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# Unsteady free convection MHD flow and heat transfer between two heated vertical paltes with heat source: an exact solution

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## Abstract

This paper is devoted for the study of effects influences by heat source on unsteady free convection flow and heat transfer characteristic of a viscous incompressible and electrically conducting fluid between two heated vertical plates in the presence of a uniform magnetic field applied transversely to the flow. The leading momentum and energy equations are solved by the Laplace transform technique and solutions are presented through graphs for velocity and temperature distribution.

**Mathematics Subject Classification:** 76W05, 76R10

**Keywords:** unsteady free convection MHD flow, heat source, Laplace transforms technique

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## 1 Introduction

Transient free convection occurs in a fluid when the temperature changes cause density variations which gives rise to buoyancy forces. A lot of free convection heat transfer problems can be seen in literature. It is due to numerous applications in metallurgical engineering such as magnetic levitation or confinement, thermonuclear fusion etc. In the metallurgical industries magnetic fields are routinely [4] used to heat pump, stir and levitate the liquid metals. Free convection flows with heat transfer rates have found a substantial and permanent place in the world of material processing through MHD processes. Moreover, this type flows has parallel application in Astrophysics, Medical sciences, Geophysics, and Aerodynamics. Workers notably Brar, Teipel, Chaoudhury [3], Soundelgekar [10], Borkakati [2], Kafoussias [7], Deka, Panton Merkin [8], Biswal etc, did work on transient as well as on steady free convection flows.

Most of them studied this type of flows in the presence of a magnetic field**.** Datta *et al*. [5] studied the problem of Magneto hydrodynamic unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous flat plate with heat sources/sinks. Ojha and Singh [9] analyzed the heat source/sink effects on free convection flow and mass transfer of visco-elastic fluid past an infinite vertical porous flat plate. Their studies has shown that the presence of heat generating sources or heat absorbing sinks in the fluid influence the flow field to a great extent as well as produce remarkable effects on the rate of heat transfer. Hence, owing to its numerous applications, in the paper of Gourla and Katoch [6], we have considered the effect of heat source for farther study. In section 2, the mathematical formulation; in section3, the solution of the equations; in section 4, results and discussion, and in section 5, the conclusions are given.

## 2 Mathematical Formulation

We assume that a viscous incompressible and electrically conducting fluid flows between two heated vertical long non-conducting plates. At time $t \le 0$, the fluid is at rest and the plates are also at temperature $T_0$ (reference temperature). At time $t > 0$, the motion of the fluids takes place and the temperature of the plate's changes according as follows -

$$T' = T_0 + \left(T_w^{/} - T_0\right)\left(1 - e^{-n't'}\right), \text{ where } n' \text{ is the decay factor.}$$

Here, $x'-$axis is taken along each plate which in the vertical upward direction and $y'-$axis is taken normal to the plate. We consider the origin of the axes at the middle point between the plates. A uniform magnetic field of strength $B_0$ is applied in a direction transverse to the direction of the vertical plates. Therefore, action of the magnetic field is in the horizontal direction and thus perpendicular to the flow while of the fluid velocity field is in the vertical upward direction.

We make the following assumptions to derive the governing equations of motion:

1. The fluid is assumed to be of low conductivity, so that the induced magnetic field is negligible.
2. The fluid is isotropic and Newtonian.
3. The strength of the magnetic field is not very large such that the generalized Ohm's law is negligible.
4. For the boundary condition it is assured that there is no-slip at the wall.
5. Viscous dissipation and Polarization effects are neglected.
6. The viscous fluid flows with constant physical properties $(\rho, \mu, k, C_p)$ in between two vertical walls, a distance 2h (- h < y < h) apart.
7. It is assumed that the plates are very long in the x-direction so the temperature $(T')$ and velocity field $(u')$ are functions of $y'$ and $t'$ alone, and velocity components $v'$ and $w'$ are zero.
8. The pressure term is balanced by gravity force term to give rise buoyancy

force term.

