Capital allocation at operational risk: a combination of statistical data and expert opinion

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Abstract

Capital management is a major concern for financial institutions. In fact, the declaration of risk appetite and capital allocation by the business line and the risk category are a means of managing the bank's activity.

In this paper, we will present an approach to capital allocation based on VaR, combining the modelling of historical data using the LDA approach and the modelling of data collected from experts using the Bayesian approach.

We will also focus on the quality of the data collected from experts by proposing an approach based on the Delphi method that ensures the reliability of experts' estimates and reduces the model risk.

Keywords: Capital Allocation, Bayesian Approach, Value at Risk, Monte Carlo, Expert opinion, Delphi method.
JEL Classification : C11, C13, C15, G21, G32

1. Introduction

Operational risk management practice is based on an approach based on four steps: identification, assessment of impact, classification of risks and implementation of action plans. Indeed, the risk management process must be able to ensure a perfect knowledge and control of operational risks at the level of the various activities exercised.

With regard to the minimum capital requirement, the legislator under the Basel II offers banks several approaches and methods for calculating operational risk depending on the degree of control and the availability of the information required for internal modelling. As a result, the regulator proposes on one hand simple, unified and standardized approaches whose characteristics are provided by him and on the other hand, complicated and sophisticated approaches whose characteristics are determined by the banks.

The Basel Committee has instituted operational risk management through three main documents. In addition to the Basel II Accord, which is the main reference for the regulation of operational risk; the committee defined best practices for operational risk management in a document published in 2003 entitled « Sound Practices for the Management and Supervision of Operational Risk ». Then, it revised these principles in 2011 with the publication of a new document entitled «Principles for the Sound Management of Operational Risk ».

In the first document, which represents the first reflections of the banks and the Basel Committee on the introduction of operational risk management at bank level, the Committee has defined the principles of best practice that can guarantee the effectiveness of the operational risk management process. Indeed, these principles are in the order of ten principles divided into four areas:

(1) The elaboration of an adequate environment for risk management.
(2) Identification, evaluation, monitoring and control and/or mitigation of risk.
(3) The role of supervisors.
(4) The role of financial communication.

In the revised document the committee proposes three lines of defence and 11 principles of best practice.

In terms of quantification, the committee presented some methods that can be used in the AMA approach in a document published in 2001 entitled « Working Paper on the Regulatory Treatment of Operational Risk ».

Following the financial crisis, the minimum capital requirements for operational risk were reviewed by the Basel Committee. Indeed, the publication in December 2017 of the document entitled « Basel III: Finalising post-crisis reforms» divulged the orientation of banking regulation after 2022, which consists in replacing existing operational risk measurement approaches with a single approach known as « Standardised Measurement Approach (SMA)» which will enter into effect in January 2022.

Before the Basel III reform enters into effect, banks continue to use their own models for calculating minimum capital requirements. Indeed, banks opt for two types of modelling approaches, namely the Top-Down approach or the Bottom-up approach.

The Top-Down approach quantifies operational risk without attempting to identify events or causes of losses. The operational losses, under this approach, are measured based on overall historical data.

The Bottom-up approach quantifies operational risk based on knowledge of events by identifying internal events and related generating factors in great detail at the level of each task and entity. The information collected is included in the overall calculation of the capital charge.

The use of internal models is essential for operational risk management despite the Basel Committee's decision to abandon the AMA approach, notably for the risk appetite process and capital allocation process.

Therefore, in this article, we will propose an approach to capital allocation based on a combination of expert opinion and the Loss Distribution Approach (LDA). Indeed, the second paragraph will be reserved for risk appetite, the third part for risk mapping and regulatory capital requirements, then, the fourth part for the LDA approach and the determination of VaR in operational risk, more over, the fifth part will be reserved for Bayesian modelling of the expert opinion and we terminate with the empirical study.

2. The risk appetite process :

Risk appetite is defined as the maximum loss that the bank accepts to support in order to achieve its profitability objectives. Indeed, the Board of Directors must define the risks that shareholders accept to support in order to achieve the objectives defined for the Senior Management.

