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Random Attractor Family for the Stochastic Higher-Order Kirchhoff Equations

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Abstract

The random attractor family of the stochastic high-order Kirchhoff equations with strong damped nonlinearity and white noise are studied. First, the original high-order Kirchhoff equation is simplified into a first-order evolution equation by using Ornstin-Uhlenbeck process and $It\hat{o}$ equation. Secondly, it is proved that there exists a bounded random absorption set in $D(E_k)$ for the first-order evolution equation. Then it is proved that there exists a compact absorption collection in stochastic dynamical system $\{S(t,w), t \geq 0\}$ when $k = 1,...,m$. Finally, a stochastic dynamical system $\{S(t,w), t \geq 0\}$ with random attractor family A_k 0f $(\mathrm{E}_0,\ E_k)$ is obtained.

Mathematics Subject Classification: 35G50; 26A18

Keywords: Stochastic higher-order Kirchhoff equations;White noise;Random absorption set;Ornstin-Uhlenbeck process;Random attractor family

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1 Introduction

In this paper, the following nonlinear strong damping higher-order Kirchhoff equations with white noise are studied:

$$u_{tt} + M(\|D^m u\|^2)(-\Delta)^m u + \beta(-\Delta)^m u_t + g(|u|^2)u = q(x)\dot{W} \quad (1)$$

$$u(x,t) = 0, \quad \frac{\partial^i u}{\partial v^i} = 0, \quad i = 1,2,\cdots,m-1, \quad x \in \partial\Omega, \quad t > 0 \quad (2)$$

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), \ x \in \Omega \subset R^n \quad (3)$$

Where $m>1$, Ω is a bounded region on R^n with smooth Dirichlet boundary $\partial\Omega$ and $g(|u|^2)u$ is a non-linear source term, the family of random attractor $A_k(k = 0,1,\cdots,m-1)$ of the stochastic dynamical system is obtained.

After introducing some results of nonlinear evolution equation and infinite dimensional dynamic system(reference [1]), Bailing Guo and Zhujun Jing[2] further introduce some important problems in infinite dimensional dynamic system.

Hans Crauel and Franco Flandoli[3] established the criterion for the existence of global attractors for stochastic systems, proved the existence of invariant Markov measures supported by stochastic attractors, and generated invariant measures for related Markov semigroups. The results were applied to the reaction-diffusion equation of additive white noise and the $Navier-Stokes$ equation of multiplicative additive white noise. ThenHans Crauel and Franco Flandoli[4] extended the reference [3], the concept of attractors for stochastic dynamical systems is introduced, and it is proved that stochastic attractors satisfy most properties of general attractors in deterministic dynamical systems theory.

Xianyun Du and Wei Chen[5] considered the dissipative KdV -type equation with additive noise

$$du + (vu_{xxxx} + \alpha uu_x + u_{xxx} + \beta u)dt = f(x)dx + \sum_{j=1}^{m}\phi_j dw_j(t)$$

the existence of global attractors is established for the asymptotic behavior in one-dimensional bounded region, and the existence of stochastic absorption set for the system is discussed.

Guigui Xu, Libo Wang and Guoguang Lin[6] studied the nonautonomous stochastic wave equation with dispersion and dissipation terms

$$u_{tt} - \Delta u - \alpha\Delta u_t - \beta u_{tt} + h(u)u_t + \lambda u + f(x,u) = g(x,t)u + \varepsilon u \cdot \frac{dW}{dt}$$

the existence of random attractor for Nonautonomous stochastic wave equations with product white noise is obtained by using the uniform estimation of solutions and the technique of decomposing solutions in a region.

For more information on random attractor, please refer to references [7-15].

On the basis of previous studies, this paper studies the family of random attractor for higher-order Kirchhoff-type equations with white noise. Firstly, some preliminary knowledge and basic assumptions are made. Then, it is proved that there are random absorption set in stochastic dynamical system. Finally, the family of random attractor is obtained.

