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Modelling Approaches for Expected Credit Losses

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Abstract

Our aim is to present two relatively simple and applicable methodologies for the consistent estimation of credit risk throughout the remaining life time of an obligor.

The first methodology relies on the cash flows expected form the obligors according to his loan schedules.

- The obligor is included in an expected non-payment evolution scheme according to his IFRS 9 – stage classification
- Subsequent adjustments are performed on the assumed non-payment frequency evolution (point-in-time), involving macro and micro characteristics, in order to arrive at the part of each scheduled cash flow that is expected not to be paid

The specific approach allows us to estimate obligor lifetime ECL and price each obligor credit risk component at the initiation of the loan granting process.

The second methodology is simpler but more intuitive as it offers a natural extension of the already implemented Basel risk parameters. Specifically, it relies on the concepts of

- PD through the economic cycle
- Worst period LGD in the economic cycle

It is established that the use of already calculated Basel risk parameters allows for a reconciliation to the IFRS 9 framework.

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1 Cash Flow Adjustment approach

Overview:

- The concept of "bad" obligor state is defined
- The probability of an obligors' transition into a state that involves even partial non- payment of a future obligation, is assumed to follow a specific process, initially on a portfolio level
- The portfolio-level process parameters are linked to macro variables
- The portfolio-level process parameters are additionally adjusted to reflect individual obligor differences
- The obligor-level curve is reduced to replicate the expected non-payment percent at each future time step

The specific modelling process

- Although it considers an expanding tree to describe obligor transition into states, it avoids the inherent computational complexity of the expanding tree by using combinatorics only to adjust for final obligor states, instead of calculating and assigning transition probabilities to every specific path
- incorporates all available past, current and projection data in a straightforward manner
- establishes obligor individual treatment
- accommodates any cash flow scheme
- eliminates the need for macro variable projection only to be included discount rates and collateral values (current values can be used as an equally base case scenario)

1.1 Visual Overview

With the assumptions expanded in sections 1.3.1, 1.3.2 the general probability curve shapes for the three stages of obligors, as seen in current time $t = 0$, are:

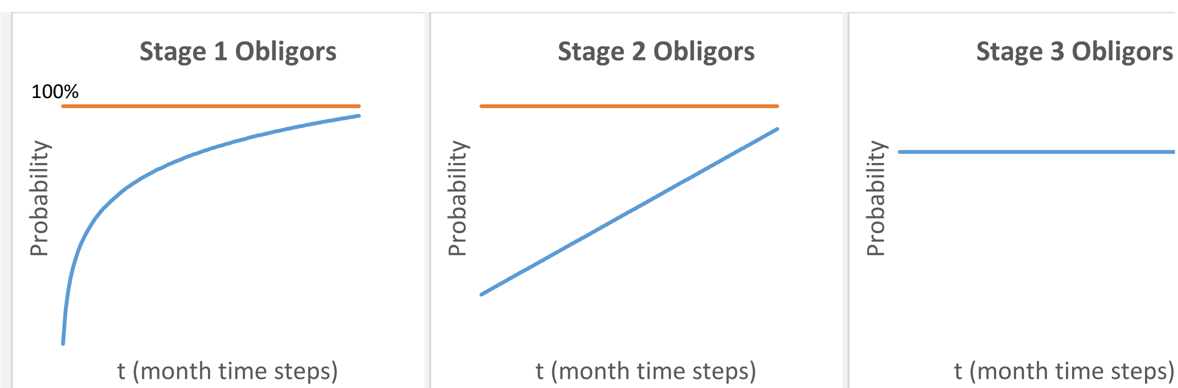

Figure 1: Assumed non-payment schemes for IFRS 9 staged obligors

The calculation process followed throughout sections 1.3.2-1.3.6 is depicted visually (stage 1 obligors curve is used as example)