Under the above assumption the governing equations of motion are as follows:

$$\nabla.\vec{q} = 0 \qquad \text{(continuity equation)} \qquad (1)$$

$$\rho\frac{\partial u'}{\partial t'} = \mu\frac{\partial^2 u'}{\partial y'^2} + \rho\beta g(T' - T_0) - \sigma B_0^2 u' \qquad \text{(momentum equation)} \qquad (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \mathrm{k}\,\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_0) \qquad \text{(energy equation)} \qquad (3)$$

Here $\rho$ is the density and $\beta$ is the coefficient of volume expansion at temperature $T_0$. Also, here, $\mu(=\upsilon\rho)$ is the viscosity coefficient, g is the acceleration due to gravity, $\sigma$ is the electrical conductivity, k is the thermal conductivity, $C_p$ is the isobaric specific heat capacity $B_0$ is the magnetic field component. In equation (2), the gradient of temperature is due to the weight of the fluid in the slit $\left(\frac{dp}{dx} = -\rho g\right)$ and viscous forces are just balanced by the buoyancy forces [1] only. These equations are to be solved with the following initial and boundary conditions:

$$\text{At} \quad t' = 0: \quad u' = 0, \quad T' = T_0 \quad \forall \quad y' \in [-h, h]$$

$$\text{At} \quad t' > 0: \quad u' = 0, \quad T' = T_0 + (T_w - T_0)\left(1 - e^{-n't'}\right) \quad \text{for} \quad y' = \mp h \qquad (4)$$

We now introduce the following dimensionless quantities

$$y = \frac{y'}{h}, \quad u = \frac{\mu u'}{\rho\beta g h^2 (T_w - T_0)}, \quad t = \frac{\mu t'}{\rho h^2}, \quad n = \frac{\rho h^2 n'}{\mu}, \quad T = \frac{T' - T_0}{T_w - T_0}$$

and dimensionless characteristic numbers [11]

$$\Pr = \frac{\mu C_p}{k} \ \text{(Prandtl number)}, \quad M = \frac{B_0^2 \sigma h^2}{\mu} \ \text{(Hartmann number)}$$

(Heat Source) (5)

in equations (2) - (3) and boundary condition (4), and have the following dimensionless forms:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + T - Mu \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + ST \tag{7}$$

$$t \le 0: \quad \text{u = 0,} \quad \text{T = 0} \qquad \forall \quad y \in [-1,+1]$$

$$t > 0: \quad \text{u = 0,} \quad T = 1 - \; e^{-nt} \quad \text{for} \quad y = \mp 1 \tag{8}$$

## 3 Solution of the equations

Taking Laplace transform [12] of equations (6) & (7), and boundary condition (8), get the following equations:

$$\frac{d^2\bar{u}}{dy^2} - (M + p)\bar{u} = \bar{T} \tag{9}$$

$$\frac{d^2\bar{T}}{dy^2} + \Pr(S - p)\bar{T} = 0 \tag{10}$$

$$\text{t = 0:} \quad \bar{u}(\pm 1, p) = 0, \qquad \bar{T}(\pm 1.p) = 0$$

$$\text{t = 0:} \quad \bar{u}(\pm 1, p) = 0 \qquad \bar{T}(\pm 1, p) = \frac{n}{p(n+p)} \tag{11}$$

where $\bar{F}(y, p) = \int_0^\infty e^{-pt} F(y,t)dt$ .

The solution of the equation (10) subject to the boundary condition (11) is

$$\bar{T} = \frac{n}{p(p+n)} \frac{\cos\sqrt{\Pr(S-p)}y}{\cos\sqrt{\Pr(S-p)}} \tag{12}$$

Again solution of (9) with the help of boundary condition (11) is

$$\bar{u} = \frac{n\cos\sqrt{\Pr\left(S-p\right)}y}{\left(\Pr\left(S-p\right)+M+p\right)p\left(p+n\right)\cos\sqrt{\Pr\left(S-p\right)}} - \frac{n\cosh\sqrt{M+p}\,y}{\left(\Pr\left(S-p\right)+M+p\right)p\left(p+n\right)\cosh\sqrt{M+p}} \tag{13}$$

