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Volatility Modeling for Forecasting Stock Index with Fixed Parameter Distributional Assumption

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Abstract

The aim of this paper is to empirically investigate the in sample and out of sample forecasting performance of several GARCH-type models such as GARCH, EGARCH and APARCH model with Gaussian, student-t, Generalized error distribution (GED), student-t with fixed DOF 10 and GED with fixed parameter 1.5 distributional assumption in case of Colombo Stock Exchange (CSE), Sri Lanka. The daily All Share Price Index (ASPI) of CSE from January 02, 1998 to December 29, 2006 for a total number of 2150 observations is used for empirical analysis. We consider first 1950 observations for in sample estimation and last 200 observations for out of sample forecasting evaluation. Our empirical study showed that fixed DOF 10 of student-t density and fixed parameter 1.5 of GED density fail to improve the in sample estimation performance compared to student-t and GED distributional assumption. Among all of these models, APARCH model with student-t density give better in sample estimation results. In case of out-of-sample forecasting performance we found that APARCH model with all distributional assumption give lower value of Mean Squared Error (MSE) and Mean Absolute Error (MAE). According to the densities student-t distribution with fixed DOF 10, student-t and Gaussian distributional assumptions give better results in case of GARCH, EGARCH and APARCH model respectively. The estimation results of SPA test suggest that APARCH model with Gaussian distributional assumption give better forecasting performance in case of all share price index of CSE, Sri Lanka.

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1 Introduction

Generally financial time series data contains the property of volatility clustering, serially correlation in the squared log return and long tailed ness i.e. the returns show normally longer tail than Gaussian distribution where volatility clustering indicate large changes tend to be followed by large changes and vice versa. To capture the volatility clustering and long tail property of financial time series data Engle [11] introduced Autoregressive Conditional Heteroskedastic(ARCH) model. Earlier empirical research shows that in order to account the dynamic of conditional variance high ARCH order is utilized i.e. we need to estimate many parameters. To solve this high ARCH order problem Bollerslev [4] proposed Generalized Autoregressive Conditional Heteroskedastic (GARCH) model. But most of time GARCH model fail to capture the thick tail property completely. This excess kurtosis naturally leads to use non normal distribution for errors. To solve this problem many authors estimate GARCH models by using student-t distribution for errors (for example, Bollerslev [5], Baillie and Bollerslev [2] and Beine et al [3]) while other authors such as Nelson [25] and Kaiser [17] suggested Generalized Error Distribution (GED). Peters [29] examined the forecasting performance of GARCH, EGARCH, GJR and APARCH models under fat tail and skewed distributional assumption for FTSE 100 and DAX 30 indices and found that asymmetric GARCH models with fat tail density give better results in case of in-sample estimation but using non normal error distribution does not clearly shows it forecasting efficiency. Although researchers showed that sometime GARCH models give better in-sample estimation but very poor forecasting performance, Anderson and Bollerslev [1] argued that GARCH models provide good volatility forecasts. There are several reasons such as using inadequate measure for volatility or choosing wrong statistical loss function lead to provide worse forecasting performance in case of GARCH models. A large number of earlier studies find out the appropriate GARCH model and their forecasting performance but they did not find any unique model for GARCH estimation which always gives better result for every stock market. Kang et al. [18] showed that component GARCH and fractionally integrated GARCH models which capture long-memory volatility provide better forecasting performance compare to simple GARCH and integrated GARCH models. On the other hand Cheong [8] showed that simple GARCH model characterize the Brent crude oil data give better results than Asymmetric GARCH models. Ramon [30] used the specifications for the mean, variance and error using ARMA, SARMA and GARCH models to predict the volatility of Philippine inflation rate. He estimate GARCH model with Gaussian, student-t with 10 DOF and GED distribution with 1.5 fixed parameter and found student-t distribution with fixed DOF 10 is the most adequate choice for the variance of the error distribution. Wei et al [34] compared the performance of different number of linear and non-linear GARCH models to capture the volatility features of two crude oil markets-Brent and West Texas Intermediate (WTI) and found that no model can outperform all of the other models for either the Brent or the WTI market across different loss functions. Liu et al [21] investigated the specification of return distribution influences the performance of volatility forecasting for two Chinese stock indexes using two GARCH models and found that GARCH model with skewed generalized error distribution give better results than GARCH model with normal distribution.

