Dynamic correlations and distributions of stock returns on China's stock markets

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Abstract

This paper investigates high frequency time-series features of stock returns and volatility on China's stock markets. The empirically observed probability distributions of log-returns are almost symmetric, highly leptokurtic, and characterized by a non-Gaussian profile for small index changes. Thus, the China's stock markets cannot be described by a random walk. We suggest that the correlation dynamics and stochastic changes of stock prices of China's stock markets are investigated by the Lorentz stable distribution. Features of stock price transiting from $Y(t)$ to $Y(t + \Delta t)$ for small time interval is presented by transition distribution. We give an explicit expression of the transition probability distribution for the China's stock price changes. Another successful model is the truncated Levy flight. It is shown that both the stable Lorentz and truncated Levy flight distribution are in agreement with empirical observations on China's stock markets. As a comparison, we also discuss the properties of probability distribution of returns for USA' stock markets. It is found that, in spite of immature and a segmented market with domestic investors dominating ownership of stocks, China's stock markets possess the same distribution of returns with other financial markets in the world.

JEL classification numbers: G15, G17, C46

Keywords: China's stock markets, Stochastic volatility, stable Lorentz distribution, transition probability distribution

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1 Introduction

China's stock markets have experienced tremendous development over the past decades. In the early stages, stocks had many of the characteristics of bonds and little similarity to the traditional concept of stocks in Western capitalist economies. Initially, these stocks were issued mainly to corporate employees. The primary share market in China began in 1984. A secondary trading market in China was initiated in 1986. At 1 December 1990, the trial operation of Shenzhen Stock Exchange was begun. At 19 December 1990, Shanghai Stock Exchange was established. Since then, the two official stock markets in China have expanded dramatically and became one of the leading equity markets. As of June 2019, stock markets in China have 3644 listed firms and a market capitalization of RMB 44302 billion (about US$ 6320 billion). It is now the second largest market and the fastest growing market in the world. China's stock market will continue to grow due to its rapid economic development and strong savings habits. China is also the world's largest investor and greatest contributor to global economic growth. China's stock market arguably has a crucial role to play in sustaining global economic growth.

China's stock markets attract foreign investors' attention because of potential opportunities. Despite tremendous growth, the Chinese stock markets is still far away in depth and maturity of stock exchange from financial markets in developed countries. China's market capitalization in 2018 as a proportion of GDP was about 54%. The corresponding figures for the US was over 148%, for the UK was over 98%, and for the Japan was over 106% [2].

The Chinese stock markets are highly criticized on legal framework and the rule of law. The percentage of institutional investors and personal investors is too low in China's stock markets. Reporting requirements for listed companies in China are neither well developed nor extensive. In particular, financial fraud of listed companies is more popular than those in the stock markets of developed countries. Due to restrictions on capital flows and holdings, the China's stock market is unique in the sense that it is a segmented market with domestic investors dominating ownership of stocks. Traders may base their actions on the decisions of others who may be more informed about market developments. Chinese investors have been shown to exhibit risk-seeking behavior. Trading behavior in China's stock markets is different from those in other markets.

In this paper, we investigate high frequency time-series features of stock returns and volatility on China's stock markets. It is shown that the empirically observed probability distribution of log-returns is not Gaussian and has power-law tails. The Lorentz stable distribution describes well the stochastic changes of stock prices. Features of stock price transiting from $S(t)$ to $S(t+\Delta t)$ for small time interval is presented

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by transition distribution. We give an explicit expression of the transition probability distribution for the China's stock price changes. The truncated Levy flight process is also used to investigate high frequency time series of stock returns and volatility on China's stock markets. It is shown that both the stable Lorentz distribution combined with transition distribution and the truncated Levy flight process are in agreement with empirical observations on China's stock markets.

The remainder of this paper is organized as follows. I make literature review in section 2. Section 3 presents the truncated Levy flight process and the stable Lorentz distribution combined with the transition distribution for index variation $Z(t) = Y(t+\Delta t) = Y(t)$. Section 4 reports main empirical results on China's stock markets. By using the stable Lorentz distribution and truncated Levy flight process, I present the best-fit of empirical data of the CSI 300 Index. In section 5, I compare the stock markets between China and USA. Section 6 gives conclusions.

2 Literature Review

The stock returns of volatility of China's stock markets were studied by Lee, Chen and Rui (2001 [1]). They examined time-series features of stock returns and volatility, as well as the relation between return and volatility in four of China's stock exchanges. Variance ratio tests were used to reject the hypothesis that stock returns follow a random walk. Evidence of long memory of returns was found. GARCH and EGARCH models were used to provide evidence of time-varying volatility. It was shown that volatility is highly persistent and predictable. Drew, Naughton, and Veeraraghavan (2003 [2]) tested the multi-factor approach to asset pricing in the China's Stock Exchange and found that mean-variance efficient investors in China can select some combination of small and low book-to-market equity firms in addition to the market portfolio to generate superior risk-adjusted returns. The market factor alone is not sufficient to describe the cross-section of average stock returns in China. Lima and Tabak (2004 [3]) tested the random walk hypothesis for stock markets in China, Hong Kong and Singapore. Using variance ratio tests, robust to heteroskedasticity and employing a bootstrap technique to customize percentiles for inference purposes, they found Class A shares for Chinese stock exchanges are weak form efficient. However, Class B shares for Chinese stock exchanges do not follow the random walk hypothesis. Huang and Zhu (2004 [4]) showed that leverage effect is not present and shocks have symmetric impact on the volatility of Chinese B-share stock returns. GARCH model is a better model to fit the Chinese B-share stock returns than EGARCH or GJR-GARCH model. Shin (2005 [5]) used parametric and semi-parametric GARCH in mean estimations and found a positive but insignificant relationship between expected stock returns and volatility in emerging stock markets. Using daily data over the period 1992 - 2007, Charles and Darne (2009 [6]) examined the random walk hypothesis for the Shanghai and Shenzhen stock markets for both A and B shares. They found that Class B shares for Chinese stock exchanges do not follow the random walk hypothesis, and therefore are significantly inefficient. The Class A shares seem more efficient. Kang, Cheong, and Yoon (2010 [7]) examined the long memory property in the volatility of Chinese stock markets and concluded that the volatility of Chinese stock markets exhibits long memory features, and that the assumption of non-normality provides better specifications regarding long memory volatility processes. Koutmos (2012 [8]) examined time-varying stock price and volatility dynamics of constituent industry sector indices in the Shanghai Stock Exchange. It was found that market beta risk is priced in the time-series movements of stock prices and responds positively to rises in non-diversifiable risk. In contrast to developed markets, there is no statistical evidence of volatility asymmetry. This suggests that good and bad news exert an equal impact on the conditional variance process on Chinese stock prices. Hou (2013 [9]) used a generalized additive nonparametric smoothing technique to examine the volatility of the Chinese stock markets. The empirical results indicated that an asymmetric effect of negative news exists in the Chinese stock markets. Chen (2015 [10]) discussed the relationship between stock returns and volatility of China's stock markets. He examined if there is any structural break affecting the risk-return relationship in the Chinese stock markets over time. Sehgal and Garg (2016 [11]) studied the cross-section of volatility in the context of economies of Brazil, Russia, India, Indonesia, China, South Korea, and South Africa (BRIICKS). They found that high unsystematic volatility portfolios exhibit high returns in all the sample countries except China. In the Chinese market, the estimated risk premium is statistically significantly negative. This negative risk premium is inconsistent with the theory that predicts that investors demand risk compensation for imperfect diversification. Jin (2017 [12]) examined the time-varying relationship between stock returns and volatility in sixteen stock markets during January 2001 to October 2014. He found the volatility to be long-term dependent with the Hurst exponent on a verge of stationarity and non-stationarity. The detrended cross-correlation coefficient was used to overcome this complication and found evidence of a significant and negative relationship between current stock market returns and current market volatility. He examined the dynamic behavior of the return-volatility relation by applying a rolling window approach and found that time-varying negative return-volatility relation is more likely to generate an asymmetric response with a greater effect when returns decline.

3 Methodology

One hypothesis widely tested is that stock prices follow a random walk, which implies returns are independent. The dynamics of a financial market containing derivative investment instruments is described by the Black-Scholes model (1973 [13][14]).

Usually, it is supposed that a stock price follows a geometric Brownian motion with constant volatility σ and constant drift r given by the stochastic differential equation $dY = Y(\sigma dB + rdt)$, where B is a Wiener process. The expression for Y, thus, is of the form

$$Y = Y_0 \exp\left(\sigma B + \left(r - \frac{\sigma^2}{2}\right) t\right). \tag{1}$$

The correction term of $-\sigma^2/2$ corresponds to the difference between the median and mean of the log-normal distribution, or equivalently for this distribution, the geometric mean and arithmetic mean, with the median (geometric mean) being lower. The solution of the geometric Brownian motion gives a familiar log-normal probability distribution for stock price changes. However, empirical observations on financial markets show that the tails of the distribution decay slower than the log-normal distribution (see 1998 [15]; 2000 [16]; 2003 [17]). A variety of models have been proposed to improve the geometric Brownian motion assumption underlying the Black-Scholes model. There is empirical evidence indicating that volatility is driven by a mean-reverting stochastic process (see 2000 [18]; 2001 [19]; 2011 [20]; 2014 [21]; 2014 [22]). It has been suggested that the volatility of an empirical financial market should be a stochastic quantity, instead of a constant as in the Black-Scholes model (see 1987 [23][24][25]; 1991 [26]; 1992 [27]; 1993 [28][29]; 1997 [30][31]; 2000 [32]). The autoregressive conditional heteroscedasticity (ARCH) model (see 1982 [33]; 1998 [34]) describes the variance of the current error term or innovation as a function of the actual sizes of the previous time periods' error terms. If an autoregressive moving average model is assumed for the error variance, the model is a generalized autoregressive conditional heteroscedasticity (GARCH) model (see 1986 [35]; 2000 [36]; 2004 [37]; 2010 [38]; 2014 [21][22]). GARCH models are commonly employed in modeling financial time series that exhibit time-varying volatility and volatility clustering.

