Statistical Evaluation of Value at Risk Models for Estimating Agricultural Risk

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Abstract

This paper develops a skewness and leptokurtic modified VaR model with a mixture weight parameter that blends the Cornish-Fisher and EWMA methods. We estimate and evaluate five existing parametric VaR specifications using weekly returns for Canadian feedlot cattle feeding margin data and Maple Leaf Foods stock return data. The estimation of VaR based on EWMA method yields the most satisfactory results particularly for returns with positive skewness or leptokurtic tails. Meanwhile, the VaR forecasts obtained using the Cornish-Fisher method provides a relatively better tracking of the observed returns compared to the other methods, and therefore, has lower forecast error. Our proposed model allows users to determine the value of VaR based on their own risk preferences.

Keywords: Value-at-risk; exponential weighted moving average; GARCH models; the Cornish-Fisher expansion method; Skewness and Leptokurtic modified VaR model.

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1 Introduction

With the growth of global trading activity and financial instability, the measurement of market risk has become an important task in today`s business decision making. As a result, empirical research on reliable market risk measurement has recently received renewed interest and growing attention among researchers. One concept that is widely used in the financial sector to measure normal market risk is the Value-at-Risk (VaR) [13]. Many financial regulatory bodies such as the Derivative Policy Group, the Bank for International Settlement, the Basel Committee on Banking Supervision, and J.P. Morgan have proposed the use of VaR as a firm-wide measure of risk [5], [13], and several banks and other financial institutions have adopted VaR for risk management [9]. The popularity of the VaR models partially arises from their appealing features, which include, but are not limited to, the following: (1) VaR models provide a single quantitative measure of risk over a specified period by aggregating all components of market risk, and (2) VaR models measure risk directly in monetary (dollar) terms [13].

The Value-at-risk (VaR) measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence level. The approaches for VaR estimation are generally grouped as the full valuation method also known as non-parametric method, and the variance-covariance method also known as the parametric method. The full valuation method includes historical simulation, Monte Carlo simulation and bootstrap simulation. The variance-covariance method may assume different specifications depending on the choice of the conditional volatility forecasting model. For instance, VaR can be estimated using the exponential weighted moving average (EWMA) as suggested by J.P Morgan. Alternatively, VaR can be estimated based on GARCH models particularly when there is evidence of volatility clustering and leverage effect in the volatility [9]. Also, the normal variance-covariance approach can be extended using the Cornish-Fisher expansion method to control for the effect of skewness and kurtosis when the returns are not normally distributed [8]. Each method has its own advantages and disadvantages. However, the covariance-variance method is widely used in practice.

As in the financial sector, agricultural financial returns are inherently risky due to production and demand uncertainty, particularly following trade liberalization and other reforms [18]. Thus, the estimation of VaR is essential for efficient decision making in the agricultural sector. For instance, farm financial sectors want to know the level of market risk faced by borrowers; farm and agribusiness managers want to estimate market risk before investing and borrowing money; and policy makers need an accurate market risk measure to design farm income policies. Despite the risky nature of agricultural businesses, the use of VaR to measure market risk is limited. Relatively few studies have applied VaR in agriculture. [17] examined the relative performance of alternative VaR techniques – both full valuation and variance-covariance methods - and found that the results were satisfactory for all methods. But, VaR estimates based on the

EWMA method yield the most satisfactory results particularly for instruments that have linear payoff structures such as cash commodity prices. Similarly, [20] investigated the performance of historical simulation (HS), variance-covariance method (CV) and extreme value theory (EVT) in measuring risk in hog production. Their findings suggest the superiority of EVT over HS and CV particularly when there is evidence of leptokurtic tails. In the literature, the question of how to estimate VaR is subject to ongoing debate. The importance of the Cornish-Fisher expansion method and the asymmetric power GARCH for VaR estimation have not previously been investigated in the context of agricultural markets, to our knowledge.

In this study, we evaluate and compare a one-week-ahead forecast performance of alternative parametric VaR specifications, namely, VaR forecasts based on EWMA as proposed by the RiskMetrics group, symmetric GARCH models (both Gaussian and non-Gaussian GARCH), asymmetric power GARCH (APARCH) and the Cornish-Fisher expansion method. The paper begins by describing the concept of VaR. After presenting the most widely used conditional volatility estimators and the Cornish-Fisher expansion method for handling skewness and excess kurtosis, the paper then discusses the methods used for evaluating VaR estimates. Then we examine the empirical performance of alternative specifications of parametric VaR models. Finally, we propose a skewness and leptokurtic modified VaR with a mixture parameter that allows users to determine their risk preferences; and we present a summary and conclusions.

2 Description of Value-at-Risk

The Value-at-risk (VaR) measure uses a probabilistic approach to predict a possible financial loss of a portfolio over a given time horizon. Let r_t be the log-return of a portfolio over a given period of time, i.e. $r_t = \ln p_t - \ln p_{t-1}$ where p_t and p_{t-1} is portfolio value at time t and t-1. Then VaR for the next period can be estimated non-parametrically as the $q-$ percentage point of the lower tail of the distribution of r_t:

$$VaR_t^{(1-q)} = f_q ,$$
                                                                     (1)

where f_q is the $q-$ lower quantile of the distribution of r_t; $(1-q)$ is the confidence level used to construct VaR (95%, say). The non-parametric approach, also known as the full valuation method, for VaR estimation includes historical simulation, Monte Carlo simulation and the bootstrap technique. But, the parametric approach also known as the variance-covariance method is widely used

for VaR estimation. The variance-covariance method, which assumes a normal distribution of r_t is given by [3]:

$$q = P(r_t \leq VaR_t^{(1-q)}) = \int_{-\infty}^{VaR_t^{(1-q)}} 1/\sqrt{2\pi} \exp(-r_t^2/2)dr_t \quad . \tag{2}$$