The Colombo Stock Exchange (CSE) is the main stock exchange in Sri Lanka which was founded at 1985. Recently CSE makes remarkable development as it present annual growth rate 41.6% in 2006 which was 30% in 2002-2004. The world Federation of Exchanges rated the CSE as the best performing stock trading place for the fiscal year of 2009. There are two indices available for CSE and these are: All Share Price Index (ASPI) and Milanka Price Index (MPI). Kumar and Mittal [20] investigated whether the common finding regarding the asymmetric impact of news on the volatility of returns and found no significant asymmetry in the volatility factors. Jaleel and Samarakoon [16] examined the impact of liberalization of the Sri Lankan Stock market on return volatility using GARCH and TGARCH models for the period from 1985 to 2000 and found that liberalization of the market to foreign investors significantly increased the return volatility in the Colombo stock Exchange. Haniffa [13] examined stock return volatility of the All Share Price Index (ASPI) using Auto Regressive Conditional Heteroskedastic (ARCH) and Generalised ARCH models that capture most common stylised facts on asset returns. Additionally, he also examined the effect of exchange rate fluctuations have any impact on the volatility of index return or not. The previous study indicate that only few researchs have been conducted based on Sri Lankan Stock Market and none of them compare the forecasting performance of different symmetric and asymmetric GARCH model with fixed and flexible parameter together for the error distribution by robust test. Since different models fit well for different stock markets and different error distribution improve the forecasting performance, so to figure out accurate forecasting models for any particular stock amrket is always interesting. For forecasting evaluation we used most commonly used two loss function and Superior Predictive Ability(SPA) test of Hansen [14].

Therefore, the aim of this paper is to empirically re-examine the in sample and out of sample forecasting performance of several GARCH-type models such as GARCH, EGARCH and APARCH model with Gaussian, student-t, Generalized error, student-t with 10 DOF and GED with fixed parameter 1.5 densities by using two loss functions and SPA test of Hansen [14] in case of CSE. This study is important because this is the first time to compare the performance of different GARCH type models with fixed parameter distributional assumption by SPA test for CSE. Rest of the paper is organized as follows: section 2 present the methodology, section 3 present the characteristic of data and empirical analysis and finally section 4 present the conclusions.

2 Methodology

2.1 Model and Error Distribution

Engle's [11] ARCH model has been widely used in financial time series analysis. In general financial return series can be formalized as:

$$\log\left(y_t / y_{t-1}\right) = \mu + \varepsilon_t h_t \tag{1}$$

where $\log\left(y_t / y_{t-1}\right)$ is the log asset price, μ is the conditional mean of the returns, h_t is the volatility process and $\varepsilon_t \sim N(0,1)$. In the case of autoregressive conditional heteroskedastic (ARCH) model we need long lag property to improve the goodness of fit. For this reason Bollerslev [4] proposed generalized ARCH (GARCH) model. The GARCH model consists of squared residuals and lag of conditional variance. Olowe [26] used six GARCH set models to investigate the volatility of Nigerian exchange rate. The Generalized ARCH (GARCH) model of Bollerslev [4] can be expressed as

$$h_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=i}^{p} \beta_j h_{t-j} \qquad (2)$$

where $w > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1,2,...,q$ and $j = 1,2,...,p$ confirm positive conditional variance and the innovation can be expressed as the product of an i.i.d process with mean 0 and variance 1. If the sum of the parameters $\alpha_i + \beta_j < 1$ then the equation (2) will be stationary and if they are close to 1 then volatility parameter will be more persistent.