In particular, the power-law tails or scaling behavior of empirical observations on financial markets have been studied extensively (1963 [39]; 1995 [40]; 1998 [41]; 1999 [42]; 2000 [43]). Mandelbrot (1963 [39]) first noticed the scaling properties of financial markets. It has been shown that the price distribution of financial assets is of the form of power law. Thus, we can say that the financial markets have obvious properties of scaling invariant (1999 [44]; 2005 [45]). Scaling properties of financial data have been analyzed from an econ-physics point of view (see 1995 [40]; 1996 [46]; 2000 [43]; 2001 [47][49]). Authors tried to establish a connection between the financial markets with critical phenomena. As a matter of fact, the dynamics of underlying prices assumed in the Black-Scholes framework is not consistent with the scaling properties of equity indices. This is a source of derivative pricing inefficiencies.

In this paper, we use the truncated Levy flight process and stable Lorentz distribution combined with transition distribution to examine the high frequency time series of China's stock markets.

3.1 Truncated Levy Flight: a Ultra-slow Convergence Process

The stable distribution is a specific type of distribution encountered in the sum of n independent identically distributed random variables that has the property that it does not change its functional form for different values of n. It is not difficult to prove that the Gaussian and Lorentzian distributions are stable. Another stable distribution is the symmetric Levy distribution of index α and scale factor γ,

$$P_L(Z) = \frac{1}{\pi} \int_0^\infty e^{-\gamma |q|^\alpha} \cos(qZ) dq \ . \tag{2}$$

The asymptotic approximation of a stable Levy distribution of index α valid for large values of Z, $P_L(Z) \sim |Z|^{-(1+\alpha)}$. Thus, the asymptotic behavior of the symmetric Levy stable distribution for large values of Z is a power-law. The property has deep consequences for the moments of the distribution. A Levy stable process has infinite variance. It does not have a characteristic scale - the variance is infinite.

A stochastic process with finite variance and characterized by scaling relations in a large but finite interval is the truncated Levy flight process (1994 [50]). The truncated Levy flight process distribution is defined by

$$P(Z) = \begin{cases} 0 \ , & Z > l \ , \\ cP_L(Z) \ , & -l \geq Z \geq l \ , \\ 0 \ , & Z < -l \ , \end{cases} \tag{3}$$

where l is the cutoff length, c is a normalizing constant.

A truncated Levy flight process has finite variance. The convergence of a sum of truncated Levy flight processes to a normal process is very slow. Ten thousands of independent events (or time intervals) may be necessary to ensure convergence to a Gaussian stochastic process. Thus, in stock markets with finite variance, one can observe a sum of a huge number of independent stochastic variables which converges to a truncated Levy flight process. A truncated Levy flight process differs from a Levy stable distribution only in the very far wings.

3.2 Lorentz Stable Distribution and Transition Distribution

The Langevin equation for a stock price $Y(t)$ is of the form

$$\frac{dY(t)}{dt} = rY + \sigma Y \eta(t) \ , \tag{4}$$

in which $\eta(t)$ is a set of stochastic functions called the noise, r is the drift coefficient and σ is volatility. The noise can be defined by a functional probability distribution

$[dP(\eta)]$. The probability distribution for a Gaussian white noise related with Markov's processes is of the form

$$[dP(\eta)] = [d\eta] e^{-\frac{1}{2\Omega(S)} \int \eta_i^2(t) dt} \ . \tag{5}$$

$\Omega(S)$ characterizes the width of the noise distribution. It is easy to verify that the 1−point and 2−point correlations for the Gaussian white noise are

$$\begin{aligned} \langle \eta(t) \rangle &= 0 \ , \\ \langle \eta(t) \eta(t') \rangle &= \Omega(S) \delta(t - t') \ . \end{aligned} \tag{6}$$

Given the value of $Y(t)$ at initio time t_0, $Y(t_0) = Y_0$, the Langevin equation generates a time-dependent probability distribution $P(Y, t)$ for the stochastic variable $Y(t)$, which can be formally written as

$$P(Y, t) = \langle \delta[Y(t) - Y] \rangle \ . \tag{7}$$

The brackets indicate average over noise.

The probability distribution $P(Y, t)$ satisfies the Fokker-Planck equation,

$$\frac{\partial P(Y,t)}{\partial t} = \frac{1}{2} \frac{\partial}{\partial Y} \left[\sigma Y \Omega(Y) \frac{\partial P(Y,t)}{\partial Y} - 2rYP(Y,t) \right] \ . \tag{8}$$

The stationary solution, $P_0(Y)$, satisfies the stationary Fokker-Planck equation,

$$\frac{\partial}{\partial Y} \left[\sigma Y \Omega(Y) \frac{\partial P_0(Y)}{\partial S} - 2rYP_0(Y) \right] = 0 \ . \tag{9}$$

After integrating this equation, we have

$$\sigma Y \Omega(Y) \frac{\partial P_0(Y)}{\partial Y} - 2rYP_0(Y) = C \ , \tag{10}$$

where C is an integral constant. It should be noticed that the equation is satisfied for any value of Y. At the case of $Y = 0$, we get $C = 0$. Thus, the above equation reduces as

$$\sigma \Omega(Y) \frac{\partial P_0(Y)}{\partial Y} - 2rP_0(Y) = 0 \ . \tag{11}$$

And then the stationary Fokker-Planck equation is solved exactly,

$$P_0(Y) \sim \exp\left(\int \frac{2r}{\sigma \Omega(Y)} dY \right) \ . \tag{12}$$

We set the width of diffusion as $\Omega(Y) = -\frac{r}{\sigma} \frac{Y^2 + \gamma^2}{Y}$ and obtain the Lorentz distribution

$$P_0(Y) = \frac{\gamma}{\pi} \frac{1}{Y^2 + \gamma^2} \ . \tag{13}$$

To discuss the stock price changes, we may define a conditional probability density distribution as the probability density distribution of the random variable Y at time $t + \Delta t$ ($Y(t + \Delta t)$) under the condition that the random variable at the time t has the sharp value $Y(t)$,

$$P(Y(t + \Delta t), t + \Delta t | Y(t), t) = \langle \delta(Y - Y(t + \Delta t)) \rangle |_{Y = Y(t)} \ . \tag{14}$$

For small Δt, the transition distribution is of the form (1979 [51]; 1984 [52]),

$$P(Z, \Delta t) = \int \frac{1}{\sqrt{2\pi\sigma\Omega(Y)\Delta t}} \exp\left(-\frac{[Z - \frac{1}{2}\left(\sigma \frac{d\Omega(Y)}{dY} - \sigma^2\right)\Delta t]^2}{2\sigma\Omega(Y)\Delta t} \right) \frac{\gamma}{\pi} \frac{1}{\gamma^2 + Y^2} dY \ . \tag{15}$$

4 High Frequency Time Series of China's Stock Markets

On April 8, 2005, the China Securities Index Company Ltd created the CSI 300 Index. The CSI 300 Index is the first broadly based stock market index of China. It is a value weighted stock market index comprising 300 large-capitalization and actively traded stocks listed on the Shanghai or Shenzhen Stock Exchange. China Securities Index Company Ltd reconstitutes the CSI 300 Index every 6 months. The CSI 300 Index constituent stocks represent approximately 70% of the total market capitalization in China's stock markets at the end of 2018. Investors generally agree that the CSI 300 Index reflects the overall performance of the entire China's stock markets. At 16 April, 2010, the CSI 300 Index futures were launched on the China Financial Futures Exchange and have become one of the most actively traded financial instruments of the Chinese financial markets.

The 1-minute frequency price data of the CSI 300 Index were obtained from the China Securities Index Company Ltd. The sample period is from April 1, 2005 to June 30, 2019. We denote the value of the CSI 300 Index as $Y(t)$, and the succussive variation of the CSI 300 Index is

$$Z(t) \equiv Y(t + \Delta t) - Y(t) \ . \tag{16}$$

To measure quantitatively the empirically observed process, we calculate the probability distribution $P(Z)$ of index variations for different values of Δt. Here, we choose $\Delta t = 1,\ 2,\ 3,\ 6,\ 10,\ 18,\ 32,\ 56,\ 100$ minutes.

Figure 2 is a semilogarithmic plot of $P(Z)$ obtained for nine different time interval Δt. It is obvious that the distributions are leptokurtic, and have more fat tails than

Figure 1: The five years of the CSI 300 Index of China's stock markets.

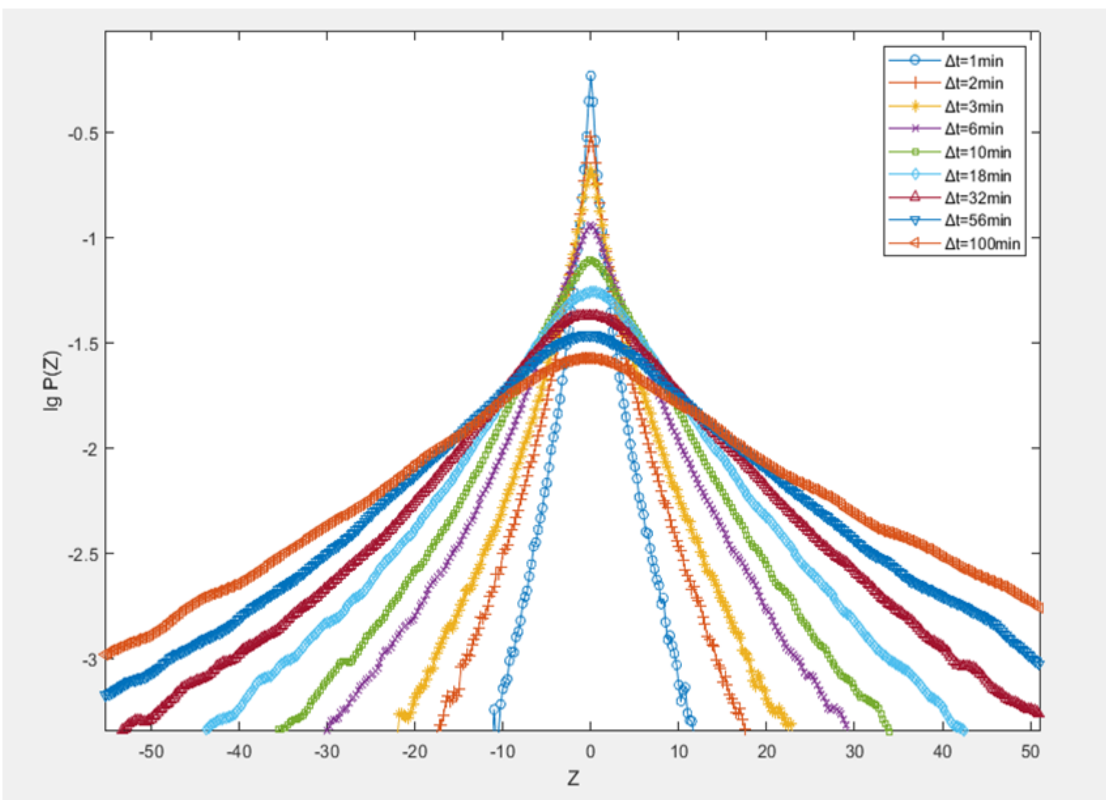

Figure 2: Probability density distributions $P(Z)$ of index variation $Z(t)$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100 minutes for high-frequency data for the CSI 300 index of China's stock markets during the period April 2005 and June 2019. The price changes is in unit of a standard σ. Probability density distributions of price changes are almost symmetric, highly leptokurtic, and characterized by a non-Gaussian profile for small index changes. By increasing Δt, a spreading of probability distribution characteristic of a random walk is observed.