2 Basic assumptions and preliminaries

In this part, we first introduce some mathematical symbols and define the inner product and norm on $L^2(\Omega)$, then give the hypothesis of Kirchhoff type stress term $M(s)$ and nonlinear source term $g(|u|^2)u$, and finally give the definition and properties of stochastic dynamical system $\{S(t,w),t\geq 0\}$ and random attractor A.

For the sake of narrative convenience, the following symbols are introduced: $\nabla = D$, $A=-\Delta$, $H=L^2(\Omega)$, $H_0^m(\Omega)=H^m(\Omega)\cap H_0^1(\Omega)$, $H_0^{m+k}(\Omega)=H^{m+k}(\Omega)\cap H_0^1(\Omega)$, $E_k = H_0^{m+k}(\Omega)\times H_0^k(\Omega),(k=0,1,2,\cdots m\text{-}1)$, $C, C_i\,(i=1,2,\cdots,6)>0$ are constants.

The inner product and norm of $L^2(\Omega)$ are respectively:

$$(u,v)=\int_\Omega uvdx,\quad \|u\|=(u,u)^{\frac{1}{2}},\quad \forall u,v\in L^2(\Omega)\ ;$$

$$(y_1,y_2)_{E_k}=(D^{m+k}u_1,D^{m+k}u_2)+(D^k v_1,D^k v_2),\quad \|y\|_{E_k}^2=\|D^{m+k}u\|^2+\|D^k v\|^2,$$

$$\forall y_i=(u_i,v_i)^T,\ y=(u,v)^T\in E_k\quad i=1,2.$$

Assume that the *Kirchhoff* stress term $M(s)$ satisfies the following condition:

(H_1) $(M(s)-\varepsilon)(D^{m+k}y_1,D^{m+k}y_2)\geq (D^{m+k}y_1,D^{m+k}y_2)$.

Assume that the non-linear source term $g(|u|^2)u$ satisfies the following condition:

(H_2) $g(s)$ exponential growth, $g(|u|^2)\leq C(1+|u|^{2p}),\quad p\leq \dfrac{2n}{n-2m}, n>2$.

Let $(B(R^+)\times F\times B(X), B_k(w)\subset D(w))$ be a probabilistic space and define a family of measures-preserving and ergodic transformation $\{\theta_t, t\in R\}$:

$$\theta_t w(\cdot)=w(\cdot+t)-w(t)\ ,$$

then $(\Omega,F,P,(\theta_t)_{t\in R})$ is an ergodic metric dynamical system.

Let $(X,\|\cdot\|_X)$ be a complete separable metric space and $B(X)$ be a $Borel\ \sigma-$algebra on X.

Definition 1[9] Let $(\Omega,F,P,(\theta_t)_{t\in R})$ be a metric dynamical system, if $(B(R^+)\times F\times B(X),B(X))$-measurable mapping

$$S:R^+\times\Omega\times X\to X,\quad (t,w,x)\mapsto S(t,w,x)$$

satisfaction:

(1) For all $s,t\ge 0$ and $w\in\Omega$, mapping $S(t,w):=S(t,w,\cdot)$ satisfies

$$S(0,w)=id,\quad S(t+s,w)=S(t,\theta_s w)\circ S(s,w);$$

(2) For each $w\in\Omega$, the mapping $(t,w)\mapsto S(t,w,x)$ is continuous.

Then S is a continuous stochastic dynamical system on $(\Omega,F,P,(\theta_t)_{t\in R})$.

Definition 2[9] A random set $B(w)\subset X$ is incremental random set if for each $w\in\Omega$, $\beta\ge 0$, there is

$$\lim_{|s|\to\infty}\inf e^{-\beta s}d(B(\theta_{-s}w))=0,$$

where $d(B)=\sup_{x\in B}\|x\|_X$ for $\forall x\in X$.