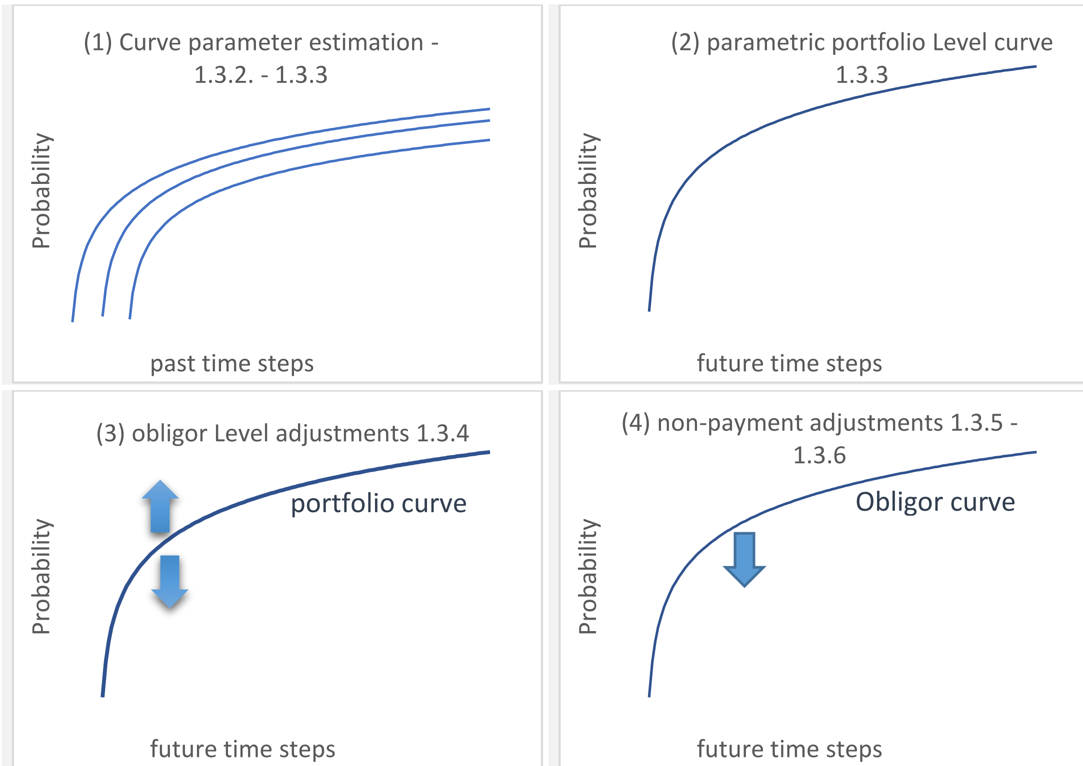

Figure 2: Non-payment curve adjustments for IFRS 9 - stage 1 obligors

1.2 Supporting data and related modelling issues

For the successful implementation of the proposed methodology the following infrastructure is required:

1. Past default data for the financial institution, preferably covering periods of economic growth and economic downturn
2. Macroeconomic variables to be used in curve parameter estimation (easy to acquire online from National Statistic Authorities)
3. A model that scores obligor behavior towards the financial institution (behavioral model section 1.3.4)
4. A model that scores obligor financial statements (financial model section 1.3.4)
5. Past due data for collateral values
6. Cost of capital approximation

Nearly all of the above certain to exist in an organization implementing the Basel Framework.

Especially for the application of 1.3.6 segment, we need to either:

- Construct collateral value and interest rate models according to different scenarios, using the available macroeconomic data
- Consider those values static, equal to current values
- Use business or expert estimations

1.3 Implementation Procedure

1.3.1 Stages classification

We define the following states for an obligor:

$E =$ Accruing obligor with no past due payments

$D =$ Obligor with past due payments of any amount, for a time period of $\geq$ 1 day

$N =$ Non-Accruing obligor, legal processes have or will be initiated on behalf of the credit institution for the collection of the remaining amounts (the irreversible state)

The mapping to IFRS 9 stages is as following:

IFRS 9 Stage	Mapped state
1	E
2	D
3	N

1.3.2 Non-payment transition curve on portfolio segment level

1.3.2.1 Stage 1 obligors

We measure an average transition percentage curve for stage 1 obligors at initiation$(t = 0)$, towards stages 2 and 3 at later times$(t > 0)$, which is a function of time. Specifically:

$$NP(t) : E_0 \rightarrow \{D_t, N_t\} \quad t \in \mathbb{N}^* \quad NP(t) \in [0,1] \quad t > 0$$

$E_0 =$ Accruing obligor at current time 0

$t =$ Month step

The shape of portfolio NP curve reflects the uncertainty for the future payments of the obligor.

The following property should hold:

$$\frac{\partial NP(t)}{\partial t} > 0$$

That is, total uncertainty should be increasing with time.

If we consider the survivorship effect, the phenomenon of diminishing non-payment probability due to the fact that better obligors of the portfolio remain in state E as the worse obligors transfer to states $\{D, N\}$, we should expect a diminishing rate of uncertainty, which may be translated as:

$$\frac{\partial^2 NP(t)}{\partial t^2} < 0$$

A logical choice, incorporating both of the afore-mentioned properties, for the non-payment of the accruing part of the portfolio E, curve would be:

$$NP(t)^E = a^E + b^E \cdot ln(t) \quad a^E, b^E \in [0,1]$$

Rewrite the bounded curve equation as:

$$NP(t)^E = min\{a^E + b^E \cdot ln(t), 1\}$$

1.3.2.2 Stage 2 obligors

$$NP(t) : D_0 \to \{D_t, N_t\} \quad t \in \mathbb{N}^* \quad NP(t) \in [0,1] \quad t \geq 0$$

For the portfolio part of stage 2 obligors, we consider the survivorship effect almost negligible, since the obligors initially included are already 'bad'

$$\frac{\partial^2 NP(t)}{\partial t^2} \to 0$$

As a result, the non-payment bounded curve would have a fixed rate of growing uncertainty

$$NP(t)^D = min\{a^D + b^D \cdot t, 1\} \quad a^D, b^D \in [0,1]$$

It would be better if we also distinguished past due payments of previous months (if there are any), as:

$$NP(t)^D = \begin{cases} min\{a^D + b^D \cdot t, 1\} & t \geq 0 \\ 1 & t < 0 \end{cases} \quad a^D, b^D \in [0,1]$$ [2]

1.3.2.3 Stage 3 obligors

It is obvious that for the non-accruing part of the portfolio

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[2] With $t < 0$ we mean past due exposures over a month past due, while past due exposures of the current month $(t = 0)$ are handled by the linear function

$$NP(t)^N = 1$$

1.3.3 Incorporating macro factors into portfolio segment non-payment curve

Using past periods data, we could fit stage 1 and 2 non-payment curves of the portfolio, at various date snapshots, obtaining a time series of calculated curve parameters.