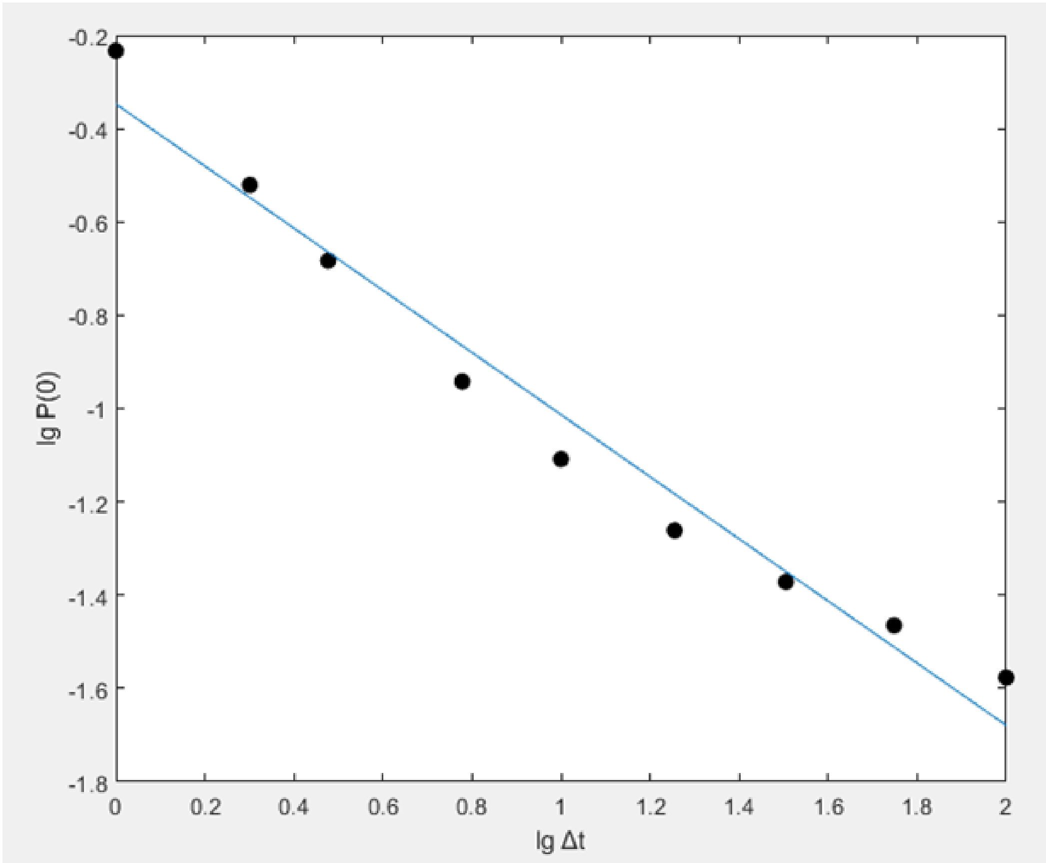

Figure 3: Probability of return $P(0)$ of $Z(t)$ for the CSI 300 Index of China's stock markets as a function of the time sampling intervals Δt. The slop of best-fit straight line is -0.6658 with $R^2 = 0.9856$.

log-normal Gaussian. That is the CSI 300 Index can not be described well by a random walk. The distributions are roughly symmetric and are spreading when the time interval Δt increases.

We first focus on the probability of return $P(Z = 0)$ as a function of the time interval Δt. The regression of $\log P(0)$ on $\log \Delta t$ gives a coefficient of -0.6658 with $R^2 = 0.9856$. In figure 3, we present the relation between the probability of return $P(Z = 0)$ and time interval Δt in a log-log plot. The scaling variables for a Levy stable process of index α are

$$\begin{aligned} Z_s &\equiv \frac{Z}{(\Delta t)^{1/\alpha}} \ , \\ P(Z_s) &\equiv \frac{P(Z)}{(\Delta t)^{-1/\alpha}} \ . \end{aligned} \tag{17}$$

Thus, the regression gives the parameter $\alpha = 1.5019$ for the CSI 300 index of China's stock markets. In Figure 4, we show that the scaled probability distributions with

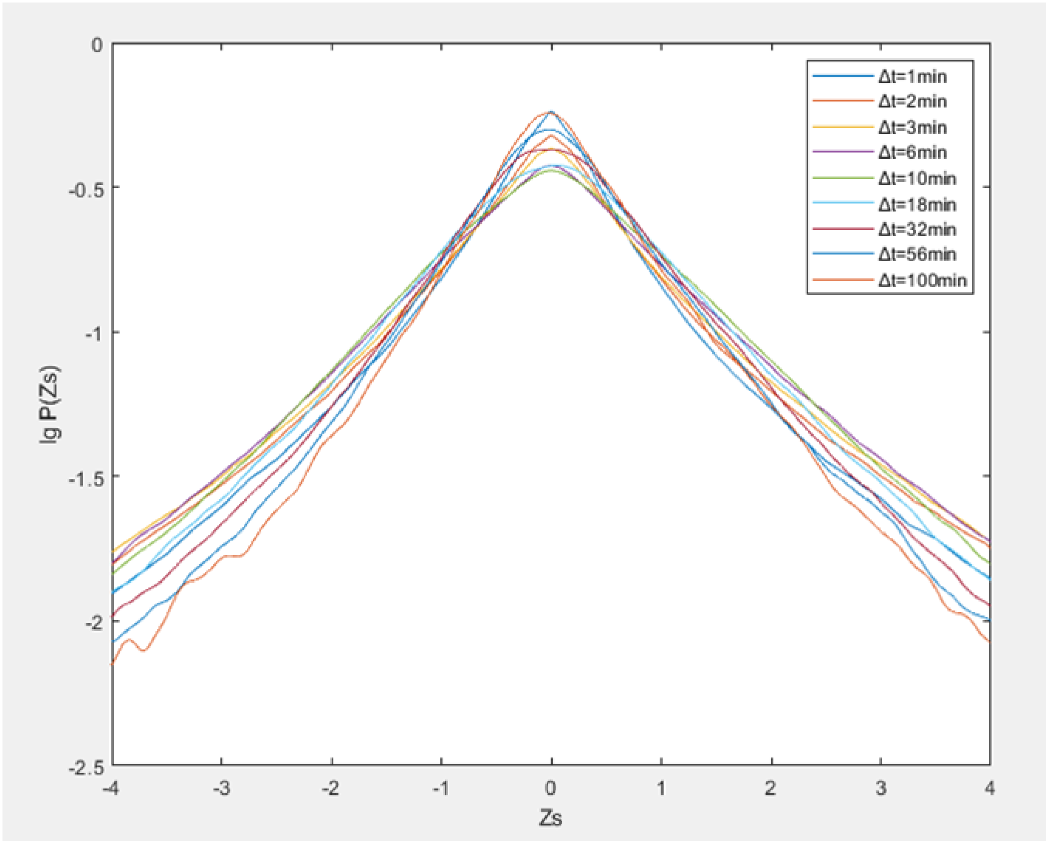

Figure 4: Scaled plot of the probability density distributions $P(Z_s)$ of index variation $Z_s(t) = Z(t)/(\Delta t)^{-0.6658}$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100 minutes for high-frequency data for the CSI 300 index of China's stock markets during the period April 2005 and June 2019.

different time horizons collapses almost all on the $\Delta t = 1$ min distribution.

We note that the mean and volatility of index variation with different time horizon are also different. The regression of the mean of index variation on the time horizon Δt gives

$$\mu = 0.0053\Delta t \ , \qquad R^2 = 0.9708 \ . \tag{18}$$

The plot of the relation between the mean of index variation and the time interval Δt is shown in figure 5.

The regression of the volatility of index variation on the time horizon Δt gives

$$\sigma^2 = 9.156\Delta t \ , \qquad R^2 = 0.9992 \ . \tag{19}$$

The plot of the relation between the volatility of index variation and the time interval Δt is shown in figure 6.

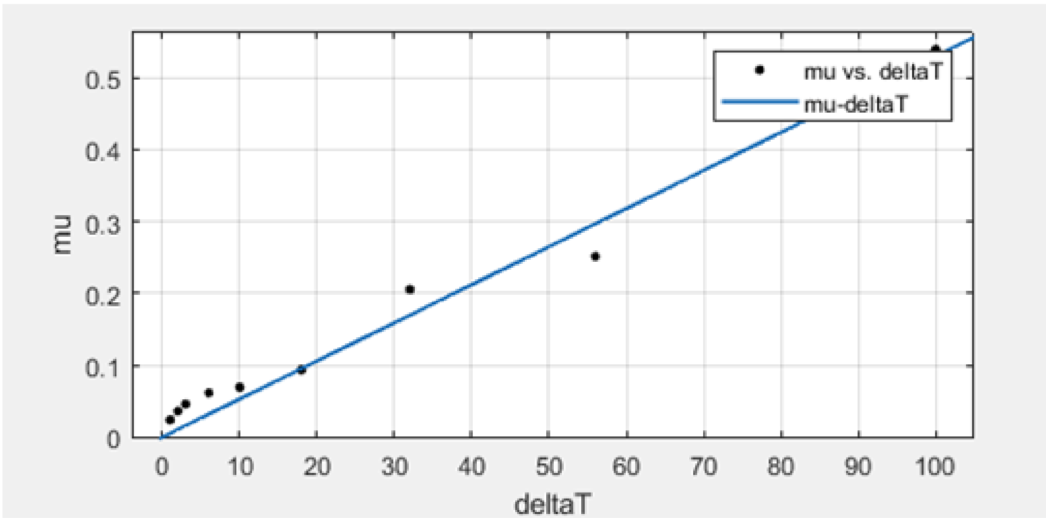

Figure 5: The regression of the mean μ for $Z(t)$ of the CSI 300 Index of China's stock markets on the time sampling intervals Δt. The slop of best-fit straight line with 95% confidence bounds is 0.0053 with $R^2 = 0.9708$.