Definition 3[9] Note the set of all random sets on $D(w)$ to X. Random set $B(w)$ is called the absorptive set on $D(w)$. For any $B(w)\in D(w)$ and $P_{a.e.w}\in\Omega$, there exists $T_{B(w)}>0$ such that

$$S(t,\theta_{-t}w)(B(\theta_{-t}w))\subset B_0(w).$$

Definition 4[9] Random set $A(w)$ becomes a random attractor for continuous stochastic dynamical system $S(t)$ on X if random set $A(w)$ satisfies:

$A(w)$ is a random compact set;

(3) $A(w)$ is an invariant set, i.e. for any $t>0, S(t,w)A(w)=A(\theta_t w)$;

(3) $A(w)$ attracts all sets on $D(w)$, i.e. for any $B(w)\in D(w)$ and $P_{a.e.w}\in\Omega$, we have limit formula:

$$\lim_{t\to\infty}d(S(t,\theta_{-t}w)B(\theta_{-t}w),A(w))=0,$$

where $d(A,B)=\sup_{x\in A}\inf_{y\in B}\|x-y\|_H$ is Hausdorff half distance(there $A,B\subseteq H$).

Definition 5[9] Let random set $B(w)\in D(w)$ be a random absorbing set of stochastic dynamical system $(S(t,w))_{t\ge 0}$, and random set $B(w)$ satisfies:

(1) Random set $B(w)$ is a closed set on Hilbert space X;

(2) For $P_{a.e.w} \in \Omega$, random set $B(w)$ satisfies the following asymptotic compactness conditions: for arbitrary sequence $x_n \in S(t_n, \theta_{-t_n} w) B_0(\theta_{-t_n} w)$ at $t_n \to +\infty$, there is a convergent subsequence in space X. For stochastic dynamical system $(S(t,w))_{t\ge 0}$, there is a unique global attractor

$$A(w) = \bigcap_{\tau \ge t(w)} \overline{\bigcup_{t\ge\tau} S(t, \theta_{-t} w) B_0(\theta_{-t} w)}.$$

Ornstein-Uhlenbeck process

In reference [2], we introduce Ornstin-Uhlenbeck process in $H_0^{m+k}(\Omega)$, which is given by Wiener process in metric system $(\Omega, F, P, \{\theta_t\}_{t\in R})$.

Let $z(\theta_t w) = -\alpha \int_{-\infty}^{0} e^{\alpha\tau} \theta_t w(\tau) d\tau$, of which $t \in R$. It can be seen that for any $t \ge 0$, stochastic process $z(\theta_t w)$ satisfies $It\hat{o}$ equation

$$dz + \alpha z dt = dW(t).$$

According to the properties of Ornstin-Uhlenbeck process, there exists a probability measure P, θ_t -invariant set $\Omega_k \subset \Omega$, and the above stochastic process $z(\theta_t w) = -\alpha \int_{-\infty}^{0} e^{\alpha\tau} \theta_t w(\tau) d\tau$ satisfies the following properties:

(1) For any given $w \in \Omega_0$, mapping $s \to z(\theta_s w)$ is a continuous mapping;

(2) Random variable $\|z(w)\|$ increases slowly;

(3) There is a slowly increasing set $r(w) > 0$, such that

$$\|z(\theta_t w)\| + \|z(\theta_t w)\|^2 \le r(\theta_t w) \le r(w) e^{\frac{\alpha}{2}|t|};$$

(4) $\lim_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_t w)|^2 d\tau = \frac{1}{2\alpha}$;

(5) $\lim_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_t w)| d\tau = \frac{1}{\sqrt{\pi\alpha}}$.

3 Existence of Random Attractor family

This part mainly turns the equation (1)-(3) into the first-order development equation, then proves that the inequality $(Ly, y)_{E_k} \ge k_1 \|y\|_{E_k}^2 + k_2 \|D^{m+k} y_2\|^2$ is established, and then uses this inequality to prove that there is a bounded compact absorption set in the stochastic dynamical system $\{S(t,w), t \ge 0\}$, and finally can get the random attractor family $A_k (k = 1, \cdots, m)$ of the stochastic dynamical system $\{S(t,w), t \ge 0\}$.