Risk appetite must be declined by the Senior Management at the level of each business line and activity, by defining risk tolerance at the intermediate level and risk limits at the operational level.

Risk appetite is directly related to the current risk profile and its evolution in correlation with the evolution of the bank's activity. As a result, the bank must determine its risk profile at the date of preparation of its risk appetite policy and must estimate the evolution of its profile in accordance with the progress of its development and expansion plan.

The risk profile is determined internally by the bank and may differ from its regulatory profile determined by the regulatory capital. Indeed, the actual profile is determined by the bank's economic capital, while the regulatory profile is defined by the minimum capital requirement according to the standard approach of the Basel III.

For the deployment of a risk appetite framework, Shang and Chen (2012) identified seven steps:

(1) Bottom-up analysis of the company's current risk profile.
(2) Interviews with the board of directors regarding the level of risk tolerance.
(3) Alignment of risk appetite with the company's goal and strategy.
(4) Formalization of the risk appetite statement with approval from the board of directors.
(5) Establishment of risk policies, risk limit and risk-monitoring processes consistent with risk appetite.
(6) Design and implementation of the risk-mitigation plan to be consistent with risk appetite.
(7) Communication with local senior management for their buy in.

Indeed, this approach should allow to define three components:

(1) The risk profile,
(2) The risk tolerance process
(3) The process for defining operational risk limits.

2.1. The process of allocation capital.

Capital allocation is the process that defines the capital allocated by the bank to a given entity to achieve the intended profitability objective. Indeed, the capital K_i allocated to unit (i) is defined according to the risk incurred by the said unit.

The definition of a risk measure ρ is an essential component in the capital allocation process. Indeed, for operational risk, two measures can be used, namely value at risk (VaR) which is a non-coherent risk measure, and the Expected Shortfall, which is a coherent risk measure. The expected Shortfal (ES) is defined by:

$$ES_\alpha = \frac{1}{\alpha}\int_0^\alpha F^{-1}(p)dp$$

with F is the cumulative distribution function of operational losses.

Let be $X_i, i = 1, \dots, n$ the random variables representing the individual losses of the n business units and $K_i, i = 1, \dots, n$ the allocation of capital for each probable individual loss (i). The total operational loss (P) and total risk capital (K) are expressed as :

$$\begin{cases} P = \sum_{i=1}^{n} X_i \\ K = \sum_{i=1}^{n} K_i \end{cases}$$

For the allocation of risk capital per unit, Dhaene et al (2012) proposes several methods that can be used to allocate equity capital as part of portfolio management. For operational risk, we use the proportional allocation method, which defines the capital K_j allocated to unit i by the formula:

$$K_j = \frac{K}{\sum_{i=1}^{n}\rho(X_i)}\rho(X_j) \tag{1}$$

with $\rho(X_j) = F_{X_i}^{-1}(\alpha) = VaR_\alpha(X_i)$ or $\rho(X_j) = ES_\alpha(X_i) = \frac{1}{\alpha}\int_0^\alpha F_{X_i}^{-1}(p)dp$

For this method, the capital allocation is based on the risk measure ρ. As a result, the use of internal models to measure operational risk at the level of each entity and at the level of all entities is essential.

Under the second pillar, the allocation of capital and the implementation of the risk appetite process strengthen the use of internal models despite the suppression of their use for the calculation of the minimum capital requirement under the first pillar. Indeed, the piloting of the activity by the risk requires an individual monitoring of the risk by business line in order to guarantee an adequacy between the risk incurred and the capital allocated.

Consequently, the bank must develop its own models for estimating the economic capital needed to develop its business independently of the regulatory constraint of measuring the solvency ratio based on the standard approach of the Basel III.

3. The risk mapping and capital requirements:

3.1. The risk mapping:

The operational risk mapping is a balance sheet of the probable risks incurred by the bank at a given date. Indeed, it represents all operational risk situations broken down by business line and risk category.

