INFORMATION FIELD THEORY

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ABSTRACT

We are able to determine that the energy content of information from a system with N equally probable states is hf $\log_2 N$ and we show that written information constitutes a microwave electromagnetic field, while spoken information constitutes a mechanical wave. The radiation flux of written information is on the order of 10^{-40} Watts per meter squared. The radiation flux of spoken information is on the order of 10^{-28} Watts per meter squared. The radiation flux of information from the average computer is on the order of 10^{-17} Watts per meter squared.

KEY WORDS: information, electromagnetic, wave equation, radiation flux

1.INTRODUCTION

The universe consists of mass, energy and information. Newton [1] discovered the fundamental relationships between mass and energy and Einstein [2] deduced the conversion of mass into energy. However, information is of growing importance and the relationship between information and energy is a subject of great interest [3].

The mathematical theory of information specifies an equation for information that is solely based on probability and is similar in form to entropy in physics [4]. However, the theory does not include energy.

The black hole information paradox results from the combination of quantum mechanics and general relativity [5]. It suggests that physical information could permanently disappear in a black hole, allowing many physical states to devolve into the same state. This is controversial because it violates a commonly assumed tenet of science—that *in principle* complete information about a physical system at one point in time should determine its state at any other time [6]. A fundamental postulate of quantum mechanics is that complete information about a system is encoded in its wave function up to when the wave function collapses [7]. The evolution of the wave function is determined by a unitary operator [8], and unitarity implies that information is conserved in the quantum sense. This is the strictest form of determinism.

Currently, the most specific relationship between energy and information is due to Landauer's limit [9], which states that the minimum amount of energy required to erase one bit of information is kTln2. The result is derived from thermodynamics [10].

In this paper, we employ set theory [11] to derive the observational energy required by information. We use that energy to determine an information field that causes a force of attraction between units of information. The field is then given spatial and temporal dependence in the form of a traveling wave with a random phase. The random phase satisfies the requirement that information is unpredictable. The wave equation is inhomogeneous [12], but it is approximately homogeneous [13] for certain relationships between wave speed and temporal frequency. The homogenous approximation creates an electromagnetic field [14]. We simply correlate this field to the way written information is read and we do an example calculation to demonstrate the level of the radiation flux [15]. We also characterize the information carried by sound and other compressional waves.

2.THEORY

Consider the following wave function with a time-variant phase:

$$\vec{\alpha}(x,t) = \frac{\alpha_0 \cos(kx - \omega t + \phi(t))\hat{x}}{x} . \qquad (1)$$

It satisfies the equation:

$$\frac{\partial^2(\vec{\alpha}x)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2(\vec{\alpha}x)}{\partial t^2}=\frac{\left(2\frac{\partial\phi}{\partial t}\omega-\left(\frac{\partial\phi}{\partial t}\right)^2\right)(\vec{\alpha}x)}{c^2}. \tag{2}$$

Let

$$\phi(t)=Xt,\ \ X\in[0,2\pi Hz],\ \ X:\Omega\to\mathbb{R}. \tag{3}$$

Equation (2) has a random phase [16] for condition (3) and becomes:

$$\frac{\partial^2(\vec{\alpha}x)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2(\vec{\alpha}x)}{\partial t^2}=\frac{\left(2X\omega-X^2\right)(\vec{\alpha}x)}{c^2}. \tag{4}$$

Equation (4) characterizes an *information wave*. The information pattern is a result of the random phase of the wave.

We can derive the following homogeneous wave from (4):

$$\frac{\partial^2(\vec{\alpha}x)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2(\vec{\alpha}x)}{\partial t^2}\approx 0,\ \left|c^2\right|\gg|\omega|. \tag{5}$$

We can consider a related field:

$$\vec{\beta}(x,t)=\frac{\alpha_0\cos(kx-\omega t+Xt)\hat{y}}{cx}. \tag{6}$$

We see that the following conditions hold:

$$\beta=\frac{\alpha}{c},\ \ \vec{\alpha}\cdot\vec{\beta}=0,\ \ \frac{\partial(\alpha x)}{\partial x}=-\frac{\partial(\beta x)}{\partial t},\ \ |c|\gg 2\pi. \tag{7}$$

