s}{(1-2^{-\alpha})}]^{s-1}d\alpha d\tau, \tag{37}$$

Using the iteration forms (34) and (35), the routine QDAGI in IMSL to compute the double and triple integrals we obtain the prediction bounds of $Y_N^\star$.

3.2 The case of even future sample size

Substituting Eqs.(18) and (28) in the integrand of (29), with $N = 2\xi, \xi = 1, 2, ..., N$, we obtain

$$h_{Y_N^\star}(y \mid \underline{x}) = \int_\Theta f_{Y_N^\star}(y \mid \underline{x})\pi^\star(\theta \mid \underline{x})d\theta, \quad y > 0, \tag{38}$$

Where

$$f_{Y_N^\star}(y \mid \underline{x})\pi^\star(\theta \mid \underline{x}) \propto \begin{cases} \int_0^y \alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}[\varepsilon_r]^{\gamma_{r+1}}(1-2^{-\alpha})^{-\gamma_1}\exp^{-\alpha(\tau+c_3)} \\ \times[\Pi_{i=s}^r \varepsilon_i^{m+1}\eta_i][(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^{\gamma_\xi^\star}\sum_{\ell=0}^{s-1}\sum_{j=0}^{\xi}\zeta_{j,\ell}^{(\xi,s)} \\ \times[(1-2^{-\alpha})^{-1}\varepsilon_z]^{(M+1)(j+1)}(1-2^{-\alpha})^{\ell(m+1)} \\ \varepsilon_s^{(m+1)(s-\ell-1)}dz, m \neq -1, M \neq -1 \\ \int_0^y \alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}\varepsilon_r^k(1-2^{-\alpha})^{-k}\exp^{-\alpha(\tau+c_3)} \\ \times[(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^K[\ln[(1-2^{-\alpha})^{-1}\varepsilon_z]]^{\xi-1} \\ \times[\Pi_{i=s}^r\eta_i][\ln\frac{\varepsilon_s}{1-2^{-\alpha}}]^{s-1}dz, m=-1, M=-1 \end{cases} \tag{39}$$

3.2.1 Order statistics case

Making use of Eq.(39), the Bayes predictive density function of the future median $Y_N^\star, N = 2\xi, \xi = 1, 2, 3, ..., N$ when $m = 0$, $k = 1$,$M = 0$, $K = 1$, for $Y_N^\star$ is

$$\begin{aligned} h_{Y_N^\star}(y \mid \underline{x}) = A_1 \int_0^\infty \int_0^\infty & \alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}(1-2^{-\alpha})^{-(n-1)}\exp^{-\alpha(\tau+c_3)} \\ & \times\varepsilon_r^{n-r+2}[\Pi_{i=s}^r\varepsilon_i\eta_i][(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^{N-\xi+1} \\ \times\sum_{\ell=0}^{s-1}\sum_{j=0}^{\xi}\omega_j(\xi)\omega_\ell(s) & \times[(1-2^{-\alpha})^{-1}\varepsilon_z]^{j+1}(1-2^{-\alpha})^\ell\varepsilon_s^{s-\ell-1}d\alpha d\tau. \end{aligned} \tag{40}$$

The Baysian prediction bounds of future order statistics in caseis case of $s=1$

$$h_{Y_N^\star}(y \mid \underline{\mathrm{x}}) = A_1 \int_0^\infty \int_0^\infty [\alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}\varepsilon_r^{n-r+2}(1-2^{-\alpha})^{-(n-1)}\exp^{-\alpha(\tau+c_3)}$$

$$\times[\Pi_{i=s}^r\varepsilon_i\eta_i][(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}]^{N-\xi+1}\sum_{j=0}^{\xi}\omega_j(\xi)[(1-2^{-\alpha})^{-1}\varepsilon_Z]^{j+1}]d\alpha d\tau, \quad (41)$$

Using the iteration forms (34) and (35), the routine QDAGI in IMSL to compute the double and triple integrals we obtain the prediction bounds of $Y_N^\star$.