Inverting (12) and (13), we get

$$T\left(y,t\right) = \frac{\cos\sqrt{S\Pr}\,y}{\cos\sqrt{S\Pr}} - \frac{\cos\sqrt{\Pr\left(S+n\right)}y}{\cos\sqrt{\Pr\left(S+n\right)}}e^{-nt} + \sum_{k=0}^{\infty}\frac{\left(-1\right)^k n\pi\left(2k+1\right)\cos\left(\frac{\pi}{2}\left(2k+1\right)y\right).e^{\left(S-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)t}}{\Pr\left(S-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)\left(S+n-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)} \tag{14}$$

$$u\left(y,t\right) = \frac{2n\cos\sqrt{\frac{\Pr\left(S+M\right)}{1-\Pr}}y.e^{-\frac{S\Pr+M}{1-\Pr}t}}{\left(S\Pr+M\right)\left(\frac{S\Pr+M}{1-\Pr}-n\right)\cos\sqrt{\frac{\Pr\left(S+M\right)}{1-\Pr}}} + \frac{1}{S\Pr+M}\left[\frac{\cos\sqrt{S\Pr}\,y}{\cos\sqrt{S\Pr}} + \frac{\cosh\sqrt{M}\,y}{\cosh\sqrt{M}}\right]$$
$$+\frac{e^{-nt}}{\left\{n\left(1-\Pr\right)-\left(S\Pr+M\right)\right\}}\left[\frac{\cos\sqrt{\Pr\left(S+n\right)}y}{\cos\sqrt{\Pr\left(S+n\right)}} - \frac{\cos\sqrt{n-M}\,y}{\cos\sqrt{n-M}}\right]$$

$$+\sum_{k=0}^{\infty}\frac{\left(-1\right)^k n\left(2k+1\right)\pi\cos\left(\frac{2k+1}{2}\pi y\right).e^{\left(S-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)t}}{\Pr\left\{\left(S-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)^2+n\left(S-\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right)\right\}\left\{M+S-\left(1-\Pr\right)\frac{\pi^2\left(2k+1\right)^2}{4\Pr}\right\}}$$
$$+\sum_{k=0}^{\infty}\frac{n\pi\left(2k+1\right)\cos\left(\frac{2k+1}{2}\pi y\right).e^{-\left(M+\frac{\pi^2\left(2k+1\right)^2}{4}\right)t}\left(-1\right)^k}{\left\{\Pr\left(S+M\right)-\left(1-\Pr\right)\frac{\pi^2\left(2k+1\right)^2}{4}\right\}\left\{\left(M+\frac{\pi^2\left(2k+1\right)^2}{4}\right)^2-n\left(M+\frac{\pi^2\left(2k+1\right)^2}{4}\right)\right\}} \tag{15}$$

## 4 Results and Discussion

Figure 1 has been obtained by plotting the temperature distribution T against y at different values of the heat source parameter S for fixed time (t = .2), decay factor (n =1), and Prandtl number (Pr = .025). As the values of S increases from 0 to 1, the values of T also increases at x – axis (i.e. the axis of parabola as the figure shows) from .607866 (approx) to .615325 (approx) (corrected upto six decimal places). Nevertheless, the curves are homogeneous parabolic with x – axis as its axis in each case. This shows that the temperature distribution is uniform – highest near the walls, and lowest in between the plates.

Figure 2 has been drawn to show the effect caused by Prandtl number (Pr) at different values for fixed values of t (= .2), n (= 5), S (= .5) on temperature distribution. It is seen that temperature distribution changes and gets its new shape according as the change of Prandtl number. For small values of Prandtl number the temperature distribution changes negligibly. But for Pr = 7, T changes remarkably, and it distributes with small change away from the plates. For Pr = 1, we have a fine parabolic curve.

Figure 3 depicts the temperature profiles for different values of n when t = .2, S = .5, Pr = .025. It is found form this figure that the temperature at any point inside the vertical channel increase uniformly with increase of n.

We have considered the figure 4 to show the temperature distribution T against y for different time at fixed values of Pr (= .025), n (= 5), S (= .5). It is seen that there is negligible change of distribution of temperature between the channel for time t = 1, 2, 3. But for t = .2, though this distribution curve is similar with earlier three, there is difference with earlier three values. In each of the above case all value of T is nearly equal to 1 and all T – curves are parallel to y - axis.