$$a_{t-k}^E, b_{t-k}^E, a_{t-k}^D, b_{t-k}^D$$

$m = m_i(t-1, \dots., -\infty) =$ Available lagged macro variables at time t

$i = 1, \dots, I$ Number of macro variables

$k =$ Time lags from current time step

Logistic regressions (or any other methodology that restricts the output in $[0,1]$ $interval$) could be used to estimate the curve coefficients $a_t^E, b_t^E, a_t^D, b_t^D$ using $m_{i,t-k}$ as explanatory variables. As a result, non-payment transition curve approximations become:

$$NP(t)^E = min\{a^E(m) + b^E(m) \cdot ln(t), 1\}$$

$$NP(t)^D = \begin{cases} min\{a^D(m) + b^D(m) \cdot t, 1\} & t \geq 0 \\ 1 & t < 0 \end{cases}$$

1.3.4 Individual non-payment transition probability curve

In order to be used at obligor level, portfolio segment curves should be adjusted with the use of:

- Obligor behavior information
- Obligor financial information

The above-mentioned information will be adequately captured with the use of two model scores

- A behavioral model that will take as input the behavior of the obligor throughout his relationship with the financial institution
- A financial model that will use elements of the obligor’s latest available financial statements. The seniority of the financial statements should also be accounted for

Both models should be calibrated with the use of a “bad” outcome as the independent variable (e.g. obligor default in 12 months’ time by Basel definition. Any other bad outcome could be used instead).

The role of the two model scores is to provide complete information about the obligor in a condensed manner. This information will be utilized into the following modelling approach, in order to obtain the obligor individual deviation from the portfolio curve.

If:

$j =$ Obligor

$B_j =$ Obligor behavioral model score at current time $t = 0$

$F_j =$ Obligor financial model score at current time $t = 0$

$B, F \in [0,1]$

The total non-payment probability for the obligor at each month (t), as represented by the portfolio segment non-payment curve, $NP(t)_j$, is comprised of an individual $\Delta NP(t)_j$, and a portfolio average component $NP(t)_{p/f}$:

$$NP(t)_j = NP(t)_{p/f} + \Delta NP_j \Rightarrow \Delta NP_j = NP(t)_j - NP(t)_{p/f}$$

$NP(t)_{p/f} =$ The curve calculated at 1.3.3

At future time $T > t$ the probability of the obligor moving to a non-payment category (or remaining at stage 2 if he is a stage 2 obligor at start), collapses to a {0,1} value.

Since $NP(t)_{p/f} \in [0,1]$ and $NP(t)_j\big|_T \in \{0,1\}$ the individual difference $\Delta NP_j \in [-1,1]$

An eligible candidate for the individual adjustment difference estimation, is a perceptron ANN with $tanh$ link function and the back-propagation algorithm.

Specifically, if:

$c_0, c_1, c_2 =$ Model coefficients will be estimated form the minimization of

$$min\left(\sum_{j=1}^{N}\left[tanh\left(c_0 + c_1 \cdot B_j + c_2 \cdot F_j\right) - \left(NP(t)_j\big|_T - NP(t)_{p/f}\right)\right]\right)$$

The individual deviation emerges as:

$$\Delta NP_j{}^E = tanh\left(c_0{}^E + c_1{}^E \cdot B_j + c_2{}^E \cdot F_j\right)$$
$$\Delta NP_j{}^D = tanh\left(c_0{}^D + c_1{}^D \cdot B_j + c_2{}^D \cdot F_j\right)$$

Thus, the individual transition curves come out as following.

For stage 1 obligors:

$$NP(t)_j{}^E = max\{min\{a^E(m) + \Delta NP_j{}^E + b^E(m) \cdot ln(t), 1\}, 0\}$$

For stage 2 obligors:

$$NP(t)_j{}^D = \begin{cases} \max\{min\{\alpha^D(m) + \Delta NP_j{}^D + b^D(m) \cdot t, 1\}, 0\} & t \geq 0 \\ 1 & t < 0 \end{cases}$$

1.3.5 Derive non-payment probability at any future time

$NP(t)_j$ = Probability of the obligor j to enter a non-payment status at future time step, t. The severity of the non-payment has not been examined yet. In the following paragraphs, we examine the procedure of reducing the frequencies given by $NP(t)_j$ curve to point-in-time non-payment coefficients.

1.3.5.1 Incremental transition schemes

Considering discrete monthly steps, the basic movement of a stage 1 obligor into different states/stages, for steps $t = 0, t = 1, t = 2$, is depicted as