For convenience, equation (1) - (3) can be transformed into

$$\begin{cases} du = u_t dt, \\ du_t + [M(\|A^{\frac{m}{2}}u\|^2)A^m u + \beta A^m u_t + g(|u|^2)u]dt = q(x)dW(t), t \in [0,+\infty), \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega. \end{cases} \tag{4}$$

Let $\varphi = (u, y)^T$, $y = u_t + \varepsilon u$,the problem (4) can be simplified as follows：

$$\begin{cases} d\varphi + L\varphi dt = F(\theta_t \omega, \varphi), \\ \varphi_0(\omega) = (u_0, u_1 + \varepsilon u_0)^T. \end{cases} \tag{5}$$

where

$$\varphi = (u, y)^T, \quad L = \begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}}u\|^2) - \beta\varepsilon)A^m + \varepsilon^2)I & (\beta A^m - \varepsilon)I \end{pmatrix},$$

$$F(\theta_t \omega, \varphi) = \begin{pmatrix} 0 \\ -g(|u|^2)u + q(x)dW(t) \end{pmatrix}.$$

Let $z = y - q(x)\delta(\theta_t \omega)$, then question (4) can be written as follows：

$$\begin{cases} \phi_t + L\phi = \overline{F}(\theta_t \omega, \phi), \\ \phi_0(\omega) = (u_0, u_1 + \varepsilon u_0 - q(x)\delta(\theta_t \omega))^T. \end{cases} \tag{6}$$

where

$$\phi = (u, z)^T, \quad L = \begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}}u\|^2) - \beta\varepsilon)A^m + \varepsilon^2)I & (\beta A^m - \varepsilon)I \end{pmatrix},$$

$$\overline{F}(\theta_t \omega, \phi) = \begin{pmatrix} q(x)\delta(\theta_t \omega) \\ -g(|u|^2)u + (\varepsilon + 1 - \beta A^m)q(x)\delta(\theta_t \omega) \end{pmatrix}.$$

Lemma 1 Let $E_k = H_0^{m+k}(\Omega) \times H_0^k(\Omega), (k = 0,1,2,\cdots,m)$,for $\forall y = (y_1, y_2)^T \in E_k$,

$0 < \varepsilon \le \frac{1}{\beta - 1}$,such that

$$(Ly, y)_{E_k} \ge k_1 \|y\|_{E_k}^2 + k_2 \|D^{m+k} y_2\|^2 \tag{7}$$

Of which $k_1 = \min\left\{ \frac{\beta\varepsilon + \varepsilon - \beta\varepsilon^2 \lambda_1^{-m}}{2\beta}, \frac{\beta\lambda_1^m - \varepsilon^2 - 2\varepsilon}{2} \right\}$, $k_2 = \frac{\beta(1 - \beta\varepsilon + \varepsilon)}{2}$.

Proof Because $L=\begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}}u\|^2)-\beta\varepsilon)A^m+\varepsilon^2)I & (\beta A^m-\varepsilon)I \end{pmatrix}$, $\forall y=(y_1,y_2)^T\in E_k$,

so from hypothesis (H_1)、Holder inequality、Young inequality and Poincare inequality, can obtain

$$(Ly,y)_{E_k}=(D^{m+k}(\varepsilon y_1-y_2),D^{m+k}y_1)+(D^k(((M(\|A^{\frac{m}{2}}u\|^2)-\beta\varepsilon)A^m+\varepsilon^2)y_1+(\beta A^m-\varepsilon)y_2),D^ky_2)$$