Furthermore, when $r_t \sim N(0,1)$, follows a standard normal distribution, then equation (2) can be written as:

$$VaR_t^{(1-q)} = z_q \quad , \tag{3}$$

where z_q is the $(100 \times q)^{th}$ percentile of the standard normal distribution. For this distribution, for instance, the VaR measured at 95% confidence level will be -1.645, which is the negative quantile of the distribution, i.e. $VaR_t^{(95\%)} = -1.645$. Note that VaR is usually reported as a positive number. In this case, the interpretation will be, there is a 95% chance that the loss over the next period will not exceed CAN$1.645. Equation (3) can be expressed in a more general form as:

$$VaR_t^{(1-q)} = \bar{r}_{t+1} + z_q \hat{\sigma}_{t+1}, \tag{4}$$

where $\bar{r}_{t+1}$ is the forecasted mean of r_t and $\hat{\sigma}_{t+1}$ is the conditional volatility of r_t. $\bar{r}_{t+1}$ can be estimated using a simple moving average method or autoregressive (AR) models. In practice, the length of the rolling window ranges between 50 and 150 [17]. In this paper we calculated $\bar{r}_t$ by taking the mean of a rolling window of 150 observations.

3 VaR Estimation Methods

3.1 Conditional volatility and the Cornish-Fisher Expansion method

The conditional volatility $\hat{\sigma}_{t+1}$ in equation (4) can be estimated using one of the following popular volatility forecast models: exponentially weighted moving average, generalized autoregressive conditional heteroskedasticity under the assumption of both normal and t-distributions and asymmetric power GARCH. In an additional model, which we study here, z_q can be adjusted for skewness and excess kurtosis using the Cornish-Fisher expansion method when r_t is not normally distributed.

3.2 The Exponentially Weighted Moving Average (EWMA)

The RiskMetrics group suggests the use of the EWMA to forecast $\hat{\sigma}_{t+1}$. The EWMA is given by:

$$\hat{\sigma}_{t+1} = \sqrt{(1-\lambda)\sum_{i=1}^{T}\lambda^{i}r_{t+1-i}^{2}} \,, \tag{5}$$

where $\lambda \in (0,1)$ is the decay factor. Assuming an infinite number of observations, equation (5) can be re-written in a recursive form as:

$$\hat{\sigma}_{t+1} = \sqrt{\lambda\hat{\sigma}_{t}^{2} + (1-\lambda)r_{t}^{2}} \quad . \tag{6}$$

The decay factor λ determines the rate of decay, and gives weight to the volatility at time t; at the extremes, when $\lambda = 0$, $\hat{\sigma}_{t+1}$ depends entirely on the current estimate of return r_{t}, but when $\lambda = 1$, $\hat{\sigma}_{t+1}$ entirely depends on the volatility $\hat{\sigma}_{t}$. The theoretically optimal λ varies by asset class and frequency of the time series data. But, for almost all asset classes with daily data, $\lambda = 0.94$ is an optimal parameter, and for monthly data $\lambda = 0.97$. Thus, in practice the RiskMetrics uses one decay factor for all series: $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data [12]. The EWMA is often criticized as an inefficient and biased estimator [12]. We used both $\lambda = 0.94$ and $\lambda = 0.97$, but in this paper we reported only the results for $\lambda = 0.94$ because the results were more satisfactory.

3.3 Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

The GARCH (p,q) model defines $\hat{\sigma}_{t+1}$ as a function of squared error from past periods, e_{t+1-j}^{2} and conditional variance σ_{t-j}^{2} [1]. The GARCH (p,q) is given by:

$$r_{t} = \bar{r} + e_{t} \tag{7}$$

$$\hat{\sigma}_{t+1}^{2} = \alpha_{0} + \sum_{j=0}^{q}\alpha_{j}e_{t-j}^{2} + \sum_{i=0}^{p}\beta_{i}\hat{\sigma}_{t-i}^{2} \,, \tag{8}$$

$$e_{t} = \hat{\sigma}_{t}z_{t} \tag{9}$$

where $\bar{r}$ and e_{t} is the mean and error of the observed returns r_{t}, respectively, z_{t} are standardized residual returns (i.e. *iid* random variable with zero mean and variance one) and the other terms are as defined previously. The coefficients must be positive, i.e. $\alpha_{0} \geq 0, \alpha_{j} \geq 0$ and $\beta_{j} \geq 0$. The values of e_{t} and $\hat{\sigma}_{t}$ are known at time t. The GARCH model can be estimated using the maximum likelihood technique

under the assumption of both Gaussian and non-Gaussian e_t. The simple GARCH(1,1) model which often provides a good fit in empirical estimation is given as:

$$\hat{\sigma}_{t+1}^2 = \alpha_0 + \alpha_1 e_t^2 + \beta \hat{\sigma}_t^2 \tag{10}$$

The GARCH model is well recognized for capturing the effect of heavy-tailed returns and volatility clustering. These models are also useful when there is evidence of skewness and excess kurtosis.

3.4 Asymmetric Conditional Volatility Models

This class of models includes a number of extensions of GARCH models such as the Exponential GARCH model (EGARCH) proposed by [19], Integrated GARCH model (IGARCH) [6], Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) model [10], Nonlinear GARCH (NGARCH) model [14] and Non-linear Asymmetric GARCH [7] and others. These models are suggested as alternatives to the symmetric GARCH models when there is evidence of leverage effect in the volatility.

The Asymmetric Power ARCH (APARCH) model developed by [4] nests several of the most popular GARCH models, and provides more flexibility compared to the standard/symmetric GARCH models.

