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In Search of Optimal Number of Bond Funds

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Abstract

Although bonds are less volatile than equities and the median bond fund holds about 200 bonds, bond investors still need to hold more than one bond fund to realize the optimal benefit of diversification. The simulation results show that three to five bond funds reduce standard deviation of terminal wealth by 50% and about 100 funds reduce the standard deviation by 90%. Given the annualized marginal cost of 0.13% for bond funds, bond investors should hold three to five funds. However, equity investors who want to diversify need only one to two bond funds, regardless of risk measures. Holding more than two bond funds does not reduce portfolio risk much further especially for portfolios with high equity weights. The portfolios mixed with government and corporate bond funds require even fewer funds than the portfolios mixed with high yield bond funds. These results are robust to different investment strategies, holding periods and time periods and not subject to survivorship bias. These findings can be generalized to portfolios with other asset mixes.

JEL classification numbers: G10, G11
Keywords: Bond diversification, optimal bond portfolio, terminal wealth

1 Introduction

The most recent financial crisis underscores the importance of diversification. In 2008, investors who held financial stocks lost 55% of their portfolios on average while investors who diversified to S&P 500 stocks lost 37% and those who added 40% bonds to their portfolios lost merely 18%. This anecdotal evidence clearly indicates that bonds, a standard diversifier, do provide the diversification benefits when needed. Concerning risk reduction, thus, bonds should be part of a well-diversified portfolio. Since mutual fund is

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the investment vehicle of choice for an average long-term U.S. investor, we ask how many bond funds are optimal for both bond and equity investors.

According to 2007 Morningstar database, the median bond fund holds close to 200 bonds.[3] Two hundred bonds seem large enough. However, the number of securities that constitute a well-diversified portfolio seems to grow larger and larger based on the results of prior studies that focus on equities and equity funds. Early work by Evans and Archer (1968) and Fisher and Lorie (1970) suggest only as few as eight stocks constitute an optimal portfolio. More recent work suggests a much larger number. Statman (2004) suggests more than 300 stocks, and Domian, Louton, and Racine (2007) suggest about 160 stocks.[4] Although equity funds hold about 80 stocks, O'Neal (1997) finds that one equity fund is not enough and suggests holding three to five equity funds. Brands and Gallagher (2005) find the optimal number of active Australian funds is six.

In this paper, through simulation, we attempt to answer the question of how many bond funds an investor should have. We address different scenarios concerning bond and equity investors who hold either diversified or non-diversified portfolios. We also analyze marginal benefits and costs of owning bond funds. The answers to these questions should provide specific guidance to financial advisors as well as individual investors. Unlike prior studies that analyze optimal portfolios on one asset class, this study examines the optimal portfolio of mixed assets specifically bonds and stocks. The results in this study should shed light on other mixed portfolios such as equity and alternative investment, and equity and hedge funds.

As a risk measure, we focus on dispersion of terminal wealth, rather than standard deviation of time series returns. Long-term investors such as those planning their retirement and their children's college savings anticipate certain wealth levels from their investments at the end of investment periods. Any shortfall from the expected wealth levels can cause disrupt of their plans. These investors are likely to be more concerned about deviation of the expected wealth levels than about dispersion of returns over time.

The sample consists of 4,528 surviving and non-surviving bond funds and 12,718 equity funds over the period of 1988 to 2007. The sample period spans over 20 years, covering periods of high and low interest rates and two cycles of multiple rate cuts followed by multiple rate increases.[5] A long sample period that covers different interest rate levels and complete cycles of interest rate changes is important for a study such as this because both levels and changes in interest rates affect bond returns and volatility and hence have impacts on portfolio diversification.

The simulation results suggest that investors benefit from holding more than one bond fund. Although standard deviation of time series returns levels off after a portfolio of five bond funds, terminal wealth volatility continues to decline after that. About three to five

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[3] This is about three times of the median number of bonds held by a bond fund in 1998. Based on 1998 Morningstar database, the median number is 68. The government bond funds hold fewer bonds than corporate and high yield bond funds.

[4] See also Bird and Tippett (1986), Elton and Gruber (1977), and Statman (1987).

[5] During late 1980s and early 1990s, interest rates were high. The one-year Treasury bill rate was as high as 9.6% and the rate for Baa-rated corporate bonds was 11.2%. The interest rates were lower during 2000s. The two cycles of multiple rate cuts and increases occurred between 1990 and 1994 and between 2000 and 2006. From July 1990 to September 1992, there were 18 interest rate cuts followed by seven rate increases from February 1994 to 1995. From January 2001 to June 2003, there were 13 interest rate cuts and 17 rate increases from June 2004 to 2006.

bond funds reduce standard deviation of terminal wealth by half and 100 bond funds reduce the standard deviation by 90%. We estimate the annualized marginal cost of diversifying to bond funds to be 0.13%. Given the estimated cost, the optimal portfolio should contain three to five bond funds, which reduces standard deviation of terminal wealth by 40-60%. The optimal portfolio of high yield bond funds require a few more funds due to the funds' high risk, and the optimal portfolio of government and corporate bond funds requires fewer funds. Investors who are concerned with only downside risk need fewer bond funds because the risk reduction based on mean shortfall and semivariance is smaller.

While bond investors need three to five bond funds in their portfolios, equity investors should have only one to two bond funds. For high equity weight portfolios such as 10/90 bond/equity mix, having more than one bond fund does not reduce risk any further. For low equity weight portfolios such as 50/50 mix, the marginal cost of holding more than two bond funds far exceeds the marginal benefit. The equity portfolios mixed with government and corporate bond funds require only one bond fund regardless of portfolio mix, while the portfolios mixed with high yield bond funds require one bond fund for high equity weight portfolios and two bond funds for low equity weight portfolios. Because bond funds reduce terminal wealth volatility of non-diversified equity portfolios more than diversified equity portfolios, non-diversified equity portfolios require even fewer bond funds. These results are robust to the assumption of buy-and-hold, different holding periods, and different time periods, and are not sensitive to survivorship bias.

The results in this study taken together with the findings of O'Neal (1997) and Brands and Gallagher (2005) can be generalized as follows: Diversifying in the same fund classes requires three to six funds. The riskier the assets are, the larger the number of funds. On the other hand, diversifying across asset classes requires very few funds. The portfolios mixed with low correlation assets need as few as one fund. The portfolios mixed with high correlation assets need as few as one to two funds, depending on portfolio mix.

The rest of this paper is organized as follows. The next section lays out the sample selection process, methodology, and return data. Section 3 explains the results of bond portfolios and bond and equity mixed portfolios. Section 4 provides robustness check on investment strategies, different holding periods, and investment periods, and analyzes the effect of survivorship bias. Section 5 concludes the study.

2 Sample, Methodology, and Returns

2.1 Sample

The sample of both bond and equity funds is obtained from CRSP Mutual Fund database. For bond funds, we select domestic bond funds that are classified as government, corporate, high-yield, or general bond funds that have at least one monthly return available during the sample period of 1988 to 2007.[6] There are 4,528 bond funds that meet the criteria. Of these, 1,105 (24.4%) are government, 2,063 (45.56%) corporate, 741 (16.36%) high-yield, and 619 (13.67%) general domestic bond funds.

[6] The classification is based on Lipper Objective Codes. If the codes are not available, we use Strategic Insights Objective Codes, Wiesenberger Objective Codes, and Policy Codes, respectively.

Panel A of Table 1 presents the number of bond funds in each category by year. There were 2,649 bond funds in 2007, an increase of 220% from 829 in 1988. During the sample period, the number of government bond funds increased from 202 to 532, while the number of corporate bond funds increased significantly from 132 to 1,242. The number of high-yield bond funds also rose significantly from 72 in 1988 to 495 in 2007.
Panel B of Table 1 presents the number of bond funds delisted over the sample period. Based on the delisting codes from CRSP Mutual Fund Database, about 12% of the bond fund sample (544 funds) was liquidated, 27.78% (1,258 funds) was merged, and 1.24% (56 funds) was delisted for other reasons. Compared across bond categories, government bond funds had the highest percentage of liquidated funds (16.92%, 187 funds) and merged funds (34.48%, 381 funds).[7] High yield bond funds had the lowest percentage of liquidated funds (8.23%) and merged funds (22.27%). Overall, the results in Panel B of Table 1 show that a significant percentage of the sample bond funds were defunct during the sample period.
For equity funds, similar to the bond fund sample, we select domestic equity funds that are classified as income, growth, large-, mid-, or small-cap funds and have at least one monthly return available during the sample period. There are 12,718 equity funds that pass the criteria. About 10% (1,243) of equity funds were liquidated and 23% (2,930) were merged, and 1% (84) was delisted for other reasons during the sample period.