Since financial time series data normally show high kurtosis value which indicate the asymmetry of the data and GARCH model with Gaussian distribution fail to account such asymmetry. The weakness of GARCH models encourages the researchers to develop different GARCH set models which can include skewness and asymmetry. The popular models of asymmetric volatility includes Exponential GARCH (EGARCH) model. This model was proposed by Nelson [25]. The specification for conditional variance is:

$$\log(h_t) = w + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{h_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{h_{t-k}} \qquad (3)$$

The left hand side of equation (3) is the log of the conditional variance. This implies that the leverage effect is exponential rather than quadratic. The presence of the leverage effects can be tested by the hypothesis $\gamma_i < 0$. The impact asymmetric if $\gamma_i \neq 0$. In the EGARCH model no restrictions are required to ensure the positive ness of the conditional variance. Another extension of Asymmetric model was APARCH model. Taylor [32] and Schwert [31] introduced the standard deviation of GARCH model, where the standard deviation is modeled rather than the variance. Ding et al [10] introduced the Asymmetric Power ARCH (APARCH) model. In this model the power parameter of the standard deviation can be estimated rather than imposed and other optional parameters are added to capture asymmetry. The APARCH (p, q) model can be expressed as:

$$h_t^{\delta} = w + \sum_{j=1}^{p} \beta_j h_{t-j}^{\delta} + \sum_{i=1}^{q} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right)^{\delta} \qquad (4)$$

where $w > 0$, $\delta \geq 0$, $\beta_j \geq 0 (j = 1,2,......., p)$, $\alpha_i \geq 0$ and $-1 < \gamma_i < 1, (i = 1,2,..., q)$.

Since APARCH model can consider the leverage effect into account so, it is interesting to use for financial time series data. Furthermore, this model includes seven other ARCH extensions as special cases (See Ding et al [10], Peters [29], Karlsson [19]).

To complete the ARCH model estimation we need the assumption of conditional distribution for the error terms. Financial time series data normally present the fat-tailed property. In order to capture this fat tail property we need to use some fat tailed distribution. EViews 5 software present the estimation of GARCH model under Gaussian, student-t, GED, student-t with fixed degrees of freedom (DOF) and GED with fixed parameter (De Swart et al [9]). GARCH models are normally estimate by considering the conditional distribution of the error terms are Gaussian. The normal or Gaussian distribution is a symmetric distribution with density function:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/\sigma^2} \tag{5}$$

Where μ is the mean value and σ^2 is the variance of the stochastic variable. The standard Gaussian distribution considers the mean value $\mu = 0$ and variance $\sigma^2 = 1$. Although Gaussian distribution hold the leptokurtic property but this leptokurtosis is not enough to explain the leptokurtosis property which is found in most of the financial data (Bollerslev, [5]). Therefore, one should take this into account and use conditionally leptokurtic distribution for the error. One alternative possibility is Student-t distribution. The density function of student-t is given by

$$f(x) = \frac{\Gamma[(v+1)/2]}{\sqrt{v\pi}\,\Gamma[v/2](1+x^2/v)^{(v+1)/2}} \tag{6}$$

Where v is the degree of freedom (df) $v > 2$. If v tends to ∞ the student-t distribution converges to normal distribution.

Another most commonly used fat tail distribution is the Generalized Error distribution (GED). The GED is a symmetric distribution and platykurtic. The GED has the following density function.

$$f(x) = \frac{v e^{\frac{1}{2}\left|\frac{x}{\lambda}\right|}}{\lambda 2^{(v+1)/v}\,\Gamma(1/v)} \tag{7}$$

Where $\lambda = \left[\frac{2^{-2/v}\,\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2}$

It includes the normal distribution if the parameter v has the value 2. For DOF v<2 indicate fat tail distribution. Last two error distributions give flexible condition for the user to used DOF and different parameter. If the DOF of student-t distribution has value about 30 or above then it can be argued that the student-t distribution is close to the normal distribution (Stoyanov et al [33]). Ramon [30] estimate GARCH set model with under Gaussian, student-t with 10 DOF and GED distribution with fixed parameter 1.5. So, at our study we consider two fix parameter distribution and check either these fix parameter can improve estimation efficiency for both in sample or out of sample or not.

