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The Research of the Periodic Features of Stock Index Volatility based on Hilbert-Huang Transformation

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Abstract

The Hilbert-Huang Transform(HHT) algorithm which proposed in recent years escape itself from the requirement of linear and smooth, and it has a clear physical meaning. The data comes from the Shanghai Composite stock index which is decomposed by HHT. It consists of two parts, the first part is empirical mode decomposition(EMD)， the second part is the Hilbert Spectrum. Firstly it gives all Intrinsic Mode Function (IMF) which is decomposed from EMD an interpretation of its physical meaning and introduces the concept of average oscillation cycle and compared the speed of between typical rise and fall times of volatility. On one hand, reconstruct the IMF and estimate its distribution for the purpose of drawing the best characterization cycle of all reconstructed IMF. On the other hand, calculate the average oscillation cycle of the treated IMF and finally derive the quantitative relationship between the two kinds of cycles. At last, to find the curve fits well with the envelope line of each IMF which has been transformed by Hilbert function.

JEL classification numbers: C6 G17
Keywords: Hilbert-Huang algorithm, EMD, IMF, average oscillation cycle, volatility

1 Introduction

Main techniques of quantitative study of stock index volatility home and abroad are based on a series of assumptions with a bias to analyze linear, stable and normal dis-

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tribution financial time series, while real financial time series embody characteristics more about nonstationary, nonlinear and sharp fluctuation; Time-frequency analysis technique includes Fourier transform and wavelet transform, etc., but their nature is all based on Fourier transform. Therefore, when analyzing non-stationary signal, aliasing and other phenomenon will appear, and wavelet transform has the choice of wavelet basis. The new method of nonlinear and non-stationary data processing, Hilbert-Huang Transform (HHT), through Empirical Mode Decomposition (EMD) which based on instantaneous frequency, firstly decomposes the signal into Intrinsic Mode Function (IMF), and then uses Hilbert spectrum analysis to transform IMF into marginal spectrum with different energy. Compared with traditional signal processing methods, HHT shakes off restraints of linear and stationarity completely and has a clear physical meaning, it can get time, frequency and energy distribution characteristics of signals. It is also a signal processing method taking on adaptability and is suitable for singular signal. The ultimate goal of time-frequency analysis is to build a distribution so that energy or strength of signal can be expressed in both time and frequency domain, and make signals that are difficult to be observed in time domain display clearly in frequency domain. Therefore, if we can use Hilbert-Huang transform on realized volatility, jump volatility, bi-power variation and other signals of Shanghai composite index and get the time-frequency distribution, then we can make comprehensive analysis, comparison and processing of various signals, and extract the feature information of signals.

The signal sequence in this research is based on the theory of realized volatility. And use Levy Separation theorem, namely, any asset price path complying with Levy Process can be separated into independent martingale process with continuous sample path and Levy Jump process with Poisson Random measure, to get nonparametric estimator of jump behavior. Andersen and Bollerslev (2003) proved that Realized Volatility is unbiased estimation of integrated volatility on the basis of time series with regular interval sequence, and put forward a new method to estimate volatility. Barndorff-Nielsen and Shephard (2006) proposed the "realized" bi-power variation method, which can be used to test the existence of jump. They achieved direct measurement of jump behavior for the first time. With the wide application of high frequency financial data, Andersen, Bollerslev and Frederiksen (2006) used nonparametric method to decompose "realized" volatility into Continuous Sample Path variance and Discontinuous Jump Variation based on Bi - Power Variation theory proposed by Barndorff-Nielsen and Shephard (2006), realized divestiture of jump behavior from high frequency data and checked intraday jump behavior of assets on real-time inspection. Zhi-jun Hu (2013) improved the sequential jump tests method of Andersen et al. (2010) to describe jump behavior of asset prices in China's stock market in detail. Lian-qian Yin et al. (2015) studied jump behavior of asset prices in China based on analysis method of high-frequency data set. Nonparametric method is a kind of direct research, its research idea is to construct statistics based on intuitive features of asset price behavior, split off jump behavior that causes volatility of asset prices, and measure jump behavior directly. As for research about signal processing, Zhi-hong

Ding and Guo-quan Xie (2009) used EMD to make multiple-time-scale decomposition on daily return time series of HS300 index in view of shortcomings of wavelet transform, and found that its fluctuation has quasi fluctuation cycles, such as 2 days, 4-5days, 15 days, 28 days, 70 days, 140-190days, 240 days and so on. They also analyzed the change trend of each component and did empirical research on multi-resolution of financial time series. Lei Wang (2008) used the advantage of Hilbert-Huang transform on high accuracy in both time and frequency domain, got marginal distribution through Hilbert-Huang transform and integration, and made over-peak analysis of each frequency energy distribution, then summarized that during 500 trading days, from October 11, 2005 to October 5, 2007, hidden fluctuation cycle of daily closing price is about 164 days. Fei Teng, Xiao-gang Dong (2008) put forward a kind of periodic signal analysis method based on Hilbert-Huang transform, by analyzing the signal nonlinear influence on frequency distribution, they found approximate corresponding relationship between frequency and periodicity of approximate periodic signal with rich high frequency. Wen-ting Yu (2014) did EMD decomposition and calculated Hilbert spectrum and marginal spectrum of HS300 index future from April 2010 to December 2010, then observed and analyzed its periodic characteristic. According to this, she put forward Brin Channel Trading strategy. Hilbert-Huang transform on the basis of EMD undeniable enriches application of signal decomposition on stock index volatility.

This paper in view of signal decomposition, selects realized volatility (RV), jump volatility (JV) and bi-power variation (BV) typically to explore volatility and periodicity of several volatilities, and analyzes instantaneous amplitude, instantaneous frequency, Hilbert marginal spectrum and other characteristics. Results show that after Hilbert-Huang transform processing and comparing, periodicity and volatility of stock index volatility have some certain relationships in different scales. And physical significance of different Hilbert spectrum in frequency domain could be explained, finally provide material for empirical study on stock index volatility.