Therefore, this yields an electromagnetic field that must satisfy Maxwell's equations [17]:

$$\nabla\cdot(\vec{\alpha}x)=\frac{\rho}{\varepsilon_0}. \tag{8}$$

$$\nabla\cdot\left(\vec{\beta}x\right)=0. \tag{9}$$

$$\nabla\times\left(\vec{\beta}x\right)=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial(\vec{\alpha}x)}{\partial t}. \tag{10}$$

$$\nabla\times(\vec{\alpha}x)=-\frac{\partial\left(\vec{\beta}x\right)}{\partial t}. \tag{11}$$

The radiation flux is given by:

$$\vec{S}=\frac{1}{\mu_0}\left((\vec{\alpha}x)\times\left(\vec{\beta}x\right)\right)=\frac{\alpha_0^2\cos^2(kx-\omega t+Xt)\hat{z}}{\mu_0 c}. \tag{12}$$

Therefore, the radiation flux of the original fields is:

$$\frac{1}{\mu_0}\left(\vec{\alpha}\times\vec{\beta}\right)=\frac{\alpha_0^2\cos^2(kx-\omega t+Xt)\hat{z}}{\mu_0 cx^2}. \tag{13}$$

If we consider that written information has a typical wavelength between 0.1 cm and 100 cm, the wavelength of microwaves, then we know that the average frequency of written information is 3 GHz. Consequently,

$$\frac{|\omega|}{|c^2|} = 2.07 \times 10^{-5} \approx 0 \quad (13)$$

This is consistent with (5) so that written information becomes an electromagnetic wave or microwave [18]. The diagram:

$$\uparrow \rightarrow \odot \quad (14)$$

shows the directions of the electromagnetic fields and their radiation flux. This is consistent with reading information from left to right and receiving the information flux perpendicular to that direction out of the page.

We can apply this to spoken information. The average frequency of the human voice is 200 Hz. The speed of sound is 340 m/s.

$$\frac{|\omega|}{|c^2|} = 0.01 \sim 0 \quad (15)$$

In this case, the information wave is not an electromagnetic wave, but a mechanical wave [19]. This is consistent with the fact that for information transported by sound, the radiation flux is parallel to the wave.

Consider the case of a wireless computer network. The average frequency of a desktop computer is 2 GHz. The wireless signal moves at the speed of light.

$$\frac{|\omega|}{|c^2|} = 7 \times 10^{-6} \approx 0 \quad (16)$$

We now want to determine the amount of energy contained in information. That will allow us to designate α_0. Consider the following theorem:

THEOREM I. There are $\log_2$N units of observational energy needed to locate one out of N equally probabie symbols.

Proof. Detection of a symbol requires observational energy, ξ. Let the total set of equally probable symbols be S. Suppose we are looking for the symbol a. The number of distinct symbols is N.

If N=1, we have located a with a cost of zero energy. If N=2, divide S into two subsets containing one element each:

$$S_1 \cup S_2 = S, \ |S_1| = |S_2| = 1$$

Observe S_1 for a. The cost is ξ.

$$a \in S_1 \Rightarrow a \notin S_2, a \notin S_1 \Rightarrow a \in S_2$$

Therefore, if N=2, we can locate a with a cost of ξ.

If N=4, divide S into two subsets containing two elements each:

$S_1 \cup S_2 = S,\ |S_1| = |S_2| = 2$.

Observe S_1 for a. The cost is ξ.

$a \in S_1 \Rightarrow a \notin S_2, a \notin S_1 \Rightarrow a \in S_2$.

If a∈S_1, divide S_1 into two subsets containing one element each:

$S_{11} \cup S_{12} = S_{1,}\ \ |S_{11}| = |S_{12}| = 1$.

Observe S_{11} for a. The cost is ξ.

$a \in S_{11} \Rightarrow a \notin S_{12}, a \notin S_{11} \Rightarrow a \in S_{12}$.

Therefore, if N=4, we can locate a with a cost of 2ξ.
If N=8, divide S into two subsets containing four elements each:

$S_1 \cup S_2 = S,\ |S_1| = |S_2| = 4$.

Observe S_1 for a. The cost is ξ.

$a \in S_1 \Rightarrow a \notin S_2, a \notin S_1 \Rightarrow a \in S_2$.

If a∈S_1, divide S_1 into two subsets containing two elements each:.