2.2 Forecasting Technique

The forecasting performance requires the minimization of the loss function property. There is no unique criterion which is always consistent for providing best forecasting performance (see Bollerslev et al [6] and Lopez [22]). Many authors argued for using real loss functions to evaluate the volatility forecasting. As for example, Engle et al [12] and

West et al [35] suggested profit based and utility based criteria for evaluating the volatility forecasts. Marcucci [23] used seven different loss functions such as Mean Squared Error, Mean Absolute Deviation, R2LOG loss function of Pagan and Schwert [27], QLIKE loss function of Bollerslev et al [6] and HMSE of Bollerslev and Ghysels [7] for evaluating complete forecasting performance of different volatility models. Later Wei et al [34] used six different loss functions to measure the difference between the realized and the estimated conditional variances. These loss functions are Mean squared Error and Mean Absolute Error, Heteroskedasticity-adjusted MSE and MAE, Logarithmic Loss (LL) function and QLIKE loss function. Therefore, we use standard loss function such as Mean Squared Error (MSE) and Mean Absolute Error (MAE) for our forecasting evaluation. These are estimated by the following equation:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(h_t - \hat{h}_t\right)^2 \quad (8)$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|h_t - \hat{h}_t\right| \quad (9)$$

Besides these two forecasting criteria we also used the SPA test of Hansen [14] to test the accuracy of forecasting models. Several authors used this SPA test with different loss function (see Hou and Suasdi [15], Medeiros and Veiga [24] etc.). Hansen [14] applied a supremum over the standardized performances and tests the null hypothesis

$$H_0 : \max_{k=1,2,\ldots m} \mu_k \le 0 \quad \text{where } \mu_k = E(f_{k,t})$$

And the test statistics

$$T^S = \max\left\{\left(\max_k \frac{n^{1/2}\bar{f}_k}{\sqrt{\hat{h}_k}}\right), 0\right\} \quad (10)$$

where $\sqrt{\hat{h}_k}$ is the standard deviation of $n^{1/2}\bar{f}_k$ then $\bar{f}_k = 1/n\sum_{t=1}^{n} f_{k,t}$ and $f_{k,t} = l_{k,t} - l_{0,t}$. The earlier identified loss function at time t is defined by $l_{0,t}$ for the benchmark model where $l_{k,t}$ indicate the value of corresponding loss function for another competing model, n is the number of out of sample data. To reduce the influence of poor performing models while preserving the influence of the alternatives with $\mu_k = 0$, Hansen [14] proposes the following consistent estimator for μ :

$$\hat{\mu}_k^c = \bar{f}_k 1_{\{n^{1/2}\hat{f}_k / \hat{\sigma}_k \ge -\sqrt{2\ln\ln n}\}, k=1,\ldots,m} \quad (11)$$

where $1_{\{.\}}$ is an indicator function. Hansen argued that the threshold rate $\sqrt{2\ln\ln n}$ ensures $\hat{\mu}_k^c$ is consistent estimator that effectively captures all alternative with $\mu_k = 0$, and this leads to a consistent estimate of the null distribution, which improves the power of the test. The distribution of the test statistic under null hypothesis can be approximated by the empirical distribution derived from the bootstrap resample based on the stationary bootstrap of Politis and Romano [28].

$$U^*_{k,b,t} = f^*_{k,b,t} - h(\bar{f}_k) \text{ for } b = 1,...,B \text{ and } t = 1,...,n \tag{12}$$

where $h(\hat{f}_k) = \bar{f}_k 1_{\{n^{1/2}\hat{f}_k/\hat{\sigma}_k \geq -\sqrt{2\ln\ln n}\}}$. The p-value of the SPA test can be obtained by the first calculation of

$$T_b^{S*} = \max\left\{\left(\max_k \frac{n^{1/2}\bar{f}_k}{\sqrt{\hat{h}_k}}\right), 0\right\} \tag{13}$$

for each $b = 1,..., B$ and then the comparing T^S to the quantiles of T_b^{S*}.