2.2 Methodology

To evaluate the number of bond funds in an optimal bond portfolio, we compare volatility of terminal wealth of one-bond fund portfolio with the volatility of multiple-bond fund portfolios. Terminal wealth is defined as ending wealth of investing \$100 at the beginning of a holding period. Three measures of variability—standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) —are used and calculated as below:

$$SD_{TW} = \sqrt{\frac{\sum_{i=1}^{N}(TW_i - E(TW))^2}{N-1}}$$

$$SF_{TW} = \frac{\sum_{i=1}^{N}(TW_i - E(TW))}{N-1} if\ TW_i < E(TW)$$

$$SV_{TW} = \frac{\sum_{i=1}^{N}(TW_i - E(TW))^2}{N-1} if\ TW_i < E(TW)$$

where TW_i is terminal wealth of portfolio i and $E(TW)$ is the expected terminal wealth. Standard deviation captures both positive and negative deviations from the expected terminal wealth, while mean shortfall and semivariance capture only negative deviations below its expected value. Mean shortfall and semivariance are appropriate for investors who do not consider upside returns as risk. We assume that investors' expected terminal wealth is equal to average ending value of all bond funds. To make semivariance numbers comparable, we report square root of semivariance.
The simulations are run using three choices of variables: the holding period (5, 10, and 20 years), the fund category (all, government, corporate, and high yield), and the number of

[7] About 60% of 187 government bond funds that were liquidated were done during the first half of the sample period (1988-1997). Only about 25% (8%) of liquidated corporate (high yield) funds were done during the first half of the sample period.

bond funds (1-10, 12, …, 20, 25, 30, 40, 50, 100 and 200). The 5-year holding period starts 2003 and ends 2007. The 10-year period is from 1998 to 2007. The 20-year period is the entire sample period of 1988-2007. For one-bond fund portfolio, we use all available bond funds. To create multiple-bond fund portfolios, we randomly select bond funds without replacement to form equally weighted portfolios. Terminal wealth is calculated assuming buy-and-hold strategy. The process is repeated 1,000 times. Standard deviation, mean shortfall and semivariance of terminal wealth are calculated for each portfolio. Average terminal wealth (TW) and standard deviation of time series returns (SD_R) of each portfolio are also calculated. For easy comparison, we standardize these numbers as percentages of the one-bond fund portfolios, and present the standardized numbers.

To analyze the optimal number of bond funds in an equity portfolio, we compare terminal wealth's volatility of an equity fund portfolio with the volatility of portfolios mixed between bond and equity funds. We first assume that equity investors have diversified equity portfolios. O'Neal suggests that three to five equity funds constitute a diversified equity portfolio. Therefore, for an equity fund portfolio, we randomly choose five equity funds to form an equally weighted portfolio. The process is repeated 1,000 times. We use these 1,000 portfolios as a base to create mixed portfolios. To create mixed portfolios, we randomly select bond funds to add to the base portfolios. The portfolio mix is 50%, 70%, and 90% of equity funds. The number of bond funds added to the base portfolios includes 1-10, 12,…, 20, 30, 40, 50, 100 and 200, equally weighted. Terminal wealth and its variability, and holding periods are defined as above. We present these numbers as percentages of the base portfolios (i.e., the equity fund portfolios). We also relax the assumption that equity investors have diversified portfolios by using one-equity fund portfolios instead of five-equity fund portfolios.

2.3 Returns

Monthly fund returns are used in this study. The returns are total returns after all administrative and trading expenses but before loads, assuming reinvestment of income and capital gain distributions. Because the results in Panel B of Table 1 show that a significant number of the sample bond funds were defunct during the sample period, we use returns of both surviving and non-surviving bond and equity funds. When non-surviving funds are merged, we follow merged funds until the end of a holding period or delisted. If the merged funds are delisted before the end of a holding period, or non-surviving funds are liquidated or delisted for other reasons, we randomly choose another fund to fill in the rest of the returns.

3 Results

3.1 How Many Bond Funds Constitute a Diversified Portfolio?

3.1.1 All bond funds

Table 2 presents terminal wealth (TW), standard deviation of time series return (SD_R), standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) of terminal wealth of all bond funds for 5-, 10- and 20-year holding periods. For each holding period,

TW is relatively constant for all portfolios and hence reported only for one-bond fund portfolios. TW for the 5-year holding period is $125.75, compared with $156.68 and $345.94 for the 10- and 20-year holding periods. While SD_R of the one-fund portfolio for the 20-year holding period is slightly higher than SD_R for 5-year holding period, SD_{TW}, SF_{TW} and SV_{TW} for the 20-year holding period are about four to five times higher than those for the 5- and 10-year holding periods. The finding of higher terminal wealth volatility of the 20-year holding period underscores the importance of diversification in long-run.

Comparing risk across bond fund portfolios, we find that SD_R declines significantly when one bond fund is added but only slightly after that, and levels off after the five-fund portfolio. This is true for all holding periods. For example, for the 20-year holding period, SD_R drops to 91% for the two-fund portfolio and to 83% for the five-fund portfolio, and stays around 80% after that.

Unlike SD_R, SD_{TW} continues to decline after the five-fund portfolio although at a much slower rate than the first five portfolios. For example, for the 20-year holding period, SD_{TW} declines to 68% for the two-fund portfolio and to 43% for the five-fund portfolio, and continues to drop to 22% for the 20-fund portfolio and to 6% for the 200-fund portfolio. To reduce SD_{TW} by half, three to four funds are needed for the 5- and 20-year holding periods and four to five funds are needed for 10-year holding period. About 100 funds reduce SD_{TW} by 90%. A similar risk reduction pattern is found for SF_{TW} and SV_{TW}. However, SF_{TW} and SV_{TW} decline at slower rates than SD_{TW}. This happens more for the 5-year holding period.

These results provide several suggestions. First, investors benefit from holding more than one bond fund. Second, although SD_R levels off after the five-fund portfolio, terminal wealth volatility continues to decline. Therefore, diversification benefits still exist after the five-fund portfolio. Third, the incremental benefits are large when a portfolio contains a few bond funds and small when a portfolio contains many bond funds. This pattern of risk reduction is the same for all holding periods. Fourth, investors need three to five funds to reduce SD_{TW} by half, and about 100 funds to reduce SD_{TW} by 90%. Fifth, investors who rely on SF_{TW} as a risk measure need a few more funds than those who use SD_{TW} to cut risk by the same percentage.

The next question is how many bond funds investors should have. Although having 100 bond funds reduces the dispersion of terminal wealth by 90%, it might not be practical because costs of buying and maintaining 100 funds might be very high. The Investment Company Institute reports that the total annualized cost of owning bond funds was 0.78% in 2007, consisting of 0.65% operating expenses and 0.13% load fees.[8] The institute also reports that the expense ratio of funds of funds is 0.93%, which includes both direct expenses and indirect expenses paid to acquire funds. For easy diversification, investors can buy funds of funds and pay the incremental cost of 0.41%, which is the difference between 0.93% and 0.65% operating expenses plus 0.13% load fees.[9] If investors diversify by directly buying bond funds, they incur transaction costs of 0.13%, which is the annualized load fee. Given the lower costs, investors are better off buying bond funds directly, especially when they need only a few bond funds. Therefore, we use 0.13% as the estimated annualized marginal cost of diversifying to bond funds.

[8] http://www.ici.org/stats/res/1fm-v18n3.pdf (Download date: May 5, 2009).
[9] About 50% of bond funds of funds charge load fees.

To compare the diversification benefits with the annualized marginal cost, we convert the amount of risk reduction to the annualized marginal benefit. We assume that the amount of risk reduction occurred at the end of a holding period can be annuitized into equal installments. For example, for the 20-year holding period, having the two-fund portfolio reduces SD_{TW} by 32%. The benefit of risk reduction is \$22.1 (32% multiplied by \$69.05) per \$100 investment or 22.1%. The annualized benefit is 1.11% given 0% interest rate and 0.67% given 5% interest rate. The 5% interest rate is the approximate one-year T-bill rate during the sample period and used as an opportunity cost paid for diversification over a holding period. As illustrated, an increase in interest rates decreases the annualized benefit.

Figures 1 and 2 show the annualized marginal benefit based on SD_{TW} at 0% and 5% rates, respectively, and the marginal cost. In Figure 1, for the 5-year holding period, the marginal benefit exceeds the marginal cost for the two-, three- and four-fund portfolios. For the 10- and 20-year holding periods, the benefit exceeds the cost for the first three- and five-fund portfolios, respectively. These findings suggest that given the estimated marginal costs of 0.13%, three to five bond funds constitute an optimal portfolio. The optimal portfolio reduces SD_{TW} by about 40-60%.

In Figure 2 where 5% interest rate is incorporated, as mentioned above, the annualized benefit declines when interest rate increases. For the 20-year holding period, the optimal number of bond funds reduces from five to four. However, the optimal portfolio for the 5- and 10-year holding periods still contains four and three bond funds. The optimal portfolio reduces SD_{TW} by 40-50%.

We also calculate the annualized benefit based on SF_{TW} and SV_{TW} at 0% and 5% rates (results not tabulated). For the 5-year holding period, the optimal portfolio based on SF_{TW} and SV_{TW} contains three and two bond funds, respectively, at both 0% and 5% rates. For the 10-year holding period, the optimal portfolio based on SF_{TW} and SV_{TW} contains three bond funds. For the 20-year holding period, the optimal portfolio contains two (two) and four (two) bond funds based on SF_{TW} and SV_{TW} at 0% (5%) rate. The optimal portfolio based on SF_{TW} and SV_{TW} generally comprises fewer bond funds than the optimal portfolio based on SD_{TW} because SF_{TW} and SV_{TW} are smaller and their risk reduction is at a slower rate. The optimal portfolio reduces SF_{TW} by 25-35% and SV_{TW} by 20-50%.

The results in Figures 1 and 2 suggest that although there are benefits of holding more than five bond funds, investors should not have more than five bond funds because the incremental costs offset the incremental benefits. Given the estimated costs of 0.13%, investors should hold only three to five bond funds, and should hold even fewer funds if they are concerned with only downside risk. The optimal portfolio reduces SD_{TW} by about 40-60%.