The operational risk situation is composed of three elements:

(1) The generating factor of the risk (hazard): it constitutes the factors that favour the occurrence of the risk incident as inexperienced personnel and the malfunction of control device.

(2) The operational risk event (incident): it constitutes the single incident whose occurrence can generates losses for the bank as internal fraud and external fraud.

(3) The impact (loss): it constitutes the amount of financial damage resulting from an event.

To normalize the identification of operational risk situation, the Basel Committee on Banking Supervision (2006) defines a generic mapping of operational risks within credit institutions, comprising 8 business lines and 7 categories of operational risks.

3.1.1. The operational risk categories.

The operational risk categories (RT_c,1≤c≤7) are : RT_1- Execution, delivery and process management, RT_2- Business disruption and system failures, RT_3- Damage to physical assets, RT_4- Clients, products and business practices, RT_5- Employment practices and workplace safety, RT_6 - External Fraud, RT_7 : Internal Fraud.

3.1.2. The business lines.

The business lines (BL_i,1≤i≤8) are :BL_1*:* Corporate finance, BL_2*:* Trading and sales, BL_3*:* Retail banking, BL_4*:* Commercial banking, BL_5*:* Payment and settlement, BL_6*:* Agency services, BL_7*:* Asset management, BL_8*:* Asset management.

3.2. Capital requirements.

The quantification of operational risk remains a major problem for the Basel Committee. Indeed, several approaches have been adopted in the Basel II framework, including the AMA approach based on internal models, considered the most important.

The research on the AMA approach focuses on the LDA approach, in these different forms, namely the classical approach, the Bayesian approach or MCMC, in particular those conducted by King (2001), Cruz (2002), Alexander (2003), Frachot et al. (2003), Chernobai et al. (2005), Bee (2006) and Schevchenko et al. (2008, 2009 and 2010).

The use of internal models has been strongly criticized by the Basel Committee. Indeed, a new orientation of the Basel Committee has been born consists in abandoning all Basel II approaches and adopting a new standard approach SMA which will replace all previous approaches.

This new approach has been criticized by various academics and professionals such as Mignola et al.(2016); Peters et al.(2016); McConnell (2017) which means that operational risk remains an incomprehensible risk according to various studies such as Cohen (2017).

The standard approach SMA (Basel 2017) is based on the Business indicateur (BI) defined as follows :

$$BI = ILDC + SC + FC \tag{2}$$

The components $ILDC$, SC and FC are calculated by the following formulas:

$$ILDC = Min\left[\left(\frac{1}{3}\sum_{i=1}^{3}|PI_i - CI_i|\right); 2{,}25\% \times \left(\frac{1}{3}\sum_{i=1}^{3} API_i\right)\right] + \frac{1}{3}\sum_{i=1}^{3} D_i \tag{3}$$

$$SC = Max\left[\left(\frac{1}{3}\sum_{i=1}^{3} ACE_i\right);\left(\frac{1}{3}\sum_{i=1}^{3} APE_i\right)\right] + Max\left[\left(\frac{1}{3}\sum_{i=1}^{3} PHC_i\right);\left(\frac{1}{3}\sum_{i=1}^{3} CHC_i\right)\right] \quad (4)$$

$$FC = \frac{1}{3}\sum_{i=1}^{3}|PLT_i| + \frac{1}{3}\sum_{i=1}^{3}|PLB_i| \quad (5)$$

with[1] :

- PI_i and CI_i are respectively the Interest Income and the Interest Expense for the year(i).
- API_i is the Interest Earning Assets for the year (i).
- D_i is the Dividend Income for the year (i).
- ACE_i and APE_i are the other operating income and the other operating expense for the year (i).
- PHC_i and CHC_i are respectively the Fee Income Fee Expense for the year (i).
- PLT_i is the Net P&L Trading Book for the year (i).
- PLB_i is the Net P&L Banking Book for year (i).