3.2.2 Record values case

Making use of Eq.(39), yields the Bayes predictive density function of the future median $Y_N^\star, N = 2\xi - 1, \xi = 1, 2, 3, ..., N$ when $m = -1$, $k = 1$,$M = -1$, $K = 1$ is

$$h_{Y_N^\star}(y \mid \underline{\mathrm{x}}) = A_2 \int_0^\infty \int_0^\infty \alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}\varepsilon_r(1-2^{-\alpha})^{-1}\exp^{-\alpha(\tau+c_3)}[\Pi_{i=s}^r\eta_i]$$

$$\times[(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}][ln[(1-2^{-\alpha})^{-1}\varepsilon_z]]^{\xi-1}[ln\frac{\varepsilon_s}{1-2^{-\alpha}}]^{s-1}d\alpha d\tau, \quad (42)$$

where

$$A_2^{-1} = \int_0^y \int_0^\infty \int_0^\infty \alpha^{\Im+2}\tau^{\wp-2}\eta_z\eta_{2y-z}\varepsilon_r(1-2^{-\alpha})^{-1}\exp^{-\alpha(\tau+c_3)}[\Pi_{i=s}^r\eta_i]$$

$$\times[(1-2^{-\alpha})^{-1}\varepsilon_{2y-z}][\ln[(1-2^{-\alpha})^{-1}\varepsilon_z]]^{\xi-1}[\ln\frac{\varepsilon_s}{1-2^{-\alpha}}]^{s-1}d\alpha d\tau dz. \quad (43)$$

Using the iteration forms (34) and (35), the routine QDAGI in IMSL Library to compute the double and triple integrals we obtain the prediction bounds of $Y_N^\star$.

Remark 3.1. (1) It may be noted that if $s = 1$, we obtain a type II censored sample technique and if we choose $s = 1$ and $r = n$, our results reduce to the complete sample case.
(2) In some situtions one can not observe some initial values of any sample, so he must be chosen the doubly type II censoring scheme.

4 Numerical computations

In this section, we obtain the interval predictors of future median OOS and ordinary upper record when the underlying population has $TTIGL$ distribution according to the following steps.

4.1 OOS $(m = 0, k = 1)$

Based on a given doubly Type-II censored sample, We will consider here the case which represents the OOS. The 95% Bayesian prediction bounds for the future median, and their actual (simulated) prediction levels, are obtained according to the following steps:
(1) A random sample of sizes (10, 30, 50) are generated from TTIGL distribution. The sample is ordered and the upper 10%, 20%, 25%,30%,40% and 50% are censored where the future sample size N is fixed and set to 5, 8. The computation are carried out for the hyper-parameters cases $(c1, c2, c3)$ are set to $(0.7, 1.3, 0.85)$, the data generated from TTIGL $(0, 0.977, 1.25)$ and $(0, 0.8, 2.239)$.
(2) Based on these doubly Type-II censored data, the 95% Bayesian prediction bounds for the future (unobserved) are then calculated by solving Eqs.(34) and (35) for the lower and upper bounds with $\tau = 95\%$ for different values of N, when $N = 2\xi - 1$ is odd and $N = 2\xi$ is even.
(3) For $10,000$ generated future ordered samples each of size N, from TTIGL density, the simulated prediction levels of $Y^{\star}_{N}$ are then calculated. The prediction are conducted on the basis of a doubly type II censored samples and type II censored samples. The results are presented in Tables 1-2.

Table 1: The lower limit (LL), the upper limit (UL), the length of the BPI and the percentage coverage of the 95% BPI for the future median Y_N^* when $N = 2\xi - 1$ is odd or $N = 2\xi$ from TTIGL($\beta = 0, \alpha = 0.977, \tau = 1.25$).