3.1.2 Bond categories

The results presented above are for all bond funds. Table 3 presents the results of the three bond fund categories. For the 5- and 20-year holding periods, the average TW and SD_{TW} are lowest for government bond funds and highest for high yield bond funds. These findings confirm conventional wisdom that high yield bond funds are risky and also support the risk-return tradeoff. For the 10-year holding period, the average TW of high yield bond funds is lower than the average TW of government and corporate bond funds while SD_{TW} is higher. This is because high yield bond funds did not perform well during the period of 1998-2007 that includes 1999 tech bubble and 2001 recession.

The pattern of risk reduction for all three fund categories is similar to the pattern of all bond funds presented in Table 2. That is, SD_{TW} declines quickly for the first few portfolios and slowly after that. It takes three to five funds to reduce SD_{TW} by half, and about 100 funds to reduce SD_{TW} by 90%. For the 5-year holding period, the optimal portfolio requires three funds for government and corporate bond portfolios, and four funds for high yield bond portfolios at both 0% and 5% rates. For the 10-year holding period, the optimal portfolio contains three, four and five funds for government, corporate and high yield bond funds, respectively. For the 20-year holding period, the optimal portfolio requires five and six (three) funds for government and corporate bond funds, and seven (five) funds for high yield bond funds at 0% (5%) rate. These results suggest that while the pattern of risk reduction is similar for all three fund categories, the optimal portfolio requires a larger number of high yield bond funds due to the funds' high risk. The optimal portfolio reduces SD_{TW} by 40-60% for all three categories.

Overall, the results in Tables 2, 3, and 4 and Figures 1 and 2 suggest that there are benefits of holding more than one bond fund. The risk reduction is large when the first few bond funds are added to a portfolio and small when a portfolio contains many bond funds. About three to five bond funds reduce SD_{TW} by half and 100 funds reduce SD_{TW} by 90%. Given the estimated marginal cost of 0.13%, three to five bond funds constitute the optimal portfolio, which reduces SD_{TW} by about 40-60%. The optimal portfolio of high yield bond funds needs to contain a few more funds, while the portfolio of government and corporate bond funds contains fewer funds because of low volatility of government and corporate bond funds.

3.2 How Many Bond Funds do Equity Investors Need?

3.2.1 All bond funds

Most investors hold stocks or equity funds in their portfolios. Do these investors also need three to five bond funds? Table 4 presents the results of adding bond funds to diversified equity portfolios. Panel A presents the results of the 5-year holding period from 2003 to 2007. As bond funds are added to equity portfolios, as expected, terminal wealth declines. This is because bond funds provide lower returns than equity funds on average. However, the wealth reduction is much less than one to one. Adding bond funds to create a 50/50 bond/equity portfolio reduces terminal wealth by only about 16%. Similarly, the wealth reduction of 30/70 and 10/90 portfolios is only 10% and 3%, respectively. SD_R reduces sharply when one bond fund is added to equity portfolios, and stays relatively constant. SD_R of the 50/50 portfolio reduces by 45% while SD_R of the 30/70 and 10/90 portfolios reduces by 28% and 10%, respectively.

The volatility of terminal wealth also declines significantly when one bond fund is added to equity portfolios. This is true for SD_{TW} as well as SF_{TW} and SV_{TW}. For example, SD_{TW} declines to 70% of the equity portfolios when one bond fund is added to create the 50/50 portfolio, 76% for the 30/70 portfolio, and 91% for the 10/90 portfolio. Adding more bond funds to the portfolios reduces the volatility further for the 50/50 portfolio, but not for the 30/70 and 10/90 portfolios. Having three bond funds in the 50/50 portfolio reduces SD_{TW} by 43%, from 30% of the one-bond fund portfolio. On the other hand, SD_{TW} of the three-bond fund, 10/90 portfolio is 90.1% of the equity portfolio, about the same as 90.7% for the one-bond fund portfolio.

Comparing the annualized benefit of risk reduction with the incremental cost of buying bond funds, we find that the optimal portfolio based on SD_{TW} contains only one bond fund for the 30/70 and 10/90 portfolios, and two bond funds for the 50/50 portfolio. The optimal portfolio reduces SD_{TW} by 40% for the 50/50 portfolio, 25% for the 30/70 portfolio and 10% for the 10/90 portfolio. The optimal portfolio based on SF_{TW} and SV_{TW} contains only one bond fund for all portfolio mixes. These results suggest that equity investors who want to diversify to bond funds need to have only one to two bond funds, not three to five bond funds as required by bond investors. Investors with high equity weight portfolios such as 10/90 mix need even fewer bond funds than those with low equity weight portfolios such as 50/50 mix. For investors with high equity weight portfolios, the incremental cost of having more than one bond fund exceeds the incremental benefit. The opposite is true for investors with low equity weight portfolios.

Panel B of Table 5 presents the results for the 10-year holding period of 1998-2007. During the period, bond funds performed well while equity funds suffered due to 1999 tech bubble and 2001 economic slowdown. Therefore, adding bond funds to equity portfolios during the 10-year holding period reduces TW much less than during the 5-year period in Panel A. TW reduces by only 7% for the 50/50 portfolio and 1% for the 10/90 portfolio, compared with 16% and 3% in Panel A.

The risk reduction in Panel B based on both time series return and terminal wealth exhibits a similar pattern to the risk reduction in Panel A. There is significant risk reduction in SD_R when one bond fund is added to equity portfolios, but the risk remains constant after that. The volatility of terminal wealth also reduces significantly when one bond fund is added. The risk reduction continues after more bond funds are added to portfolios with low equity weights although at a much slower rate. The optimal portfolio contains only one bond fund for all portfolio mixes.

In Panel C of Table 5 for the 20-year holding period from 1987-2007, the average TW of equity portfolios is $550.8, about three times of TW of equity portfolios for the 5-year period presented in Panel A. While the TW increases about three times, the volatility of terminal wealth increases about 8-9 times. SD_{TW} in Panel C is $160.4, compared with $17.41 in Panel A. SF_{TW} and SV_{TW} in Panel C are $-118.58 and $141.66, compared with -$13.17 and $16.48 in Panel A. An increase in risk that is out of proportion to an increase in returns, once again, underscores the importance of diversification in long-run.

When bond funds are added to equity portfolios, TW of the portfolios for the 20-year holding period reduces slightly more than TW for the 5-year holding period presented in Panel A. TW of the 50/50 portfolio reduces by 18% while TW of the 30/70 and 10/90 portfolios reduces by 11% and 4%, respectively. On the other hand, SD_R for the 20-year holding period declines less than SD_R for the 5-year holding period. For example, SD_R of the 50/50 portfolio declines 36% for the 20-year holding period, compared with 45% for the 5-year holding period.

The risk reduction based on volatility of terminal wealth for the 20-year holding period shows the same pattern as that for 5-year holding period. That is, the volatility of terminal wealth declines significantly when one bond fund is added to the portfolios. The risk continues to drop when more bond funds are added but only for the portfolios with low equity weights. Compared the risk reduction of Panel C with Panel A, the risk reduction of the 50/50 and 30/70 portfolios declines more for the 20-year period than for the 5-year period, while the risk reduction of the 10/90 portfolio is essentially the same. The optimal portfolio contains two bond funds for the 50/50 portfolio and one bond fund for the 30/70 and 10/90 portfolios.

The results in Table 4 provide many suggestions for portfolio management. First, well-diversified equity investors still benefit from having bond funds. Second, the optimal portfolio should contain only one to two bond funds, not three to five bond funds as required by bond investors. Third, investors with high equity weight portfolios need only one bond fund, while investors with low equity weight portfolios benefit from having more than one bond fund. Fourth, the benefit of having bond funds is larger for longer holding periods and for portfolios with lower equity weights.

3.2.2 Bond categories

In this section, we analyze the diversification benefits of adding different bond fund categories to equity portfolios. Table 5 presents the results for the 30/70 portfolio mix. For all holding periods, the decline in TW and SD_R is relatively constant for all bond portfolios. Therefore, only TW and SD_R of the one-bond fund portfolio are reported. For the 5- and 20-year periods, TW for portfolios with high yield bond funds is higher than that for portfolios with government and corporate bond funds due to higher return provided by the funds. The opposite is true for the 10-year period, due to poor performance of high yield bonds during the period.

For all holding periods, the risk reduction based on SD_R is greatest for portfolios with government bond funds and lowest for portfolios with high yield bond funds. This finding can be explained by low correlation between government bond funds and equity funds. The correlation between equity funds and government bond funds is 0.02, compared with 0.16 between equity funds and corporate bond funds and 0.59 between equity funds and high yield bond funds.

The risk reduction based on SD_{TW} is slightly greater for government bond funds and smaller for high yield bond funds. For example, for the 5-year holding period, the one-government bond fund portfolio reduces SD_{TW} by 30%, compared with 25% of the one-high yield bond fund portfolio. Adding more than one government and corporate bond fund does not reduce risk much further. SD_{TW} of the government (corporate) bond fund portfolio stays at about 70% (72%) of the equity fund portfolios for the 5-year holding period, 70% (70%) for the 10-year holding period and 71% (71%) for the 20-year holding period. However, adding more than one high yield bond fund reduces risk slightly more.