$$=\varepsilon\|D^{m+k}y_1\|-(D^{m+k}y_2,D^{m+k}y_1)+(M(\|A^{\frac{m}{2}}u\|^2)-\varepsilon)(D^{m+k}y_1,D^{m+k}y_2)+(\varepsilon-\beta\varepsilon)(D^{m+k}y_1,D^{m+k}y_2)$$
$$+\varepsilon^2(D^ky_1,D^ky_2)+\beta\|D^{m+k}y_2\|-\varepsilon\|D^ky_2\|^2$$
$$\geq\varepsilon\|D^{m+k}y_1\|-(D^{m+k}y_2,D^{m+k}y_1)+(D^{m+k}y_1,D^{m+k}y_2)-(\beta\varepsilon-\varepsilon)(D^{m+k}y_1,D^{m+k}y_2)+\varepsilon^2(D^ky_1,D^ky_2)$$
$$+\beta\|D^{m+k}y_2\|-\varepsilon\|D^ky_2\|^2$$
$$\geq\varepsilon\|D^{m+k}y_1\|-\frac{(\beta-1)\varepsilon}{2\beta}\|D^{m+k}y_1\|^2-\frac{\beta(\beta-1)\varepsilon}{2}\|D^{m+k}y_2\|^2-\frac{\varepsilon^2}{2}\|D^ky_1\|^2-\frac{\varepsilon^2}{2}\|D^ky_2\|^2+\beta\|D^{m+k}y_2\|-\varepsilon\|D^ky_2\|^2$$
$$\geq\frac{\beta\varepsilon+\varepsilon}{2\beta}\|D^{m+k}y_1\|^2+\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2}\|D^{m+k}y_2\|^2-\frac{\varepsilon^2\lambda_1^{-m}}{2}\|D^{m+k}y_1\|^2+\frac{\beta\lambda_1^m-\varepsilon^2-2\varepsilon}{2}\|D^ky_2\|^2$$
$$=\frac{\beta\varepsilon+\varepsilon-\beta\varepsilon^2\lambda_1^{-m}}{2\beta}\|D^{m+k}y_1\|^2+\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2}\|D^{m+k}y_2\|^2+\frac{\beta\lambda_1^m-\varepsilon^2-2\varepsilon}{2}\|D^ky_2\|^2$$
$$\geq k_1(\|D^{m+k}y_1\|^2+\|D^ky_2\|^2)+k_2\|D^{m+k}y_2\|^2$$
$$=k_1\|y\|_{E_k}^2+k_2\|D^{m+k}y_2\|^2,$$

where $k_1=\min\left\{\frac{\beta\varepsilon+\varepsilon-\beta\varepsilon^2\lambda_1^{-m}}{2\beta},\frac{\beta\lambda_1^m-\varepsilon^2-2\varepsilon}{2}\right\}$, $k_2=\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2}$ .Lemma 1 is proved.

Lemma 2 Let φ be a solution of the problem (5), then there exists a bounded random absorption set $\tilde{B}_{0k}\in D(E_k)$, so that for any slowly increasing random set $B_k(w)\in D(E_k)$, there exists a random variable $T_{B_k(w)}>0$, making

$$\varphi(t,\theta_t w)B_k(\theta_{-t}w)\subset\tilde{B}_{0k}(w),\qquad \forall t\geq T_{B_k(w)}, w\in\Omega \tag{8}$$

Proof Let ϕ be a solution of the problem (6). By taking the inner product of the two sides of $\phi=(u,z)^T\in E_k$ and equation (6) on E_k, can get

$$\frac{1}{2}\frac{d}{dt}\|\phi\|_{E_k}^2+(L\phi,\phi)=(\overline{F}(\theta_t w,\phi),\phi) \tag{9}$$

Which can be obtained by lemma 1

$$(L\phi,\phi)\geq k_1\|\phi\|_{E_k}^2+k_2\|D^{m+k}z\|^2 \tag{10}$$

according to the inner product defined on E_k ,have

$$(\overline{F}(\theta_t w,\phi),\phi)=(D^{m+k}q(x)\delta(\theta_t w),D^{m+k}u)+(D^k(-g(|u|^2)u+(\varepsilon+1-\beta A^m)q(x)\delta(\theta_t w)),D^k z) \tag{11}$$

according to Holder inequality、 Young inequality and Poincare inequality, can obtain

$$(D^{m+k}q(x)\delta(\theta_t w),D^{m+k}u)\leq\frac{\varepsilon}{2}\|D^{m+k}u\|^2+\frac{\lambda_1^{-m}}{2\varepsilon}\left\|A^{m+\frac{k}{2}}q(x)\right\|^2|\delta(\theta_t w)|^2 \tag{12}$$

$$(D^k\varepsilon q(x)\delta(\theta_t w)),D^k z)\leq\frac{\varepsilon\lambda_1^{-m}}{2}\|D^{m+k}z\|^2+\frac{\varepsilon}{2}\|D^k q(x)\|^2|\delta(\theta_t w)|^2 \tag{13}$$

$$(D^k(1-\beta A^m)q(x)\delta(\theta_t w),D^k z)\leq\frac{\varepsilon\lambda_1^{-m}}{2}\|D^{m+k}z\|^2+\frac{1}{2\varepsilon}(\|D^k q(x)\|^2+\beta^2\left\|A^{m+\frac{k}{2}}q(x)\right\|^2)|\delta(\theta_t w)|^2 \tag{14}$$