Figure 5 has been obtained by plotting the value of the velocity u against y at different magnetic Hartmann number M. Series1 & 2 are for t = .1 and n = 1. Series 3 & 4 are for t = .5 and n = 10. In every case we have considered Pr = .025, S = .05. It is seen that for small time (t = .1) the curves 1 & 2 are almost parallel to the y-axis between the channel, and the values of u are positive zeros. For time t = .5 and n = 10, the distribution of velocity is like circular arc. At y = 0, the value of u is minimum, and at $y = \pm 1$, it is maximum.

Figure 6 is drawn for different values of heat source parameter S and for fixed n, Pr, M. Series1 & 2 for t = .1, and series 3 & 4 for t = .5. It is seen that for small time the flow of the velocity is not fully developed, but for time t = .5, the velocity field is seemed to be developed, and its shape are homogeneous right circular arc. In each case, near the walls the velocity is the highest while at the centre of the channel the fluid velocity is lowest.

The velocity profiles have been plotted against y for M = 1, S = .05, t = .1, n = 1 and for various values of Prandtl number Pr as shown in figure 7. This figure shows that the velocity profiles takes the shape of positive parabolic curves with x-axis as its axis in each case. It is seen that as Prandtl number increases the velocity curves turns from just forming parabolic to fine uniform and symmetric parabolic curves. Theses are symmetric in x-axis. This signifies the effect caused by different values of Prandtl number.

Figure 8 has been drawn for different values of decay factor n and at fixed values of Pr (= .025), S (= .05) and M (= .5). In series1 & 2, we have considered t = .1 whereas in series3 & 4, this value is .5. It is seen that for small time the value of the velocity is slightly greater than zero. But for time t = .5, the velocity distribution curve is right circular arc. Again, in each case, the value of the velocity is highest at the walls while it is lowest at the centre of the channel.

Figure 9 depicts the velocity profiles for different combinations of values of Prandtl number, magnetic Hartmann number and Heat Source Parameter at fixed values of t (= .5) and n (= 10). It is seen that in each case the curve is a right circular arc. This means that the velocity is highest near the walls and lowest at the centre position of the walls.

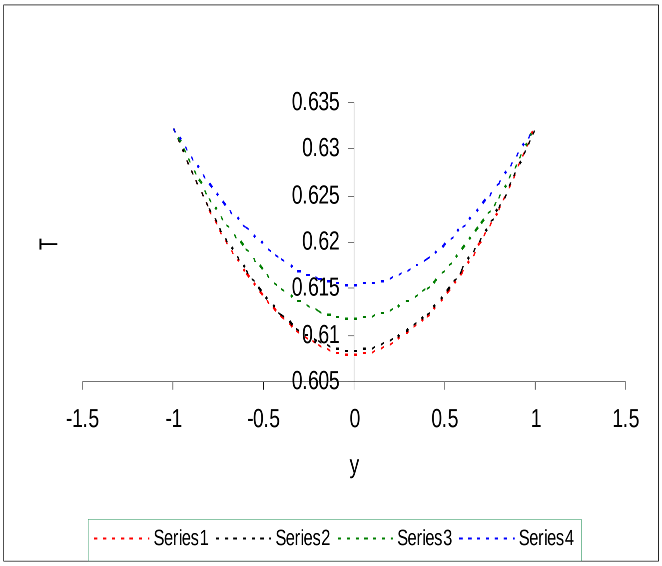


Figure 1: T vs. y series1 for S=0, series2 for S=.05, series3 for S=.5, series4 for S=1 at t = .2, n = 5, Pr = .025

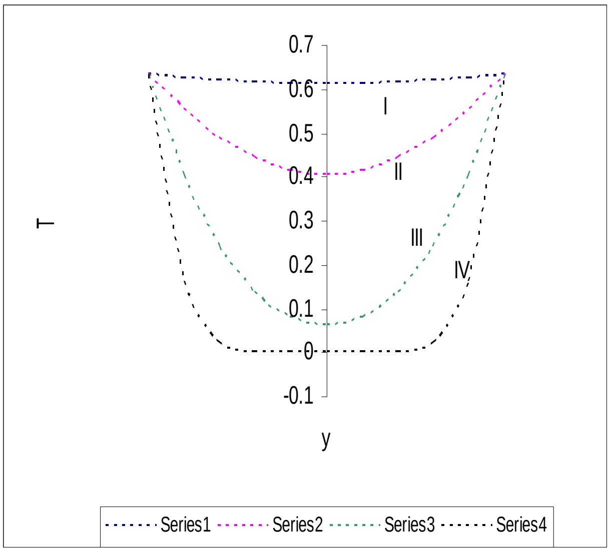


Figure 2: T vs. y I for Pr = .025, II for Pr = .25, III for Pr= 1, IV for Pr = 7 at t = .2, n = 5, S=.5