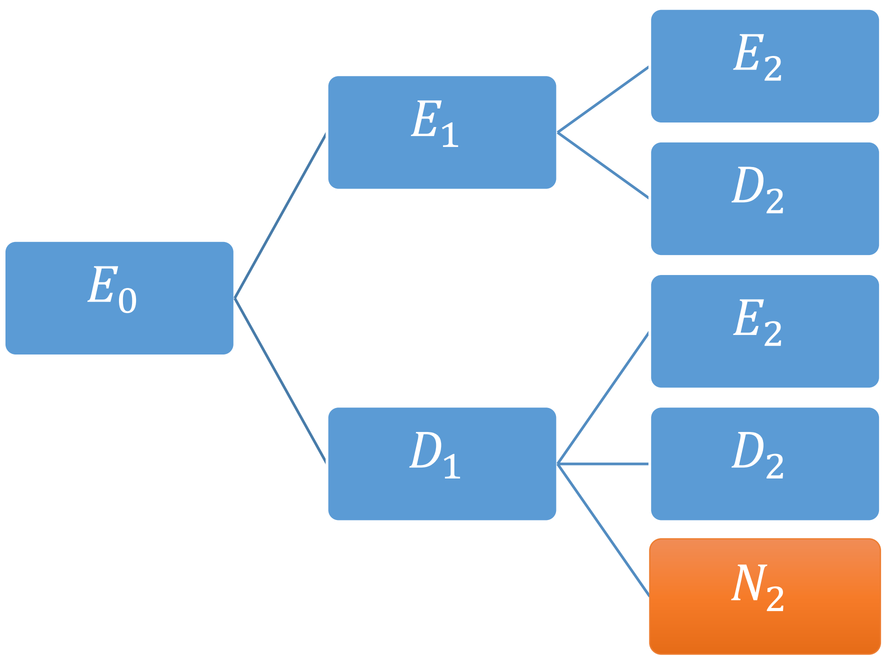

Figure 3: Marginal state transition for IFRS 9 - stage 1 obligors

The minimum time interval during which an accruing obligor may fall into irreversible state is 2 months. The basic movement of a stage 2 obligor for steps $t = 0, t = 1$ is:

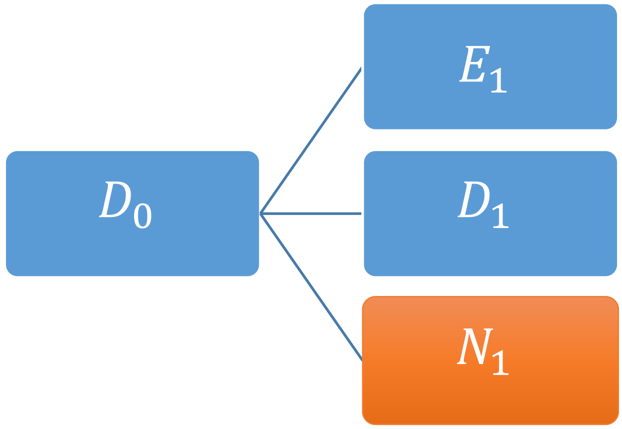

Figure 4: Marginal state transition for IFRS 9 - stage 2 obligors

Through the following sections, we shall use an adjustment approach based on these incremental transition schemes.

1.3.5.2 Total outcomes at specific time step for stage 1 obligors

We use the following notation for defining a specific path

$t =$ Discrete future monthly steps

$E =$ Stage 1 state at any future step $t > 0$

$i =$ Multiplicity of E state during a specific path

$D =$ Stage 2 state at any future step $t > 0$

$j =$ Multiplicity of D state during a specific path

$N =$ Stage 3 state at any future step $t > 1$

$k =$ Multiplicity of N state during a specific path

A specific path (without regarding the order of states) is symbolized as:

$$E^i D^j N^k \quad i + j + k = t > 0$$

Since the order of states appearance is not considered, each path (as depicted above) may appear multiple times. Path multiplicity is denoted with the combinations coefficient:

$$c(t, i, j, k) = \frac{t!}{i! \cdot j! \cdot k!}$$

The subsequent logical adjustments should be applied to the coefficient $c(t, i, j, k)$ to reflect the assumed transition schemes of 1.3.5.1

- An obligor cannot enter state N without having previously entered state D.

  This leads to the adjustment
  $$\left.\begin{matrix} j = 0 \\ k > 0 \end{matrix}\right\} \Rightarrow c(t, i, j, k) = 0$$
- State N is irreversible. Reaching irreversible state can be done with $\frac{[i+(j-1)]!}{i! \cdot (j-1)!}$ ways.
  $$\left(E^i \cdot D^{j-1}\right) \cdot \underbrace{D \cdot N^k}_{1\ ending}$$
- Irreversible state N can be reached only after $t \geq 2$.

The adjusted multiplicity coefficient, for every path is:

$$c^E(t, i, j, k) = \begin{cases} 0 & k > 0, j = 0 \\ \dfrac{[i + (j-1)]!}{i! \cdot (j-1)!} & k > 0, j > 0 \\ \dfrac{t!}{i! \cdot j! \cdot k!} = \dfrac{(i+j)!}{i! \cdot j!} & k = 0 \end{cases}$$

At any future time-step $t > 0$ exist $O(t)^E$ possible outcomes

$$O(t)^E = \sum c^E(t, i, j, k)$$

1.3.5.3 Total outcomes at specific time step for stage 2 obligors

Implementing the same rules as in previous section 1.3.5.2 we end up with:

$$c^D(t, i, j, k) = \begin{cases} \dfrac{[i + (j-1)]!}{i! \cdot (j-1)!} & k > 0 \\ \dfrac{(i+j)!}{i! \cdot j!} & k = 0 \end{cases}$$

Irreversible state N can be reached from $t = 1$. The total outcomes at time step t are:

$$O(t)^D = \sum c^D(t, i, j, k)$$

1.3.5.4 Total bad outcomes at specific time step for stage 1 & 2 obligors

As unfavorable outcomes, we consider:

- Partial non-payment end state D with and ending sequence

$$E^{i-1} \cdot D^{(j-j^*)} \cdot \underbrace{E^1 \cdot D^{j^*}}_{ending} \quad j^* = 1, \dots, j \quad t = i + j \,, k = 0$$

The sum of partial non-payment states is

$$SD(t) = \sum_{j^*=1,\ \ t=i+j}^{j} \frac{[(j-j^*)+(i-1)]!}{(j-j^*)! \cdot (i-1)!}$$

$j^* =$ The continuous ending months where the obligor is in state D, out of the total j appearences of D state in the specific path

- Total non-payment states N, which total

$$SN(t) = \sum_{k>0, j>0} c^*(t,i,j,k) = \sum_{k>0, j>0} \frac{[i+(j-1)]!}{i! \cdot (j-1)!} \quad c^* = \{c^E, c^D\}$$

Obligor transition curve to non-payment status $NP(t)$ that indicates non-payment, at any future time step $t > 0$, is approximated by:

$$\frac{SD(t) + SN(t)}{O(t)} \rightarrow NP(t)$$

1.3.5.5 Non-payment severity

Since not all outcomes that compose $NP(t)$ refer to 100% non-payment, a downward adjustment is essential to reflect the true non-payment severity.

First, the adjustment should be made upon partial non-payment states $SD(t)$.

$j_{max} =$ The number of continuous month in state D where the obligor reaches 100% non-payment. This will be decided with the use of statistical analysis and / or company policy.

The adjustment for each outcome is

$$c_{j^*} = \begin{cases} \dfrac{j^*}{j_{max}} & j^* \leq j_{max} \\ 1 & j^* > j_{max} \end{cases}$$

$$SD^*(t) = \sum_{j^*=1,\ \ t=i+j}^{j} \frac{[(j-j^*)+(i-1)]!}{(j-j^*)! \cdot (i-1)!} \cdot c_{j^*} < SD(t)$$

$ac(t) =$ Adjustment coefficient t

$$ac(t) = \frac{SD^*(t) + SN(t)}{SD(t) + SN(t)}$$

The probability of the obligor not paying at time t (without considering recoveries) is:

$$NP^*(t) = ac(t) \cdot NP(t)$$

Of course, adjustment coefficient $ac(t)$ needs to be calculated separately for E_0 and D_0 obligors.

1.3.6 LGD for state *N* obligors

All outcomes that compose obligor $NP(t)$ and refer to 100% non-payment should take into account the inflows that will emerge at times $T > t$ as a result of legal actions against the obligor.

The inflows that will arise as a consequence of legal actions will be directly connected to the collateral coverage of the obligor exposures.

$m =$ Available and statistically significant lagged macro variables at time step t (same notation as in 1.3.3)

$col_t(m) =$ Collateral value at time t, as a function of lagged macro variables

$E_t =$ Obligor exposure at time t. We will only use E_0.

If E_t is covered by $i = 1, \dots, I$ collaterals, percentage cover by each collateral type at $t = 0$ is defined as:

$$cov_0^i = \frac{col_{0,i}}{E_{0,i}} \quad cov_0^i \in [0,1]$$

$rec_{0,i} =$ Recovery percent provided by the collateral value, $rec_{0,i} \in [0,1]$

Due to expenses and legal & other fees we have to assume that

$$rec_{0,i} < cov_0^i \Rightarrow rec_{0,i} = dr_i \cdot cov_0^i \quad dr_i \in [0,1]$$

$dr_i =$ Downward adjustment for real collateral coverage (statistically or expert derived)

It is also assumed that recovery percent changes at future times $t > 0$ with the same analogy to collateral value

$$\frac{\Delta rec_{t,i}}{rec_{0,i}} = \frac{\Delta col_{t,i}}{col_{0,i}} \Rightarrow rec_{t,i} = rec_{0,i} \cdot \left(1 + \frac{col_{t,i}(m) - col_{0,i}}{col_{0,i}}\right)$$

$rec_{0,i}, col_{0,i} =$ Values available at current time

$col_{t,i}(m) =$ Estimation for future time according to collateral value model (or assumptions). The case of $col_{t,i}(m) = col_{0,i}$ yields the base case scenario.

Furthermore:

$\Delta t_{min,i} =$ The minimum amount of time (monthly steps) since entrance into irreversible state, after which cash inflows are expected from collateral coverage i (company policy or law process restriction variable).

$\Delta t_{max,i} =$ The maximum amount of time (monthly steps) after which cash inflows are expected from collateral coverage i (company policy or law process restriction variable).

$$\Delta t_{min,i}\,, \Delta t_{max,i} \in \mathbb{N}$$

Since the recovery rec_t will take place after the obligor will be traced in an irreversible state N at future time, at an uncertain time step, we consider the value rec_t evenly distributed, during months $\{t + \Delta t_{min}, t + \Delta t_{max}\}$.

$k =$ The number of monthly steps the obligor is already in irreversible state N , $k \geq 1$.