The APARCH(p,q) model is given as follows:

$$\sigma_{t+1}^\delta = \omega + \sum_{i=0}^q \alpha_i (|\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{i=0}^p \beta_i \sigma_{t-i}^\delta \ , \tag{11}$$

where $\delta \succ 0$ and $-1 \prec \gamma_i \prec 1$. The APARCH model is equivalent to the linear GARCH(p,q) model when $\delta = 2$ and $\gamma_i = 0$, the TS-GARCH(p,q) model when $\delta = 1$ and $\gamma_i = 0$, the NGARCH(p,q) model when $\delta = 1$ and $0 \leq \gamma_i \leq 1$. When $\delta \rightarrow 0$ and $\gamma_i = 0$ the model reduces to the log-GARCH(p,q). The model implies that when $\gamma = 0$ there is no leverage effect, when $\gamma \succ 0$ the leverage effect is as expected, i.e. volatility is higher for negative returns than positive returns of the same magnitude, and vice-versa when $\gamma \prec 0$.

3.5 The Cornish-Fisher Expansion Method (CFE)

Sometimes returns r_t exhibit asymmetry and heavy tails (i.e. skewness and excess kurtosis) which cannot be fully described by the GARCH/APARCH process alone. In such situations, the estimation of VaR using z_q results in

overestimation of the true risk. To avoid this bias, z_q is adjusted for the excess skewness and kurtosis by applying the Cornish-Fisher expansion method (1937) as follows (see [8] and [9]):

$$z_{q,cf} = z_q + (1/6)(z_q^2 - 1)s + (1/24)(z_q^3 - 3z_q)k - (1/36)(2z_q^3 - 5z_q)s^2 \tag{12}$$

where $z_{q,cf}$ is the modified critical value of the inverse of the cumulative standard normal distribution, s and k are skewness and the excess kurtosis of the empirical distribution, respectively.

4 Evaluation of VaR Estimates

To evaluate the VaR estimates given by the different models, we employed three different backtesting methods: the theoretical versus realized probability as suggested by the Basel Committee of Banking Supervision, the unconditional coverage test proposed by [15] and the conditional coverage test proposed by [2].

4.1 The Theoretical versus Realized Probability

The method proposed by the Basel Committee for assessing the predictive performance of VaR models recommend comparing the significance level q used for VaR estimation with the actual percentage of forecast violations of the model [11]. Violation occurs when VaR forecasts underestimate the actual loss, i.e. $VaR_q^t \prec r_{t+1}$. The Basel Committee theoretical versus realized probability method can be summarized as follows:

(1) Compute VaR_q^t for time t

(2) Compare VaR_q^t with the actual return r_{t+1} on time $t+1$. If $VaR_q^t \prec r_{t+1}$ then a violation occurs.

(3) The realized probability of violation p_v is calculated by dividing the number of violations V by the total number of the observations T, i.e.,
$$p_v = \frac{V}{T}.$$

(4) If a VaR model has p_v close to (the significance level) theoretical probability q, then it is said to be adequate.

4.2 The Unconditional Coverage Test

The unconditional coverage test proposed by [15] implies testing the null hypothesis that the true probability of a violation q*, say, equals the nominal VaR significance level q. We wish to test the null hypothesis, Ho: q*= q. The probability of observing V violations for a given sample size T is represented by a binomial process $(1-q)^{T-V} q^V$, under the null hypothesis and the likelihood ratio (LR) statistics for testing the null hypothesis is given by:

$$LR_{uc} = -2\ln[(1-q)^{T-V} q^V] + 2\ln[(1-V/T)^{T-V} (V/T)^V] \sim \chi_1^2 \quad .$$
(13)

The LR test statistic (13) has a chi-squared distribution with one degree of freedom, under the null hypothesis that q*=q. A rejection of the null hypothesis signifies that a given model is inadequate.

4.3 The Conditional Coverage Test

The conditional coverage test developed by [2] tests the null hypothesis that the occurrence of a violation at time $t+1$ is independent of the occurrence of a violation at time t. Christofferson`s LR test, which follows a chi-squared distribution with one degree of freedom under the null hypothesis that the violations are independent, is given by:

$$LR = 2(\ln L_a + \ln L_b) \sim \chi_1^2$$
$$L_a = (1-\pi_{01})^{T_{00}} (\pi_{01})^{T_{01}} (1-\pi_{11})^{T_{10}} (\pi_{11})^{T_{11}}$$
$$L_b = (1-\pi)^{T_{00}+T_{10}} \pi^{T_{01}+T_{11}}$$
$$\pi_{ij} = \frac{T_{ij}}{\sum_j T_{ij}}$$
$$\pi = (T_{01} + T_{11}) / (T_{00} + T_{01} + T_{10} + T_{11}),$$
(14)

where T_{ij} is the number of observations in state j in time t after having been in state i in the previous period $t-1$. Here i and j can assume only the values 1 and 0. If actual returns are less than the forecasted VaR, this is denoted as state 1, otherwise it is denoted as state 0. Therefore, under this test, a VaR model is considered to be inadequate if the violations are somehow interrelated.

5 Data Description

We implemented these methods using the most recent weekly data on cash prices for corn, fed cattle, feeder cattle, and cattle feeding margin for the period

from January 07, 2007 to June 29, 2012, and weekly stock prices for Maple Leaf for the period from December 14, 2000 to December 14, 2012. Apart from being the most recent, the period covered in our analysis includes the years 2007-2009, when volatility and prices were high for agricultural commodity in general, and corn in particular. Furthermore, we choose these variables because the cattle feeding industry is a very risky business due to high volatility in the prices of feeder cattle, feed grains and fed cattle. We also included Maple Leaf Foods stock returns because it is one of the largest agribusiness firms in Canada.