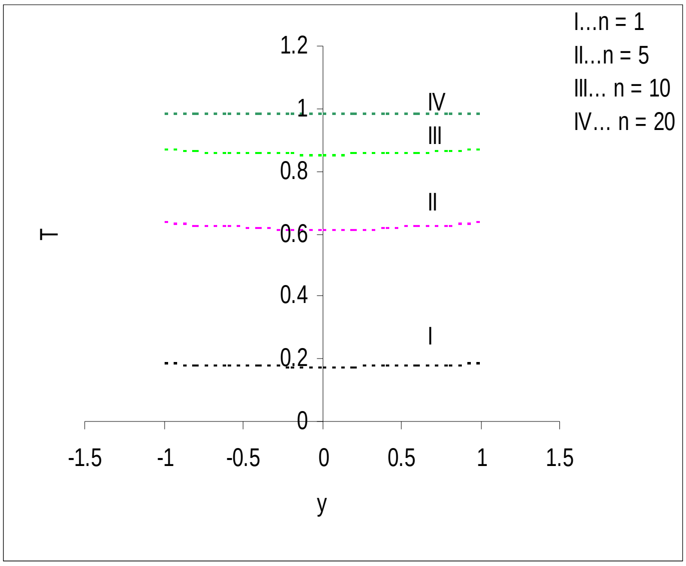


Figure 3: T vs. y for n = 1, 5, 10, 20 at Pr = .025, S = .5, t = .2

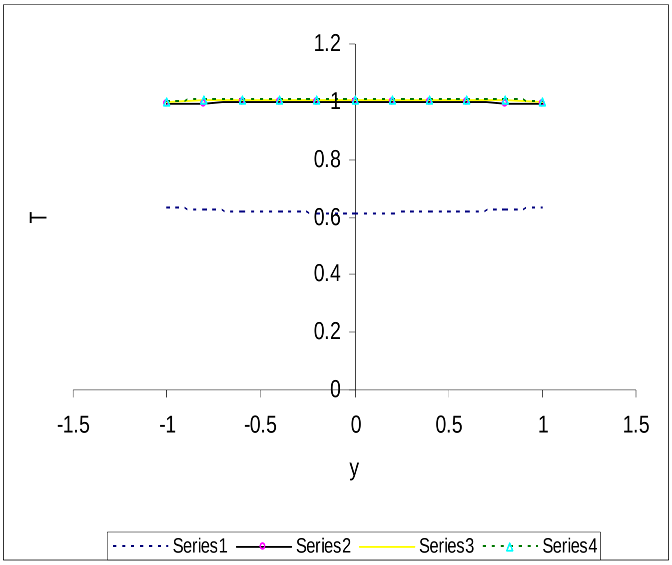


Figure 4: T vs. y Series1 for t = .2, Series2 for t = 1, Series3 for t = 2, Series4 for t=3 at Pr = .025, S = .5, n = 5