For all holding periods, the optimal portfolio contains only one bond fund for all three categories. For the 10/90 bond/equity portfolio, the optimal portfolio also contains one bond fund for all three bond categories (results not tabulated). For the 50/50 portfolio, the optimal portfolio contains one bond fund for government and corporate bond portfolios and two bond funds for high yield bond portfolios. These results lead us to conclude that although high yield bond funds have higher correlation with equity funds than government bond funds, the optimal portfolios with high equity weights should contain only one bond fund for both high yield and government bond funds. For portfolios with low equity weights, the incremental benefit of having more than one high yield bond fund exceeds the incremental cost.

3.2.3 Non-diversified equity portfolios

This section analyzes whether the above results change if investors have non-diversified equity portfolios (i.e., one-equity fund portfolios), instead of diversified equity portfolios (i.e., five-equity fund portfolios). Table 6 presents the results of all bond funds over the 5-year holding period. The average TW, SD_R, and terminal wealth volatility of non-

diversified equity portfolios are higher than those of diversified equity portfolios presented in Panel A of Table 4. SD_{TW} for non-diversified equity portfolios is \$43.27, about twice of \$17.41 for diversified equity portfolios. SF_{TW} and SV_{TW} are -\$26.51 and \$34.43 for non-diversified equity portfolios, compared with -\$13.17 and \$16.48 for diversified equity portfolios.

Interestingly, when bond funds are added, TW and SD_R for non-diversified equity portfolios reduce by about the same percentages as for diversified equity portfolios. The pattern of risk reduction of terminal wealth volatility is also similar. That is, for all combinations of portfolios, the risk reduction is greatest when one bond fund is added. The risk reduction is close to zero when more bond funds are added to portfolios with high equity weights such as 10/90. However, the terminal wealth volatility of non-diversified portfolios declines more than the volatility of diversified equity portfolio. This is especially true for the one-bond fund portfolio and portfolios with 50/50 mix. For the 50/50 mix, SD_{TW} of one-bond fund portfolio drops 46% for non-diversified equity portfolios, compared with 30% for diversified equity portfolios in Panel A of Table 4.

For all portfolio mixes, the optimal portfolio contains only one bond fund, compared with one to two bond funds for diversified equity portfolios. The results for the 10- and 20-year holding periods are similar to those for the 5-year holding periods (results not tabulated). For the 10- and 20-year holding periods, the optimal portfolio contains only one bond fund, compared with one to two bond funds for diversified equity portfolios. These results suggest that bond funds provide more diversification benefits to non-diversified equity portfolios than to diversified equity portfolios. Investors who have non-diversified equity portfolios with low equity weights need only one bond fund.

We also analyze whether adding bond funds from different categories to non-diversified equity portfolios provide more risk reduction than to diversified equity portfolios (results not tabulated). As above, for all three categories, the risk reduction is larger for non-diversified equity portfolios than for diversified equity portfolios. This is especially true for high yield bond funds. For government and corporate bond funds, the optimal portfolio still contains one bond fund. The optimal portfolio for high yield bond funds contains two bond funds for 50/50 portfolio at the 10- and 20-year holding periods, and one bond fund for the rest. These results confirm the above conclusion that adding bond funds to non-diversified equity portfolios reduce risk more than adding bond funds to diversified equity portfolios.

4 Robustness Check

In this section, we present the results of annual rebalance and different time periods of bond fund portfolios.[10] Further, we analyze the effects of survivorship bias on bond diversification.

[10] The assumption of annual rebalance and the survivorship bias have the same effects on the mixed portfolios as bond portfolios. Further, the results of bond and equity portfolios presented in Section 3 hold for different time periods. Therefore, for brevity, we do not present the results of bond and equity portfolios in this section.

4.1 Annual Rebalance

The results in Section 3 are based on the assumption that investors buy and hold funds until the end of a holding period. The advantages of buy-and-hold strategy are low cost and easy to implement. The disadvantage is that it could lead to overweight outperforming funds and underweight poor performing funds. To analyze the impacts of buy-and-hold strategy, we re-run simulations assuming that investors rebalance their portfolios yearly. At the beginning of each year, investors divide money equally among funds by selling overweight funds and buying underweight funds.

The simulation results show that the annual rebalance has minimal impacts for the 5- and 10-year holding periods, but results in higher TW and SD_{TW} for the 20-year holding period (results not tabulated). For bond portfolios, TW and SD_{TW} of the rebalance strategy are \$347.6 and \$72.27, compared with \$345.94 and \$69.05 of buy-and-hold strategy. This translates to \$1.66 in higher TW over the 20-year period or the annualized gain of \$0.08. The gain, however, does not offset the transaction costs such as the annualized load fees of 0.13%, capital gain taxes and higher SD_{TW}. Further, the rebalance strategy does not change the number of bond funds that constitutes an optimal portfolio for the 20-year holding period as well as the 5- and 10-year holding periods.

4.2 Different Time Periods

Do the results in Section 3 hold for different time periods? Table 7 presents the results of bond funds for 10-year rolling periods. The average TW varies from period to period due to interest rate changes over the sample period. The TW from 1988 to 1997 is \$221.44, compared with \$156.68 from 1998 to 2007. The lower terminal wealth from 1998 to 2007 is due to lower interest rates and hence lower bond returns during 2000s. Similarly, SD_{TW} varies; it is generally higher during early periods when TW is high and lower during later periods when TW is low.

The pattern of risk reduction based on SD_{TW} is the same across different time periods. The risk declines quickly when the first few bond funds are added to a portfolio, and declines slowly when the number of bond funds increases. To reduce SD_{TW} by half, investors need three to five bond funds. At both 0% and 5% rates, the optimal portfolio contains four to five bond funds for the first eight periods and two to three bond funds for the rest.[11] The optimal portfolio reduces SD_{TW} by 40-50%. We also analyze SF_{TW} and SV_{TW} of 10-year rolling periods and the results of 5-year rolling periods and find similar results to those reported in Table 2. These results suggest that the earlier findings are robust to different time periods.

4.3 Survivorship Bias

To study the effect of survivorship bias, we compare the above results with the results that include only surviving bond funds. Table 8 presents the results that include only surviving

[11] The Investment Company Institute shows that the ownership cost as well as load fees of mutual funds decline over time. For example, the total fee for bond funds is 0.95% in 2003, compared with 0.78% in 2007. The load fee in 2003 is 0.21%, compared with 0.13% in 2007. Adjusted for higher load fees in early periods, the optimal portfolio also requires two to three bond funds in early periods.

bond funds. The average TW in Table 8 is higher than the TW of both surviving and non-surviving bond funds in Table 2 for all holding periods, and the difference is largest for the 20-year holding period. This finding is expected because non-surviving funds tend to be poor performers and the number of poorly performing funds increases over a longer time period. The volatility of terminal wealth in Table 8 is also higher than that in Table 2 for the 5- and 20-year holding periods, while the volatility in Table 8 is less for the 10-year holding period.

The pattern of risk reduction in Table 8 is similar to that in Table 2. Specifically, the risk based on both SD_R and the terminal wealth volatility reduces when bond funds are added. While SD_R levels off after the five-fund portfolio, the terminal wealth volatility continues to decline after that. To reduce SD_{TW} by about half, investors need three to five bond funds, the same number as in Table 2. About 100 bond funds would reduce SD_{TW} by about 90%.

Based on SD_{TW}, the optimal portfolio contains four to six bond funds at 0% rate and three to four bond funds at 5% rate, compared with three to five and two to four bond funds, respectively, of Table 2. The optimal portfolio reduces SD_{TW} by 50-60%. The optimal portfolio based on SF_{TW} and SV_{TW} contains two to three bond funds, the same as in Table 2. The results in Table 8 suggest that while survivorship bias affects TW and its volatility, it does not significantly change the risk reduction pattern and the optimal number of bond funds.

5 Conclusion

Given the benefits and costs of diversification, prior studies examine a number of optimal securities that constitute a diversified portfolio. However, they focus on only stocks and equity funds. In this study, we extend the analysis to bond funds and portfolios of mixed assets as many investors, especially retirees, hold bonds in their portfolios. The sample obtained from CRSP Mutual Fund Database consists of 4,528 surviving and non-surviving bond funds and 12,718 equity funds over the sample period of 1998 to 2007.

The simulation results show that although bonds are less volatile than equities and the median bond fund holds about 200 bonds, exceeding the optimal number of securities suggested by prior studies, investors still benefit from holding more than one bond fund. Three to five bond funds reduce standard deviation of terminal wealth by half, and 100 funds reduce the standard deviation by 90%. Given the annualized marginal cost of 0.13%, the optimal portfolio should contain three to five bond funds. Due to high risk, the portfolio of high yield bond funds requires a larger number of funds than the portfolio of corporate and government bond funds. Investors who are concerned with only downside risk need fewer funds because the risk reduction based on shortfall and semivariance is smaller.

The simulation results also show that while bond investors need three to five bond funds, equity investors need only one to two bond funds. For high equity weight portfolios such as 10/90 bond/equity mix, having more than one bond fund does not reduce risk any further. For low equity weight portfolios such as 50/50 bond/equity mix, the marginal costs of having more than two bond funds exceed the marginal benefits. The optimal portfolios mixed with government bond funds require fewer funds than the portfolios mixed with high yield bond funds because of low correlations between government and equity funds. Non-diversified equity fund portfolios also require fewer bond funds than

diversified portfolios because the diversification benefit is larger for non-diversified portfolios.