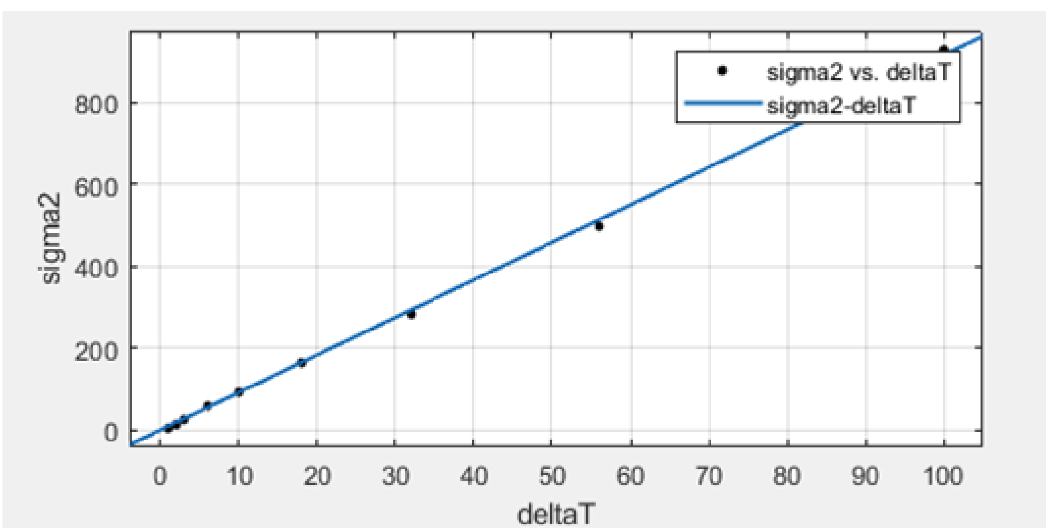

Figure 6: The regression of the volatility σ^2 for $Z(t)$ for the CSI 300 Index of China's stock markets on the time sampling intervals Δt. The slop of best-fit straight line with 95% confidence bounds is 9.156 with $R^2 = 0.9992$.

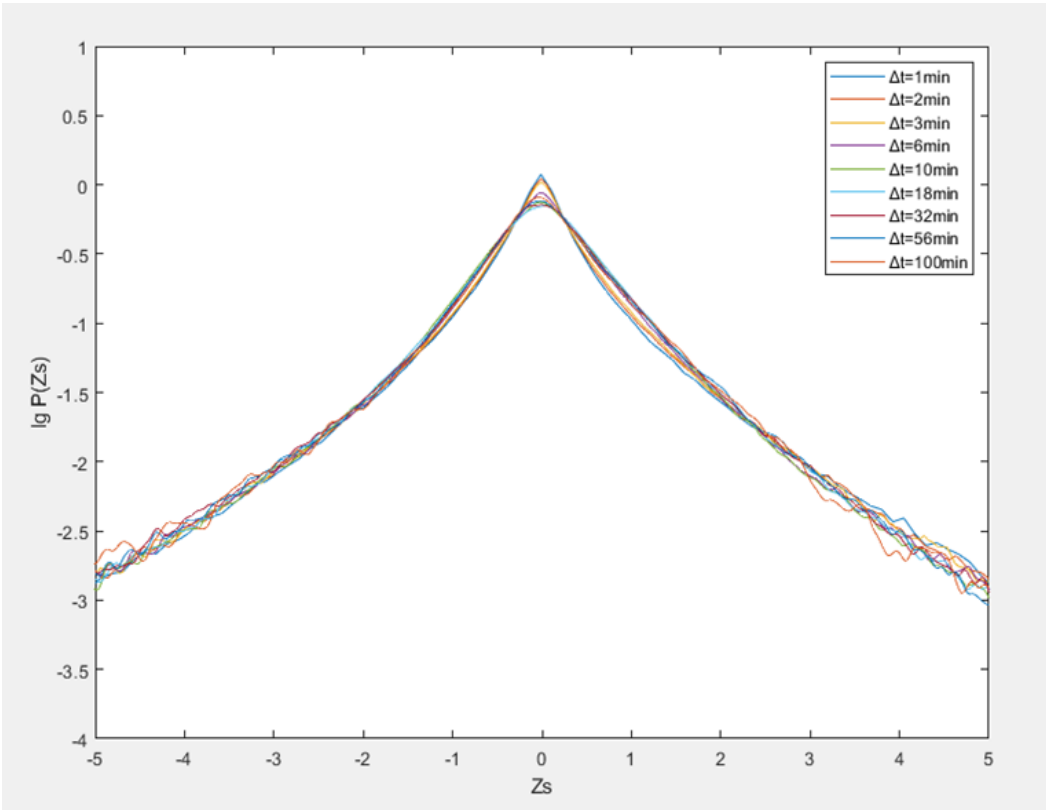

Figure 7: Scaled plot of the probability density distributions $P(Z_s)$ of index variation $Z_s(t) = (Z(t) - \mu)/\sigma$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100 minutes for high-frequency data for the CSI 300 index of China's stock markets during the period April 2005 and June 2019.

Taking the dependence of the mean and volatility of the index variations on the time interval Δt into account, we can introduce another scaled variable,

$$Z'_s \equiv \frac{Z - \mu}{\sigma} \ . \tag{20}$$

In Figure 7, it is shown that the scaled probability distributions $\log P(Z'_s)$ for the CSI 300 index of China's stock markets with different time horizons collapses all on the $\Delta t = 1$ min distribution.

We now at the position to give a best-fit of the empirical data by using the truncated Levy flight and stable Lorentz distribution. For the truncated Levy flight process, we get the precise form as

$$P(Z) = \begin{cases} 0 \ , & Z > 100 \ , \\ \dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-0.5269|q|^{1.5019}} \cos(qZ) dq \ , & -100 \geq Z \geq 100 \ , \\ 0 \ , & Z < -100 \ . \end{cases} \tag{21}$$

For the stable Lorentz distribution, we obtain the transit probability distribution of the index variation as

$$P(Z) = \int \frac{1}{\sqrt{2\pi * 0.005847\Omega(Y)\Delta t}} \exp\left(-\frac{[Z - \frac{1}{2}\left(0.005847 * \frac{d\Omega(Y)}{dY} - 0.005847^2)\Delta t\right)]^2}{2 * 0.005847\Omega(Y)\Delta t}\right)$$
$$* \frac{0.003304}{\pi} \frac{1}{0.003304^2 + Y^2} dY \ , \tag{22}$$

where $\Omega(Y) = \frac{1}{0.005847} \frac{0.003304^2+Y^2}{2Y}$.

In Figure 8, 9, 10 and 11, we show a best-fit of the empirical data with time horizon Δt=1, 10, 32, 100 minutes, respectively, with the truncated Levy flight process (25) and the stable Lorentz transition distribution (22). Both truncated Levy flight process and the stable Lorentz transition distribution describe well returns of the CSI 300 Index of China's stock markets. They give almost symmetric and highly leptokurtic probability distributions for index variations. In particular, both truncated Levy flight process and the stable Lorentz transition distribution are characterized by a non-Gaussian profile for small index changes.

5 Comparisons with Stock Markets in USA

Stocks have been traded in the United States of America (USA)for more than two hundred years. The past few decades have witnessed a sharp rise in both the number of companies that have issued stocks to the public, and in the volume of shares traded. The USA's stock market is the largest in the world. More than 10,000 companies are traded in a number of different exchanges. Included in these, are some of the largest corporations in the world, for example, Microsoft Corporation - the largest software company; the Coca-Cola Company - the largest beverage company; Citigroup - the worlds largest bank.

It is interesting to compare the correlation dynamics and distributions of index returns of USA's stock markets and China's stock markets. We repeat our previous analysis on the S&P 500 Index of USA's stock markets. The 1-minute frequency price data of the S&P 500 Index were obtained from the Chicago Mercantile Exchange. The sample period is from April 2, 2007, to June 8, 2019.

Figure 12 is a semilogarithmic plot of $P(Z)$ for S&P 500 Index obtained for twelve different time interval Δt. It shows almost same behaviors with the CSI 300 Index of China stock markets. The distributions are leptokurtic, and fat tailed. They are also spreading when the time interval Δt increases.

The regression of $\log P(0)$ on $\log \Delta t$ for the S&P 500 Index gives a coefficient of

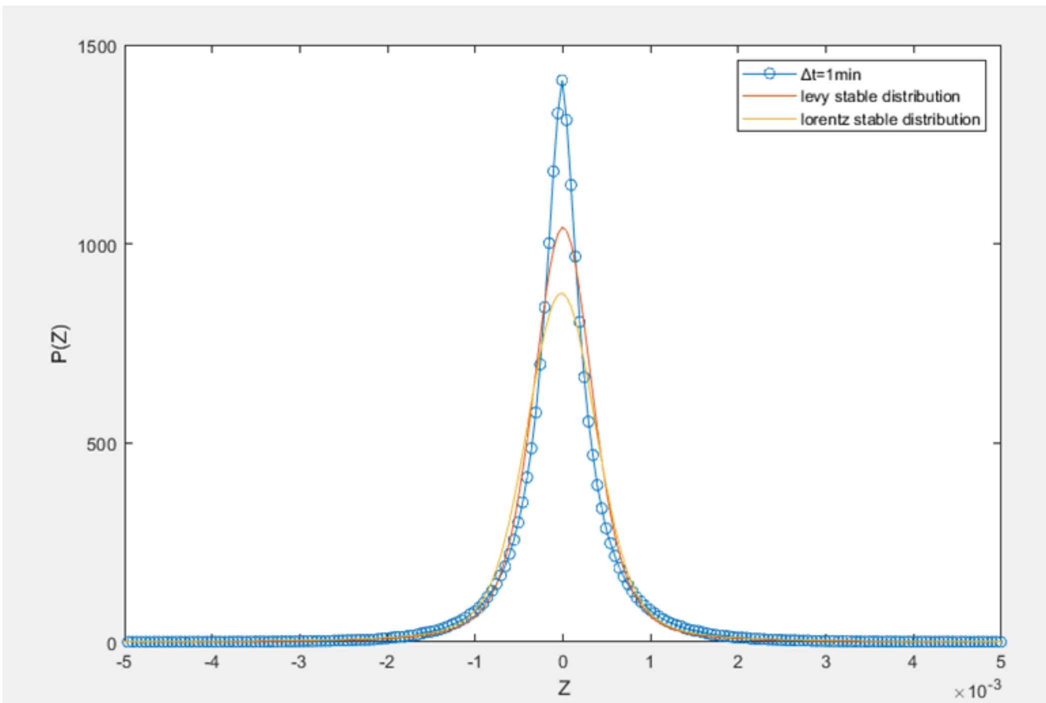

Figure 8: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the CSI 300 Index of China's stock markets with time interval $\Delta t = 1$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 1.5019$, $\gamma = 0.5269$. For stable Lorentz distribution, best-fit parameters are $\sigma = 0.005847$, $\gamma = 0.003304$.