2 Jump signal and Hilbert-Huang Transform

2.1 Volatility

Merton (1980) pointed out when interval number, m, of trading time divided on the first t day tends to be infinity, Realized Volatility (RV) of quadratic sum of daily rate of return of asset logarithmic prices will be consistent uniform convergence in probability of quadratic variation (QV). Alizadeh, Brandt and Deibold et al. (2002) further developed the research idea of Merton and obtained the incremental theory of RV, namely:

$$RV_t = \sum_{i=1}^{m} r_{t,i}^2 = \xrightarrow{\Delta \to 0, p} \int_{t-1}^{t} \sigma_s^2 dW_s + \sum\nolimits_{t-1}^{t} \kappa_s^2 \tag{2.1}$$

In which, $r_{t,i}^2$ is the square sequence of daily yield of each interval, $r_{t,i}(i=1,2,L,m)$, on the first t day. When m tends to be infinity, that is, each time interval of section, Δ, tends to be zero, RV will be consistent uniform convergence in probability of QV on the first t day. Realized bi-power variation is actually the absolute value of product of asset yields on adjacent two days before and after the first t day, and the formula expression is:

$$BV_t = \frac{\pi}{2}\frac{m}{m-1}\sum_{i=2}^{m}|r_{t,i}||r_{t,i-1}| \xrightarrow{\Delta\to 0,p} \int_{t-1}^{t}\sigma_s^2 dW_s \tag{2.2}$$

In which, $r_{t,i-1}$ is daily yield on the first t-1 day. When m tends to be infinity, realized Bi-power variation (BV) will be consistent uniform convergence in probability of integral variance (IV) on the first t day.

Combine equation 2.1 and 2.2, we can get, the difference between RV and BV is consistent uniform convergence in probability of JV estimator on the first t day,

$$RV_t - BV_t \xrightarrow{\Delta\to 0,p} JV_t = \sum_{t-1}^{t}\kappa_s^2 \tag{2.3}$$

So far, we directly measured jump behavior, and jump variance is the part of asset price jump behavior leading to fluctuation. Thus, volatility caused by jump behavior can be defined as realized jump variation:

$$JV_t = I_{\{ZJ_t > \Phi_{1-\alpha}\}}(RV_t - BV_t) \tag{2.4}$$

In which, $I_{\{\cdot\}}$ is indicator function. If $ZJ_t > \Phi_{1-\alpha}$, then $I_{\{\cdot\}}=1$, or it will be zero. $\Phi_{1-\alpha}$ is the $1-\alpha$ quantile of standard normal distribution, α is the selected confidence level when estimating JV_t.

2.2 Empirical Mode Decomposition (EMD)

The essence of EMD is a screening process, it uses the average of upper and lower envelopes obtained by fitting to get the instantaneous equilibrium position, and extract Intrinsic Mode Function (IMF). To determine IMF, it must satisfy following two conditions: First, the number of extreme value point (maximum or minimum) of signal is equal to, or at most a difference to the number through zero point; Second, average of upper envelope composed of local maximum value and lower envelope composed of local minimum is zero. And the basic process of EMD can be summarized as follows:

1. Find out all maximum points of original signal JV(t) and use cubic spline interpolation function to fit and form upper envelope of original data. Similarly, find out all minimum points, and use minimum points to get lower envelope.

2. Calculate average of upper and lower envelopes, denoted by m1. Subtract m1 from JV (t) and get a new data sequence h1, namely, X (t) - ml = h1.

3. If h1 still has negative local maximum value and positive local minimum value, it shows that this is not a nature modal function and still need "filter". Repeat steps above and obtain h2, h3.... If there is h(t) meet two conditions of IMF, then they will get the h(t) as IMF1 and do the next step.

4. Subtract IMF1 from original signal JV(t), and the rest signal start from (1) as original signal to calculate rest IMF (t).

Finally we get indecomposable signals, such as when the condition is monotone sequence or constant sequence, screening process comes to an end. Residual signal e(t) represents average and RES of the signal. Thus, original sequence is composed of one RES and many IMFs. As mentioned above, the whole process is like a screening process, we extract intrinsic mode function from the signal according to time characteristic. It is important to note that according to the above, extracted IMF should satisfy two conditions, but in real practice, signal that can strictly meet the two conditions does not exist, so if judge IMF by the two conditions, we may not get the result or take lengthy program execution time as expense. In order to ensure that amplitude modulation and frequency modulation of IMF have physical meaning, and considering the feasibility of application, we must make criteria to end screening. Traditionally, standard deviation can be used here to finish. We control SD of screening process through the accuracy in actual situation.

2.3 Hilbert Spectrum Analysis

Time domain analysis is mainly concerned about signal spectrum varies with time. Instantaneous frequency is the characteristic representing transient of signal on the local time point, and instantaneous frequency on the whole duration reflects time-varying regularity of signal frequency. For JV (t), we make Hilbert transform, and obtain Y (t):

$$Y(t) = \frac{1}{\pi} PV\left(\int_{-\infty}^{\infty} \frac{X(\tau)}{t-\tau} d\tau\right) \tag{2.5}$$

In which, PV is Cauchy principal value. The formula means that Y (t) is convolution of X (t) and $\frac{1}{\pi\tau}$. According to this definition, X (t) and Y (t) form a pair of conjugate complex number, and we can get a parsing signal Z (t),

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} \tag{2.6}$$

In which
$$\begin{cases} a(t) = \left[X^2(t) + Y^2(t)\right]^{1/2} \\ \theta(t) = \arctan\left(\dfrac{Y(t)}{X(t)}\right) \end{cases} \tag{2.7}$$

In theory, there are many methods to define imaginary part. But Hilbert transform offers the only imaginary value, making its results become an analytic function. After getting the phase, instantaneous frequency can be obtained, because it is derivative of phase.

$$\omega = \frac{d\theta(t)}{dt} \tag{2.8}$$

Amplitude and frequency obtained from Hilbert transform are functions of time. If we use 3d graphic to express relationship among amplitude, frequency and time, or display amplitude in the form of grayscale on frequency - time plane, Hilbert spectrum $H(w,t)$ can be obtained.

If computing the integral of $H(w,t)$ to time, we can get Hilbert marginal spectrum $h(w)$:

$$h(w) = \int_0^T H(w,t)dt \tag{2.9}$$

Marginal spectrum provides total amplitude measuration of each frequency, expressing accumulating amplitude throughout the whole timespan. If computing integral of the square of amplitude to time, we can get the Hilbert energy spectrum:

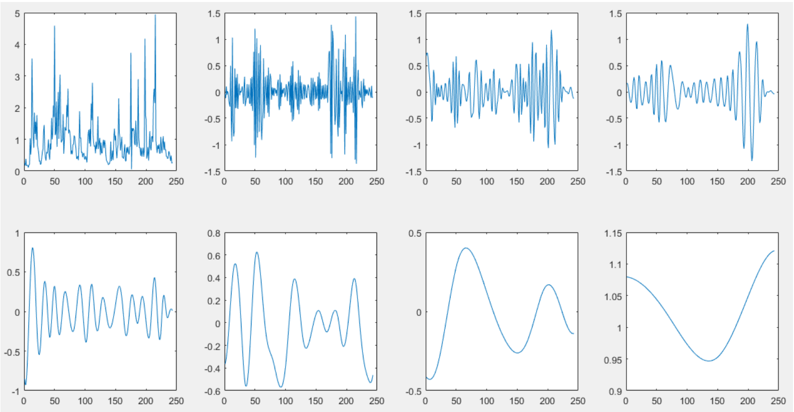
$$ES(w) = \int_0^T H^2(w,t)dt \tag{2.10}$$

Hilbert energy spectrum provides energy measuration of each frequency, expressing accumulating energy of each frequency throughout the whole timespan.