The results above are robust to different investment strategies, holding periods and time periods, and are not sensitive to survivorship bias. These results taken together with the findings of O'Neal (1997) and Brands and Gallagher (2005), who study equity funds, can be generalized to other asset classes. Diversifying within the same fund classes requires more funds than diversifying across fund classes. Diversifying across fund classes requires as few as one fund.

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Appendices

Appendix 1: Figures 1 and 2

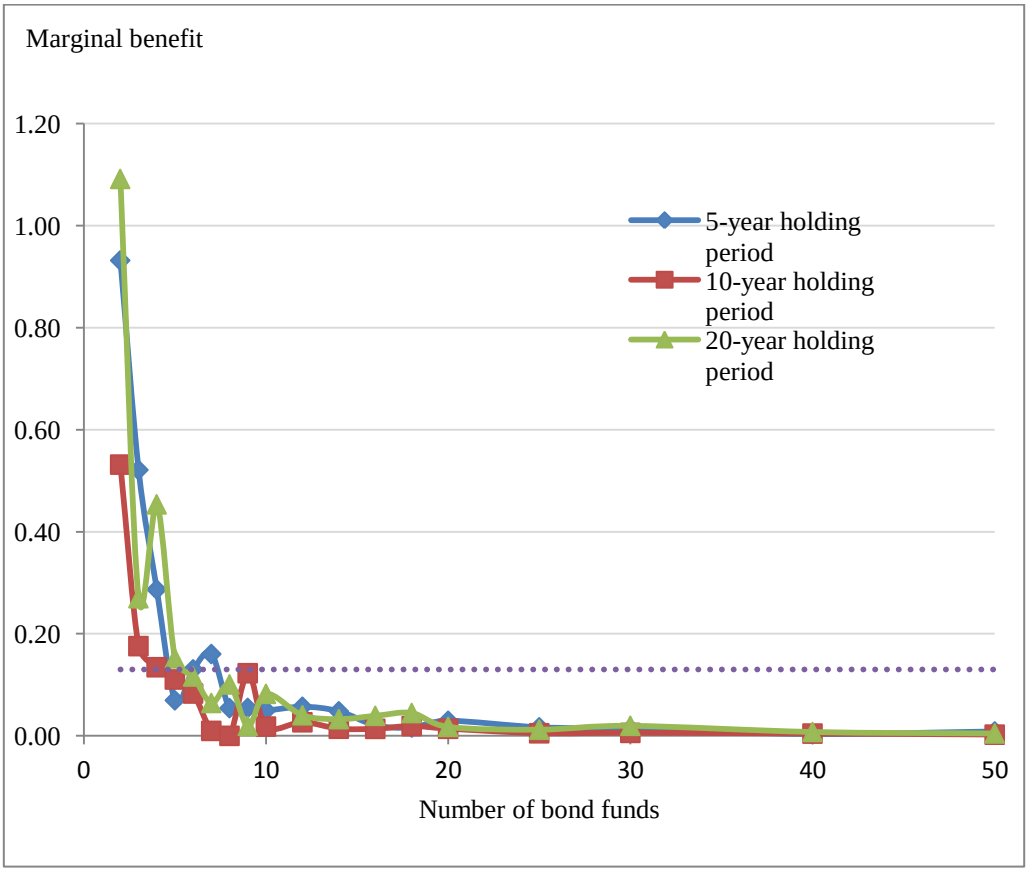

Figure 1: The annualized marginal benefit based on standard deviation of terminal wealth over 5-, 10-, and 20-year periods for all bond funds at 0% interest rate
Note: The plot starts at two-bond fund portfolios.

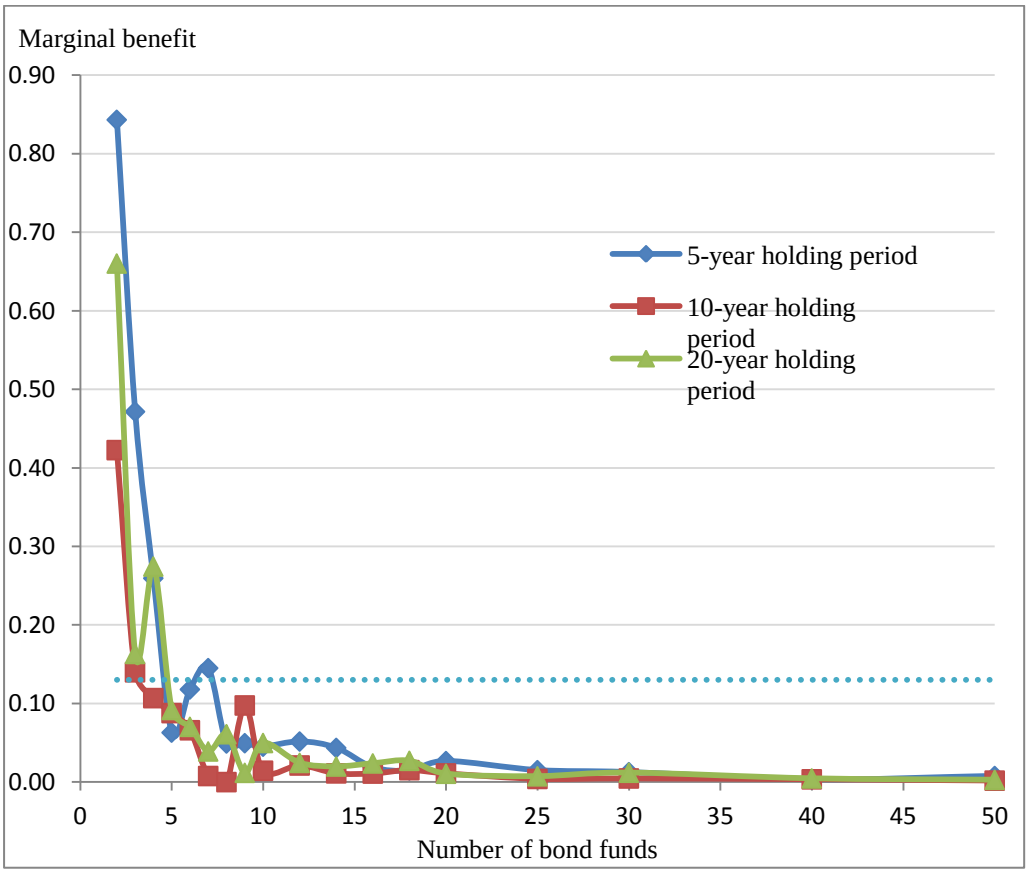

Figure 2: The annualized marginal benefit based on standard deviation of terminal wealth over 5-, 10-, and 20-year periods for all bond funds at 5% interest rate
Note: The plot starts at two-bond fund portfolios.

Appendix 2: Panel A of this table presents the number of government (GB), corporate (CB), high yield (HY), and general (BD) bond funds that have at least one monthly return available during the sample period of 1988-2007, and the number of bond funds that existed at the beginning of each year during the sample period. The sample is obtained from CRSP Mutual Fund Database. Panel B of this table presents the number of bond funds that were liquidated, merged, or delisted for other reasons during the sample period. The numbers in parentheses are presented as percentages of all bond funds in Panel A.

Table 1: Number of bond funds and delisting reasons

Panel A: Number of bond funds

	Number of bond funds				
Year	**Total**	**GB**	**CB**	**HY**	**BD**
1988	829	202	132	72	423
1989	942	242	161	80	459
1990	979	251	188	84	456
1991	1,000	266	210	87	437
1992	1,057	296	250	85	426
1993	1,207	361	341	88	417
1994	1,457	475	459	106	417
1995	1,747	587	606	132	422
1996	1,812	585	649	162	416
1997	1,903	579	718	194	412
1998	2,027	600	768	253	406
1999	2,221	599	851	352	419
2000	2,417	625	958	397	437
2001	2,454	614	1,003	426	411
2002	2,484	577	1,071	442	394
2003	2,587	580	1,162	450	395
2004	2,702	599	1,237	468	398
2005	2,736	583	1,273	473	407
2006	2,680	561	1,228	496	395
2007	2,649	532	1,242	495	380
All	4,528	1,105	2,063	741	619

Panel B: Delisting reasons

	Number of bond funds				
Delisting	**Total**	**GB**	**CB**	**HY**	**BD**
Liquidated	544	187	239	61	57
% of all	(12.01)	(16.92)	(11.59)	(8.23)	(9.21)
Merged	1258	381	533	165	179
% of all	(27.78)	(34.48)	(25.84)	(22.27)	(28.92)
Others	56	18	28	5	5
% of all	(1.24)	(1.63)	(1.36)	(0.67)	(0.81)

Appendix 3: This table presents average terminal wealth (TW), average standard deviation of monthly returns (SD_R, %), and terminal wealth volatility of all bond funds (i.e., one-bond fund portfolio) over different holding periods. This table also presents SD_R and terminal wealth volatility of portfolios with different numbers of bond funds as percentages of SD_R and terminal wealth volatility of the one-bond fund portfolio. TW is the ending wealth of investing $100 at the beginning of a holding period. Terminal wealth volatility includes standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) of terminal wealth. The holding periods include 5 years, from 2003 to 2007; 10 years, from 1998 to 2007; 20 years, from 1988 to 2007. N is the number of bond funds that exist at the beginning of a holding period, and # is the number of bond funds in a portfolio.