$$P^S = \sum_{b=1}^{B} \frac{1_{\{T_b^{S*} > T^S\}}}{B} \tag{14}$$

3 Data, Results and Discussions

3.1 Characteristics of Data

For our empirical study we use the daily All Share Price Index (ASPI) of Colombo Stock Exchange, Sri Lanka. The data of the range from January 02, 1998 to December 29, 2006 for a total 2150 observations. The first 1950 observations are taken for in sample estimation and last 200 observations consider for out of sample forecasting performance. For analysis we used EViews5.0 and MATLAB 7.0. In order to obtain the stationary series we transformed these data into their returns. Daily returns of ASPI are plotted at Figure 1 which indicates that the data is more volatile in the period of July, 03 to March 04. The descriptive statistics of ASPI returns are displayed in Table-1. From Table-1 we found that mean returns of the CSE is 0.0626. Volatility which is measured by standard deviation is 1.355. The returns hold the property of leptokurtosis and positive skewness. The normality of the returns is rejected based on the Jarque-Berra statistics. The ARCH test confirms the presence of ARCH effect. Overall these results clearly support for the rejection of the hypothesis that CSE time series of daily ASPI returns are time series with independent daily values.

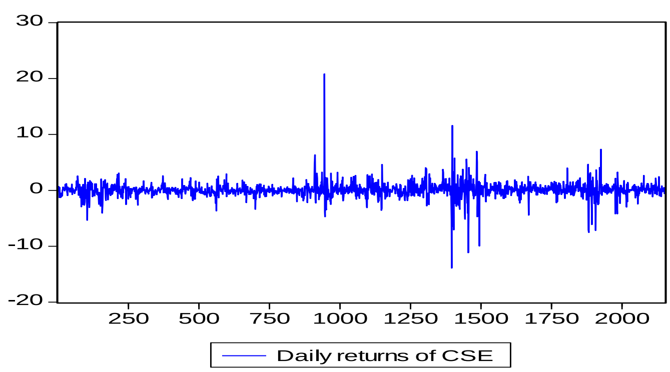

Figure 1: Daily returns of Colombo Stock Exchange

Table 1: Descriptive Statistics for ASPI daily returns

Sample size	Mean	Min.	Max.	Standard Deviation	Skewness	Kurtosis	Jarque-Bera test	ARCH test
2149	0.0626	-13.893	20.829	1.355	0.7966	42.688	141267 (0.000)	0.0128 (0.965)

3.2 In Sample Estimation

In our study we consider two types of GARCH model such as symmetric and asymmetric GARCH models (i.e. EGARCH and APARCH) with different distributional assumptions with fixed and flexible parameter distribution of student-t and generalized error distribution. For fixed parameter we use student-t distribution with fixed DOF 10 and GED distribution with fixed parameter 1.5. Parameter estimation results for GARCH model with Gaussian, student-t, GED, student-t with fixed degrees of freedom (DOF) 10 and GED with fixed parameter 1.5 distributional assumptions are given at Table-2. From Table-2 we found that all of the parameters of GARCH model under all distributional assumption are significant at 5% level of significance.

Table-2: GARCH model estimation with different distributional assumption

	μ	w	α_i	β_j	υ
Gaussian	0.0488 (3.361)	0.1593 (16.845)	0.6241 (48.407)	0.3027 (33.107)	
Student-t	0.0531 (3.425)	0.1256 (6.014)	0.4397 (7.905)	0.5405 (15.244)	3.854 (11.757)
GED	0.0474 (3.440)	0.1186 (6.654)	0.4921 (10.204)	0.5063 (15.936)	1.084 (37.666)
Student-t Fixed DOF	0.0545 (3.207)	0.1067 (8.773)	0.4062 (12.086)	0.5353 (20.480)	
GED Fixed Parameter	0.0510 (3.158)	0.1279 (10.345)	0.5392 (23.538)	0.4041 (23.227)	

Table-3: EGARCH model estimation with different distributional assumption

	μ	w	α_i	β_j	γ_k (or γ_i)	υ
Gaussian	0.0391 (2.751)	-0.5915 (-40.10)	0.8498 (42.806)	0.0971 (6.672)	0.7962 (77.848)	
Student-t	0.0490 (3.159)	-0.4110 (-13.43)	0.5905 (12.834)	-0.0251 (-1.002)	0.8752 (49.230)	3.824 (11.949)
GED	0.0468 (3.400)	-0.4381 (-14.94)	0.6024 (15.020)	-0.0073 (-0.223)	0.8728 (49.712)	1.074 (39.758)
Student-t Fixed DOF	0.0479 (2.841)	-0.4723 (-20.017)	0.5793 (19.603)	0.8583 (60.639)	-0.0110 (-0.5903)	
GED Fixed Parameter	0.0477 (2.974)	-0.5144 (-25.083)	0.6808 (27.164)	0.8414 (62.267)	0.0230 (1.296)	