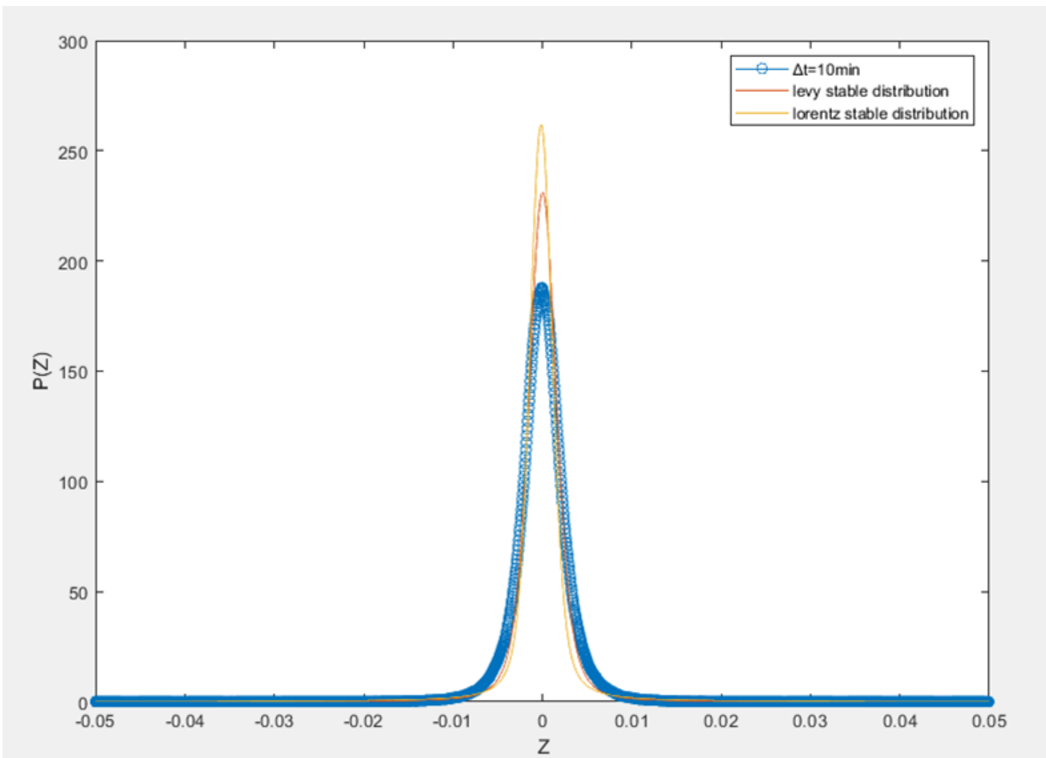

Figure 9: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the CSI 300 Index of China's stock markets with time interval $\Delta t = 10$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 1.5019$, $\gamma = 0.5269$. For stable Lorentz distribution, best-fit parameters are $\sigma = 0.005847$, $\gamma = 0.003304$.

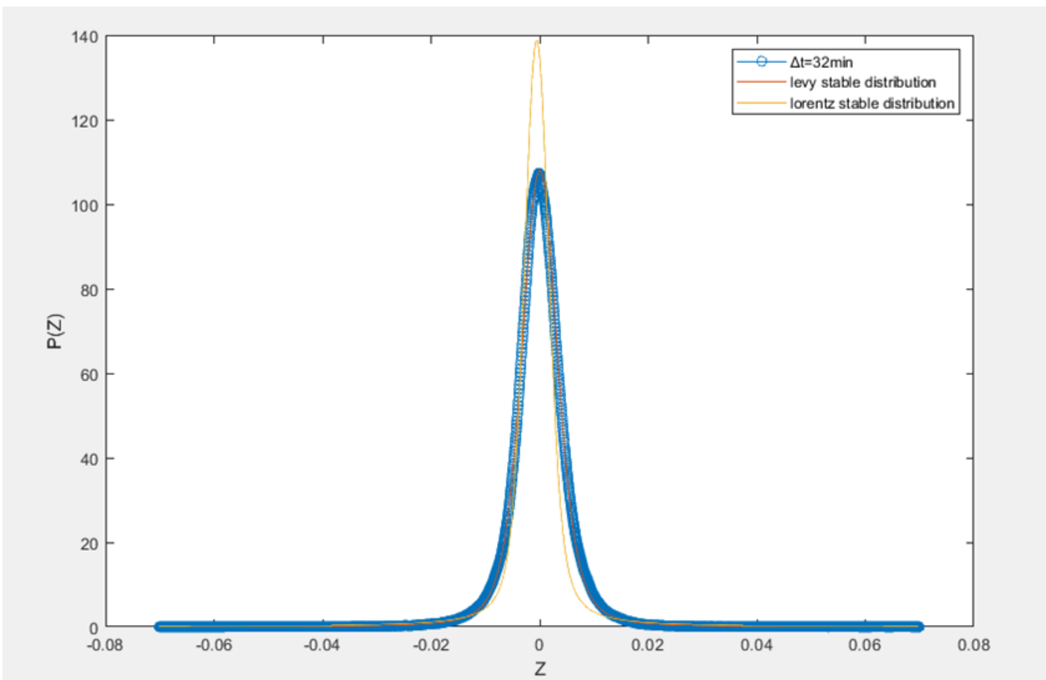

Figure 10: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the CSI 300 Index of China's stock markets with time interval $\Delta t = 32$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 1.5019$, $\gamma = 0.5269$. For stable Lorentz distribution, best-fit parameters are $\sigma = 0.005847$, $\gamma = 0.003304$.

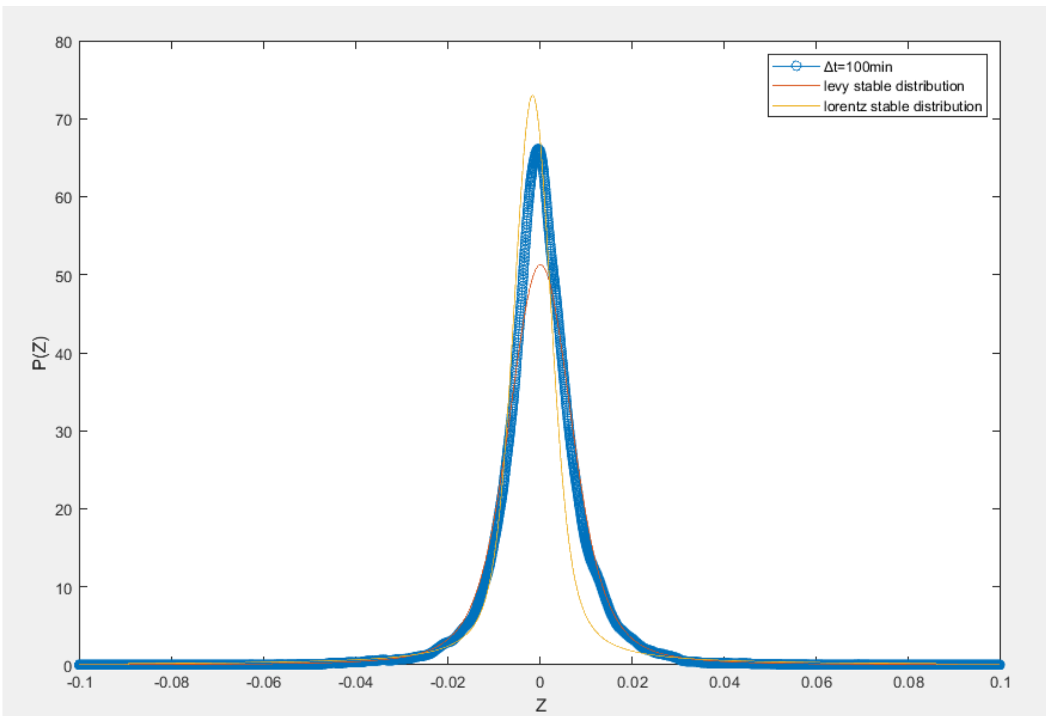

Figure 11: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the CSI 300 Index of China's stock markets with time interval $\Delta t = 100$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 1.5019$, $\gamma = 0.5269$. For stable Lorentz distribution, best-fit parameters are $\sigma = 0.005847$, $\gamma = 0.003304$.