4. The LDA approach and the VaR of operational risk.

4.1. The loss distribution Approach LDA.

The LDA approach uses distributions of the frequency and the severity of operational losses occurred to determine operational losses over a time horizon T.

4.1.1. The Classical LDA model.

4.1.1.1. Mathematical formulation of the model

In the LDA approach, the operational loss in horizon T is considered as a random variable P defined as follows:

$$P_N = \sum_{i=1}^{N} X_i \quad (6)$$

with:

- X_i : is the random variable that represents the individual impact of operational risk incidents.
- N : is the random variable that represents the number of occurrences on a horizon T.

The random variables X_i are independent and identically distributed. The random variable N is independent with the variables X_i.

The mathematical expectation and the variance of the compound random variable P are defined as follows:

- $E(P) = E(X) \times E(N) = \lambda \; E(X)$ (7)
- $\mathrm{VAR}\,(\mathrm{P}) = \mathrm{E(N)} \times \mathrm{var(X)} + \mathrm{var(N)} \times E(X)^2$ (8)

4.1.1.2. Presentation of the classic LDA approach.

The classical LDA approach considers that severity and frequency can be modelled by usual theoretical laws whose parameters are estimated from these data

4.1.1.2.1. Modelling the individual severity of losses X_i.

Several distributions can be used to represent the severity random variable X as the LogNormal distribution, the Beta distribution, the Weibull distribution or other distributions which are detailed in Chernoubai et al. (2007). In our study, we limit ourselves to LogNormal distribution $LN(\mu, \sigma)$ defined as follows:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log(x)-\mu)}{2\sigma^2}} \; si \; x \geq 0 \quad (9)$$

with

$$E(X) = e^{\mu+\frac{\sigma^2}{2}} \text{ and } Var(X) = (\sigma^2 - 1)e^{2\mu+\sigma^2} \quad (10)$$

4.1.1.2.2. Modelling the frequency of losses N.

With regard to the modelling of the loss frequency N, we use the Poisson distribution $P(\lambda)$ or the Negative Binomial distribution $BN(a, b)$ defined as follows:

- $\mathcal{P}\,(\lambda)$: $P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

[1] The rubrics for calculating the BI are detailed in Appendix : definition of the components of the BI of the Basel III reform

- $BN(a,b)$: $P(X=K)=\frac{\Gamma(a+k)}{\Gamma(a)k!}\times\frac{b^a}{(b+1)^{k+a}}$; $\Gamma(r)=\int_0^\infty x^{r-1}e^{-x}dx$ if r∈N then : $\Gamma(r)=(r-1)!$

4.1.2. The Pure Bayesian LDA Approach.

In the pure Bayesian LDA approach, the parameters of the distribution of the frequency N and the individual loss X_i are considered as random variables with a probability density function.

The pure Bayesian approach considers the parameters (μ,σ) and λ of the density functions of X_i and N as the random variables whose the density are respectively π_μ, π_σ and π_λ

4.1.2.1. Description of the Pure Bayesian LDA Approach.

Let $Y=(Y_1,\dots Y_m)$ be a vector of random variables independent and identically distributed (i.i.d). Let $(y_1,\dots y_m)$ be a realization of the vector Y and let $\theta=(\theta_1,\theta_2,\dots,\theta_p)$ be a vector of the random variables of the parameters of the density of the vector Y.

The density function $f(Y,\theta)$ of the vector $(Y,\theta)=(Y_1,\dots Y_m,\theta_1,\theta_2,\dots,\theta_p)$ is defined by:

$$f(Y,\theta)=f(Y/\theta)\pi(\theta)=\pi(\theta/Y)f(Y) \qquad (11)$$

where :

- $\pi(\theta)$ is the probability density of the parameter θ, called "prior density function".
- $\pi(\theta/Y)$ is the conditional probability density function of the parameter θ knowing Y, called "posterior density";
- $f(Y,\theta)$ is a probability density function of the couple (Y,θ) ;
- $f(Y/\theta)$ is the conditional density function of Y knowing θ, it is the likelihood function $f(Y/\theta)=\prod_{i=1}^m f_i(Y_i/\theta)$

with $f_i(Y_i/\theta)$ is the conditional probability density function of Y_i.