The cattle feeding margin is a common measure of profitability in the cattle feeding industry. Assuming a fixed feeding technology and other variable costs constant, it is defined as the revenue generated from the sale of fed cattle minus the costs incurred for feeder cattle and feeding. For an average efficient feedlot, the average weight for feeder cattle is 1,100 pounds and 650 pounds for fed cattle, and an average of 45 bushel of corn is consumed [17]. Therefore, the cattle feeding margin is given as:

$$cfm = (11 * p_{fed}) - (6.5 * p_{feeder}) - (45 * p_{corn}) \qquad (15)$$

where cfm is the cattle feeding margin (CAD\$/head), p_{fed} and p_{feeder} are the price of fed (CAD\$/cwt) and feeder cattle (CAD\$/cwt) respectively, and p_{corn} is the price of corn (CAD\$/bushel). The data were obtained from CanFax (http://www.canfax.ca).

Table 1 gives summary statistics of the returns of the variables considered. Overall, the results suggest that the returns are not normally distributed, but are characterized by skewness and positive excess kurtosis/leptokurtosis. The Jarque-Bera (JB) normality test suggests that the null hypothesis of normally distributed returns is rejected for all variables at 99 per cent confidence level.

Table 1: Summary statistics of the returns

Prices	Corn	Cattle fed	Cattle feeder	Margin	Maple Leaf
Mean	0.007	-0.002	-0.000	0.250	0.000
Minimum	-2.079	-4.812	-5.924	-13.360	-0.092
Maximum	1.792	5.897	5.130	42.410	0.067
Volatility	0.303	0.856	0.966	3.444	0.014
Skewness	-0.429	0.603	0.027	9.596	-0.391
Kurtosis	18.460	21.860	2.113	118.920	5.235
JB test	3908.200*	5822.700*	2305.900*	221481.700*	738.130*

5.1 Testing for Autoregressive Conditional Heteroscedasticity (ARCH) Effect

Another important statistical property of returns is to investigate the residuals for evidence of heteroscedasticity, also known as the ARCH/GARCH effect. In particular, characterizing the nature of the volatility of the returns, i.e. testing for the existence of volatility clustering and leverage effects, is essential for selecting an appropriate volatility forecasting model, which in turn can potentially improve VaR estimation. To do this, we employed the APARCH model of [4], which nests several of the most popular GARCH models and is more flexible compared to the other GARCH form. The parameters are estimated using the maximum likelihood procedure.

The APARCH results are given in Table 2. The results show the ARCH coefficients $\hat{\alpha}$ are significant at the 5 per cent level for all variables including Maple Leaf stock returns, suggesting that the conditional volatility is influenced by the previous period`s squared disturbance/error. Similarly, the GARCH coefficients $\hat{\beta}$ are significant in all cases, indicating that the conditional volatility is determined by the magnitude of the previous period`s conditional volatility, which suggests the existence of volatility clustering. Furthermore, $\hat{\alpha} + \hat{\beta}$ is very close to one for feeder cattle returns and margin returns, which implies that volatility shocks are persistent for these variables, i.e., a one time volatility shock is more likely to endure for longer periods for these two variables, which in turn implies longer period of risk exposure. The leverage effect coefficients $\hat{\gamma}$ are significantly different from zero for fed cattle returns, and therefore, for this variable, conditional volatility is higher when there are negative shocks (negative returns) than positive shocks (positive returns) of the same magnitude. In other words, the risk is higher for downstream (when prices are decreasing) than upstream (when prices are increasing) for fed cattle returns.

The results also show two useful diagnostic tests. The Ljung-Box test for squared residuals is insignificant in all cases, which suggest that a Gaussian GARCH model fits the data well. The Lagrange multiplier test for ARCH (LM-ARCH) is insignificant in all cases, and therefore, the model fits the data well since there is no ARCH effect remaining in the residuals of the mod.

6 Results

For illustrative purposes, we estimated a one-week VaR for the Canadian cattle feeding margin using the variance-covariance, historical simulation, Monte Carlo simulation and bootstrap methods for a variety of confidence levels. The results are reported in Table 3. The VaR estimates given by the different approaches show some differences, and the difference becomes wider as the confidence level increases. For instance, the VaR estimate given by the normal

variance-covariance method at 5% is 2.6, which suggest that there is 0.05 probability that the margin return will fall in value by more than CAN\$ 2.6 per head over a period of a week. There are numerous empirical studies which have examined the relative performance/accuracy of these approaches. Several of these empirical studies suggest that no single approach is dominantly superior to the other methods (see, for instance [13]). Furthermore, each method has its own advantages and disadvantages, as discussed earlier.

Table 2: Results of APARCH(1,1) model

Parameter	Corn	Fed	Feeder	Margin	Maple Leaf
Mean Equation					
$\hat{\mu}$	-0.010	-0.021	-0.002	0.231***	0.001
Variance Equation					
$\hat{\omega}$	0.003	0.082***	0.040*	0.000	0.000
$\hat{\alpha}$	0.459***	0.200**	0.703***	0.603***	0.067**
$\hat{\gamma}$	0.078	1.000***	0.006	-0.415(na)	0.101
$\hat{\beta}$	0.687***	0.570***	0.390***	0.397***	0.193***
$\hat{\delta}$	1.251	2.000* **	2.000*	0.005(na)	1.188**
Diagnostic Tests					
Ljung-Box Test	13.470	14.040	25.730	7.413	104.033
LM ARCH Test	5.930	0.913	0.190	0.622	6.286

Note: ***, ** and * denote significance at the 0.1%, 1% and 5% levels, respectively.