Table-4: APARCH model estimation with different distributional assumption

	μ	w	α_i	β_j	γ_k (or γ_i)	δ	υ
Gaussian	0.0635 (3.569)	0.1496 (13.393)	0.6721 (24.816)	0.3722 (18.986)	-0.0713 (-3.946)	2.364 (11.879)	
Student-t	0.0496 (3.196)	0.1303 (6.299)	0.4172 (8.985)	0.5884 (17.050)	0.0215 (3.005)	1.299 (6.088)	3.837 (11.918)
GED	0.0471 (3.418)	0.1266 (6.577)	0.4504 (10.425)	0.5061 (16.272)	0.0171 (2.397)	1.475 (6.005)	1.081 (36.837)
Student-t Fixed DOF	0.0511 (2.985)	0.1177 (8.566)	0.3816 (13.931)	0.5797 (22.094)	0.0230 (0.6867)	1.424 (8.439)	
GED Fixed Parameter	0.0526 (3.142)	0.1319 (9.654)	0.4039 (19.871)	0.5089 (21.274)	-0.0189 (-0.698)	1.6610 (9.026)	

Parameter estimation results of EGARCH and APARCH model with different distributional assumptions are given at Table-3 and Table-4 respectively. From Table-3 we found that most of the parameters of EGARCH model with different distributional assumptions are significant at 5% level of significance except parameter β_j in case of student-t and GED distribution and parameter γ_k or γ_i in case of student-t with fixed DOF 10 and GED with fixed parameter 1.5. Table-4 showed that most of the parameters of APARCH model with all distributional assumptions are significant at 5% level except the asymmetric parameter γ_k or γ_i in case of student-t with fixed DOF and GED with fixed parameter. The sum of GARCH parameters under all distributional assumptions are less than one which suggest that the volatility are limited and the data are stationary which explain that the models are fitted well. In order to find out the best performing model in case of in sample estimation we used some model comparison criteria such as Box-Pierce statistics for both residuals and squared residuals, Akaike Information Criteria (AIC), Log Likelihood value. These estimation results are given at Table-5. From this table we found that all the models seem to do a good job in describing the dynamic of the first two moments of the series based on Box-Pierce statistics for both of the residuals, which are all non-significant at 5% level. The Akaike Information Criteria (AIC) and the log-likelihood values suggest that APARCH model give better results than GARCH and EGARCH models under all distributional assumption. Among these models EGARCH model showed the worst in sample estimation result in case of CSE. Regarding the densities, Student-t and Generalized error distributions clearly outperform than student with fixed DOF 10 and GED with fixed parameter 1.5 and all of these density show better performance than Gaussian density. In the case of student-t distribution the AIC value for the model GARCH, EGARCH and APARCH are less than other densities. The log likelihood value is strictly increasing in case of student-t distributional assumptions where fixed parameter of student-t density can not improve the log likelihood value. Similar results also found in case of GED and GED with fixed parameter. So, finally from Table-5 we found that in case of in sample performance APARCH model with student-t distribution give better results than other models for CSE, Sri Lanka.