- $f(Y)$) is a marginal density of Y that can be written as $\int f(Y/\theta)\pi(\theta)d\theta$.

The Bayes formula allows to determine $\pi(\theta/Y)$ of the parameter θ knowing Y as follows:

$$\pi(\theta/Y)=\frac{f(Y/\theta)\pi(\theta)}{f(Y)} \qquad (12)$$

Hence

$$\pi(\theta/Y)\propto f(Y/\theta)\pi(\theta) \qquad (13)$$

$f(Y)$ is a normalization constant and the posterior distribution $\pi(\theta/Y)$ can be viewed as a combination of a prior knowledge $\pi(\theta)$ with a likelihood function $f(Y/\theta)$ for observed data. Since $f(Y)$ is a normalization constant, the posterior distribution is often written with the form (13) where the symbol ∝ signified "is proportional" with a constant of proportionality independent of the parameter θ.

4.1.2.2. The Bayesian Estimator $\hat{\theta}_{Bay}$

The estimate of Bayesian posterior mean $\hat{\theta}_{Bay}$ of θ is defined as follows:

- **The parameter θ is univariate :**

The estimate of the Bayesian posterior mean of θ noted $\hat{\theta}_{Bay}$ is a conditional expectation of θ knowing Y:

$$\hat{\theta}_{Bay}=E(\theta/Y)=\int\theta\times\pi(\theta/Y)d\theta=\frac{\int\theta\times f(Y/\theta)\pi(\theta)d\theta}{f(Y)} \qquad (14)$$

- **The parameter θ is multivariate**

In a multidimensional context where $\theta=(\theta_1,\theta_2,\dots,\theta_p)$, the estimate of the Bayesian posterior mean of θ noted $\hat{\theta}_{Bay}$ is a conditional expectation of the vector θ knowing Y defined by :

$$\hat{\theta}_{Bay}=E(\theta/Y)=\left(E(\theta_1/Y),E(\theta_2/Y),\dots,E(\theta_p/Y)\right)$$
$$=\left(\int\theta_1\times\pi(\theta_1/X)d\theta_1,\int\theta_2\times\pi(\theta_2/X)d\theta_2,\dots,\int\theta_p\times\pi(\theta_p/X)d\theta_p\right) \qquad (15)$$

4.1.2.3. Calculation of the estimate of the Bayesian posterior mean.

To determine the estimate of Bayesian posterior mean defined by the formulas (14) and (15), we must determine the prior law and the posterior law of the random variable θ.

In fact, we will limit our study to the Lognormal distribution for loss severity $X_i \sim LN(\mu, \sigma)$,$1 \leq i \leq m$ and to the Poisson distribution for the frequency of the losses $N \sim P(\lambda)$. The parameters μ, σ and λ are considered random variables.

Therefore, we have to determine the following estimate of the Bayesian posterior mean:

$$\hat{\theta}_{Bay} = (\hat{\mu}, \hat{\sigma}) = E(\mu, \sigma / X_1, \dots X_n) \quad (16)$$

$$\hat{\theta}_{Bay} = \hat{\lambda} = E(\lambda / N) \quad (17)$$

4.1.2.4. Determination of the prior law of the parameters.

The Bayesian approach depends on the accuracy of the information provided by experts on the parameters of the prior law. In fact, we will present below the approach adopted:

- **The prior law of the parameter $\boldsymbol{\lambda}$ with $\boldsymbol{N \sim P(\lambda)}$**

In our study, we will consider that the prior law is a gamma distribution Γ with the parameters (a, b) to be determined by the experts.