Table 2: Diversification benefits of all bond funds over different holding periods

	5-year holding period N = 2,587; TW = $125.75				**10-year holding period** N = 2,027; TW = $156.68				**20-year holding period** N = 829; TW = $345.94			
#	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
1	1.01	$16.67	-$8.23	$9.83	1.15	$17.38	-$11.66	$17.78	1.28	$69.05	-$40.97	$55.37
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
2	90	72	89	83	89	69	77	72	91	68	74	71
3	87	56	79	77	84	59	65	58	87	61	68	65
4	86	48	74	71	81	52	60	52	85	47	54	51
5	85	46	67	66	79	45	52	45	83	43	52	48
6	82	42	61	60	77	40	48	42	83	40	45	41
7	83	37	56	56	76	40	45	39	81	38	45	40
8	82	35	51	51	76	40	45	39	81	35	40	38
9	81	34	50	51	74	33	38	32	81	34	41	38
10	81	32	48	48	74	32	36	30	80	32	39	35
12	80	29	44	45	74	29	34	29	80	30	37	33
14	80	26	40	40	73	27	32	27	79	28	35	32
16	80	25	39	39	72	25	30	25	79	25	31	28
18	80	24	35	37	72	23	27	23	79	23	30	27
20	79	22	34	35	72	22	26	22	79	22	27	25
25	80	19	31	31	71	20	24	20	79	20	25	23
30	79	17	27	28	71	19	23	19	78	17	22	20
40	79	16	25	26	71	16	20	16	78	15	19	17
50	79	14	21	22	71	15	18	15	78	14	18	16
100	79	10	16	17	70	10	12	10	78	9	12	11
200	78	7	11	11	70	7	8	6	78	6	8	7

Appendix 4: This table presents average terminal wealth (TW, $) and standard deviation of terminal wealth (SD_{TW}, $) of three categories of bond funds (i.e., one-bond fund portfolio) over 5-, 10-, and 20-year holding periods. This table also presents SD_{TW} of portfolios with different numbers of bond funds as percentages of SD_{TW} of the one-bond fund portfolio. TW is the ending wealth of investing $100 at the beginning of a holding period. Three categories of bond funds are government (GB), corporate (CB) and high yield (HY) bond funds. N is the number of bond funds in each category that exist at the beginning of a holding period, and # is the number of bond funds in a portfolio.

Table 3: Diversification benefits of bond funds classified by bond fund categories

	5-year holding period: 2003-2007			**10-year holding period: 1998-2007**			**20-year holding period: 1988-2007**		
	GB	**CB**	**HY**	**GB**	**CB**	**HY**	**GB**	**CB**	**HY**
N	580	1,162	450	600	768	253	202	132	72
TW	116.43	120.46	154.95	158.46	160.43	143.57	339.08	368.47	392.98
SD_{TW}	5.69	8.60	15.79	13.04	13.48	27.72	52.72	70.23	104.20
#	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**
1	100%	100%	100%	100%	100%	100%	100%	100%	100%
2	71	70	73	75	70	67	65	69	71
3	58	54	59	60	59	59	57	56	57
4	50	50	48	51	48	51	50	50	50
5	45	44	45	44	44	44	44	44	44
6	40	39	43	40	39	40	42	40	40
7	37	36	39	39	39	38	38	37	36
8	35	33	35	33	37	35	35	34	35
9	33	31	32	33	34	33	33	32	31
10	32	33	30	31	30	31	30	31	28
12	28	28	28	28	27	29	27	29	26
14	27	27	26	26	27	27	27	25	24
16	24	26	25	26	25	24	24	24	22
18	24	24	23	23	23	24	23	22	20
20	22	22	22	22	21	21	22	21	19
25	20	20	19	20	19	19	20	18	16
30	18	17	18	18	19	18	17	16	14
40	15	15	16	16	15	14	14	13	11
50	14	14	13	14	14	13	13	11	8
100	9	10	9	9	10	8	7	5	-
200	6	6	5	6	6	3	1	-	-

Appendix 5: This table presents average terminal wealth (TW), average standard deviation of monthly returns (SD_R) and terminal wealth volatility of 1,000 diversified equity fund portfolios (i.e., the equity fund portfolio) over different holding periods. This table also presents TW, SD_R and terminal wealth volatility of the portfolios that mix between equity funds and bond funds as percentages of TW, SD_R, and terminal wealth volatility of the equity fund portfolio. The mix is 50% bond and 50% equity for 50/50 portfolio, 30% bond and 70% equity for 30/70 portfolio, and 10% bond and 90% equity for 10/90 portfolio. # is the number of bond funds in a mixed portfolio. TW is the ending wealth of investing $100 at the beginning of a holding period. Terminal wealth volatility includes standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) of terminal wealth. A diversified equity fund portfolio is created from five randomly chosen equity funds, equally weighted. The holding periods include 5 years, from 2003 to 2007, for Panel A; 10 years, from 1998 to 2007, for Panel B; 20 years, from 1988 to 2007, for Panel C.

Table 4: Diversification benefits of bond funds in diversified equity portfolios

Panel A: 5-year period from 2003-2007; TW = $184.95; SD_R = 2.63%; SD_{TW} = $17.41; SF_{TW} =$-13.17; SV_{TW} = $16.48

	50/50 mix					**30/70 mix**					**10/90 mix**				
#	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
0	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	83.9	55.5	69.7	61.0	60.5	90.4	72.1	76.2	75.8	75.0	96.8	90.5	90.7	89.7	90.2
2	83.9	54.7	60.4	59.6	59.0	90.3	71.8	72.7	74.2	73.7	96.8	90.4	90.1	90.7	90.7
3	83.8	54.4	57.0	58.1	57.6	90.3	71.7	71.9	70.9	71.8	96.8	90.4	90.1	89.0	89.7
4	83.9	54.5	56.2	58.0	57.3	90.4	71.8	71.8	74.3	73.7	96.8	90.4	90.2	92.2	91.4
5	84.0	54.5	54.9	54.2	54.4	90.4	71.9	71.5	69.3	70.5	96.8	90.5	90.2	90.2	90.2
6	83.9	54.3	53.1	53.8	54.1	90.4	71.8	70.4	71.1	71.2	96.8	90.4	89.9	90.9	90.5
7	83.9	54.2	53.2	51.9	52.6	90.3	71.7	70.9	71.1	71.0	96.8	90.4	90.1	90.9	90.5
8	83.9	54.3	53.1	52.0	52.5	90.4	71.8	70.8	71.4	71.3	96.8	90.4	90.1	90.9	90.6
9	83.9	54.3	52.9	53.1	52.5	90.4	71.8	71.0	71.0	70.6	96.8	90.4	90.2	91.0	90.4
10	83.9	54.2	52.1	52.3	52.3	90.3	71.7	70.5	70.6	70.6	96.8	90.4	90.0	90.1	90.1
12	83.9	54.2	51.6	49.8	50.3	90.3	71.7	70.3	70.4	70.1	96.8	90.4	90.0	90.9	90.3
14	84.0	54.3	51.6	52.3	51.8	90.4	71.8	70.4	72.1	71.2	96.8	90.4	90.0	90.3	90.1
16	83.9	54.2	51.3	50.7	50.7	90.4	71.7	70.3	70.3	70.1	96.8	90.4	90.0	90.3	90.1
18	83.9	54.2	51.5	50.8	51.4	90.3	71.7	70.5	70.2	70.5	96.8	90.4	90.1	90.6	90.4
20	83.9	54.1	50.9	49.6	50.3	90.4	71.7	70.1	69.4	69.7	96.8	90.4	90.0	90.4	90.2
25	84.0	54.2	51.3	50.0	50.8	90.4	71.8	70.4	68.8	69.7	96.8	90.4	90.1	89.9	90.0
30	83.9	54.1	50.8	50.7	50.7	90.4	71.7	70.2	70.6	70.4	96.8	90.4	90.0	90.3	90.2
40	83.9	54.1	50.6	49.9	50.4	90.3	71.7	70.2	70.4	70.4	96.8	90.4	90.0	90.7	90.4
50	83.9	54.1	50.4	49.7	50.0	90.4	71.7	70.1	69.5	69.7	96.8	90.4	90.0	90.9	90.4
100	83.9	54.1	50.1	50.0	49.9	90.4	71.7	70.0	70.3	70.1	96.8	90.4	90.0	90.4	90.1
200	83.9	54.1	50.1	49.8	49.9	90.4	71.7	70.0	70.2	70.1	96.8	90.4	90.0	90.4	90.2

Panel B: 10-year period from 1998-2007; TW = \$182.76; SD_R = 3.75%; SD_{TW} = \$43.85; SF_{TW} =\$-20.29; SV_{TW} = \$24.50