$Case$	$Y_N^\star$	$s=1$		$s=2$		$s=3$	
		(LL,UL) Length	%	(LL,UL) Length	%	(LL,UL) Length	%
$n=10$ $r=5$	$Y_3^\star$	(0.0115,1.5640) 1.5525	91.7	(0.0059,1.7099) 1.7040	88.5	(0.00342,1.8180) 1.8146	85.7
	$Y_4^\star$	(0.01203,1.4843) 1.4843	85.7	(0.0095,1.8716) 1.8673	86.5	(0.01203,1.9940) 1.9820	84.5
$n=10$ $r=8$	$Y_3^\star$	(0.02450,1.4091) 1.3846	91.8	(0.0115,1.5916) 1.5801	92.8	(0.00368,1.7085) 1.7048	90.7
	$Y_4^\star$	(0.01004,1.3105) 1.30054	89.5	(0.01281,1.63011) 1.6173	92.3	(0.0125,1.8145) 1.7420	87.3
$n=30$ $r=18$	$Y_5^\star$	(0.9042,1.9700) 1.06583	88.7	(0.7163,1.8803) 1.1640	87.4	(0.7542,1.9650) 1.2108	89.3
	$Y_6^\star$	(0.08505,0.88645) 0.80340	84.4	(0.0963,0.94552) 0.84927	85.5	(0.08501,0.9885) 0.90342	86.4
$n=30$ $r=22$	$Y_5^\star$	(0.0421,0.9848) 0.9424	92.7	(0.0050,1.0096) 1.0046	92.5	(1.003,2.0093) 1.0063	84.5
	$Y_6^\star$	(0.0904,0.8765) 0.7861	89.5	(0.0800,0.8614) 0.7814	88.2	(0.0904,0.9165) 0.8761	87.2
$n=30$ $r=27$	$Y_5^\star$	(0.0082,0.7944) 0.8026	94.7	(0.0065,0.8570) 0.8505	93.5	(1.0042,1.9199) 0.9157	86.5
	$Y_6^\star$	(0.08014,0.76142) 0.6810	95.5	(0.07404,0.8302) 0.7562	97.2	(0.08605,0.8940) 0.8079	97.2
$n=50$ $r=25$	$Y_5^\star$	(0.0641,1.0930) 1.0866	92.7	(0.0441,1.2135) 1.1694	92.5	(0.0613,1.2187) 1.2800	91.5
	$Y_6^\star$	(0.08001,0.86142) 0.7814	84.5	(0.07114,0.9103) 0.8392	88.2	(0.0850,0.9885) 0.9035	84.2
$n=50$ r=35	$Y_5^\star$	(0.0416,0.9289) 0.9705	92.7	(1.0041,2.1350) 1.1309	86.5	(1.0041,2.0408) 1.0367	84.5
	$Y_6^\star$	(0.06499,0.7723) 0.7073	93.5	(0.0963,0.94552) 0.8442	96.2	(0.07114,0.9103) 0.8392	92.2
$n=50$ r=40	$Y_5^\star$	(0.0910,0.8485) 0.7575	95.7	(0.0631,1.0237) 0.9606	96.5	(1.0043,2.0064) 1.0021	87.5
	$Y_6^\star$	(0.09625,0.7455) 0.6493	92.5	(0.0800,0.7614) 0.6814	92.2	(0.0672,0.7863) 0.7191	84.2

Table 2: The lower limit (LL), the upper limit (UL), the length of the BPI and the percentage coverage of the 95% BPI for the future median Y_N^* when $N = 2\xi - 1$ is odd or $N = 2\xi$ from TTIGL ($\beta = 0, \alpha = 0.8, \tau = 2.239$).