For collateral i the recoveries (inflows) will be

$$Inflow_i = \frac{rec_{t,i}}{\Delta t_{max,i} - \Delta t_{min,i} + 1}$$

$$\sum Inflow_i = rec_{t,i}$$

$$PV(Inflow_i, j) = \frac{Inflow_i}{\left(1 + c_{t+j}(m)\right)^{1+j}}$$

$c_{t+j}(m) =$ Appropriate interest rate that is expected to reflect the financial institution cost of money during the interval $\left[0, \Delta t_{max,i}\right]$

Adjustments according to the months the obligor is already in irreversible state

$$rec^*{}_{t,i} = \begin{cases} \displaystyle\sum_{j=\Delta t_{min,i}}^{\Delta t_{max,i}} PV(Inflow_i, j) & k = 1 \\ \displaystyle\sum_{j=\Delta t_{min,i}-k}^{\Delta t_{max,i}-k} PV(Inflow_i, j) & 1 < k \leq \Delta t_{min,i} \\ \displaystyle\sum_{j=0}^{\Delta t_{max,i}-k} PV(Inflow_i, j) & \Delta t_{min,i} < k \leq \Delta t_{max,i} \end{cases}$$

Of course

$$rec^*{}_{t,i} = \{0 \quad k > \Delta t_{max,i}$$

$rec^*{}_{t,i} =$ Total recovery sum expected at time t

At obligor level, for each state N that implies 100% non-payment, we expect to recover

$$R_t = \sum_{i=1}^{I} rec^*{}_{t,i}$$

Total non-payment states for the obligor would be adjusted as

$$SN^*(t) = \sum_{k>0,j>0} c^*(t,i,j,k) \cdot (1 - R_t) = \sum_{k>0,j>0} \frac{[i + (j-1)]!}{i! \cdot (j-1)!} \cdot LGD_t \quad c^* = \{c^E, c^D\}$$

Using $SN^*(t)$ and the notation of 1.3.5.5, the adjustment coefficient is extended to account for possible recoveries also by redefining $SD^*(t)$ as

$$c_{j^*} = \begin{cases} \dfrac{j^*}{j_{max}} & j^* \leq j_{max} \\ 1 - R_t & j^* > j_{max} \end{cases}$$

$$SD^*(t) = \sum_{j^*=1,\ t=i+j}^{j} \frac{[(j-j^*) + (i-1)]!}{(j-j^*)! \cdot (i-1)!} \cdot c_{j^*} < SD(t)$$

So the final adjustment coefficient becomes

$$ac^{**}(t) = \frac{SD^*(t) + SN^*(t)}{SD(t) + SN(t)}$$

The total result is a non-payment probability for each obligor of current state in $\{E_0, D_0\}$, at any future time step is:

$$NP_{E,D}{}^{**}(t) = ac^{**}(t) \cdot NP(t)^{E,D}$$

For obligors of current state N we should adjust accordingly

$$NP_N{}^{**}(t) = NP(t)^N \cdot (1 - R_t) = 1 - R_t$$

1.3.7 Pricing

The use of $NP^{**}(t)$ curve, for each expected cash flow at future time t, enables us to extract the part that is expected not to be paid

$CF_t =$ Loan cash inflow expected at future time t that includes capital plus interest

$CF_t \cdot NP^{**}(t)$ = Part of the inflow expected not to be paid

We may utilize the above separation to obtain the minimum interest rate the financial institution has to charge any existing or candidate client during the loan granting process.

L = Loan value, amount given at $t = 0$

The main concept is that the loan amount will be recovered by the part of the cash flows which are expected to be paid:

$$L = \sum PV\left(CF_t \cdot \left(1 - NP^{**}(t)\right)\right)$$

If

r_{min} = The minimum annual interest rate that will be charged

$r_{min,12} = \frac{r_{min}}{12}$ Is the period (month) interest rate

m = Available and statistically significant lagged macro variables at time step t (same notation as in 1.3.3)

$c_t(m)$ = Appropriate period (month) interest rate that is expected to reflect the financial institution cost of money at future time t[3]

T = Loan duration (months)

Some basic loan types are presented below as examples.

1.3.7.1 Amortizing loan

The minimum interest rate emerges with the iterative solution of the equation

$$\sum_{t=1}^{T} \frac{\left(\frac{r_{min,12} \cdot L}{1 - \left(\frac{1}{1 + r_{min,12}}\right)^T}\right) \cdot \left(1 - NP^{**}(t)\right)}{\left(1 + c_t(m)\right)^t} = L$$

1.3.7.2 Limit with one final payment

$$\frac{L \cdot \left(1 + r_{min,12}\right)^T \cdot \left(1 - NP^{**}(T)\right)}{\left(1 + c_T(m)\right)^T} = L$$

[3] It should be noted that m reflects the lagged past variables in relation to time t, that is m may include projected values also (unless kept constant), according to a macro model, in relation to current time $t = 0$

1.3.7.3 Limit with intermediate interest payments

$$\sum_{t=1}^{T} \frac{\left(r_{min,12} \cdot L\right) \cdot \left(1 - NP^{**}(t)\right)}{\left(1 + c_t(m)\right)^t} + \frac{L \cdot \left(1 - NP^{**}(T)\right)}{\left(1 + c_T(m)\right)^T} = L$$

1.3.7.4 Customized payment scheme

If we assume that a customized payment scheme is designed for a particular customer, with

$L_t =$ The amount of capital agreed to be paid at time t

$$\sum_{t=1}^{T} L_t = L$$

$B =$ The set of future times for the agreed payments

$$\sum_{t \in B} \frac{L_t \cdot \left(1 + r_{min,12}\right)^t \cdot \left(1 - NP^{**}(t)\right)}{\left(1 + c_t(m)\right)^t} = L$$

The final interest rate to be charged is

$$r = r_{min} + (other\ cost) + premium$$

$other\ cost =$ Part of the interest rate reflecting costs + other types of risk, since the methodology we have applied accounts for credit risk in r_{min}.