Table 3: VaR for the Canadian cattle feeding margin using alternative techniques

Type of Method	$q = 1\%$	$q = 5\%$	$q = 10\%$
Normal variance-covariance method	7.980	2.600	1.390
Historical simulation method	15.970	3.180	1.830
Monte Carlo simulation method	7.397	2.386	1.282
Bootstrapping method	13.700	2.828	2.048

However, the focus of this paper is on the empirical performance of alternative specifications of parametric VaR, namely: EWMA-based VaR (VaR_{ewma}), GARCH-based VaR both under the assumption of normal (VaR_{garch}) and t-distribution (VaR_{tgarch}), APARCH-based VaR (VaR_{aparch}) and CF-based VaR (VaR_{cf}). Table 4 reports the results for one-week ahead VaR forecasts at the 99% confidence level for corn, fed cattle, feeder cattle and cattle feeding margin returns. The first column gives the name of the variables; the second column shows the method used for VaR estimation and columns 3-5 shows the three different backtesting methods. For corn, both the percentage of violations and Kupiec`s LR test suggest that all the models are adequate and have roughly equal forecast performance, except for the VaR_{CF} model.

However, a further investigation among the adequate models using Chistoffersen`s LR test suggests that the null hypothesis that the violations are independent can be rejected for all models, except for the VaR_{ewma} model. Thus, the VaR_{ewma} model shows the most satisfactory performance. By following the same argument, the VaR_{garch}, VaR_{aparch} and VaR_{ewma} models show relatively the most satisfactory performance for fed cattle, feeder cattle and margin returns respectively. Particularly, in the case of margin returns, which are characterized by a very large positive kurtosis, Kupiec`s LR test suggests that only the VaR_{ewma} and VaR_{garch} models are adequate, suggesting the superiority of these models in capturing leptokurtic tails compared to the other models. On the other hand, the VaR_{CF} model exhibits the most unsatisfactory result. The VaR_{ewma} also shows the most satisfactory result for the Maple Leaf stock returns.

Table 4: Backtesting a 99 % one-week VaR forecasts.

Variable	VaR Method	% of Violation	Kupiec`s LR Test	Christoffersen`s LR Test
Corn	VaR_{ewma}	**0.017**	**0.967(0.33)**	**0.14(0.71)**
	VaR_{garch}	0.017	0.967(0.33)	3.99(0.04)*
	VaR_{tgarch}	0.017	0.967(0.33)	3.99(0.04)*
	VaR_{aparch}	0.017	0.967(0.33)	3.99(0.04)*
	VaR_{CF}	0.042	13.920(0.00)*	0.62(0.62)
Fed cattle	VaR_{ewma}	0.000	1.750(0.19)	0.078(0.78)
	VaR_{garch}	**0.000**	**0.690(0.41)**	**0.139(0.71)**
	VaR_{tgarch}	0.043	3.500(0.06)***	0.139(0.71)
	VaR_{aparch}	0.043	1.750(0.19)	0.034(0.64)
	VaR_{CF}	0.000	6.310(0.01)**	0.139(0.71)

Feeder cattle	VaR_{ewma}	0.013	0.167(0.68)	0.000(1.00)
	VaR_{garch}	0.017	0.967(0.33)	0.009(0.93)
	VaR_{tgarch}	0.017	0.967(0.33)	0.009(0.93)
	VaR_{aparch}	**0.009**	**0.080(0.81)**	**0.000(1.00)**
	VaR_{CF}	0.017	0.967(0.33)	0.000(1.00)
Margin	VaR_{ewma}	**0.009**	**0.056(0.40)**	**0.03(0.85)**
	VaR_{garch}	0.043	0.999(0.31)	0.009(0.92)
	VaR_{tgarch}	0.000	4.720(0.03)**	0.00(1.00)
	VaR_{aparch}	0.000	4.720(0.03)**	0.00(1.00)
	VaR_{CF}	0.490	72.000(0.00)*	119(0.00)*
Maple Leaf	VaR_{ewma}	**0.0252**	**7.420(0.006)**	**0.43(0.62)**
	VaR_{garch}	0.0210	10.670(0.00)*	1.67(0.20)
	VaR_{tgarch}	0.0210	10.670(0.00)*	1.67(0.20)
	VaR_{aparch}	0.0210	10.670(0.00)*	1.67(0.20)
	VaR_{CF}	0.0399	1.091(0.30)	12.54(0.00)*

Notes: Values in brackets are p-values; the number shown in bold indicate models which are relatively adequate; and the asterisk *,** and *** reject the null hypothesis of adequate model at 1%, 5% and 10% significance level.

Table 5 reports the results for VaR forecasts using 97.5% confidence level. For corn, both Kupiec`s LR test and Christoffersen`s LR test indicate that all the models are adequate, except for VaR_{tgarch}. The VaR_{ewma} and VaR_{garch} models have roughly equal forecast performance. In terms of the relative forecast performance of the models the VaR_{CF} outperformed the other models given its large p-value. For fed cattle, the VaR_{ewma} and VaR_{aparch} show the most satisfactory results. In the case of feeder cattle, all the models are adequate. The VaR_{garch} model has the largest p-value both for Kupiec`s LR test and Christoffersen`s LR test, suggesting that VaR_{garch} model has the most satisfactory performance. As before, only the VaR_{ewma} and VaR_{garch} models are adequate, and the VaR_{ewma} shows the most satisfactory result for margin returns. Similarly, the VaR_{ewma} also shows the most satisfactory result for the Maple Leaf stock returns.

Table 6 reports the results for VaR forecasts using a 95% confidence level. The VaR_{CF} model outperforms the other models in the case of corn, and the VaR_{garch} and VaR_{tgarch} models are inadequate. All the models are adequate in the case of fed cattle, but the VaR_{ewma} model yields the most satisfactory result. For feeder cattle, the VaR_{ewma}, VaR_{tgarch} and VaR_{CF} models have roughly equal

performance, and the VaR_{garch} and VaR_{aparch} models show the most unsatisfactory result. Again, all of the models are inadequate in the case of margin, except the VaR_{ewma} which is adequate at 1% significance level, which suggest that the VaR models yield poor results when there is excess positive kurtosis or leptokurtic tails. For Maple Leaf stock, VaR_{ewma} shows the most satisfactory result.