	50/50 mix					**30/70 mix**					**10/90 mix**				
#	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
0	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	92.8	53.1	53.2	60.0	62.2	95.7	71.1	70.6	73.1	73.2	98.6	90.2	89.9	90.5	90.2
2	92.8	52.5	52.0	55.9	56.7	95.7	70.9	70.5	72.4	72.3	98.6	90.2	90.1	91.2	90.7
3	93.0	52.3	51.5	54.1	54.9	95.8	70.9	70.5	71.5	71.7	98.6	90.2	90.1	90.2	90.3
4	92.9	52.3	50.9	53.7	54.2	95.8	70.8	70.2	71.1	71.3	98.6	90.2	90.0	90.3	90.3
5	93.0	52.2	50.9	54.3	53.9	95.8	70.8	70.3	72.3	71.9	98.6	90.2	90.1	90.2	90.3
6	93.0	52.3	51.1	51.6	52.0	95.8	70.9	70.4	70.9	70.7	98.6	90.2	90.1	90.1	90.1
7	93.0	52.1	51.1	51.8	52.0	95.8	70.8	70.5	71.0	70.9	98.6	90.2	90.1	90.4	90.3
8	92.9	52.1	50.5	51.5	51.7	95.8	70.8	70.2	70.6	70.5	98.6	90.2	90.0	90.4	90.2
9	93.0	52.0	50.8	52.5	52.4	95.8	70.8	70.3	70.2	70.5	98.6	90.2	90.1	90.3	90.2
10	93.0	52.1	50.7	52.4	52.3	95.8	70.8	70.3	71.7	71.2	98.6	90.2	90.1	90.7	90.4
12	93.0	52.0	50.4	50.3	50.5	95.8	70.8	70.1	68.9	69.4	98.6	90.2	90.0	90.5	90.2
14	93.0	52.1	50.3	51.5	51.4	95.8	70.8	70.1	70.4	70.4	98.6	90.2	90.0	89.8	90.0
16	92.9	52.0	50.2	50.0	50.4	95.8	70.8	70.0	70.2	70.2	98.6	90.2	90.0	89.6	89.8
18	92.9	52.0	50.0	49.9	50.0	95.8	70.8	69.9	70.1	69.9	98.6	90.2	90.0	90.1	90.0
20	93.0	51.9	50.5	50.2	50.6	95.8	70.8	70.3	70.3	70.3	98.6	90.2	90.1	90.0	90.0
25	93.0	52.0	50.2	50.2	50.4	95.8	70.8	70.1	69.7	70.0	98.6	90.2	90.0	89.9	89.9
30	93.0	52.0	50.2	51.1	50.7	95.8	70.8	70.1	70.4	70.2	98.6	90.2	90.0	89.9	89.9
40	93.0	52.0	50.0	50.4	50.3	95.8	70.8	70.0	70.4	70.2	98.6	90.2	90.0	90.1	90.1
50	93.0	51.9	50.2	50.4	50.3	95.8	70.8	70.1	70.3	70.2	98.6	90.2	90.0	90.3	90.2
100	93.0	51.9	50.1	49.9	50.0	95.8	70.8	70.0	70.1	70.1	98.6	90.2	90.0	89.7	89.8
200	92.9	51.9	50.0	50.3	50.2	95.8	70.8	70.0	70.1	70.1	98.6	90.2	90.0	90.3	90.2

Panel C: 20-year period from 1988-2007; TW = \$550.80; SD_R = 2.37%; SD_{TW} = \$160.41; SF_{TW} =\$-118.58; SV_{TW} = \$141.66

	50/50 mix					**30/70 mix**					**10/90 mix**				
#	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
0	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	81.6	63.6	54.7	52.5	53.2	89.0	76.4	71.3	71.7	71.4	96.3	91.7	90.1	90.8	90.4
2	81.5	62.2	51.9	53.1	53.1	88.9	75.9	70.4	71.7	71.1	96.3	91.6	90.0	90.5	90.2
3	81.9	62.2	52.3	51.9	52.4	89.1	76.0	70.7	70.4	70.5	96.4	91.7	90.1	90.4	90.2
4	81.6	61.6	51.2	50.7	51.4	88.9	75.7	70.3	69.9	70.2	96.3	91.6	90.0	90.5	90.3
5	81.5	61.4	50.8	51.0	51.2	88.9	75.6	70.1	70.4	70.4	96.3	91.6	90.0	90.3	90.2
6	81.7	61.4	50.5	51.5	51.4	89.0	75.7	70.0	70.3	70.4	96.3	91.6	90.0	90.8	90.4
7	81.7	61.4	50.8	50.8	50.8	89.0	75.6	70.2	70.5	70.3	96.3	91.6	90.0	90.4	90.2
8	81.5	61.3	50.6	51.3	51.0	88.9	75.6	70.2	70.3	70.2	96.3	91.6	90.0	90.5	90.3
9	81.5	61.2	50.6	50.6	50.6	88.9	75.6	70.2	71.0	70.5	96.3	91.6	90.0	90.5	90.2
10	81.6	61.2	50.4	50.5	50.6	89.0	75.6	70.1	70.3	70.2	96.3	91.6	90.0	90.2	90.1
12	81.5	61.2	50.9	51.0	51.2	88.9	75.7	70.4	70.8	70.7	96.3	91.6	90.1	90.6	90.4
14	81.5	61.1	50.6	50.8	50.8	88.9	75.6	70.3	70.5	70.4	96.3	91.6	90.1	90.5	90.3
16	81.5	61.0	50.2	50.4	50.2	88.9	75.5	70.0	70.2	70.1	96.3	91.6	90.0	90.2	90.1
18	81.6	61.0	50.2	50.2	50.3	88.9	75.5	70.0	69.9	70.0	96.3	91.6	90.0	90.3	90.2
20	81.6	61.1	50.2	50.1	50.1	88.9	75.6	70.0	69.7	69.8	96.3	91.6	90.0	90.1	90.1
25	81.6	61.0	50.3	50.4	50.4	89.0	75.5	70.1	70.6	70.4	96.3	91.6	90.0	90.2	90.1
30	81.6	61.0	50.3	50.2	50.3	89.0	75.6	70.1	70.5	70.4	96.3	91.6	90.0	90.2	90.1
40	81.6	61.0	50.3	50.5	50.6	89.0	75.5	70.1	70.8	70.6	96.3	91.6	90.0	90.4	90.2
50	81.5	61.0	50.1	51.0	50.6	88.9	75.6	70.0	70.5	70.3	96.3	91.6	90.0	90.5	90.3
100	81.6	61.0	50.2	50.5	50.4	89.0	75.5	70.1	70.5	70.3	96.3	91.6	90.0	90.2	90.1
200	81.6	61.0	50.1	50.2	50.2	89.0	75.5	70.0	70.2	70.1	96.3	91.6	90.0	90.0	90.0

Appendix 6: This table presents average terminal wealth (TW), average standard deviation of monthly returns (SD_R) and standard deviation of terminal wealth (SD_{TW}) of 1,000 diversified equity fund portfolios (i.e., the equity fund portfolio) over 5-, 10-, and 20-year holding periods. This table also presents TW, SD_R and SD_{TW} of the portfolios that mix between 30% bond funds from three bond fund categories and 70% equity funds as percentages of TW, SD_R, and SD_{TW} of the equity fund portfolio. Three categories of bond funds are government (GB), corporate (CB) and high yield (HY) bond funds, and # is the number of bond funds in a mixed portfolio. TW is the ending wealth of investing $100 at the beginning of a holding period. A diversified equity fund portfolio is created from five randomly chosen equity funds, equally weighted.

Table 5: Diversification benefits of bond funds in diversified equity portfolios classified by categories of bond funds

	5-year holding period TW = \$184.95; SD_R = 2.63%; SD_{TW} = \$17.41			**10-year holding period TW = \$182.76; SD_R = 3.75%; SD_{TW} = \$43.85**			**20-year holding period TW = \$550.80; SD_R = 2.37%; SD_{TW} = \$160.41**		
	GB	**CB**	**HY**	**GB**	**CB**	**HY**	**GB**	**CB**	**HY**
%TW	89.0	89.7	95.2	96.0	96.4	93.6	88.6	90.2	91.3
$\%SD_R$	69.7	70.9	79.7	68.9	70.3	80.9	74.2	76.1	86.5
#	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**	**% SD_{TW}**
0	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	70.4	72.0	74.5	70.3	70.4	73.3	71.1	70.7	72.4
2	70.5	72.1	72.3	70.3	70.1	71.1	70.6	70.3	71.8
3	70.2	70.2	72.2	70.2	70.3	70.7	70.1	70.2	70.8
4	70.1	70.9	71.8	70.2	70.1	70.5	70.3	69.8	70.8
5	70.3	70.4	71.3	70.3	70.0	70.6	70.3	70.2	70.8
6	70.2	70.5	70.7	70.1	70.2	70.6	70.2	70.3	70.2
7	70.1	70.8	70.8	70.1	70.0	70.6	69.9	70.2	70.7
8	69.9	70.5	70.5	69.9	70.1	70.6	69.9	70.1	70.0
9	70.0	70.4	70.9	70.0	69.9	70.2	70.0	70.2	70.2
10	70.3	70.3	70.0	69.9	70.0	70.5	70.1	70.2	70.1
12	70.1	70.4	70.3	70.2	69.9	70.1	70.0	70.0	70.1
14	70.0	70.2	70.0	70.0	70.1	70.4	70.2	70.1	70.2
16	69.9	70.0	70.4	70.2	70.1	70.3	69.9	70.1	69.9
18	70.2	70.1	70.6	70.0	70.0	70.2	70.0	70.0	70.5
20	70.0	70.2	70.1	70.0	69.9	70.3	70.0	70.0	70.0
25	70.0	70.0	70.2	70.1	70.0	70.2	70.0	70.1	69.9
30	70.0	70.2	70.1	70.0	70.0	69.9	70.1	69.9	70.0
40	70.0	70.1	69.9	70.0	70.1	70.1	69.9	70.0	70.1
50	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.1	70.0
100	70.0	70.0	70.1	70.0	70.0	70.1	70.0	70.0	
200	70.0	70.0	70.0	70.0	70.0	70.0	70.0		