By using hypothesis (H_2)、Embedding theorem and Poincare inequality,we get

$$\left\|g(|u|^2)\right\|^2\leq\left\|C(1+|u|^{2p})\right\|^2\leq C_1\|u\|^{4p}+C_2\leq C_1\lambda_1^{-(m+k)}\|D^{m+k}u\|^{4p}+C_2\,,$$

From $\phi=(u,z)^T\in E_k$ to $\|D^{m+k}u\|^{4p}$ is bounded, then $\left\|g(|u|^2)\right\|^2\leq C_3$, so according Holder inequality、Young inequality and Poincare inequality, we have

$$(D^k(-g(|u|^2)u),D^k z)\leq\frac{1}{2}\left\|g(|u|^2)\right\|^2\|u\|^2+\frac{1}{2}\|D^{2k}z\|^2\leq C_4+\frac{\lambda_1^{-(m-k)}}{2}\|D^{m+k}z\|^2 \tag{15}$$

Collecting with (9)-(15) and lemma 1,we get that

$$\frac{d}{dt}\|\phi\|_{E_k}^2+2k_1\|\phi\|_{E_k}^2+(2k_2-2\varepsilon\lambda_1^{-m}-\lambda_1^{-(m-k)})\|D^{m+k}z\|^2\leq\varepsilon\|D^{m+k}u\|^2+2C_4+\frac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{k}{2}}q(x)\right\|^2|\delta(\theta_t w)|^2$$
$$+(\varepsilon+\frac{1}{\varepsilon})\|D^k q(x)\|^2|\delta(\theta_t w)|^2 \tag{16}$$

take $\eta=2k_1$, $M=\frac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{k}{2}}q(x)\right\|^2+(\varepsilon+\frac{1}{\varepsilon})\|D^k q(x)\|^2$, then have

$$\frac{d}{dt}\|\phi\|_{E_k}^2+\eta\|\phi\|_{E_k}^2\leq C_5+M\,|\delta(\theta_t w)|^2 \tag{17}$$

By the Gronwall inequality, $P_{a.e.w}\in\Omega$,conclude that

$$\|\phi(t,w)\|_{E_k}^2\leq e^{-\eta t}\|\phi_0(w)\|_{E_k}^2+\int_0^t e^{-\eta(t-s)}(C_5+M\,|\delta(\theta_s w)|^2)ds \tag{18}$$

Because $\delta(\theta_t w)$ is slowly increasing and $\delta(\theta_t w)$ is continuous with respect to t, according to literature [15], a slowly increasing random variable $r_1:\Omega\to R^+$ can be obtained, so that for $\forall t\in R, w\in\Omega$, there is

$$\left|\delta(\theta_t w)\right|^2 \le r_1(\theta_t w) \le e^{\frac{\eta}{2}|t|} r_1(w) \tag{19}$$

Replacing w in formula (18) with $\theta_{-t} w$, we can get

$$\|\phi(t,\theta_{-t}w)\|_{E_k}^2 \le e^{-\eta t}\|\phi_0(\theta_{-t}w)\|_{E_k}^2 + \int_0^t e^{-\eta(t-s)}(C_5 + M\left|\delta(\theta_{s-t}w)\right|^2)ds \tag{20}$$

where（let $s-t=\tau$）

$$\int_0^t e^{-\eta(t-s)}(C_5+M\left|\delta(\theta_{s-t}w)\right|^2)ds = \int_{-t}^0 e^{\eta\tau}(C_5+M\left|\delta(\theta_\tau w)\right|^2)d\tau \le \frac{C_5}{\eta}+\frac{2}{\eta}M\, r_1(w) \tag{21}$$