Table 5: Backtesting a 97.5 % one-week VaR forecasts

Variable	VaR Method	% of Violation	Kupiec`s LR Test	Christoffersen`s LR Test
Corn	VaR_{ewma}	0.038	1.470(0.23)	0.920(0.34)
	VaR_{garch}	0.038	1.470(0.23)	0.920(0.34)
	VaR_{tgarch}	0.043	3.030(0.08)***	0.620(0.43)
	VaR_{aparch}	0.034	2.460(0.11)	1.280(0.26)
	VaR_{CF}	*0.055*	*0.710(0.40)*	*0.120(0.74)*
Fed cattle	VaR_{ewma}	*0.013*	*1.750(0.19)*	*0.080(0.78)*
	VaR_{garch}	0.017	0.690(0.41)	3.990(0.04)*
	VaR_{tgarch}	0.001	3.50(0.06)***	0.034(0.85)
	VaR_{aparch}	*0.013*	*1.750(0.19)*	*0.080(0.78)*
	VaR_{CF}	0.004	6.310(0.01)**	0.008(0.92)
Feeder cattle	VaR_{ewma}	0.021	0.140(0.71)	0.220(0.64)
	VaR_{garch}	*0.021*	*0.140(0.71)*	*0.120(0.74)*
	VaR_{tgarch}	0.017	0.690(0.41)	0.140(0.71)
	VaR_{aparch}	0.021	0.140(0.71)	0.220(0.64)
	VaR_{CF}	0.021	0.140(0.71)	0.220(0.64)
Margin	VaR_{ewma}	*0.017*	*0.690(0.40)*	*0.140(0.71)*
	VaR_{garch}	0.013	1.750(0.19)	0.080(0.78)
	VaR_{tgarch}	0.000	11.890(0.00)*	0.000(1.00)
	VaR_{aparch}	0.000	11.890(0.00)*	0.000(1.00)
	VaR_{CF}	0.350	32.000(0.00)*	21.720(0.00)*
Maple Leaf	VaR_{ewma}	*0.042*	*0.670(0.41)*	*0.030(0.86)*
	VaR_{garch}	0.032	3.920(0.04)**	0.480(0.49)
	VaR_{tgarch}	0.032	3.920(0.05)***	0.480(0.49)
	VaR_{aparch}	0.034	3.920(0.05)***	0.340(0.55)
	VaR_{CF}	0.057	0.430(0.51)	12.920(0.00)*

Notes: Values in brackets are p-values; the number shown in bold and indicate models which are relatively adequate; and the asterisk *,** and *** reject the null hypothesis of adequate model at 1%, 5% and 10% significance level.

Overall, our results show that the estimation of VaR based on EWMA method yields the most satisfactory results particularly where there is evidence of positive kurtosis or leptokurtic tails. This is because the EWMA model overestimate actual losses compared to the other alternative models (i.e. it is conservative). Meanwhile, the VaR forecasts obtained using the Cornish-Fisher method provide a relatively better tracking of the actual losses as compared to the other methods (see for example Figure 1), and therefore, has lower forecast error, a desirable property of a good forecast model, which is not shown by the violation criteria. The results for the GARCH-based VaR forecasts (VaR_{garch}, VaR_{tgarch} and VaR_{aparch}) are relatively close to the VaR_{ewma} results as opposed to the VaR_{CF}.

Table 6: Backtesting a 95% one-week VaR forecasts.

Variable	VaR Method	% of Violation	Kupiec`s LR Test	Christoffersen`s LR Test
Corn	VaR_{ewma}	0.064	0.870(0.35)	0.002(0.97)
	VaR_{garch}	0.077	3.030(0.08)***	0.140(0.71)
	VaR_{tgarch}	0.077	3.030(0.08)***	0.140(0.71)
	VaR_{aparch}	0.068	1.460(0.23)	0.010(0.92)
	VaR_{CF}	*0.055*	**0.140(0.71)**	**0.110(0.74)**
Fed cattle	VaR_{ewma}	*0.055*	*0.006 (0.94)*	*2.310(0.13)*
	VaR_{garch}	0.043	0.290(0.59)	3.590(0.06)
	VaR_{tgarch}	0.038	0.730(0.39)	0.920(0.34)
	VaR_{aparch}	0.034	1.410(0.23)	1.280(0.25)
	VaR_{CF}	0.038	0.730(0.39)	0.730(0.39)
Feeder cattle	VaR_{ewma}	*0.030*	**2.350(0.13)**	**0.430(0.51)**
	VaR_{garch}	0.026	3.580(0.06)***	0.320(0.57)
	VaR_{tgarch}	0.030	*2.350(0.13)*	0.430(0.51)
	VaR_{aparch}	0.026	3.580(0.06)***	0.320(0.57)
	VaR_{CF}	0.030	**2.350(0.13)**	0.430(0.51)
Margin	VaR_{ewma}	*0.021*	**5.16(0.02)***	**0.080(0.78)**
	VaR_{garch}	0.013	9.65(0.002)*	3.040(0.08)*
	VaR_{tgarch}	0.000	24.11(0.00)*	0.000(1.00)
	VaR_{aparch}	0.000	24.11(0.00)*	0.000(1.00)
	VaR_{CF}	0.0043	17.08(0.00)*	0.009(0.96)
Maple Leaf	VaR_{ewma}	0.0651	2.100(0.15)	12.780(0.00)*
	VaR_{garch}	0.0609	1.120(0.29)	14.860(0.00)*
	VaR_{tgarch}	*0.0567*	**2.510(0.43)**	**12.920(0.00)***
	VaR_{aparch}	0.0609	1.120(0.29)	14.860(0.00)*
	VaR_{CF}	0.0778	6.640(0.01)	13.970(0.00)*