$Case$	$Y_N^\star$	$s=1$		$s=2$		$s=3$	
		(LL, UL) Length	%	(LL, UL) Length	%	(LL, UL) Length	%
$n=10$ $r=5$	$Y_3^\star$	(1.7371,5.0102) 4.2730	90.7	(1.6711,5.8874) 4.2163	91.5	(2.5870,6.5371) 3.9501	84.11
	$Y_4^\star$	(1.4135,5.5819) 3.1684	91.7	(1.5289,5.1217) 3.5928	91.5	(2.0009,6.7622) 4.7613	85.5
$n=10$ $r=8$	$Y_3^\star$	(2.6371,5.6468) 3.0097	88.3	(1.9007,5.0090) 3.1083	85.8	(3.7371,6.4809) 3.1438	96.7
	$Y_4^\star$	(2.10621,5.4787) 3.3725	90.5	(2.5581,6.0617) 3.5036	86.3	(3.0618,6.5305) 3.4732	83.7
$n=30$ $r=18$	$Y_5^\star$	(1.7986,5.3097) 3.5111	86.7	(1.9800,6.0062) 4.0262	89.4	(3.7986,8.6071) 4.8085	87.7
	$Y_6^\star$	(1.01074,5.9458) 4.9351	88.3	(0.01075,4.14680) 4.1361	90.5	(3.0309,7.0049) 3.9740	84.1
$n=30$ $r=22$	$Y_5^\star$	(2.0905,6.0007) 3.9102	87.2	(2.7987,6.3096) 3.5109	92.5	(2.7301,7.1700) 4.3714	81.5
	$Y_6^\star$	(1.6902,4.6411) 2.9509	93.5	(1.0102,4.1037) 3.0936	92.2	(2.0131,6.7056) 4.6925	97.2
$n=30$ $r=27$	$Y_5^\star$	(1.8960,4.0076) 2.1116	92.7	(1.0780,3.5090) 2.4310	92.5	(2.7180,5.2182) 2.5002	92.5
	$Y_6^\star$	(1.0183,3.0450) 2.0267	94.5	(1.1709,3.6460) 2.4751	93.2	(2.0160,5.0060) 2.9900	93.2
$n=50$ $r=25$	$Y_5^\star$	(1.7006,5.0096) 3.3090	96.7	(2.7987,6.4007) 3.6020	96.5	(4.0310,10.3098) 4.2788	96.5
	$Y_6^\star$	(2.01043,5.1247) 3.11414	94.5	(2.02075,5.1068) 3.08605	95.2	(5.01107,9.3580) 4.3469	94.2
$n=50$ $r=35$	$Y_5^\star$	(1.9003,3.9093) 2.009	96.7	(2.8087,6.0076) 3.1989	96.5	(2.9007,6.2969) 3.3962	96.5
	$Y_6^\star$	(1.01178,3.2128) 2.2010	94.5	(2.01074,5.1458) 2.1351	96.2	(2.09572,6.9641) 4.8684	97.2
$n=50$ $r=40$	$Y_5^\star$	(1.7987,3.3096) 1.5109	96.7	(1.1008,4.3607) 2.5620	95.5	(2.8087,5.5666) 2.7579	96.5
	$Y_6^\star$	(1.01074,3.1458) 2.1351	96.5	(1.01142,3.1899) 2.1785	97.2	(2.01075,4.1458) 2.4405	83.2

4.2 OURV $(m = -1, k = 1)$

Our interest is in the median future recored, because generating of record sample takes a relatively longer time than the OOS, we use $n = 10, 15$ from TTIGL $(\alpha = 0.8, \tau = 2.239)$ distribution and apply 20% and 50% censoring. The future sample size N is fixed. The vector of hyperparameters (c_1, c_2, c_3) are same as in OOS example $(0.7, 1.3, 0.85)$. Based on these doubly Type-II censored data, the 95% Bayesian prediction bounds for the future (unobserved) are then calculated by solving Eqs.(3.6) and (3.7) for the lower and upper bounds.

For $100,000$ generated future recored samples each of size $N = 4, 5$, from TTIGL density, the percentage coverage for the simulated prediction levels of $Y_N^\star$ are then calculated. The results are presented in Table 3.

Table 3: The lower limit (LL), the upper limit (UL), the length of BPI and the percentage coverage of the 95% BPI for the future median Y_N^* of ordinary upper record values when $N = 2\xi - 1$ is odd or $N = 2\xi$ from TTIGL $(\beta = 0, \alpha = 0.8, \tau = 2.239)$.

$Case$	$Y_N^\star$	$s = 1$		$s = 2$	
		(LL, UL) Length	%	(LL, UL) Length	%
$n = 10$ $r = 5$	$Y_3^\star$	(3.6035,7.5723) 3.9688	88.7	(3.1134,7.1229) 4.0095	89.1
	$Y_2^\star$	(2.914889,7.09231) 4.1774	92.5	(2.15007,6.4831) 4.3330	90.4
$n = 10$ r=8	$Y_3^\star$	(2.9617,6.7875) 3.8258	93.6	(2.4493,6.1373) 3.6880	89.7
	$Y_2^\star$	(3.9838,7.6229) 3.6391	93.1	(3.1440,6.6493) 3.5053	89.4
$n = 15$ $r = 7$	$Y_3^\star$	(1.2967,4.2712) 2.9745	84.7	(2.2593,5.4408) 3.1815	92.5
	$Y_2^\star$	(1.3983,4.5923) 3.1940	85.6	(2.8332,5.1774) 3.3442	92.4
$n = 15$ r=12	$Y_3^\star$	(2.1489,4.8712) 2.7223	92.6	(2.6580,5.6841) 3.0261	92.7
	$Y_2^\star$	(3.2004,5.3824) 2.1820	94.1	(2.9058,5.7516) 2.8458	93.5