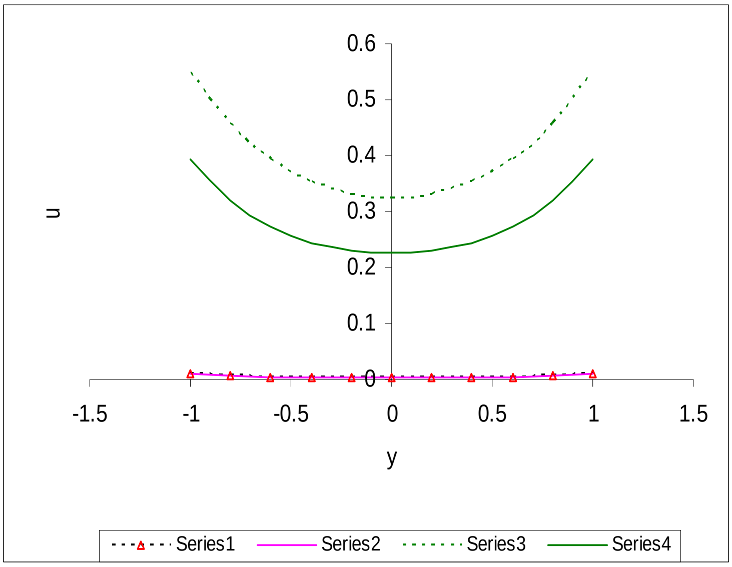


Figure 5: u vs. y; series1 for M = .5, series2 for M = 1 at t = .1, n = 1, S = .05, Pr = .025, Series3 for M = 2, series4 for M = 4 at t = .5, n =10, S = .05, Pr = .025

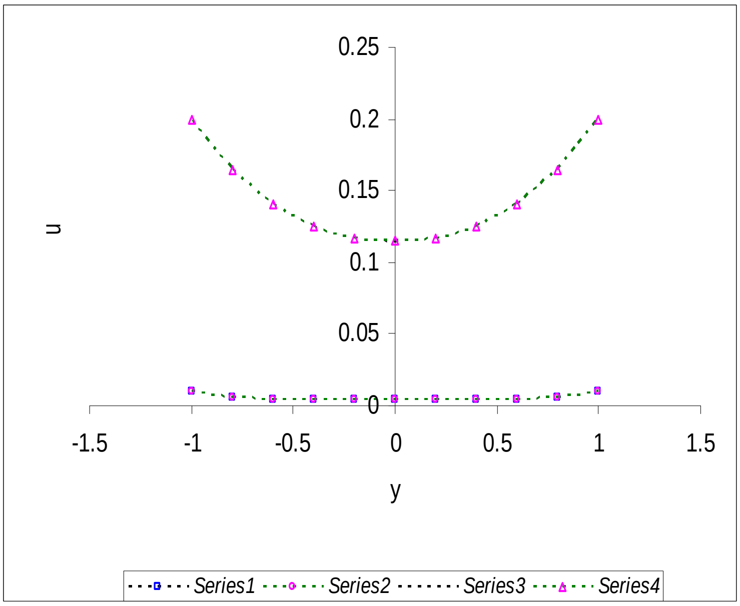


Figure 6: u vs. y; series1 for S = 0, series2 for S = .05 at t = .1, M = .5, n = 1, Pr = .025, series3 for S = .5, series4 for S = 1 at t = .5, M = .5, n = 1, Pr = .025