Table-5: Model comparison based on in sample

Error Distribution	Model	$Q(20)$	$Q^2(20)$	AIC	Log-likelihood
Gaussian	GARCH	181.24	10.725	2.8572	-3066.09
	EGARCH	173.63	13.539	2.8686	-3077.41
	APARCH	186.31	13.665	2.8553	-3062.12
Student-t	GARCH	195.56	6.822	2.6645	-2858.05
	EGARCH	187.13	4.883	2.6703	-2863.26
	APARCH	190.49	5.521	2.6627	-2854.17
GED	GARCH	194.34	7.209	2.6873	-2882.58
	EGARCH	186.90	5.521	2.6945	-2889.25
	APARCH	190.99	6.177	2.6872	-2881.13
Student-t Fixed DOF	GARCH	193.84	6.934	2.7073	-2905.23
	EGARCH	182.64	5.444	2.7132	-2910.76
	APARCH	190.02	5.838	2.7067	-2902.21
GED Fixed parameter	GARCH	188.28	8.544	2.7365	-2936.50
	EGARCH	179.98	6.797	2.7454	-2945.11
	APARCH	188.62	7.730	2.7362	-2935.63

3.3 Out of Sample Forecasting Performance

Since the out of sample test can control the possible over fitting or over parameterization problems, therefore many empirical researchers became interested to have good volatility forecast based on out of sample estimation instead of good in sample estimation. In our paper we use out of sample evaluation of one step ahead volatility forecast based on the loss function MSE and MAE. To better assess the forecasting performance of the various models we use the Superior Predictive Ability (SPA) test of Hansen [14]. The estimation results are given at Table-6.

Table-6: Forecasting performance comparison based on out of sample

Error Distribution	Model	MSE	MAE	SPA(p-value)	
				MSE	MAE
Gaussian	GARCH	1.02697	0.67343	0.178	0.234
	EGARCH	1.02645	0.67358	0.051	0.094
	APARCH	1.02507	0.67246	0.988	0.966
Student-t	GARCH	1.02596	0.67325	0.574	0.634
	EGARCH	1.02629	0.67354	0.328	0.289
	APARCH	1.02535	0.67311	0.901	0.889
GED	GARCH	1.02671	0.67362	0.320	0.460
	EGARCH	1.02665	0.67376	0.201	0.190
	APARCH	1.02655	0.67359	0.707	0.743
Student-t Fixed DOF	GARCH	1.02556	0.67305	0.427	0.365
	EGARCH	1.02627	0.67352	0.234	0.158
	APARCH	1.02528	0.67299	0.634	0.534
GED Fixed parameter	GARCH	1.02596	0.67330	0.334	0.432
	EGARCH	1.02639	0.67354	0.122	0.204
	APARCH	1.02592	0.67318	0.523	0.661

From Table-6 we found that APARCH model with all distributional assumption give the lowest value of MSE and MAE where EGARCH model with all distributional assumptions provide poorest forecasting performance. The comparison among densities suggest that GARCH model, student-t distribution with fixed DOF 10 give better results than other densities and for EGARCH model, student-t density give better results and for APARCH model, Gaussian density give lowest value of MSE and MAE. We also found that models with student-t density with fixed DOF 10 and GED with fixed parameter 1.5 give better forecasting performance than student-t and GED with flexible parameter densities respectively. Among these models APARCH model with Gaussian density give better results. Since we found that various distributional assumptions give better results for different models. So, in order to find out unique forecasting model for ASPI index of CSE we use SPA test of Hansen [14]. At this table we only reported the p-value of the SPA test under MSE and MAE loss function. Under the null hypothesis the base model is not outperformed by all of the other models, the higher p-value indicate the superiority of the forecasting performance. The P-value for the APARCH model with Gaussian distribution is 0.988 and 0.968 for MSE and MAE loss function respectively which is virtually close to 1 suggesting that APARCH model with Gaussian density presents the highest forecasting accuracy than other models in case of CSE.