$premium =$ The net desired return from the loan

1.3.8 Provisions

Once the loan granting process has taken place r interest rate is taken as an input and provisions at current time can be calculated at obligor level:

$CF_t =$ Total obligor cash inflow expected at time t (including missed payments for stage 2 obligors as CF_0 with $NP(t) = 1$)

$B =$ The set of time steps for payments that emerge from all obligor loans (including $t = 0$ missed payments for stage 2 obligors)

$T =$ Maximum time step up to which the obligor has payment obligations

$$Provisions_{t=0}^{T} = \sum_{t \in B} \frac{CF_t \cdot NP^{**}(t)}{\left(1 + c_t(m)\right)^t}$$

1.4 IFRS 9 Issues

IFRS 9 expected credit losses (ECL) may easily be calculated with the use of the proposed methodology.

1.4.1 Stage 1 obligors

It is evident that the 12-month part of the total provisions is

$$ECL_{12} = \sum_{t=1}^{12} \frac{CF_t \cdot NP^{**}(t)}{\left(1 + c_t(m)\right)^t}$$

1.4.2 Stage 2 & 3 obligors

$$ECL_{life} = Provisions_{t=0}^{T} = \sum_{t \in B} \frac{CF_t \cdot NP^{**}(t)}{\left(1 + c_t(m)\right)^t}$$

1.4.3 Credit impairment

In our opinion, the provisions increase is sufficient to deal with the augmented credit risk. However, if desired by the financial institution, a logical indicator for characterizing impaired obligors would be

$$\frac{Provisions_{t=0}^{T}}{E_0} > Imp$$

$E_0 =$ Obligor exposure at current date

$Imp =$ Critical value (percent) for impairment classification

1.4.4 Implied relation to Basel Framework parameters

We equate the Basel calculated expected credit losses to the estimated provisions of the proposed methodology (using the notation in 1.3.6)

$$PD \cdot LGD \cdot EAD = Provisions_{t=0}^{T}$$

Approximating

$LGD = 1 - R_0$

$EAD = E_0$

An estimation of a forward-looking Basel PD measure may be derived from the already calculated provisions

$$PD_T = \frac{Provisions_{t=0}^{T}}{E_0 \cdot (1 - R_0)}$$

If:

$T = 12 \Rightarrow$ We get the 1 year PD

$T = Obligor\ maximum\ contractual\ obligation\ date \Rightarrow$ We get the expected Lifetime PD

Forward looking unexpected losses could also be derived with the implementation of the $PD_T, 1 - R_0$ values into the Basel formulas.

2 Basel extension approach

The implementation of IFRS9 concepts, can be based on a simple Basel II framework extension. Provisioning for long term assets requires the extension of "bad" definition (default) up to maturity. Given the availability of one year probability of default for any IRB compliant financial institution, probabilities for longer maturities can be intuitively estimated according to the methodology that follows.

2.1 Long term Probability of Default

$N =$ The years of remaining life for the obligor

$t =$ Annual time steps, $t \in [0, N]$

Keeping the Basel default definition, we define the Bernoulli variable Y_t as

$Y_t =$ Default event up to annual step t

$$Y_t = \begin{cases} 1 & p_t \\ 0 & 1 - p_t \end{cases}$$

With $E(Y_t) = p_t$

It is easy to establish that for the one year period $t = 1$

$$E(Y_1) = p_1 = p = PD_{Basel}{}^{4}$$

The PD is approximated by the average through the cycle frequency of default. Since it is an average value, at every future time step the above relationship also holds

$$E(p_{t,t+1}) = p$$

[4] It is the PD of the relevant rating assigned to the obligor

$p_{t,t+1}$ = The average probability of default expected at time t for the next year, estimated now $(t = 0)$

If d_t = the number of default events up to year t, $d_t \in \mathbb{N} \quad d_t \le t$, the probability of default up to t may be calculated with the use of the Binomial distribution

$$p_t = Prob(d_t > 0) = 1 - Prob(d_t = 0) = \binom{t}{d_t} . p^{d_t} . (1-p)^{t-d_t}\Big|_{d_t=0}$$
$$= 1 - (1-p)^t$$

Furthermore, we define the survival (non-default) probability up to t, S_t

$$S_t = Prob(d_t = 0) = 1 - p_t = (1-p)^t$$

For a 25-year credit obligation, with Basel 1-year PD 20%, the expected values p_t and S_t are given below:

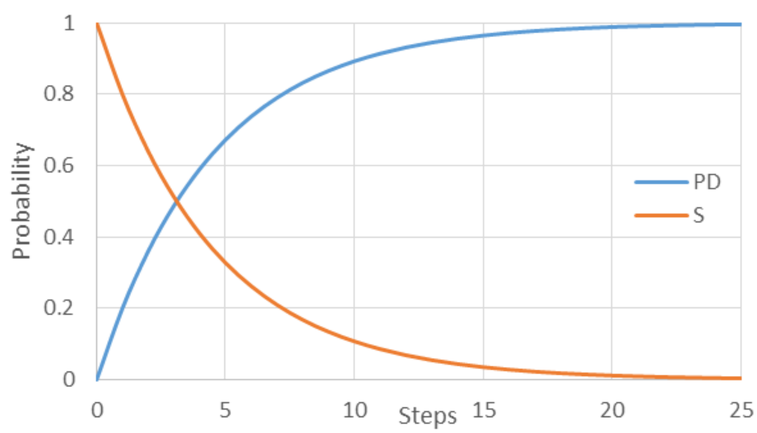

Figure 5: PD & Survival probability example

The derivative of p_t expresses the probability of an obligor to default through step $t + 1$, after he has survived up to step t

$$\frac{dp_t}{dt} = p \cdot (1-p)^t = p_{t,t+1} \cdot S_t$$

2.2 Life time ECL modeling

Lifetime ECL is calculated as the sum of all ECLs taken at discrete annual non-intervening time steps.