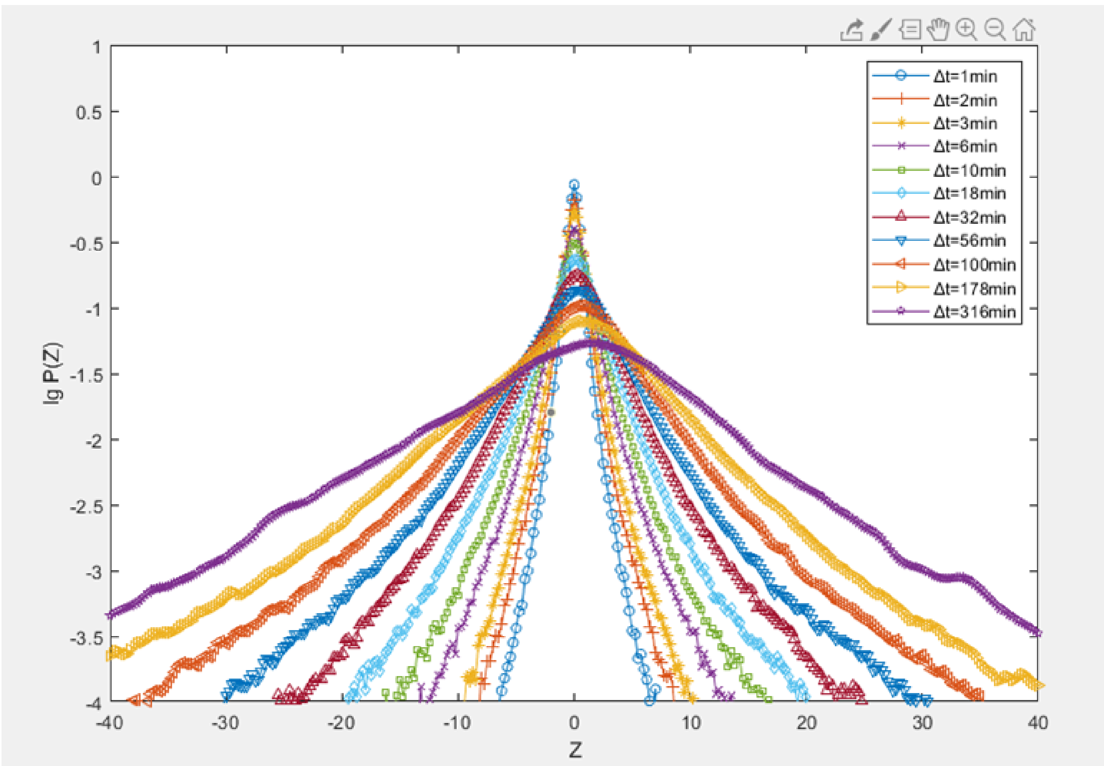

Figure 12: Probability density distributions $P(Z)$ of index variation $Z(t)$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100, 178, 316 minutes for high-frequency data for the S&P 500 Index of USA's stock markets during the period April 2, 2007 and June 8, 2019. The price changes is in unit of a standard σ. Probability density distributions of price changes are almost symmetric, highly leptokurtic, and characterized by a non-Gaussian profile for small index changes. By increasing Δt, a spreading of probability distribution characteristic of a random walk is also observed.

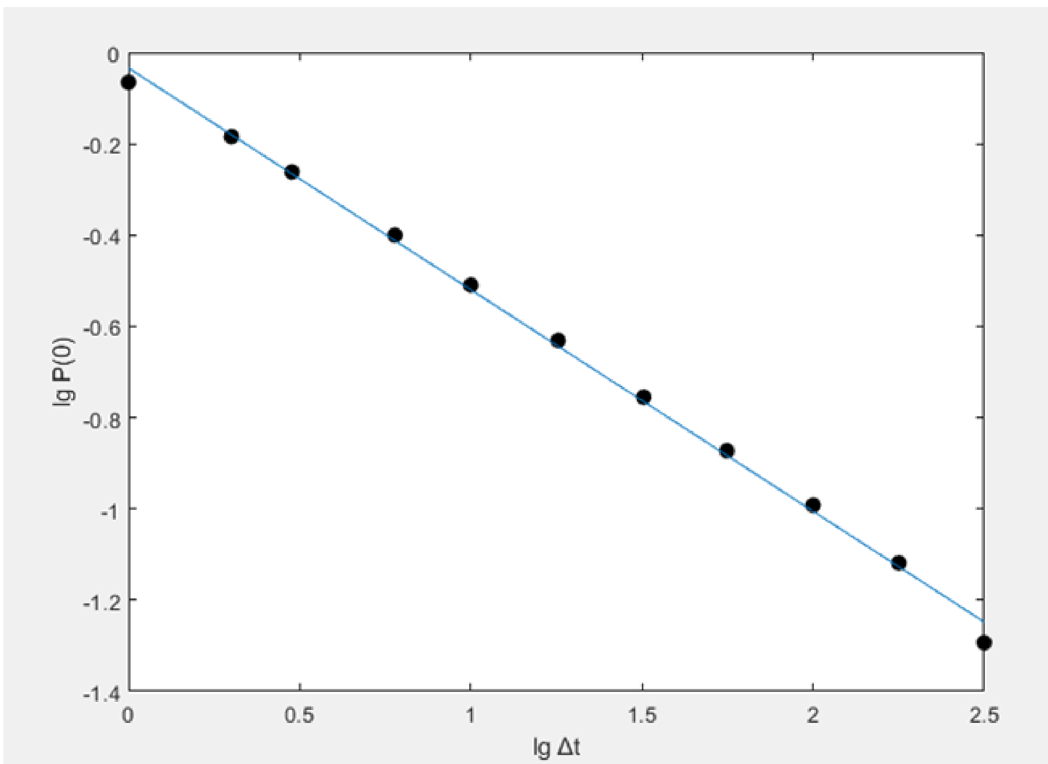

Figure 13: Probability of return $P(0)$ of $Z(t)$ for the S&P 500 Index as a function of the time sampling intervals Δt. The slop of best-fit straight line is -0.4855 with $R^2 = 0.9988$.

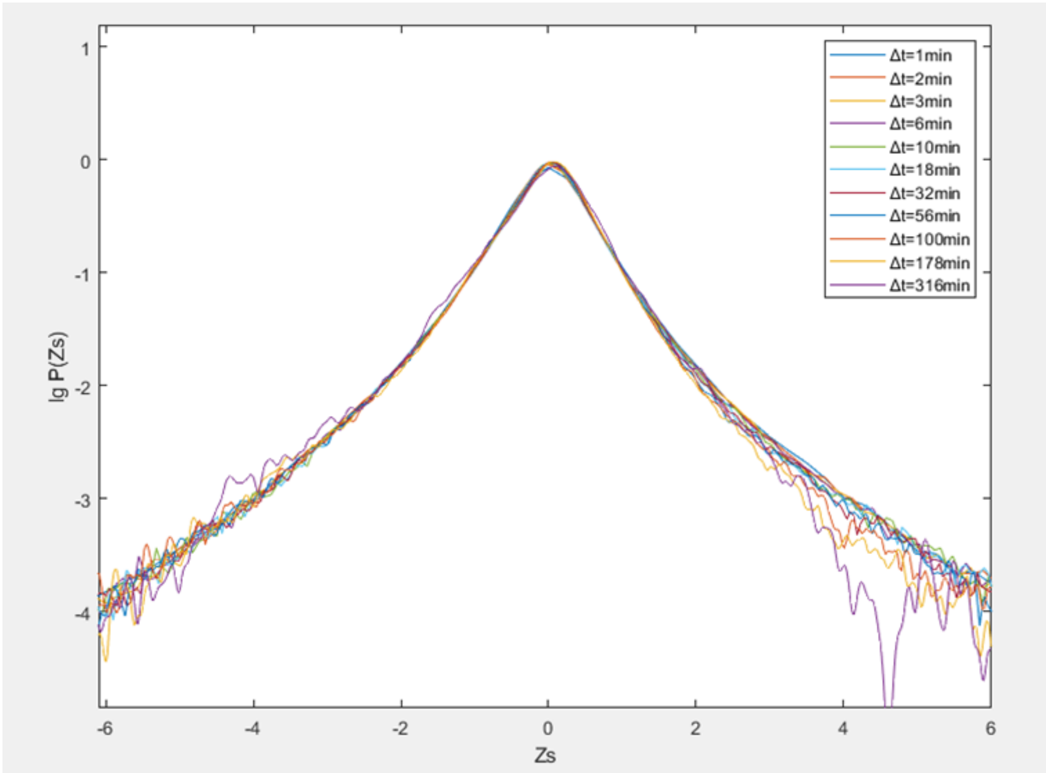

Figure 14: Scaled plot of the probability density distributions $P(Z_s)$ of index variation $Z_s(t) = Z(t)/(\Delta t)^{-0.4855}$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100, 178, 316 minutes for high-frequency data for the S&P 500 Index of USA's stock markets during the period April 2, 2007 and June 8, 2019.

-0.4866 with $R^2 = 0.9988$. In figure 13, we present the relation between the probability of return $P(Z = 0)$ and time interval Δt in a log-log plot. The scaling variable for the Levy stable process is $\alpha = 2.0596$. It is a bit bigger than $\alpha = 1.5019$ for the CSI 300 index of China's stock markets. In Figure 14, we show that the scaled probability distributions with different time horizons collapses also almost all on the $\Delta t = 1$ min distribution.

The regression of the mean of index variation for the S&P 500 Index of USA's stock markets on the time horizon Δt gives

$$\mu = 0.0004217\Delta t \ , \qquad R^2 = 0.997 \ . \tag{23}$$

The plot of the relation between the mean of index variation for the S&P 500 index and the time interval Δt is shown in figure 15. The mean of index variation for the S&P 500 Index of USA's stock markets is less than one tenth of that for the CSI 300 index of China's stock markets. Thus, the probability distribution of returns for USA' stock markets is more symmetric than that of China's stock markets.