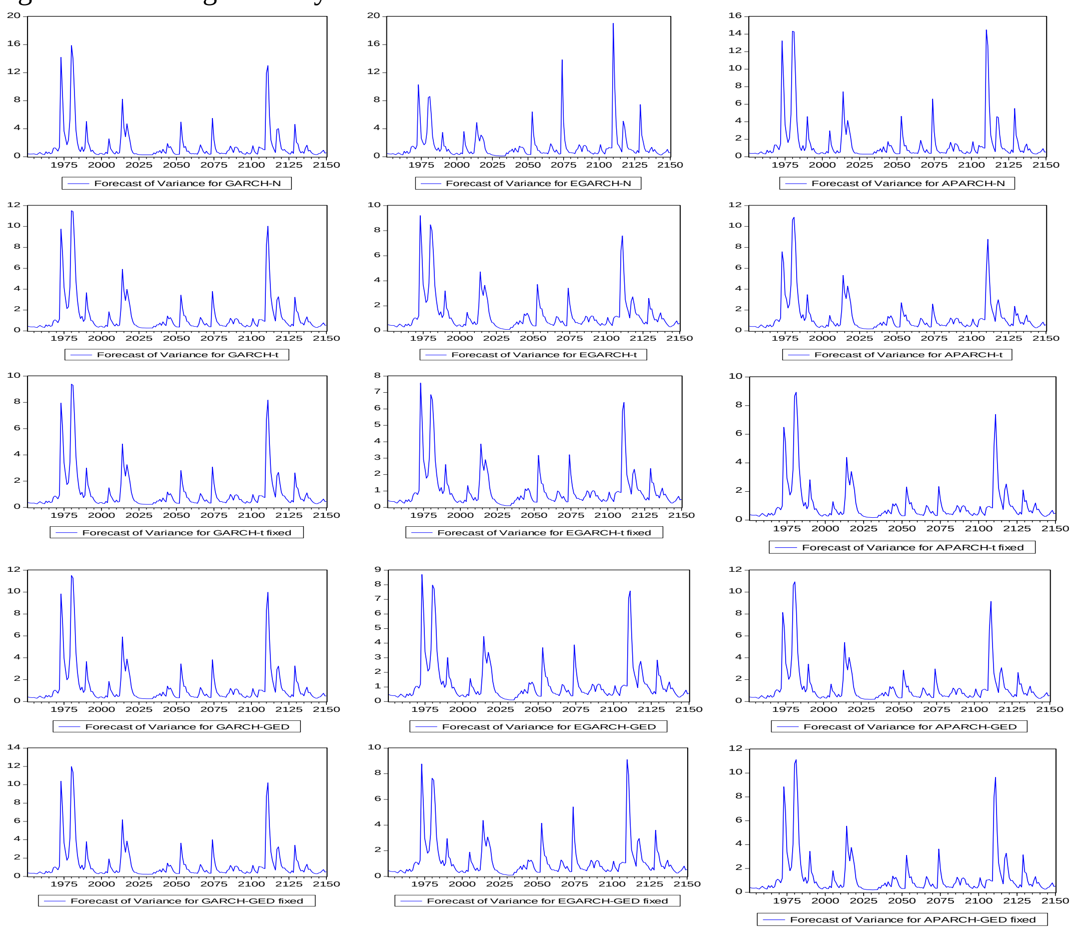

Figure 2: Forecast of variance

4 Conclusions

In this paper we compared in sample and out of sample forecasting performance of several GARCH-type models such as GARCH, EGARCH and APARCH model with Gaussian, student-t and generalized error distribution (GED), student-t with fixed DOF 10 and GED with fixed parameter 1.5 in case of Colombo Stock Exchange. Our empirical results show that noticeable improvements can be made when using asymmetric GARCH model with non normal distributional assumptions in case of in-sample estimation. The log likelihood value is strictly increasing in case of student-t distributional assumptions where fixed parameter of student-t and GED density fail to improve the log likelihood value compared to both student-t and GED distribution. Among these models APARCH model with student-t distributional assumption give better in sample estimation results of ASPI index of CSE. In the case of out-of-sample forecasting performance we found that APARCH model with all distributional assumption give the lowest value of MSE and MAE. According to the densities student-t distribution with fixed DOF 10, student-t and Gaussian distributional assumptions give better results in case of GARCH, EGARCH and APARCH model respectively. It is also observed that fixed parameter of student-t and GED improves the forecasting performance than student-t and GED density. The estimation results of SPA test suggest APARCH model with Gaussian distributional assumption give better forecasting performance in case of CSE for this study period. Therefore, the result suggests that APARCH model with student-t distributional assumption give better in sample estimation results and APARCH model with Gaussian distributional assumption is the best forecasting model in case of ASPI index of Colombo stock market, Sri Lanka.

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