Notes: Values in brackets are p-values; the number shown in bold and indicate models which are relatively adequate; and the asterisk *,** and *** reject the null hypothesis of adequate model at 1%, 5% and 10% significance level.
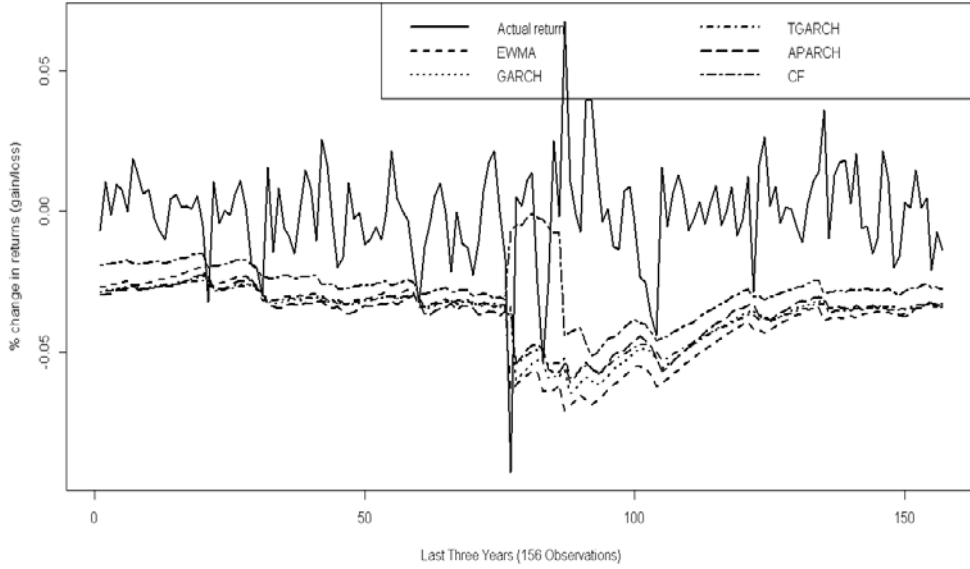

Figure 1: Actual returns versus a 99% one-week VaR forecasts for
Maple Leaf stock returns.

The above results suggest a tradeoff between lower forecast error and the expected percentage of violations. Given the tradeoffs between accuracy of VaR forecast and VaR violation, we propose a Skewness and Leptokurtic modified VaR model (henceforth called SL-modified VaR) with a mixture weight parameter that blends the Cornish-Fisher method and the EWMA model. The SL-modified VaR is given by:

$$VaR_{t,sl}^{(1-q)} = \omega(VaR_{t,ewma}) + (1-\omega)VaR_{t,cf} \,. \tag{16}$$

Equation (16) can be expressed as:

$$VaR_{t,sl}^{(1-q)} = \omega(\bar{r} + \hat{\sigma}_t z_q) + (1-\omega)(\bar{r} + \hat{\sigma}_t z_{q.cf}) \,, \tag{17}$$

which, in turn, can be written as follows:

$$VaR_{t,sl}^{(1-q)} = \bar{r} + \hat{\sigma}_t(\omega z_q + (1-\omega)z_{q.cf}) \,, \tag{18}$$

In all three equations, $VaR_{t,sl}^{(1-q)}$ is the SL-modified VaR, $\hat{\sigma}_t$ is the conditional volatility estimated using EWMA, z_q is the $(100*q)^{th}$ percentile of the standard normal distribution, $z_{q,cf}$ is the skewness and leptokurtic modified critical value of the inverse of the cumulative standard normal distribution, $\omega \in (0,1)$ is the mixture weight parameter which is determined by the users based on their risk preferences. At the extremes, when $\omega = 0$ the SL-modified VaR is equivalent to CF-based VaR, i.e., $VaR_{t,sl}^{(1-q)} = VaR_{t,cf}^{(1-q)}$, but when $\omega = 1$ the SL-modified VaR is equivalent to EWMA-based VaR, i.e., $VaR_{t,sl}^{(1-q)} = VaR_{t,ewma}^{(1-q)}$.

Alternatively, the value of ω can be determined in such a way as to penalize the VaR models for their forecast error and percentage of violations:

$$\omega = \frac{v_{cf} + \varepsilon_z}{(v_{cf} + v_z) + (\varepsilon_{cf} + e_z)}, \tag{19}$$

where v_{cf} and v_z are the number of violations for the CF-based VaR model and the EWMA-based VaR, respectively, ε_{cf} and ε_z are forecast errors for CF-based VaR model and the EWMA-based VaR, respectively. The forecast error is given by:

$$\varepsilon_t = g(r_t - VaR_{t-1}), \tag{20}$$

where r_t is an actual return on week t, VaR_{t-1} is the VaR forecast on week $t-1$ and $g(\cdot)$ is an error function. Although there are different forecast error functions $g(\cdot)$, the Root Mean Square Forecast Error (RMSFE) is widely used in empirical analysis, given its desirable properties. A thorough discussion on the different forms of forecast error functions is provided in [22] and [23]. The RMSFE is given by:

$$\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - VaR_{t-1})^2} \quad , \tag{21}$$