The choice of the prior distribution of the parameter λ depends on the description of the characteristics of the random variable given by the experts. In our study, we will consider that the prior law is a Gamma distribution (Γ) of parameter (a, b). Indeed, the Gamma distribution is defined by:

$$\Gamma(\lambda) = \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} \times e^{-\frac{\lambda}{b}}$$

- **The prior law of $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ with $\boldsymbol{X_i \sim LN(\mu, \sigma)}$**

In this paper we limit ourselves to the case where μ is a gaussian random variable $\mu \sim N(\mu_0, \sigma_0)$ and σ a known constant.

However, Schevchenko PV (2011), represented σ^2 by the inverse Chi-square distribution (Inv.Chi.Sq) of parameters (α, β) whose the probability density function is defined by :

$$f(\sigma^2) = \frac{\left(\frac{\sigma^2}{\beta}\right)^{-1-\frac{\alpha}{2}}}{2^{\frac{\alpha}{2}} \times \Gamma\left(\frac{\alpha}{2}\right) \times \beta} \times e^{-\frac{\beta}{2\sigma^2}}$$

4.1.2.5. Determination of the posterior law of the parameters $\boldsymbol{\lambda}$ and .

The posterior distribution is determined from the likelihood function and the prior distribution by the formula (13). Thereby, we will calculate the posterior law of frequency and severity:

- **The posterior law of the parameter $\boldsymbol{\lambda}$ with $\boldsymbol{N \sim P(\lambda)}$**

Let $N = (N_1, \dots N_l)$ be a vector of random variables of the frequency. Let $(n_1, \dots n_l)$ be a realization of the vector N. We suppose that $N_j \sim P(\lambda)$ and we consider that $\lambda \sim \Gamma(a, b)$.

The posterior law conjugated at the prior law λ is defined by:

$$\pi(\lambda/N) \propto f(N/\lambda)\pi(\lambda) \propto f(N/\lambda) \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} \times e^{-\frac{\lambda}{b}}$$

We have :

$$f(N/\lambda) = \prod_{j=1}^{l} f_j(N_j/\lambda) = \prod_{j=1}^{l} \frac{\lambda^{n_j}}{n_j!} e^{-\lambda}$$

Thus

$$\pi(\lambda/N) \propto \prod_{j=1}^{l} \frac{\lambda^{n_j}}{n_j\,!} e^{-\lambda} \times \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} \times e^{-\frac{\lambda}{b}} \propto \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} \times e^{-\frac{\lambda}{b}} \times \prod_{j=1}^{l} e^{-\lambda} \frac{\lambda^{n_j}}{n_j\,!}$$

$$\propto \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} e^{-\frac{\lambda}{b}} \prod_{j=1}^{l} \left(e^{-\lambda} \frac{\lambda^{n_j}}{n_j\,!}\right) \propto \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a) \times b} \prod_{j=1}^{l} \frac{\lambda^{n_j}}{n_j\,!} \times \left(e^{-l \times \lambda} e^{-\frac{\lambda}{b}}\right)$$

$$\propto \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a)\times b}\prod_{j=1}^{l}\frac{\lambda^{n_j}}{n_j\,!}\times\left(e^{-l\times\lambda}e^{-\frac{\lambda}{b}}\right)\propto\frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a)\times b}\prod_{j=1}^{l}\frac{\lambda^{n_j}}{n_j\,!}\times e^{-\lambda\left(\frac{1}{b}+l\right)}$$

$$\propto \lambda^{a-1}\lambda^{\sum_{j=1}^{l}n_j}\times e^{-\lambda\left(\frac{1}{b}+l\right)}\propto\lambda^{\left(a+\sum_{j=1}^{l}n_j\right)-1}\times e^{-\lambda\left(\frac{1+b\times l}{b}\right)}$$

We pose $a_l = a + \sum_{j=1}^{l} n_j$ and $b_l = \frac{b}{1+b\times l}$. Thus

$$\pi(\lambda/N) \propto \lambda^{a_l-1}e^{-\frac{\lambda}{b_l}} \tag{18}$$

From the formula (18) we deduct that the posterior law is a gamma law $\Gamma(a_l, b_l)$.