4.3 Concluding remarks

Based on the numerical results in Tables (1-3), it may be observed that:
(1) The generalized results obtained for the prediction of GOS enable us to specialize to any of the other cases which are included in the GOS by appropriate choice of m and k.
(2) As τ increases the lengths of the intervals increase.
(3) Whether n is odd or even, shorter predictive intervals BPI and its percentage coverage could be obtained by increasing r, since more information is introduced to the informative sample.
(4) from the tables we realize that for fixed value of n the percentage coverage probability improve by using a large number of observed values.
IMSL Reference manual, institute of mathematical statistics library, IMSL, Inc; Houston, TX, 1984.

References

[1] A.A. Ahmad, Relations for single and product moments of generalized order statistics from doubly truncated Burr type XII distribution., *J. Egypt. Math. Soc.*, **15**(1), (2007), 117-128.

[2] A.A. Ahmad, Single and product moments of generalized order statistics from linear exponential distribution., *Commun. Statist.-Theor. Meth.*, **37**(8), (2008), 1162-1172.

[3] A.A. Ahmad, On Bayesian interval prediction of future generalized order statistics using doubly censoring., *Statistics*, **45**(5), (2011), 413-425.

[4] A.A. Ahmad and T.A. Abu-Shal, Recurrence relations for moment generating functions of generalized order statistics from a class of doubly truncated continuous distributions., *J. Prob. and Statist. Sci.*, **6**(2), (2008), 243-257.

[5] A.A. Ahmad and M. Fawzy, Recurrence relations for single moments of generalized order statistics from doubly truncated distributions., *J. Statis. Plann. and Inference*, **117**, (2003), 241-249.

[6] M. Ahsanullah, *Introduction to Record Values*, Ginn Press, Needham Heights, Massachusetts, 1994.

[7] M. Ahsanullah, Generalized order statistics from exponential distribution, *J. Statist. Plann. and Infer.*, **85**, (2000), 85-91.

[8] A. AL-Angary, *Truncated logistic distributions as lifetime models*, M. Sc. Thesis, Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Kindom of Saudi Arabia, 1997.

[9] E.K. AL-Hussaini, Predicting observables from a general class of distributions, *J. Statist. Plann. Infer.*, **79**, (1999), 79-91.

[10] E.K. AL-Hussaini, On Bayes Prediction of Future Median, *Commun. Statist.-Theor. Meth.*, **30**(7), (2001), 1395-1410.

[11] E.K. AL-Hussaini, Generalized order statistics: Prospective and Applications, *J. of Appl. Statist. Sci.*, **13**(1), (2004), 59-85.

[12] E.K. AL-Hussaini and A.A. Ahmad, On Bayesian interval prediction of future records, *Test*, **12**(1), (2003a), 79-99.

[13] E.K. AL-Hussaini and A.A. Ahmad, On Bayesian predictive distributions of generalized order statistics, *Metrika*, **57**, (2003b), 165-176.

[14] E.K. AL-Hussaini, G.R. AL-Dayan and A. AL-Angary, Bayesian prediction bounds under the truncated type I generalized logistic model, *J. Egyptian. Math. Soc.*, **14**, (2006), 55-67.

[15] E.K. AL-Hussaini and S.F. Ateya, Maximum likelihood estimations under a mixture of truncated type I generalized logistic components model, *J. Statist. Th. Appl.*, **2**(1), (2003), 47-60.

[16] E.K. AL-Hussaini and S.F. Ateya, Bayes estimations under a mixture of truncated type I generalized logistic components model, *J. Statist. Th. Appl.*, **4**(2), (2005), 183-208.