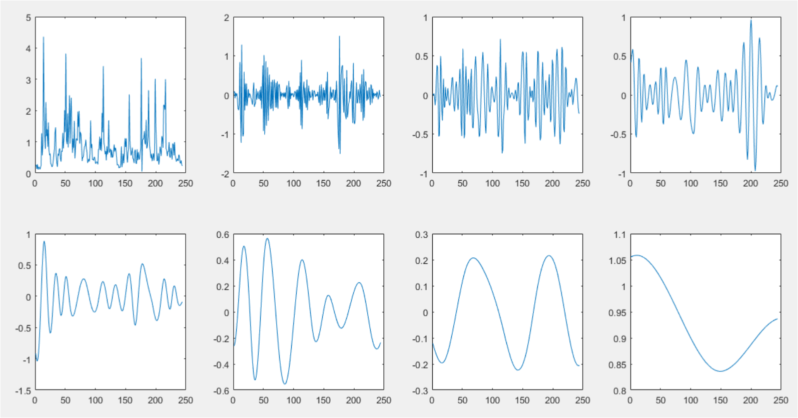
3 Empirical Analysis

3.1 EMD signal decomposition processing

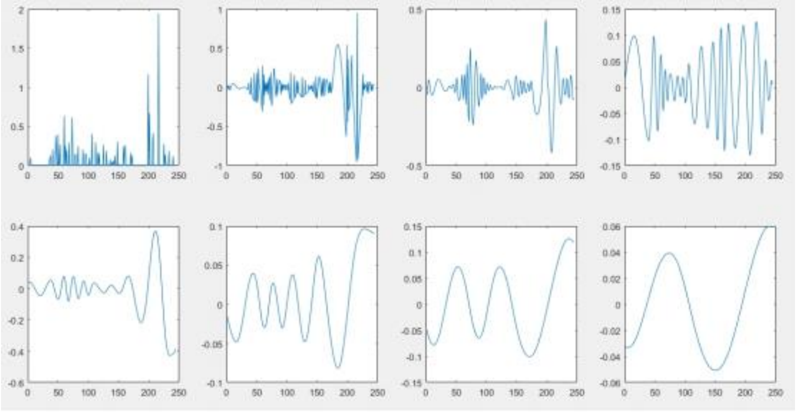
Take RV, BV and JV data of Shanghai Composite Index in 2008 as example, we use EMD decomposition to decompose IMF of each dimension. They are separately original signal, IMF1~ IMF6 and residual (RES) from left to right, from top to bottom:



(a) EMD results of RV in 2008



(b) EMD results of BV in 2008



(c) EMD results of JV in 2008

Figure1: EMD results of variation

From the details of fluctuation, IMF1 is the most volatile component than any other components and shows a significantly high frequency shock. Its fluctuation details are the closest to original signal, so it represents the highest frequency signal, and keeps details of original signal well. IMF2 ~ IMF3 are still fluctuating, but their frequencies are lower that IMF1, and also show a high frequency characteristic. Fluctuation information of IMF4 ~ IMF6 begin to decrease obviously. Only when the original signal expresses a sudden change, they retain fluctuation details. Otherwise, it expresses a low-frequency trend. The trend of RES is consistent with the original signal, explaining the overall decline and rise trend of volatility. And the lowest position of RES is corresponding to the lowest volatility of original signal, illustrating RES retains more energy and is enough to affect the original signal. As for the quantitative relation, RV = BV + JV. Comparing IMF for every dimensions of these three volatilities, we can find the signal of BV and RV are almost the same except some details. From IMF6 and RES, the trend of JV is opposite to that of BV and RV. On other dimensions, when JV is large, BV and RV are relatively small. On the contrary, when JV is small, BV and RV are relatively large.

3.2 Explore average oscillation cycle of each IMF

In order to have enough data, we take daily volatility data from 2001 to 2008 as example to calculate average oscillation cycle of IMF. Define average oscillation cycle = total number of days/(the maximum number of days + the minimum number of days) / 2.

Table1: Average oscillation cycle of IMF

Total number of days	Maximum number of days			Minimum number of days			Average oscillation cycle		
Signal	m	BV	JV	RV	BV	JV	RV	BV	JV
Original signal	584	701	161	584	701	22	3.280	2.733	6.818
IMF1	635	626	281	635	625	281	3.017	3.063	10.164
IMF2	359	340	188	360	341	189	5.330	5.627	15.027
IMF3	209	183	128	209	183	127	9.167	10.469	23.801
IMF4	129	102	81	129	102	80	14.853	18.784	45.082
IMF5	65	52	43	64	52	42	29.705	36.846	66.069
IMF6	36	32	29	36	32	29	53.222	59.875	132.138
IMF7	19	14	14	18	14	15	103.568	136.857	147.385
IMF8	8	7	13	9	8	13	225.411	255	255.467
IMF9	3	2	8	4	3	7	547.429	766.4	319.333
IMF10	2	1	6	1	1	6	1277.33	1916	766.4

(1) From IMF1 ~ IMF10, decomposition scale is larger, number of extreme value point is less, and average oscillation cycle is longer. Component which has a greater decomposition scale expresses a lower frequency change, so it is a long-term change.

(2) It can be seen from days of the table, for RV, IMF1 represents frequency variation of 3 days, IMF2 represents average change trend of a week. While for IMF10, because data is not enough, it has no practical significance. It represents that the number of days will appear error because EMD has inherent boundary problems when solving its extreme value point, the amount of data is not enough and other reasons, so it is only for reference.

(3) Although total numbers of days are the same, compared decomposition results of these three kinds of volatility, JV has one more dimension. Comparing days and average oscillation cycle of extreme value, we can find that, in high frequency phase, because data density of JV is small, most of them are 0. Therefore, the number of extreme value point is low. The extreme value of days can be sorted as: RV > BV > JV, so average oscillation cycle of JV is the biggest. But after IMF7, because its total decomposition scale is once more than its volatility, moderate degree of its is not better than that of BV, its fluctuation cycle is short, and extremum days of JV gradually increases.

3.3 Compare average oscillation cycle of typical rising and decline period

Choose typical rising period (2006/6/22 ~ 2007/6/12) and decline period (2007/5/8 ~ 2008/1/11) of RV according to the trend of volatility, typical rising period (2006/6/22 ~ 2007/6/21) and decline period (2008/3/14 ~ 2009/5/8) of BV, typical rising period (2007/9/11 ~ 2008/11/24) and typical decline period (2008/11/17 ~ 2009/7/2) of JV.