Because $\varphi_0(\theta_{-t}w)\in B_k(\theta_{-t}w)$ is increasing slowly and $\left|\delta(\theta_{-t}w)\right|$ is increasing slowly, set

$$R_0^2(w) = \frac{C_5}{\eta}+\frac{2}{\eta}M\, r_1(w) \tag{22}$$

Then $R_0^2(w)$ is also slowly increasing. Note $\hat{B}_{0k}=\{\phi\in E_k \,\big|\, \|\phi\|_{E_k}\le R_0(w)\}$ is a random absorption set, and due to

$$\tilde{S}(t,\theta_{-t}w)\phi_0(\theta_{-t}w)=\varphi(t,\theta_{-t}w)(\phi_0(\theta_{-t}w)+(0,q(x)\delta(\theta_{-t}w))^T)-(0,q(x)\delta(\theta_{-t}w))^T,$$

so let

$$\tilde{B}_{0k}(w)=\{\varphi\in E_k \,\big|\, \|\varphi\|_{E_k}\le R_0(w)+\|D^k q(x)\delta(w)\|=\overline{R}_0(w)\},$$

Then $\tilde{B}_{0k}(w)$ is the random absorption set of $\varphi(t,w)$, and $\tilde{B}_{0k}(w)\in D(E_k)$. The proof is complete.

Lemma 3 When $k=1,...,m$, for $\forall \mathrm{B}_k(w)\in D(E_k)$, let $\varphi(t)$ be the solution of equation (5) in initial value $\varphi_0=(u_0,u_1+\varepsilon u_0)^T\in B_k$, it can be decomposed into $\varphi=\varphi_1+\varphi_2$, where φ_1,φ_2 satisfies respectively

$$\begin{cases} d\varphi_1 + L\varphi_1 dt = 0, \\ \varphi_{01}(w) = (u_0, u_1 + \varepsilon u_0)^T. \end{cases} \tag{23}$$

$$\begin{cases} d\varphi_2 + L\varphi_2 dt = F(w,\varphi), \\ \varphi_{02}(w) = 0. \end{cases} \tag{24}$$

then $\left\|\varphi_1(t,\theta_{-t}w)\right\|_{E_k}^2 \to 0, (t \to \infty)$, $\quad \forall \varphi_0(\theta_{-t}w) \in B_k(\theta_{-t}w)$, （25）

and there exists a slowly increasing random radius of $R_1(w)$, which makes $w \in \Omega$ satisfy

$$\left\|\varphi_2(t,\theta_{-t}w)\right\|_{E_k} \le R_1(w) \tag{26}$$

Proof Let $\phi = \phi_1 + \phi_2 = (u_1, u_{1t} + \varepsilon u_1)^T + (u_2, u_{2t} + \varepsilon u_2 - q(x)\delta(\theta_t w))^T$ be the solution of equation (6), then according to equation (23) and (24), we can see that ϕ_1, ϕ_2 satisfy respectively

$$\begin{cases} \phi_{1t} + L\phi_1 = 0, \\ \phi_{01} = \phi_0 = (u_0, u_1 + \varepsilon u_0 - q(x)\delta(\theta_t w))^T. \end{cases} \tag{27}$$

$$\begin{cases} \phi_{2t} + L\phi_2 = \overline{F}(\phi_2, \theta_t w), \\ \phi_{02} = 0. \end{cases} \tag{28}$$

Taking the scalar product in E_k of equation (27) with $\phi_1 = (u_1, u_{1t} + \varepsilon u_1)^T$,we have that

$$\frac{1}{2}\frac{d}{dt}\left\|\phi_1\right\|_{E_k}^2 + (L\phi_1, \phi_1) = 0 ,$$

then due to lemma 1 and Gronw

$$\left\|\phi_1(t,w)\right\|_{E_k}^2 \le e^{-2k_1 t}\left\|\phi_{01}(w)\right\|_{E_k}^2 \tag{29}$$

all inequality,can get

If w in Formula (29) is replaced by $\theta_{-t}w$, and $\delta(\theta_{-t}w)\in B_k$ increases slowly, then

$$\left\|\phi_1(t,\theta_{-t}w)\right\|_{E_k}^2 \le e^{-2k_1t}\left\|\phi_0(\theta_{-t}w)\right\|_{E_k}^2 \to 0,(t\to\infty)\ , \qquad \forall\phi_0(\theta_{-t}w)\in B_m\ .$$

Thus (25) is established.