The regression of the volatility of index variation for the S&P 500 Index of USA's

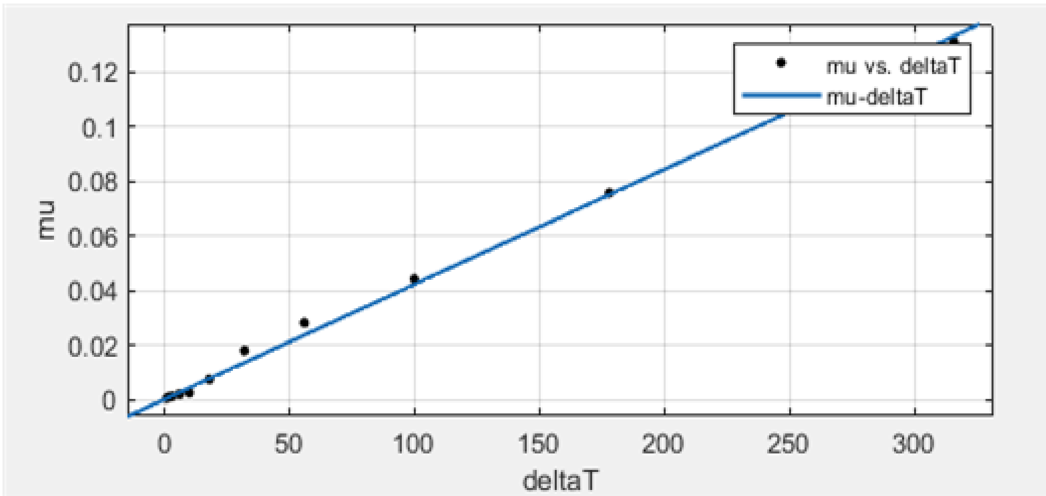

Figure 15: The regression of the mean μ for $Z(t)$ of the S&P 500 Index of USA's stock markets on the time sampling intervals Δt. The slop of best-fit straight line with 95% confidence bounds is 0.0004217 with $R^2 = 0.997$.

stock markets on the time horizon Δt gives

$$\sigma^2 = 0.414\Delta t \ , \qquad R^2 = 0.9975 \ . \tag{24}$$

The plot of the relation between the volatility of index variation for the S&P 500 Index and the time interval Δt is shown in figure 16. The volatility of USA's stock markets is less than one twentieth that of China's stock markets. It is in agreement with our expectations.

By using the scaled variable Z'_s $(\frac{Z-\mu}{\sigma})$, we show in Figure 17 that the scaled probability distributions $\log P(Z'_s)$ for the S&P 500 Index with different time horizons collapses all on the $\Delta t = 1$ min distribution also.

For the truncated Levy flight process, we get the precise form for the S&P 500 Index of USA's stock markets as

$$P(Z) = \begin{cases} 0 \ , & Z > 100 \ , \\ \dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-0.0871|q|^{2.0596}} \cos(qZ)dq \ , & -100 \geq Z \geq 100 \ , \\ 0 \ , & Z < -100 \ . \end{cases} \tag{25}$$

It is really remarkable to note that both stock markets of China and USA obey the same distribution of index returns. In Figure 18, 19, 20 and 21, we show a best-fit of the empirical data for the S&P 500 Index with time horizon Δt=1, 10, 32, 100 minutes, respectively, with the truncated Levy flight process (25).

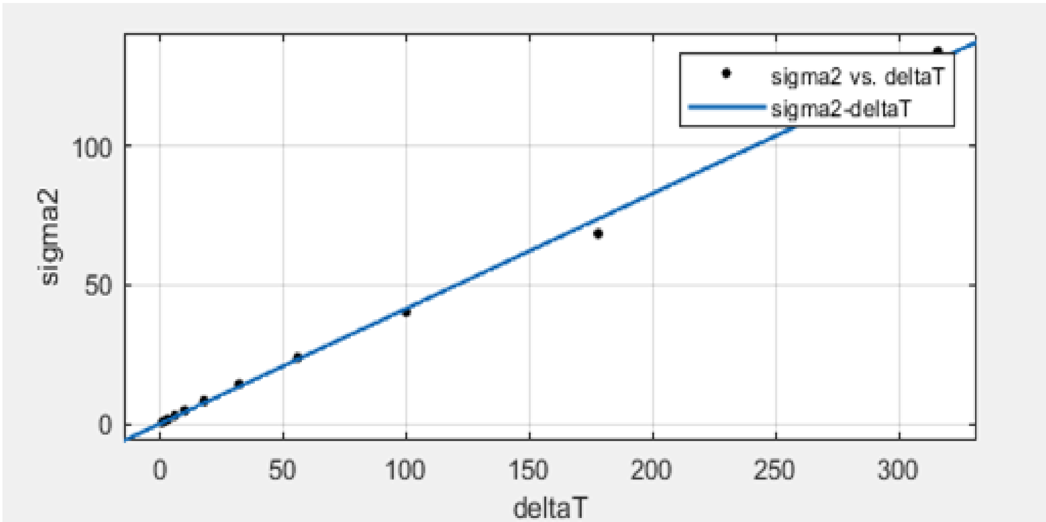

Figure 16: The regression of the volatility σ^2 for $Z(t)$ for the S&P 500 Index of USA's stock markets on the time sampling intervals Δt. The slop of best-fit straight line with 95% confidence bounds is 0.414 with $R^2 = 0.9976$.

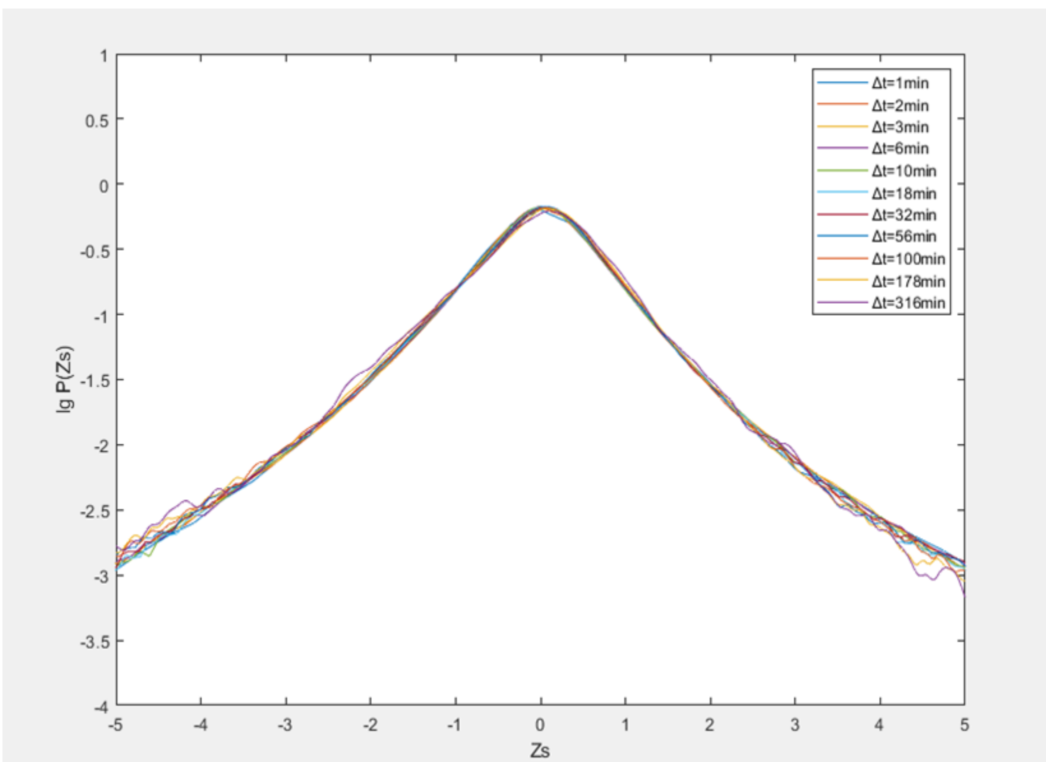

Figure 17: Scaled plot of the probability density distributions $P(Z_s)$ of index variation $Z_s(t) = (Z(t) - \mu)/\sigma$ measured at different time horizons (Δt) 1, 2, 3, 6, 10, 18, 32, 56, 100, 178, 316 minutes for high-frequency data for the S&P 500 Index of USA's stock markets during the period April 2, 2007 and June 8, 2019.

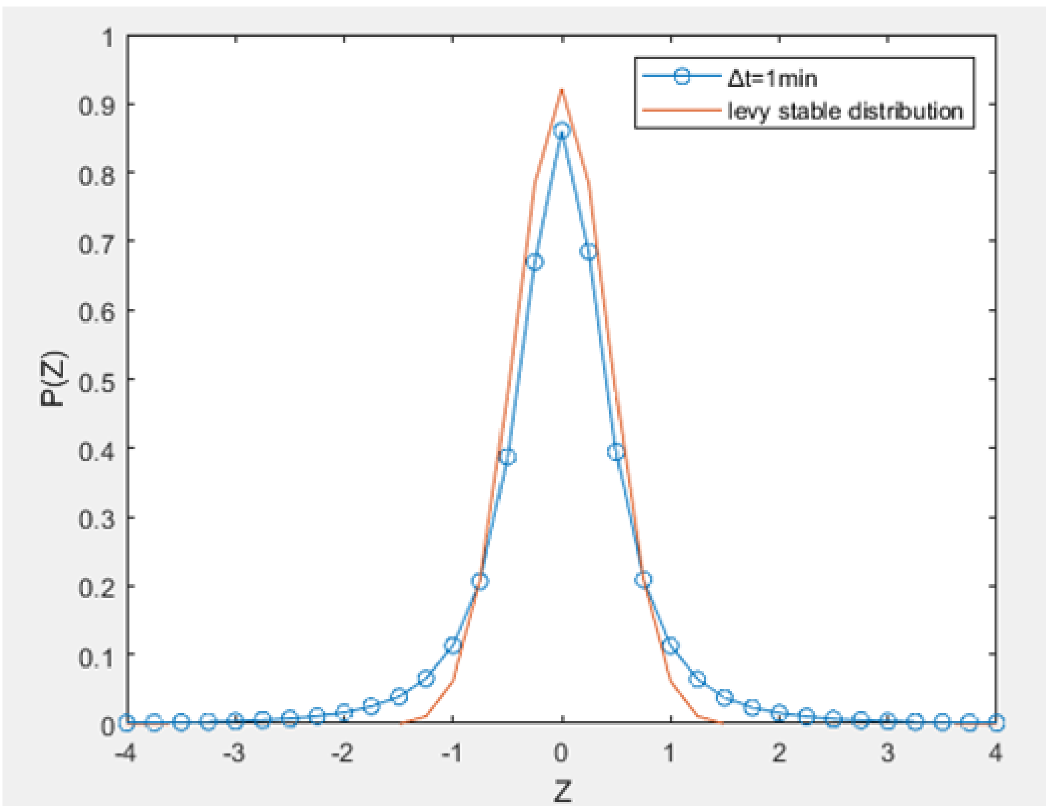

Figure 18: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the S&P 500 Index of USA's stock markets with time interval $\Delta t = 1$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 2.0596$, $\gamma = 0.0871$.