Table2: Average oscillation cycle of RV during typical rising and decline period

	Typical rising period（171 天）			Typical decline period（236 天）		
RV	maximum days	minimum days	Average oscillation period	maximum days	minimum days	Average oscillation period
Original signal	49	49	3.48	75	75	3.147
IMF1	56	56	3.054	78	78	3.026
IMF2	28	27	6.218	44	44	5.364
IMF3	13	13	13.154	23	24	10.043
IMF4	7	6	26.308	12	13	18.88
IMF5	3	3	57	5	5	47.2
IMF6	2	1	114	3	2	94.4
IMF7		1	171		1	236

Table3: Average oscillation cycle of BV during typical rising and decline period

BV	Typical rising period			Typical decline period		
	maximum days	minimum days	Average oscillation period	maximum days	minimum days	Average oscillation period
Original signal	76	76	3.105	89	89	3.157
IMF1	77	77	3.065	95	95	2.958
IMF2	40	40	5.900	44	43	6.460
IMF3	22	21	10.976	22	21	13.070
IMF4	13	12	18.880	12	12	23.417
IMF5	4	4	59.000	2	5	70.250
IMF6	2	3	94.400	2	2	140.50

Table4: Average oscillation cycle of JV during typical rising and decline period

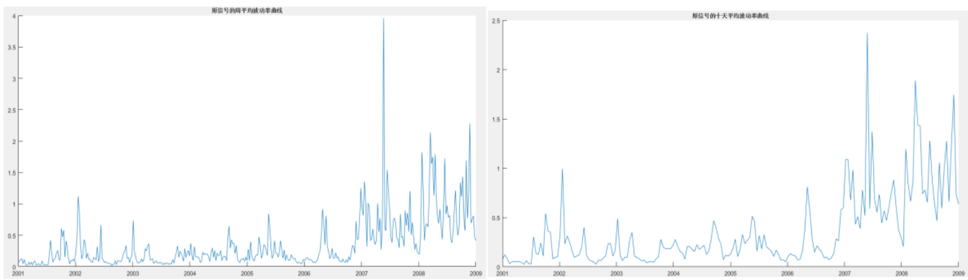
	Typical rising period			Typical decline period		
JV	maximum days	minimum days	Average oscillation period	maximum days	minimum days	Average oscillation period
Original signal	41	7		27	9	
IMF1	58	57	5.06	36	35	4.310
IMF2	31	32	9.238	20	21	7.463
IMF3	17	17	17.118	9	9	17
IMF4	9	9	32.333	6	6	25.5
IMF5	6	5	52.909	2	3	61.2
IMF6	3	4	83.142	1	2	102
IMF7	1	2	194	1	1	153

By comparison with changing speed of average oscillation cycle, the shorter the average oscillation cycle is, the faster changing speed is. First of all, for BV, each IMF component is compared separately, average oscillation cycle of typical rising period is shorter than that of typical decline period, and we can conclude that rising speed is faster than decline speed. RV is just the opposite. For JV, comparing each IMF component separately, in the high frequency phase, average oscillation cycle of typical rising period is longer than typical decline period, the high frequency phase is slow in rising period, and IMF5 - IMF6 of low frequency shifts slowly in decline period. It can be seen from the extreme value point comparison analysis, extreme value points of IMF1 of RV and BV are bigger than original signal, and the maximum and minimum of original signal are the same. Absolute value of the maximum and minimum difference of original signal of JV is not 1, but very big. This is because JV is not continuous, thus, we lose some minimum values, its average oscillation cycle has not any meaning.

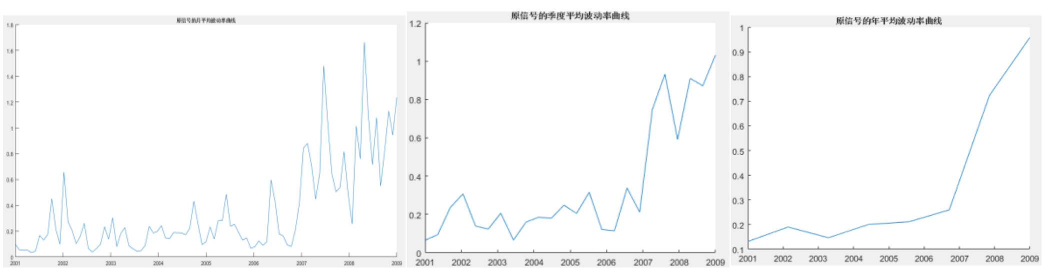
3.4 Explore relationship among average volatilities of different fluctuation cycles and IMF after reconstruction

3.4.1 Average volatilities of different fluctuation cycles

We explore oscillation cycle of different IMF above, but for stock index volatility, it has its own cycles, such as, fluctuation for week cycle and for month cycle. The longer the cycle is, the more moderate volatility is, and the closer it gets to the characteristics of low frequency. So whether the volatility cycle of stock index itself is related to the IMF? First, calculate average index number for each cycle, and get intuitive figure of volatility:

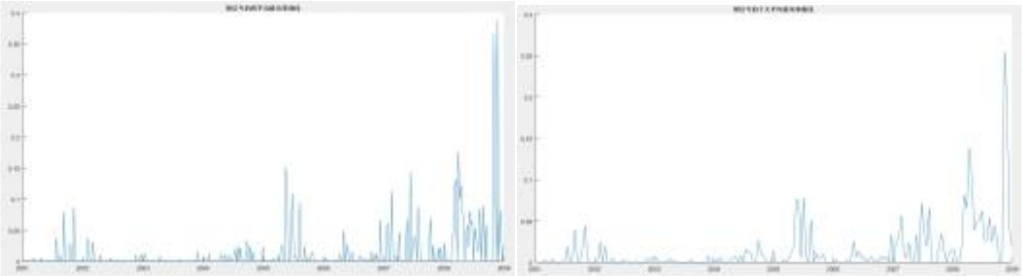


(a) average volatility for week (b) average volatility for half-a-month

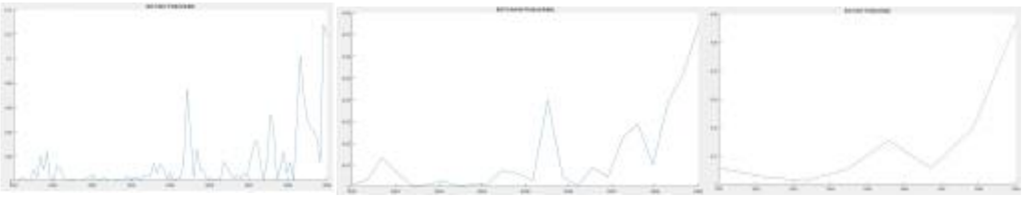


(c) average volatility for month(d) average volatility for quarter (e) average volatility for year

Figure2: Curves of each cycle of original signal of RV



(a) average volatility for week (b) average volatility for half-a-month



(c) average volatility for month(d) average volatility for quarter (e) average volatility for year

Figure3: Curves of each cycle of original signal of JV

3.4.2 Reconstruct IMF in different scales

Because each IMF only represents fluctuation of that component, but stock index represents original signal, therefore, we need to reconstruct IMF by adding them gradually from residual, low frequency to high frequency, and then compare cycle of signal after reconstruction with cycle of stock index. This reconstruction idea refers to a master's degree paper. Then reconstruct RV and JV. Reconstruction formula is $IMFi\_c = \sum IMFi + RES$ $(i= 1, 2, 3...10)$.