By taking the inner product of $\phi_2=(u_2,u_{2t}+\varepsilon u_2-q(x)\delta(\theta_t w))^T$ and equation (28) on E_{k}, and by lemma 1 and lemma 2, can obtain

$$\frac{d}{dt}\left\|\phi_2\right\|_{E_{\mathrm{k}}}^2+\eta\left\|\phi_2\right\|_{E_{\mathrm{k}}}^2\le C_6+M_1\left|\delta(\theta_t w)\right|^2 \tag{30}$$

where $\eta=2k_1$, $M_1=\dfrac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{\frac{3\mathrm{k}}{2}}q(x)\right\|^2+(\varepsilon+\dfrac{1}{\varepsilon})\left\|D^{\mathrm{k}}q(x)\right\|^2$,

substituting w by $\theta_{-t}w$ in（30）and using Gronwall inequality, we know

$$\left\|\phi_2(t,\theta_{-t}w)\right\|_{E_m}^2\le e^{-\eta t}\left\|\phi_{02}(\theta_{-t}w)\right\|_{E_m}^2+\int_0^t e^{-\eta(t-s)}(C_6+M_1\left|\delta(\theta_{r-s}w)\right|^2)ds\le\frac{C_6}{\eta}+\frac{2}{\eta}M_1r_1(w) \tag{31}$$

So there's a slowly increasing random half so that for 2, there's

$$R_1^2(w)=\frac{C_6}{\eta}+\frac{2}{\eta}M_1r_1(w)\ ,$$

such that for $\forall w\in\Omega$, have

$$\left\|\varphi_2(t,\theta_{-t}w)\right\|_{E_{\mathrm{k}}}\le R_1(w)\ .$$

Lemma 3 is proved.

Lemma 4 The stochastic dynamical system $\{S(t,w),t\ge 0\}$ determined by equation (5) has a compact absorption set $K(w)\subset E_k$ in $t=0,P_{a.e.w}\in\Omega$.

Proof Let $K(w)$ be a closed sphere with radius $R_1(w)$ in space E_k. According to embedding relation $E_k\subset E_0$, then $K(w)$ is a compact set in E_k. For arbitrary incremental random set $B_k(w)$,for $\forall\phi(t,\theta_{-t}w)\in B_k$,according lemma 3, have $\phi_2=\phi-\phi_1\in K(w)$, so for $\forall t\ge T_{B_k(w)}>0$,due to lemma 3, there is

$$d_{E_k}(S(t,\theta_{-t}w)B_k(\theta_{-t}w),K(w))=\inf_{\vartheta(t)\in K(w)}\|\phi(t,\theta_{-t}w)-\vartheta(t)\|_{E_k}^2$$
$$\le\|\phi_1(t,\theta_{-t}w)\|_{E_k}^2$$
$$\le e^{-2k_1t}\|\phi_{01}(t,\theta_{-t}w)\|_{E_k}^2\to 0,(t\to\infty).$$

The proof is complete.

According to Lemma 1-Lemma 4, there is the following theorem:

Theorem 1 The stochastic dynamical system $\{S(t,w),t\ge 0\}$ has a family of random attractor $A_k(w)\subset K(w)\subset E_k$, $w\in\Omega$, and there exists a slowly increasing random set $K(w)$, which makes $P_{a.e.w}\in\Omega$,

$$A_k(w)=\bigcap_{t\ge 0,\tau\ge t}\overline{\bigcup S(t,\theta_{-\tau}w,K(\theta_{-\tau}w))},$$

and $S(t,w)A_k(w)=A_k(\theta_t w)$.

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