where T is the number of observations in the data set used to compute the forecast error. Figure 2 shows the results for CF-based VaR, EWMA-based VaR and SL-modified VaR using different values (0.2, 0.5 and 0.8) for the mixture parameter ω. As shown in the figure, SL-modified VaR forecasts fall between VaR_{CF} and VaR_{ewma}.
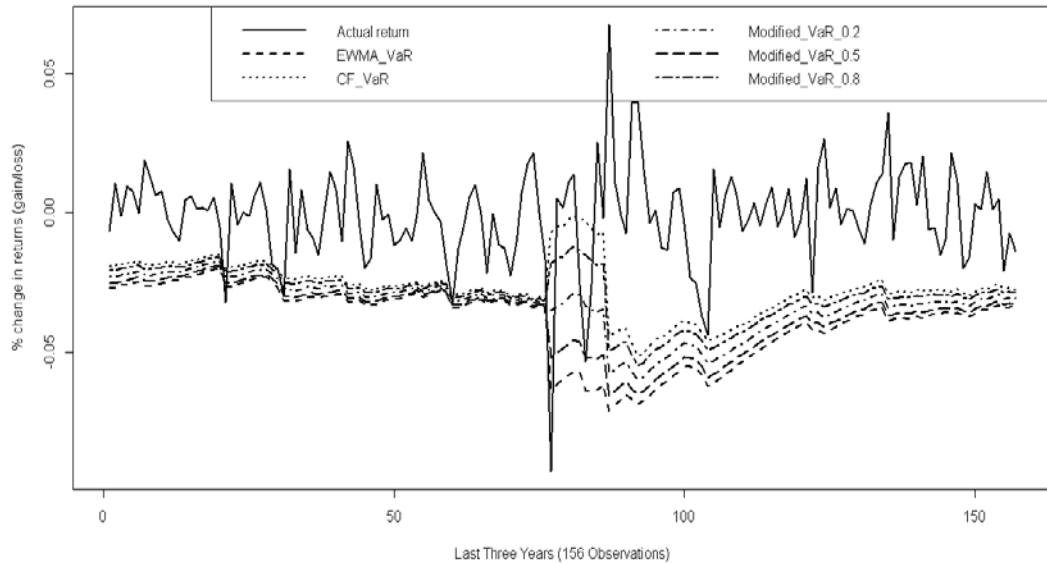
Figure 2: Actual returns and VaR forecasts obtained from EWMA, CF and SL
modified VaR for Maple Leaf stock returns.

7 Summary and Conclusions

With the increasing pressure towards the deregulation and liberalization of
agricultural business, the need for evaluating the exposure to market risks, which
arises from variations in prices of commodities, exchange rates and interest rates,
has become very important. One concept that is widely used in the financial sector
to measure normal market risk is Value-at-Risk (VaR). VaR measures the
potential loss in value of a risky asset or portfolio over a defined period for a given
confidence interval. Despite the risky nature of agricultural businesses, the use of
VaR to measure market risk in this area has been limited.

In this paper, we consider novel applications of this risk measure in
agribusiness,   with evidence from the analysis of the degree of risk exposure of
feedlot industry in the Canadian province of Ontario and stock price of a major
Canadian publicly traded food processing company. We also propose a novel
mixture VaR model that allows for variations in firm managers' "risk
preferences".

We investigated the empirical performance of alternative VaR estimation
techniques to measure the agricultural market risk. We measured VaR using full-
valuation methods including historical simulation, Monte Carlo simulation and the
bootstrap technique; we also studied parametric methods. The VaR estimates

given by the different approaches show some differences, and the difference becomes wider as the confidence level increases.

The parametric method is widely used for measuring market risk in practice. Thus, we examined a one-week ahead forecast performance of five alternative specifications of parametric methods of VaR estimation: EWMA, GARCH under the assumption of normal and t-distributions, APARCH and the Cornish-Fisher expansion methods. The GARCH model is well recognized for modeling time-varying volatility and useful when there is volatility clustering. Similarly, the APARCH model is a suitable model when there is a leverage effect on volatility. This is important because many financial data show volatility clustering and leverage effects. The Cornish-Fisher expansion method is used to correct for the effect of skewness and excess kurtosis which is a common phenomenon in many financial data.

The VaR can be measured using the changes in the portfolio value, or profits and losses. We implemented these methods using weekly data on cash prices for corn, fed cattle, feeder cattle, and cattle feeding margin for the period from January 07, 2007 to June 29, 2012, and weekly stock prices for Maple Leaf Foods for the period from December 14, 2000 to December 14, 2012. We choose these variables because the cattle feeding industry is a very risky business due to high volatility in the prices of major inputs (feeder cattle and feed grains) and final product (fed cattle) [21]. We also used returns on stock prices of Maple Leaf Foods – one of the largest agribusiness firms in Canada. Three backtesting methods were used to evaluate the forecast performance based on the violation criteria: 1) the theoretical versus the realized probability as suggested by the Basel Committee of Banking Supervision, 2) the unconditional coverage test proposed by [16] and 3) the conditional coverage test proposed by [2].

Our empirical findings show that the estimation of VaR based on the EWMA method yields the most satisfactory results despite the evidences for the presence of volatility clustering, leverage effect, skewness and excess positive kurtosis/ leptokurtic tails. This is contrary to our expectation that GARCH-based VaR models would yield better results in the presence of volatility clustering and leverage effect compared to the other models. Similarly, one would expect that in the presence of skewness and excess positive kurtosis/ leptokurtic tails the CF-based VaR would improve estimation. The reason for the superiority of VaR forecasts based on EWMA model is that this method overestimates actual losses compared to the estimates obtained using the other methods (i.e. the EWMA model is conservative). The results for the GARCH-based VaR estimates (VaR_{garch}, VaR_{tgarch} and VaR_{aparch}) are only slightly lower than the VaR_{ewma} results. Meanwhile, the VaR estimates obtained using the Cornish-Fisher method provide a relatively better tracking of the actual losses as compared to the other methods, and therefore, have lower forecast error. The above results suggest a tradeoff between lower forecast error and the expected percentage of violation. Given the tradeoffs between accuracy of VaR forecast and VaR violation, we propose a skewness and

leptokurtic modified VaR model with a mixture weight parameter ω that blends the Cornish-Fisher based VaR model and the EWMA based VaR model. The mixture weight parameter ω can be determined by the user based on their risk preferences. Alternatively, the value of mixture weight parameter ω can be calculated in such a way as to penalize the Cornish-Fisher based VaR model and the EWMA based VaR model for their forecast error and percentage of violation.

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