Appendix 7: This table presents terminal wealth (TW), standard deviation of monthly returns (SD_R) and terminal wealth volatility of the portfolios that mix between equity and bond funds as percentages of TW, SD_R and terminal wealth volatility of 1,000 randomly chosen equity funds (i.e.., the equity fund portfolio) over the 5-year holding period 2003 to 2007. The mix is 50% bond and 50% equity for 50/50 portfolio, 30% bond and 70% equity for 30/70 portfolio, and 10% bond and 90% equity for 10/90 portfolio, and # is the number of bond funds in a mixed portfolio. TW is the ending wealth of investing \$100 at the beginning of a holding period. Terminal wealth volatility includes standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) of terminal wealth. The statistics for the equity fund portfolio are: *TW = \$185.66; SD_R = 2.91%; SD_{TW} = \$43.27; SF_{TW} = -\$26.51; SV_{TW} = \$34.43.*

Table 6: Diversification benefits of bond funds in non-diversified equity portfolios over 5-year holding period

	50/50					**30/70**					**10/90**				
#	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**TW**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
0	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	83.7	55.4	53.6	55.5	53.9	90.2	72.3	70.8	71.3	70.9	96.7	90.5	90.0	89.4	89.6
2	83.9	54.9	52.1	54.0	53.2	90.3	72.2	70.6	70.6	70.7	96.8	90.5	90.1	90.2	90.2
3	83.8	54.5	51.5	54.9	53.5	90.3	72.0	70.5	72.1	71.6	96.8	90.5	90.1	91.1	90.7
4	83.8	54.5	50.6	52.4	51.6	90.3	72.0	70.0	70.3	70.3	96.8	90.5	90.0	90.3	90.2
5	83.7	54.3	50.9	50.9	51.2	90.2	71.9	70.3	70.5	70.5	96.7	90.5	90.1	89.8	90.0
6	83.8	54.3	50.6	51.2	50.8	90.3	71.9	70.1	69.2	69.5	96.8	90.5	90.0	89.9	89.9
7	83.7	54.3	50.9	51.6	51.7	90.2	71.9	70.4	70.0	70.5	96.7	90.5	90.1	90.0	90.1
8	83.8	54.3	50.3	50.8	50.6	90.3	71.9	70.0	69.4	69.7	96.8	90.5	90.0	90.0	90.0
9	83.8	54.2	50.4	50.1	50.4	90.3	71.9	70.1	69.8	70.0	96.8	90.5	90.0	90.2	90.1
10	83.8	54.2	50.4	51.3	50.9	90.3	71.9	70.1	70.2	70.2	96.8	90.5	90.0	89.8	89.9
12	83.8	54.1	50.1	50.1	50.0	90.3	71.9	70.0	70.2	70.0	96.8	90.5	90.0	90.4	90.1
14	83.8	54.1	50.2	50.6	50.1	90.3	71.9	70.0	69.4	69.6	96.8	90.5	90.0	90.3	90.1
16	83.8	54.1	50.2	49.9	50.2	90.3	71.9	70.0	70.1	70.1	96.8	90.5	90.0	90.1	90.1
18	83.8	54.1	50.3	50.6	50.5	90.3	71.9	70.1	70.5	70.3	96.8	90.5	90.0	90.2	90.1
20	83.7	54.0	50.3	50.2	50.4	90.2	71.8	70.1	69.8	70.1	96.7	90.5	90.0	89.8	89.9
25	83.8	54.0	50.1	49.2	49.6	90.3	71.8	70.0	69.3	69.6	96.8	90.5	90.0	89.8	89.9
30	83.8	54.1	49.9	51.3	50.7	90.3	71.8	69.9	70.6	70.3	96.8	90.5	90.0	90.3	90.2
40	83.8	54.0	50.1	50.0	50.2	90.3	71.8	70.0	70.0	70.1	96.8	90.5	90.0	90.0	90.0
50	83.8	54.1	50.1	49.9	50.0	90.3	71.8	70.0	70.3	70.2	96.8	90.5	90.0	89.8	89.9
100	83.8	54.1	50.1	50.5	50.4	90.3	71.8	70.1	70.2	70.2	96.8	90.5	90.0	89.7	89.9
200	83.8	54.0	49.9	50.5	50.3	90.3	71.8	70.0	70.0	70.0	96.8	90.5	90.0	90.0	90.0

Appendix 8: This table presents average terminal wealth (TW, $) and standard deviation of terminal wealth (SD_{TW}, $) of all bond funds (i.e., one-bond fund portfolio) over 10-year holding periods. This table also presents SD_{TW} of portfolios with different numbers of bond funds as percentages of SD_{TW} of the one-bond fund portfolio. TW is the ending wealth of investing $100 at the beginning of a 10-year holding period. N is the number of all bond funds that exist at the beginning of a holding period, and # is the number of bond funds in a portfolio.

Table 7: Diversification benefits of bond funds over 10-year rolling periods

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Begin	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
End	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
N	829	942	979	1,000	1,057	1,207	1,457	1,747	1,812	1,903	2,027
TW	221.44	213.12	192.63	198.29	178.87	176.87	169.43	183.29	162.36	163.13	156.68
SD_{TW}	33.55	29.09	34.37	35.61	20.40	21.53	18.54	21.78	16.51	18.05	17.38
#	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**	**SD_{TW}**
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
2	69	72	77	72	68	72	72	69	74	66	69
3	59	60	58	60	58	58	60	57	58	59	59
4	52	49	50	48	51	48	49	46	50	50	52
5	49	44	45	42	44	44	44	42	46	44	45
6	41	41	42	39	41	40	40	37	42	42	40
7	37	38	38	39	37	38	37	39	37	37	40
8	35	35	35	34	35	35	36	36	35	36	40
9	32	33	32	32	33	33	33	32	32	34	33
10	30	31	31	33	32	31	31	31	32	32	32
12	28	29	28	29	29	28	29	28	29	29	29
14	27	27	28	26	27	26	26	27	27	27	27
16	25	25	24	24	25	25	24	25	25	25	25
18	22	24	23	23	24	23	24	24	23	24	23
20	22	22	22	22	22	22	22	23	22	22	22
25	20	21	19	20	20	21	19	20	20	20	20
30	18	18	17	19	18	18	18	18	18	17	19
40	15	15	16	15	16	16	16	16	16	16	16
50	14	13	14	13	14	14	14	14	14	14	15
100	9	9	10	9	10	10	10	9	10	10	10
200	6	6	6	6	7	7	7	7	7	7	7

Appendix 9: This table presents average terminal wealth (TW), average standard deviation of monthly returns (SD_R, %), and terminal wealth volatility of all surviving bond funds (i.e., one-bond fund portfolio) over different holding periods. This table also presents SD_R and terminal wealth volatility of portfolios with different numbers of bond funds as percentages of SD_R and terminal wealth volatility of the one-bond fund portfolio. TW is the ending wealth of investing $100 at the beginning of a holding period. Terminal wealth volatility includes standard deviation (SD_{TW}), mean shortfall (SF_{TW}) and semivariance (SV_{TW}) of terminal wealth. The holding periods include 5 years, from 2003 to 2007; 10 years, from 1998 to 2007; 20 years, from 1988 to 2007. N is the number of bond funds that exist at the beginning of a holding period, and # is the number of bond funds in a portfolio.

Table 8: Diversification benefits of surviving bond funds over different holding periods

	5-year holding period: 2003-2007 N = 1,984; TW = $126.51				**10-year holding period: 1998-2007** N = 1,217; TW = $158.27				**20-year holding period: 1988-2007** N = 460; TW = $354.66			
#	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**	**SD_R**	**SD_{TW}**	**SF_{TW}**	**SV_{TW}**
1	1.02	$17.26	-$8.73	$10.44	1.14	$16.25	-$10.49	$15.57	1.25	$72.98	-$39.61	$53.83
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
2	91	70	87	85	89	71	76	72	92	71	79	75
3	88	57	82	78	83	58	64	59	89	59	67	62
4	86	52	76	75	81	49	55	49	86	51	63	59
5	85	46	66	66	78	45	53	48	86	46	54	50
6	83	41	61	61	77	42	48	43	84	40	50	46
7	82	38	57	57	77	39	45	40	84	39	51	45
8	82	36	51	52	76	35	42	36	83	36	46	42
9	81	32	48	48	75	32	39	34	83	34	43	39
10	81	33	47	48	75	33	40	35	83	32	41	38
12	81	29	42	43	74	29	34	30	82	29	39	35
14	80	26	40	41	73	26	32	27	82	26	35	32
16	80	24	37	37	73	25	31	26	81	24	32	29
18	80	24	36	37	73	24	29	25	81	23	31	28
20	79	22	33	34	73	24	28	24	81	22	30	27
25	79	20	30	31	72	19	23	20	81	19	26	23
30	79	18	26	28	72	19	23	19	81	18	25	22
40	79	16	25	26	71	16	19	16	81	15	21	18
50	79	14	21	21	71	14	17	14	81	13	18	17
100	78	10	14	15	71	10	12	10	80	9	13	11
200	78	7	10	11	70	7	8	7	80	5	8	7