$S_{11} \cup S_{12} = S_{1,}\ \ |S_{11}| = |S_{12}| = 2$.

Observe S_{11} for a. The cost is ξ.

$a \in S_{11} \Rightarrow a \notin S_{12}, a \notin S_{11} \Rightarrow a \in S_{12}$.

If a∈S_{11}, divide S_{11} into two subsets containing one element each:

$S_{111} \cup S_{112} = S_{11,}\ \ |S_{111}| = |S_{112}| = 1$.

Observe S_{111} for a. The cost is ξ.

$a \in S_{111} \Rightarrow a \notin S_{112}, a \notin S_{111} \Rightarrow a \in S_{112}$.

Therefore, if N=8, we can locate a with a cost of 3ξ.

We can make a table of our results:

N	$\cos t$
1	0
2	ξ
4	2ξ
8	3ξ

.

This correlates to the formula:

$E = \xi \log_2 N$.

□

The observational energy is equivalent to the electromagnetic energy used for observation. Therefore, the energy contained within information is:

$$E = hf \log_2 N \quad (17)$$

Equation (16) implies that we can write:

$$Fx = hf \log_2 N \quad (18)$$

Or:

$$F = \frac{hf \log_2 N}{x} \quad (19)$$

We propose that information is attractive so that if there are two information systems with N_1 and N_2 states, then the force between them is:

$$\vec{F}_{21} = -\frac{hf\sqrt{\log_2 N_1 \log_2 N_2}}{x_{21}}\hat{x}_{21} \quad (20)$$

Equation (19) suggests that the field due to the second system can be written as:

$$\vec{\alpha}_2(x) = \frac{hf\sqrt{\log_2 N_2}}{x}\hat{x} \quad (21)$$

This is consistent with *positive information current* moving from left to right and *negative information current* moving from right to left. The *charge* to the left is positive, while the *charge* to the right is negative. [This is the case for English, but in general, the polarity is dependent upon the language. In either case, the radiation flux is still in the positive z direction or out of the page.]

A time-dependent, random phase modulation of (21) yields:

$$\vec{\alpha}_2(x,t) = \frac{hf\sqrt{\log_2 N_2}\cos(kx - \omega t + Xt)}{x}\hat{x} \quad (22)$$

If we suppress the indices, we get:

$$\vec{\alpha}(x,t) = \frac{hf\sqrt{\log_2 N}\cos(kx - \omega t + Xt)}{x}\hat{x} \quad (23)$$

Equation (23) gives the wave function given in (1) with:

$$\alpha_0 = hf\sqrt{\log_2 N} \quad (24)$$

The radiation flux can therefore be written as:

$$\frac{1}{\mu_0}\left(\vec{\alpha} \times \vec{\beta}\right) = \frac{(hf)^2\left(\log_2 N\right)\cos^2(kx - \omega t + Xt)\hat{z}}{\mu_0 c x^2} \quad (25)$$

[The units are corrected by multiplying the result by 1 C^2. Note that the radiation flux is in Watts per square meter.]

EXAMPLES:

1.Calculate the radiation flux of the information wave due to written information that has a separation of one centimeter:

$$\frac{1}{\mu_0}(\vec{\alpha}\times\vec{\beta})\approx 10^{-40}W/m^2 \quad (26)$$

2.Calculate the radiation flux of the information wave due to the human voice over a separation of one meter:

$$\frac{P}{m^2}=\frac{hf^2\log_2 N}{A}\approx 10^{-28}W/m^2 \quad (27)$$

3.Calculate the radiation flux of the information wave due to a wireless computer network at a separation of 10 meters:

$$\frac{P}{m^2}=\frac{hf^2\log_2 N}{A}\approx 10^{-17}W/m^2 \quad (28)$$

3.CONCLUSION

We have determined the energy contained in information and derived the information field equations. The equations specify that written information is an electromagnetic field that is comprised of microwaves. The radiation flux of the information field is consistent with the reading of information because the information content is collected in a perpendicular direction to the flow of information symbols. We also characterize the information wave carried by sound and other compressional waves and we find that the radiation flux of information carried by a computer is orders of magnitude greater than the radiation flux of sound and written information.

Although the radiation flux of information is extremely small, the results of this paper might be used to measure the radiation flux of information in different environments.

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