- **The posterior law of the parameter $\mu \sim \mathcal{N}(\mu_0, \sigma_0)$ with σ a constant**

Let $x_1, \dots, x_m$ be the realizations of random variables $X_1, \dots, X_m$ representing the collected losses. We suppose here for the Bayesian modelling of the severity that $\mu \sim \mathcal{N}(\mu_0, \sigma_0)$ and σ a constant, which we estimate from the sample by the maximum likelihood method. We pose $Z_i = ln(X_i)$. Thus $Z_i \sim \mathcal{N}(\mu, \sigma)$.

We consider the random vector $Z = (Z_1, \dots, Z_m)$.The prior distribution of μ is given by :

$$\pi(\mu) = \frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0{}^2}}$$

The conditional distribution of the random vector Z is given by:

$$f(Z/\mu,\sigma) = \prod_{i=1}^{m}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(Z_i-\mu)^2}{2\sigma^2}}$$

Hence the posterior law of μ:

$$\pi(\mu/Z) \propto f(Z/\mu)\pi(\mu)$$

$$\pi(\mu/Z=(z_1,\dots,z_m)) \propto \prod_{i=1}^{m}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(z_i-\mu)^2}{2\sigma^2}}\times\frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0{}^2}}$$

$$\pi(\mu/Z=(z_1,\dots,z_m)) \propto e^{-\frac{(\mu-\mu_{0m})^2}{2\sigma_{0m}{}^2}} \tag{19}$$

with :

$$\begin{cases}\mu_{0m} = \frac{\mu_0+m\times\varepsilon\times\bar{Z}}{1+m\times\varepsilon}\\ \sigma_{0m}^2 = \frac{\sigma_0^2}{1+m\times\varepsilon}\end{cases} \quad \text{where} \quad \begin{cases}\bar{Z} = \frac{1}{m}\sum_{I=1}^{m} z_i\\ \varepsilon = \frac{\sigma_0^2}{\sigma^2}\end{cases}$$

The formula (19) shows that the posterior law of μ is a gaussian law $\mathcal{N}(\mu_{0m}, \sigma_{0m})$.

4.1.2.6. Calculation of the Bayesian estimator $\hat{\mu}_{Bay}$ and $\hat{\lambda}_{Bay}$

- **The Bayesian estimator $\hat{\lambda}_{Bay}$ of the parameter λ :**

The Bayesian estimator $\hat{\lambda}_{Bay}$ is given by:

$$\hat{\lambda}_{Bay} = E(\lambda/N).$$

The result (18) shows that the posterior law of λ is a $\Gamma(a_l, b_l)$ distribution with $(a_l, b_l) = \left(a + \sum_{j=1}^{l} n_j, \frac{b}{1+b\times l}\right)$. Consequently, the estimator $\hat{\lambda}_{Bay}$ is the mathematical expectation of the posterior law of λ:

$$\hat{\lambda}_{Bay} = a_l \times b_l = \left(a + \sum_{j=1}^{l} n_j\right)\times\frac{b}{1+b\times l}$$

$$= \frac{a\times b + b\times l\times\left(\frac{\sum_{j=1}^{l}n_j}{l}\right)}{1+b\times l} = \frac{\lambda_0 + b\times l\times\left(\frac{\sum_{j=1}^{l}n_j}{l}\right)}{1+b\times l} = \frac{\lambda_0+b\times l\times(\bar{N})}{1+b\times l}$$

$$\hat{\lambda}_{Bay} = \varepsilon_0\times\lambda_0 + (1-\varepsilon_0)\times\bar{N} = \varepsilon_0\times\lambda_0+(1-\varepsilon_0)\times\lambda_{observed} \tag{20}$$

with $\varepsilon_0 = \frac{1}{1+b\times l}, \lambda_{observed} = \bar{N} = \frac{\sum_{j=1}^{l} n_j}{l}$ *and* $\lambda_0 = E(\lambda)$. The parameter λ_0 is estimated by the experts.