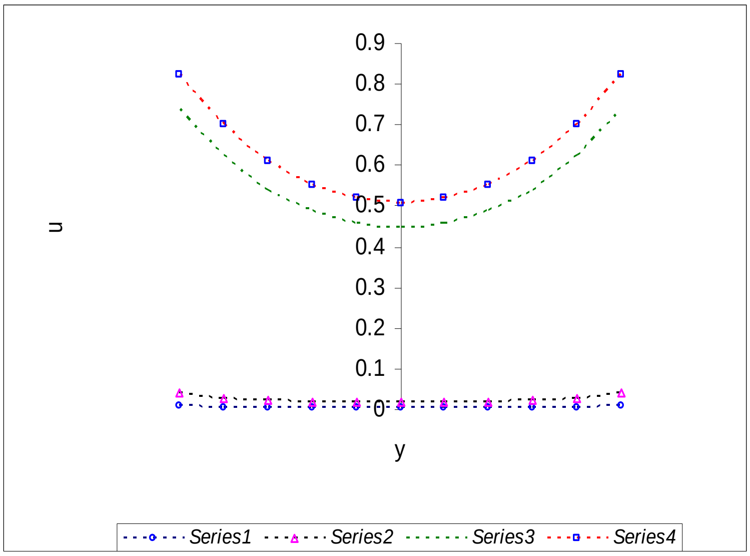


Figure 7: u vs. y for Pr = .025, .25, .5, .71 at M = 1, S = .05, t = .1, n = 1

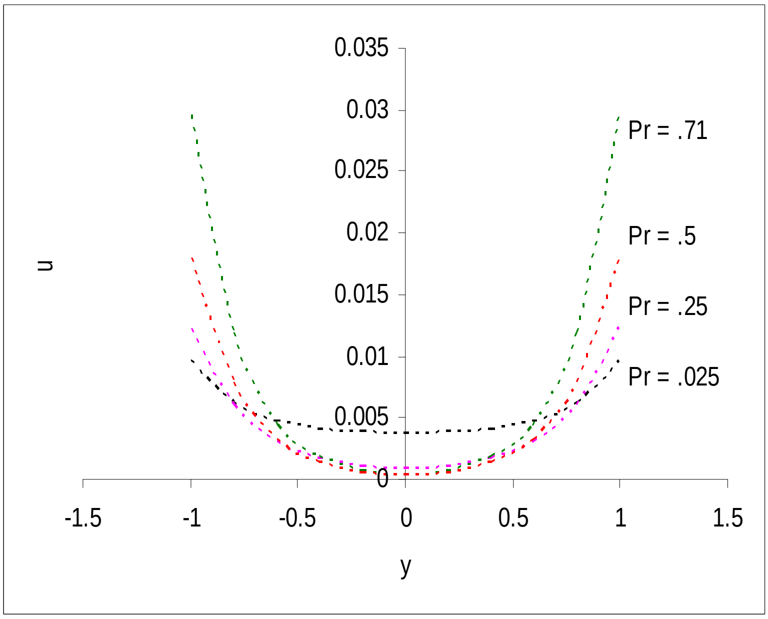


Figure 8: u vs. y; series1 for n = 1, series2 for n = 5 at t = .1, M = .5, Pr = .025, S = .05, series3 for n = 10, series4 for n = 20 at t = .5, M = .5, Pr = .025, S = .05

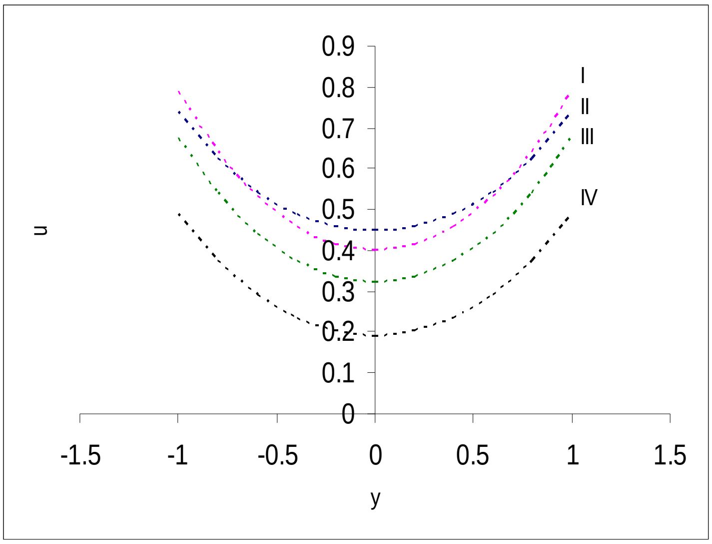


Figure 9: u vs. y; I for Pr = .25, M = 1, S = .5; II for Pr = .025, M = .5, S = .05 III for Pr = .5, M = 2, S = 1; IV for Pr = .71, M = 4, S = 0 at t =.5, n = 10

## 5 Conclusions (0bservations)

(1) Decay factor ‘n’ and magnetic field parameter ‘M’ has a balancing relation. The graphs drown in figure 5 for n = 1 and M = .5, 1, and also for n = 10 and M = 2, 4; has shown this characteristic. When n is fixed and at the same time M increases, the values of the velocity decreases. We can also get the values of the velocity for equal values of n and M. But we cannot get a real value of u for the values of M which is higher than n. So, this is a restriction in our problem.

(2) Naturally, the fluids are to flow in such a way that its velocity is the highest at the middle position of the channel. But in our case this phenomena is completely opposite. For ready reference the paper of Gourla and Katoch (that we have investigated), can be cited. We think that this is due to the *Heat Source* applied at the plate.

(3) The velocity distribution is sharp near the two plates than at the centre between the plates.

(4) As time increases the temperature is also increases. As a result the value of the velocity also increases. This is seen in all figures.

(5) In each case, the investigation is carried out for small time (t = .1, .5). So, the solution can be thought of as the onset of free convection.

(6) We have studied the problem for the fluid with prandtl number less than unity.

(7) This problem left scope for further study.

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