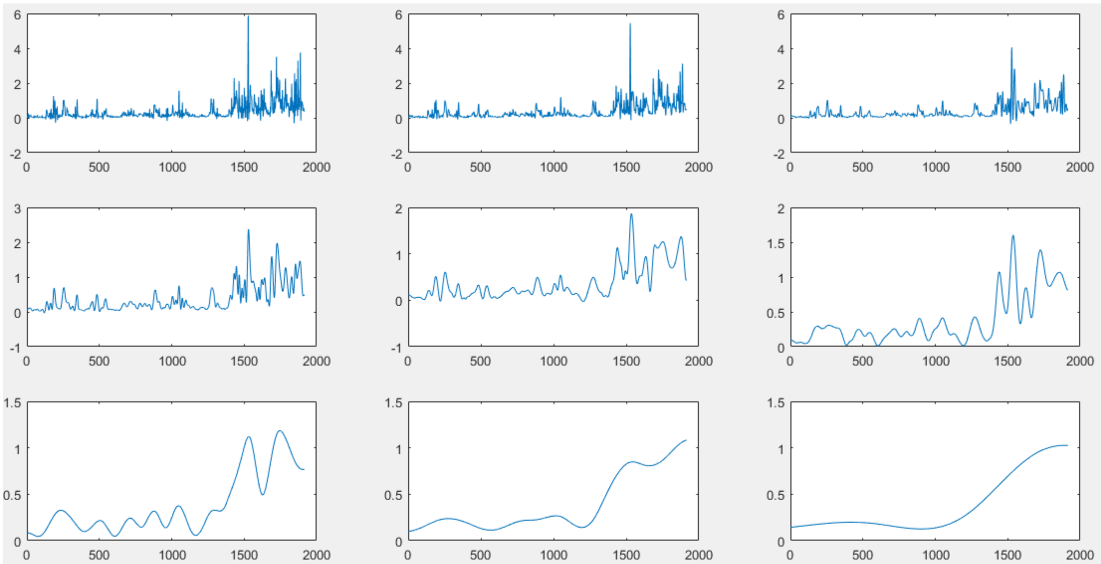


Figure4: From left to right, from top to bottom is respectively IMF2_c, IMF3_c, ..., IMF9_c, IMF10_c of RV.

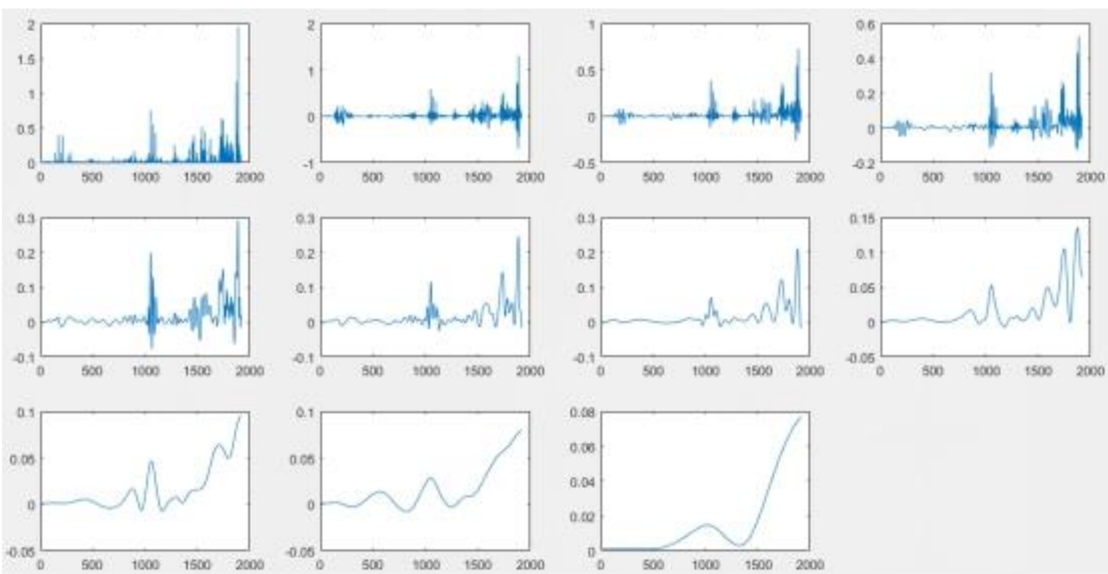


Figure5: From left to right, from top to bottom is respectively the original signal, IMF2_c, IMF3_c, ..., IMF10_c, IMF11_C of JV.

3.4.3 Explore volatility cycle represented by each IMF after reconstruction and get quantitative relationship.

Similarly, explore JV as above process, compare the two different volatilities and find that, to explore best explanation cycle of each IMF after reconstruction, the significance of RV is greater than that of JV. This is because IMF of JV after reconstruction has a large negative value, while original signals are all positive, and due to the low frequency of JV in earlier stage, they cannot be consistent. But JV can be basically consistent in addition to some very big negative values. Through the same method above, narrow the range of date gradually, determine how long the most consistent volatility cycle with IMF after reconstruction, and get the following table: (after the test, separated IMF cannot represent original signal of any period.)

Table5: IMF of JV after reconstruction and its represented volatility cycle, etc.

IMF after reconstruction	Represented volatility cycle	Average oscillation cycle	IMF in each dimension	Average oscillation cycle
IMF2_c	2	10.413	IMF2	10.164
IMF3_c	3	15.514	IMF3	15.027
IMF4_c	4	24.408	IMF4	23.801
IMF5_c	6	43.545	IMF5	45.082
IMF6_c	18	67.228	IMF6	66.069
IMF7_c	30	123.613	IMF7	132.138
IMF8_c	40	166.609	IMF8	147.385
IMF9_c	60	273.714	IMF9	255.467
IMF10_c	240	425.778	IMF10	319.333
IMF11_c		958	IMF11	766.4

Table6: IMF of RV after reconstruction and its represented volatility cycle, etc.