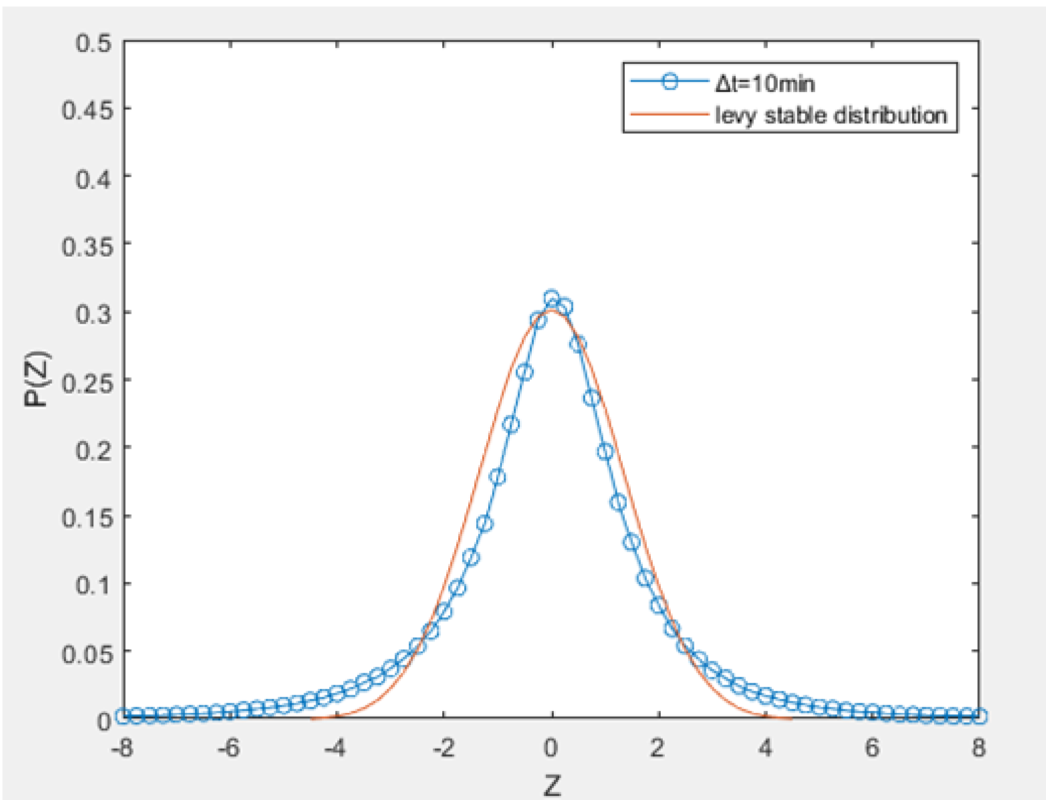

Figure 19: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the S&P 500 Index of USA's stock markets with time interval $\Delta t = 10$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 2.0596$, $\gamma = 0.0871$.

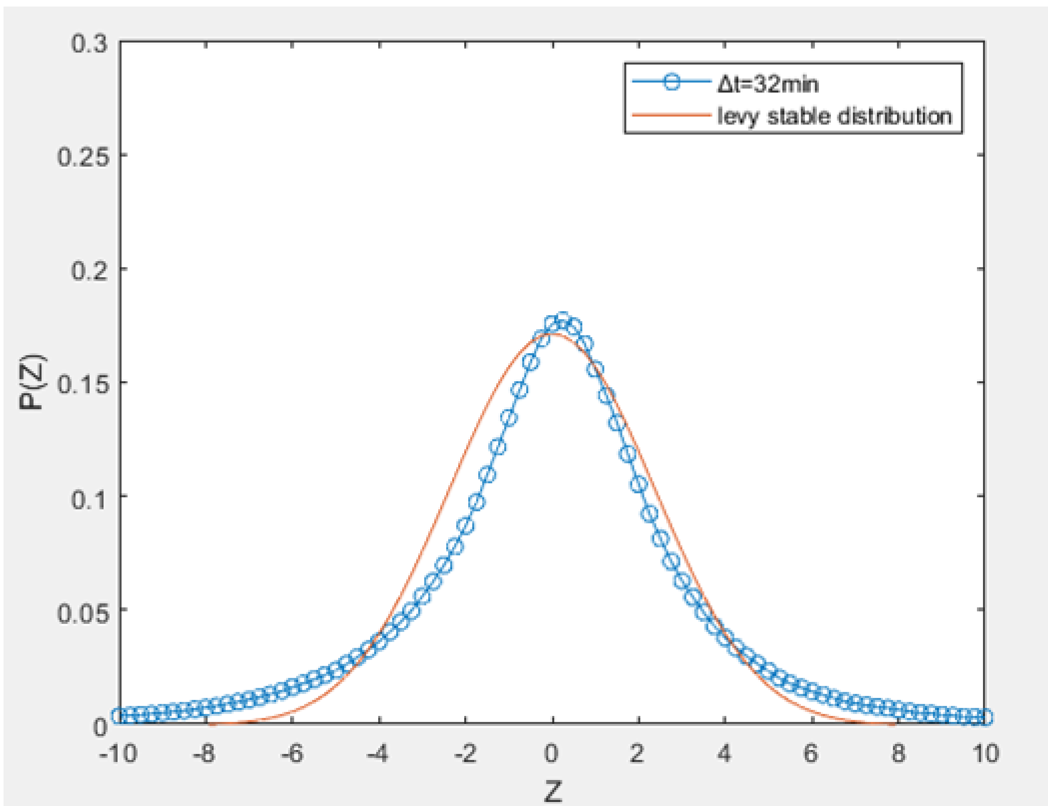

Figure 20: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the S&P 500 Index of USA's stock markets with time interval $\Delta t = 32$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 2.0596$, $\gamma = 0.0871$.

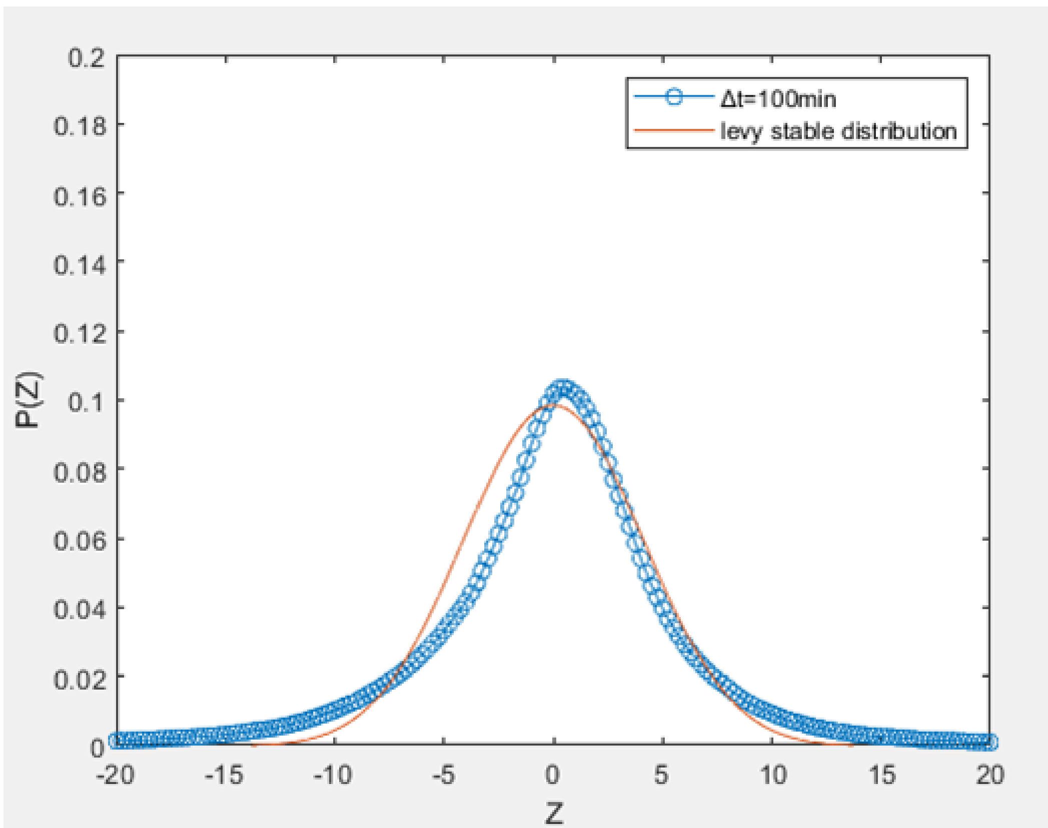

Figure 21: The best-fit of the probability density distributions $P(Z)$ of index variation $Z(t)$ for the S&P 500 Index of USA's stock markets with time interval $\Delta t = 100$ min. For the truncated Levy flight, best-fit parameters are $\alpha = 2.0596$, $\gamma = 0.0871$.

6 Conclusions

China's stock markets have experienced tremendous development over the past decades. As of June 2019, stock markets in China have 3644 listed firms and a market capitalization of RMB 44302 billion (about US$ 6320 billion). It is now the second largest market and the fastest growing market in the world. China's stock markets attract foreign investors' attention because of potential opportunities. Despite tremendous growth, the Chinese stock markets is still far away in depth and maturity of stock exchange from financial markets in developed countries. China's market capitalization in 2018 as a proportion of GDP was about 54%. The corresponding figures for the US was over 148%. The Chinese stock markets are highly criticized on legal framework and the rule of law. Reporting requirements for listed companies in China are neither well developed nor extensive. Due to restrictions on capital flows and holdings, the China's stock market is unique in the sense that it is a segmented market with domestic investors dominating ownership of stocks. Traders may base their actions on the decisions of others who may be more informed about market developments. Thus, trading behaviors in China's stock markets are different from those in other markets.

In this paper, we have investigated high frequency time-series features of stock returns and volatility on China's stock markets. It is shown that the empirically observed probability distribution of log-returns is not Gaussian and has power-law tails. By using the stable Lorentz distribution and truncated Levy flight process, I have presented a best-fit of empirical data of the CSI 300 Index. Comparisons between the stock markets of China and USA were discussed. They all bear the specified truncated Levy flight process. Of course, the stock market in USA has lower mean and volatility of returns than stock market in China. This is in agreement with our expectation. In spite of immature and a segmented market with domestic investors dominating ownership of stocks, China's stock markets possess the same distribution of returns with other financial markets in the world. They are almost symmetric, highly leptokurtic, and characterized by a non-Gaussian profile for small index changes.

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