[17] E.K. AL-Hussaini and Z.F. Jaheen, Bayesian prediction bounds for the Burr type XII failure model, *Commun. Statist.-Theor. Meth.*, **24**(7), (1995), 1829-1842.

[18] E.K. AL-Hussaini and Z.F. Jaheen, Bayesian prediction bounds for the Burr type XII distribution in the presence of outliers, *J. Statist. Plain. Inference*, **55**, (1996), 23-37.

[19] E.K. AL-Hussaini and Z.F. Jaheen, Parametric prediction bounds for the future median of the exponential distribution, *Statistics*, **32**, (1999), 267-275.

[20] M.A.M. Ali Mousa, Bayesian prediction Based on Pareto doubly censored data, *Statistics*, **37**(1), (2003), 65-72.

[21] B.C. Arnold, N. Balakrishnan and H.N. Nagaraja, *A First Course in Order Statistics*, Wiley, New York, 1992.

[22] B.C. Arnold, N. Balakrishnan and H. Nagaraja, *Records*, Wiley, New York, 1998.

[23] N. Balakrishnan, Order statistics from half-logistic distribution, *J. Statist. Comput. Simul.*, **20**, (1985), 287-309.

[24] N. Balakrishnan, *Handbook of the Logistic Distribution*, Marcel Dekker, New York, 1992.

[25] N. Balakrishnan and P.S. Chan, Estimation for the scaled half-logistic distribution under type II censoring, *Comput. Statist. and Data Analy.*, **13**, (1992), 123-141.

[26] N. Balakrishnan and K.H.T. Wong, Approximate MLEs for the location and scale parameter of the half-logistic distribution with type II censoring IEEE Trans, *On Reliab.*, **40**, (1991), 140-145.

[27] E. Cramer and U. Kamps, Sequential order statistics and k-out-of-n systems with sequentially adjusted failure rates, *Ann. Inst. Statist. Math.*, **48**, (1996), 535-549.

[28] E. Cramer and U. Kamps, Relations for expectations of functions of generalized order statistics, *J. Statist. Plann. and Inference.*, **89**(1-2), (2000), 79-89.

[29] H.A. David and H.N. Nagaraja, *Order Statistics*, John Wiley Sons, Canada, 2003.

[30] Hafiz M.R. Khan, Ahmed N. Albatineh, Saeed Alshahrani, Nadine Jenkins and Nasar U. Ahmed, Sensitivity analysis of predictive modeling for responses from the three-parameter Weibull model with a follow-up doubly censored sample of cancer patients, *Comput. Stat. Data Anal.*, **55**(12), (2011), 3093-3103.

[31] Hafiz M.R. Khan, Serge B. Provost and Ashima Singh, Sensitivity analysis of predictive modeling for responses from the three-parameter Weibull model with a follow-up doubly censored sample of cancer patients, *Commun. Statist. The. Meth.*, **39**(7), (2010), 1237-1246.

[32] M. Habibullah and M. Ahsanullah, Predictive inference from a two parameter Rayleigh life model given a doubly censored sample, *Commun. Statist. The. Meth.*, **29**(7), (2000), 1597-1609.

[33] Z.F. Jaheen, On Bayesian prediction of generalized order statistics, *J. Statist. Th. Appl.*, **1**(3), (2002), 191-204.

[34] Z.F. Jaheen, Estimation based on generalized order statistics from the Burr model, *Commun. Statist. The. Meth.*, **34**, (2005), 785-794.

[35] U. Kamps, *A Concept of Generalized Order Statistics*, Teubner Stuttgart, 1995.

[36] U. Kamps and U. Gather, Characteristic property of generalized order statistics for exponential distributions, *Appl. Math. (Warsaw).*, **24**(4), (1997), 383-391.

[37] C. Keseling, Conditional distributions of generalized order statistics and some characterizations, *Metrika*, **49**(1), (1999), 27-40.

[38] P. Pawlas and D. Szynal, Recurrence relations for single and product moments of generalized order statistics from Pareto, generalized Pareto and Burr distributions, *Commun. Statist.-Theor. Meth.*, **30**(4), (2001), 739-746.