IMF after reconstruction	Represented volatility cycle	Average oscillation cycle	IMF in each dimension	Average oscillation cycle
IMF2_c	2-3	5.904	IMF2	5.330
IMF3_c	3-4	9.826	IMF3	9.167
IMF4_c	5-6	15.087	IMF4	14.853
IMF5_c	10-12	30.903	IMF5	29.705
IMF6_c	28-30	61.806	IMF6	53.222
IMF7_c	45-50	116.121	IMF7	103.568
IMF8_c	60-80	225.411	IMF8	225.411
IMF9_c	150-240	638.667	IMF9	547.429

We know that the greater the decomposition scale of IMF is, the smoother the curve is, and the longer volatility cycle represented by IMF is. So, represented cycle after reconstruction is longer and longer. For average oscillation cycle of each IMF and IMF after reconstruction: whether RV or JV, they are basically equal (when decomposition scale is less than IMF6), or the one after reconstruction is slightly larger than the original IMF, it shows that after reconstruction, the number of extreme value point slightly decreases, the image becomes more peaceful, negative value is less, and is closer to the original signal, so the cycle is slightly longer. Because each IMF represents volatility cycles of different frequency bands, and in the same way, we can't infer fluctuation cycles of original signals represented by each IMF, but IMF after reconstruction is closer to the original signal through palliative treatment, and we can infer the fluctuation cycle of original signal represented by IMF after reconstruction. IMF after reconstruction will have a higher frequency, because they have absorbed details of several IMFs. For RV, days represented by IMF after reconstruction will be shorter than single IMF, roughly 2 ~ 3 times than that, and JV is roughly 4 ~ 5 times. For RV and JV: the accuracy of fluctuation cycle of original signal represented by IMF after reconstruction by JV decreases a lot, because it doesn't match in the early stage, only when in the late high frequency period is identical to the trend.

3.5 Explore the correlation and variance contribution rate of original signal and IMF in different scales.

Table7: The correlation and variance contribution rate of original signal and IMFs.

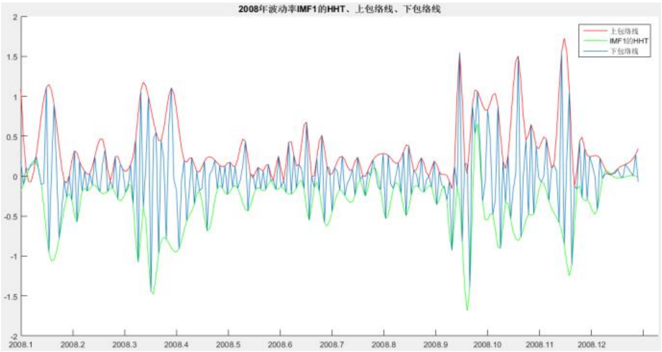
Statistics	Correlation of each IMF component, RES of the rest components and the original signal		The variance of each IMF component: mean of square minus square of mean		The percentage of variance of each IMF component, that is the variance contribution rate	
Volatility	RV	JV	RV	JV	RV	JV
IMF1	0.309	0.3696	0.050	0.0064	14.044	44.1415
IMF2	0.212	0.2513	0.026	0.0042	7.335	29.1022
IMF3	0.180	0.3203	0.037	0.0010	10.329	6.8653
IMF4	0.319	0.2431	0.058	0.0014	16.340	9.5596
IMF5	0.265	0.1189	0.027	0.0004	7.462	2.5049
IMF6	0.147	0.0676	0.012	0.0002	3.274	1.2566
IMF7	0.262	0.1166	0.020	0.0003	5.592	2.0169
IMF8	0.193	0.1330	0.015	0.0002	4.158	1.2267
IMF9	-0.003	0.0660	0.006	0.0001	1.689	0.5801
IMF10	0.224	0.0274	0.026	0.0000	7.317	0.3023
RES/IMF10	0.474	0.1408	0.080	0.0001	22.458	0.3902
RES		0.2026		0.0003		2.0536

We can see from the correlation of each IMF and original signal that, for RV, the correlation of RES and original signal is the highest, that is, RES represents the overall trend of RV. While for JV, correlation of IMF1 and original signal is the highest, which is different from RV. This is because JV is a jump degeneration, that is, frequency changes a lot, details occupy a large proportion, so the correlation and variance contribution rate of IMF1 are very big, but BV as a kind of common signal, RES represents the overall trend of RV, which is a normal phenomenon. For RV, IMF1 is most volatile and represents the highest frequency component, so its variance and variance contribution rate are also the biggest. For JV, its variance is reduced in an order of two, that must because the distribution of signals in each IMF is not so average as that of RV.

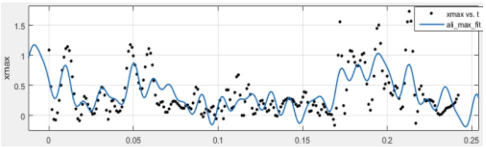
3.6 Volatility signal processing based on Hilbert transform.

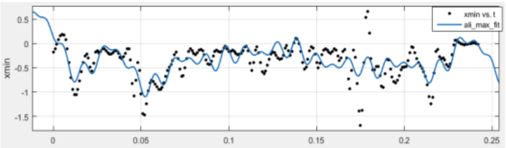
3.6.1 Fit on the envelopes after Hilbert transform.

Use data of 2008, in MATLAB, by searching for extreme value points and draw upper and lower envelope for each IMF, it can be found that maximum or minimum points cannot appear on the boundary at the same time, so there will be border effect. In MATLAB, we fit by carve fitting tool, for example, the figure below is the fitting situation of upper and lower envelopes after Hilbert transform of IMF1 of BV:



(a) Upper and lower envelopes of IMF1





(b) Upper envelope fitting of IMF1 (c) Lower envelope fitting of IMF1

Figure7: The fitting situation of envelopes of IMF1 after Hilbert transform

We can find by fitting that, whether RV or JV, the best fit distribution of each IMF is the sum of sine. When it is the 8th order, the fitting effect comes to the best, so we can say that, using sum of sine to fit IMF is the best choice, and the specific fitting effect is shown in the following table:

Table8: The upper and lower envelopes fitting effect of each IMF of volatilities

	RV				JV			
Sum of sine(8)	SSE	R2	Ad-R2	RMSE	SSE	R2	Ad-R2	RMSE
a1i_max_fit	17.68	0.4753	0.4202	0.2842	2.511	0.8082	0.7882	0.1068
a1i_min_fit	14.11	0.5087	0.4571	0.2538	3.568	0.6689	0.6343	0.1273
a2i_max_fit	0.6093	0.9725	0.9696	0.05275	0.04748	0.9839	0.9822	0.01469
a2i_min_fit	0.7285	0.9046	0.9562	0.05768	0.6019	0.8352	0.818	0.5023
a3i_max_fit	0.1064	0.9955	0.9951	0.02204	0.004631	0.9888	0.9877	0.004588
a3i_min_fit	0.09523	0.9966	0.9963	0.02085	0.000881	0.9974	0.9971	0.002001
a4i_max_fit	0.3274	0.9797	0.9776	0.03867	0.00834	0.9989	0.9988	0.006157
a4i_min_fit	0.05091	0.9953	0.9948	0.01525	1.902	0.968	0.9646	0.09298
a5i_max_fit	0.005239	0.9996	0.9995	0.00489	0.000480	0.9999	0.9999	0.001478
a5i_min_fit	0.000687	0.9999	0.9999	0.00177	0.001518	0.9959	0.9955	0.002627
a6i_max_fit	0.04702	0.9964	0.996	0.01465	0.005954	0.9968	0.9965	0.005202
a6i_min_fit	0.426	0.9825	0.9807	0.0441	0.01018	0.9882	0.9869	0.006802

By observing actual fitting lines, we can know: in low frequency, its envelope effect is not credible because the algorithm itself has inherent drawbacks, such as endpoint effect, etc. While in high frequency, effect of lower envelope is bad because some existing special small data affect the fitting effect. So, according to data validity, the overall situation is effect of lower envelopes will be better than that of upper envelopes. Therefore, in actual situation, the fitting of lower envelope in intermediate frequency will give a better reference, and the best distribution part is the sum of sine(8), fitting effect at IMF5 is the best, while at IMF6, it turns over.

3.6.2 Time - amplitude curves obtained by Hilbert transform

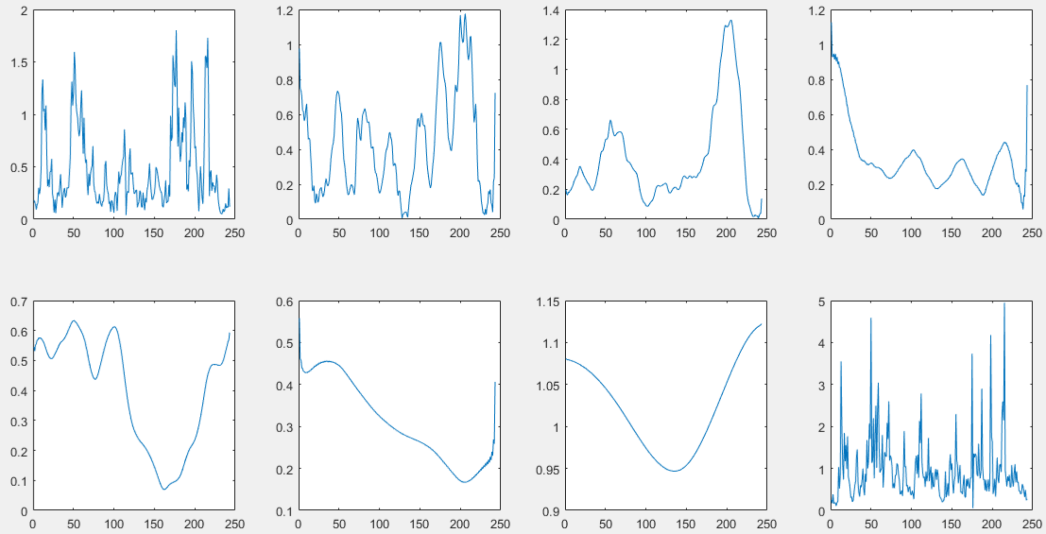


Figure8: Time - amplitude figure of each IMF of RV after Hilbert transform

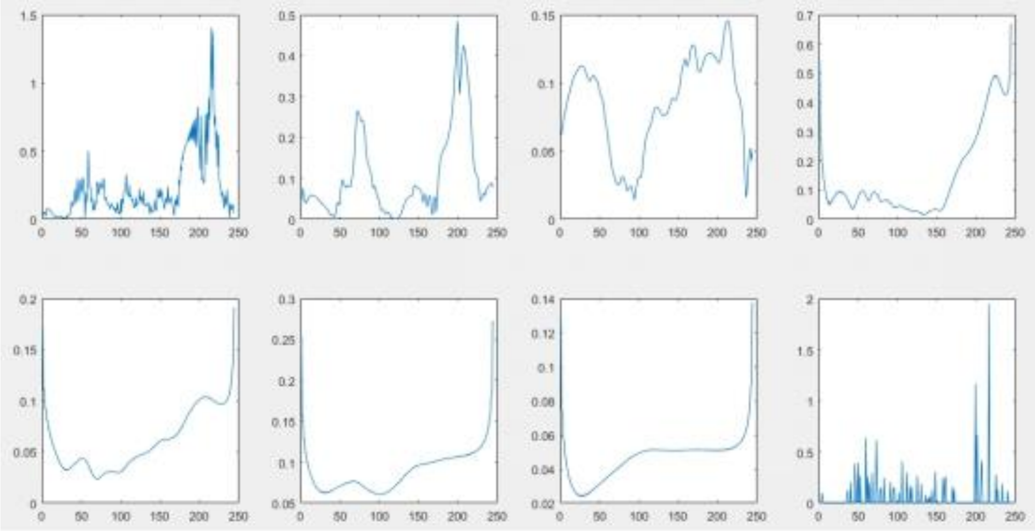


Figure9: Time - amplitude figure of each IMF of JV after Hilbert transform

Figure1 ~ Figure7 are instantaneous amplitudes of IMF1~ IMF7 after Hilbert transform. Figure8 is the original signal. Amplitude curve is similar to IMF's original signal. The one which has a small decomposition scale represents a high frequency component, so its amplitude changes will become fast. For example, IMF1-IMF3 represent positions of details that amplitude of original signal changes fast. Since maximum values of each IMF represent mutations on that scale, and mutations of high frequency component are more likely to be preserved, so if we find maximum value position keeping arising in the first three IMF of the highest frequency, that position will be the breakpoint (Maximum is not necessarily the breakpoint, only the point which suddenly rises to maximum can be regarded as mutation point.). From the picture above, maximum values appear in the first three IMFs which are close to 200, and that is a breakpoint of original signal (at the bottom right corner).

3.6.3 Time - frequency curves after Hilbert transform

The concept of frequency is from mechanical rotary motion, and is defined as angular velocity. For periodic motion, angular velocity is angular frequency. We usually regard anticlockwise of θ as positive, so positive frequency of rotation is anticlockwise rotation angular velocity, negative frequency is clockwise rotation angular velocity. This is its physical meaning, positive and negative sign do not affect its physical meaning. But usually, because Hilbert transform simply calculates instantaneous amplitude, frequency and phase of signal, and is likely to appear negative frequency. So for researches on stock index volatility, it may be meaningless. If making EMD decomposition of signal before Hilbert transform, getting components in different scales, and making Hilbert transform of each component, we can obtain instantaneous frequency with practical significance.