$$ECL_{Life} = \sum_{t=0}^{N} ECL_t = \sum_{t=0}^{N} E_t \cdot PD_{t,t+1} \cdot LGD_{t,t+1}$$ [5]

$E_t =$ The expected capital amount outstanding at time step t, according to the loan payment schedule, based on the assumption that the obligor has survived up to $t > 0$

The expected $PD_{t,t+1}$ as already seen, is:

$$PD_{t,t+1} = p$$

If we define the loan-schedule dependent variable λ_t as:

$$\lambda_t = \frac{E_t}{E_0}$$

$$ECL_{Life} = \sum_{t=0}^{N} (S_t \cdot E_t) \cdot p \cdot LGD_w = \sum_{t=0}^{N} (S_t \cdot \lambda_t \cdot E_0) \cdot p \cdot LGD_w$$
$$= (E_0 \cdot p \cdot LGD_w) \cdot \left(\sum_{t=0}^{N} \lambda_t \cdot S_t \right) = ECL_0 \cdot \left(\sum_{t=0}^{N} \lambda_t \cdot (1-p)^t \right)$$

$LGD_w =$ The loss given default (LGD) according to Basel II specifications (calculated at the worst part of the economy cycle).

Thus, for every obligor considered necessary[6], the expected loss produced by the Basel Framework will have to be extended using the factor f, in order to arrive to a lifetime estimation.

$$ECL_{Life} = f \cdot ECL_0 \quad f = \sum_{t=0}^{N} \lambda_t \cdot (1-p)^t$$

2.3 Incorporating macro-economic information

The calculation in 0 is valid under the assumption that for the remaining part of the obligor life, the Basel risk parameters PD, LGD will not change significantly. Indeed, this is a valid assumption as

- PD refers to through the cycle average
- LGD is the worst-cycle period estimation

However, in the case that

[5] No present value adjustment is applied here
[6] Stage 1 and 2 obligors of the IFRS 9 framework

- We have indications that the risk parameters will worsen during our portfolio remaining life and will affect significantly the average and worst cycle values
- We wish to put more pressure on our ECL estimations, for some reason

Stressed risk parameters may be used $p^* \quad LGD^*$, which

- May arise as a product of expert judgment $p^* > p \quad LGD^* > LGD_w$
- Develop a macroeconomic scenario and express the risk parameters directly as estimated functions of the projected macro variables $p^* = p(m) \quad LGD^* = LGD(m) \quad m = macro\ variables$
- Adopt a stochastic process that will describe the risk parameters evolution

As far as the PD is concerned, a mean reverting process would be appropriate to describe its incremental evolution. For the LGD we could apply an approach similar to the one described in 1.3.6.

The key point is that all the stressing and projection required comes down to marginally worsening the Basel risk Parameters.

2.4 Unexpected losses

Since

$$ECL_{Life} = f \cdot ECL_0 \Rightarrow PD_{Life} \cdot LGD_{Life} = f \cdot PD_{Basel} \cdot LGD_{Basel}$$
$$= \underbrace{(f_{PD} \cdot PD_{Basel})}_{PD_{Life}} \cdot \underbrace{(f_{LGD} \cdot LGD_{Basel})}_{LGD_{Life}}$$

If the current ratio of PD to LGD for the rating category that the obligor belongs to is

$$pl = \frac{PD}{LGD}$$

The following definition could be inferred

$$\left.\begin{array}{l} \dfrac{f_{PD}}{f_{LGD}} = pl \Rightarrow f_{PD} = pl \cdot f_{LGD} \\ f = f_{PD} \cdot f_{LGD} \end{array}\right\} \Rightarrow \begin{array}{l} f_{PD} = \sqrt{pl \cdot f} \\ f_{LGD} = \sqrt{\dfrac{f}{pl}} \end{array}$$

$$PD_{Life} = \left(\sqrt{pl \cdot f}\right) \cdot PD_{Basel} \quad LGD_{Life} = \left(\sqrt{\frac{f}{pl}}\right) \cdot LGD_{Basel}$$

And the Basel formulas for the calculation of unexpected losses could be applied with the insertion of the inferred Life-Risk parameters.

2.5 Impairment

Impairment may be defined in an analogous manner to 1.4.3 part as

$$\frac{ECL_{Life}}{E_0} > Imp$$

E_0 = Obligor exposure at current date

$Imp =$ Critical value (percent) for impairment classification

Disclaimer

The proposed methodologies reflect the author's views only and have no relation to any practices implemented in National Bank of Greece (NBG).

To the best of my knowledge, up to the time the current document is written (April 2017), there is no publication describing similar methodologies.

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