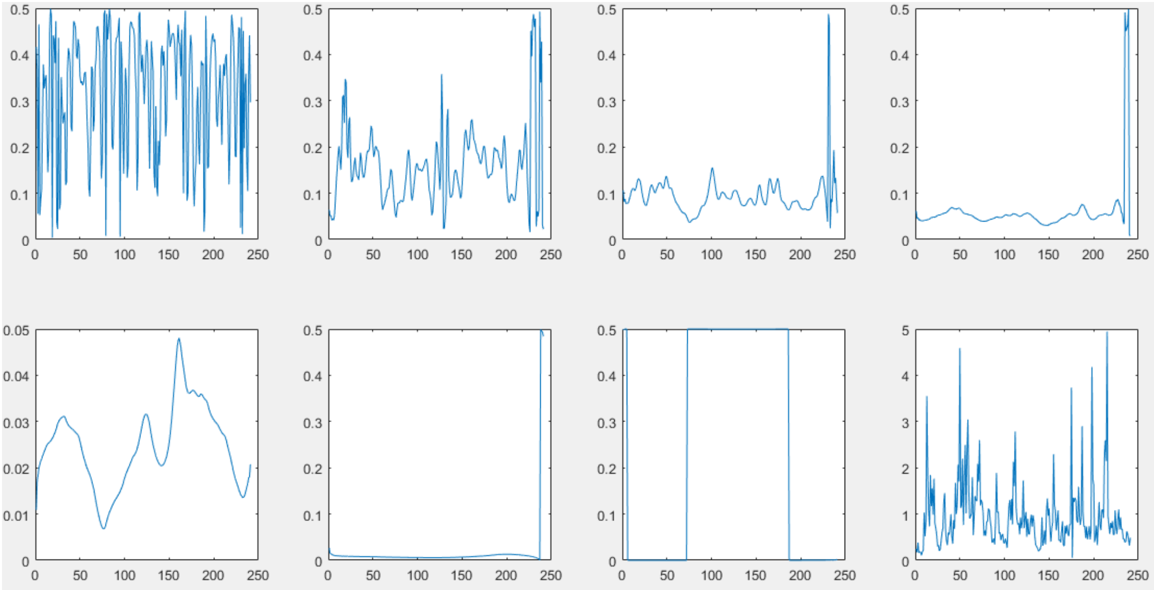


Figure10: Instantaneous frequency of RV after EMD decomposition and Hilbert transform

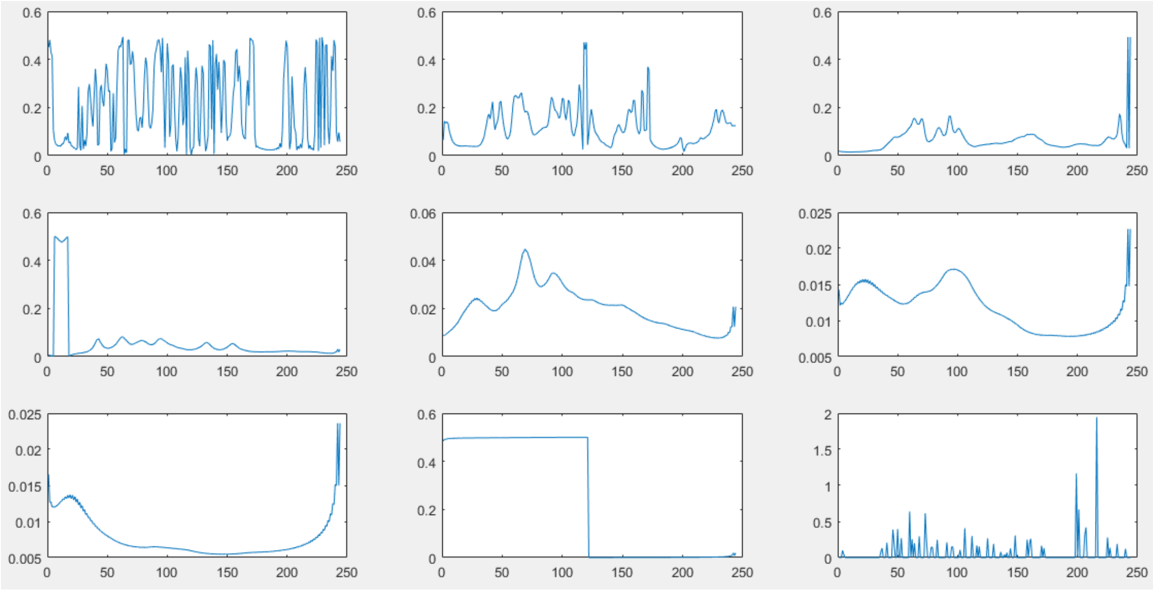


Figure11: Instantaneous frequency of JV after EMD decomposition and Hilbert transform

Figure1 ~ Figure7 are instantaneous frequencies of IMF1 ~ IMF6+RES of RV after Hilbert transform. Figure8 is the original signal. Figure1 ~ Figure8 are instantaneous frequencies of IMF1 ~ IMF7+RES of JV after Hilbert transform. Figure9 is the original signal. For time-frequency diagram, the horizontal axis shows change speed of volatility on different frequencies. For RV, from IMF1 ~ IMF6, obviously, in IMF component of high frequency, because of constantly changing details, namely quickly changing in high frequency, instantaneous frequencies are all the biggest. When it comes to IMF5, it presents downtrend in low frequency and doesn't change much, the frequency becomes very low. JV is the same, it changes slow in low frequency, so the frequency is low. While for RES of the two volatilities, the frequency truncation appears, that is, from high frequency to basically 0 frequency. That is because RES represents trend, while it is high frequency that represents fast changing frequency part, it is low frequency that represents slow changing part. For example, numerical value of JV is not big in the earlier stage, but changes quickly, so frequency is high. And later it changes slowly at basically 0 frequency. RV is the same, it changes quickly in the middle, and the frequency of RES is big.

4 Conclusions

Researches on stock index volatility involve many theories, methods, researches and technologies, in which properties of volatility are numerous. We process stock index volatility only through Hilbert-Huang Transform algorithm and from the signal decomposition processing point of view in this article. We select typical RV, JV and BV, and complete analysis and comparison of these several volatilities under the framework of this algorithm process, and also get some unique properties from the signal decomposition point of view, for example, by calculating extreme value points, shape and goodness of fit of signals to speculate cycles and get quantitative relations. However, there are still some problems to be solved in this paper, such as limited data, lack of many calculations, conclusions and without considering endpoint effect of Hilbert-Huang Transform algorithm, etc. So in practical application, if you need reference ideas of this paper, or continue to deepen, you still need accumulation and perfection.

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