- **The Bayesian estimator $\hat{\mu}_{Bay}$ of the parameter μ:**

The Bayesian estimator $\hat{\mu}_{Bay}$ is given by $\hat{\mu}_{Bay} = E(\mu/(\sigma; X_1, \dots, X_m)) = E(\mu/(\sigma; x_1, \dots, x_m))$.
With σ is a constant and $x_1, \dots, x_m$ are realizations of the random variables $X_1, \dots, X_m$.

The result (19), shows that the posterior law of μ is a gaussian distribution $\mathcal{N}(\mu_{0m}, \sigma_{0m})$. Consequently, the estimator $\hat{\mu}_{Bay}$ is the mathematical expectation of the posterior law of μ. Thus :

$$\hat{\mu}_{Bay} = \mu_{0m} = \frac{\mu_0 + m \times \varepsilon \times \bar{Z}}{1 + m \times \varepsilon}$$

which can be written :

$$\hat{\mu}_{Bay} = \varepsilon_2 \times \mu_0 + (1 - \varepsilon_2) \times \bar{Z} = \varepsilon_2 \times \mu_0 + (1 - \varepsilon_2) \times \mu_{observed} \quad (21)$$

With

$$\varepsilon_2 = \frac{1}{1+m\times\varepsilon} \; ; \; \varepsilon = \frac{\sigma_0^2}{\sigma^2} \; ; \; \bar{Z} = \frac{1}{m}\sum_{I=1}^{m} z_i = \mu_{observed} \; ; \quad z_i = ln(x_i) \; ; \; \mu_0 = E(\mu)$$

The parameter μ_0 is estimated by the experts. Consequently, the parameters of the LogNormal law used in the simulation are $\hat{\mu}_{Bay}$ and σ.

4.2. Value at Risk of operational risk:

Value at Risk (VaR) is a measure adopted by the Basel Committee on Banking Supervision under Basel II to measure credit risk, market risk and operational risk in the framework of advanced approaches based on internal models. Indeed, the committee requires that the internal model be very robust and meet a very high requirement by fixing the threshold for the VaR of operational risk at 99,9%.

In terms of operational risk, the VaR model is a main component for the calculation of capital requirements by the LDA approach because it is based on the determination of the distribution of aggregate operational losses and the determination of the 99,9% percentile of this distribution.

The determination of VaR is dependent on the determination of the aggregate operational loss distribution because it can be calculated analytically, determined by numerical algorithms or calculated by Monte Carlo simulation.

4.2.1. Presentation of Value at Risk (VaR).

Let be $X_t, t = 1, \dots, n$, a series of stationary data of cumulative distribution function F. The value at risk (VaR) for a given probability α is defined mathematically by :

$$VaR_\alpha = inf\{u/F(u) \geq \alpha\}$$

4.2.2. Definition of the capital at operational risk.

We consider the aggregated loss $P_N = \sum_{i=1}^{N} X_i$ in a given horizon T. We fix the level of confidence $1 - \alpha = 99.9\%$.

The requirement of capital to cover the operational risk is measured by the Value At Risk (VaR). The VaR is the quantile of order $1 - \alpha$ of the aggregated loss P_N defined by:

$$F_{P_N}(VaR) = F_{\sum_{i=1}^{N} X_i}(VaR) = P(P_N \leq VaR) = 1 - \alpha \quad (22)$$

Where F_{P_N} is the cumulative distribution function of P_N. The VaR is given by:

$$VaR = F_{P_N}^{-1}(1 - \alpha) \quad (23)$$

4.2.3. Simulation of aggregate operational losses.

To simulate the losses, we use the appropriate estimator. For the classical LDA approach, we use the maximum likelihood estimator $(\hat{\lambda}, \hat{\alpha}_i, \hat{\beta}_i)$ of (λ, α, β) respectively the parameters of P(λ) and $LN(\alpha, \beta)$. For the Bayesian approach we use the Bayesian estimators $(\hat{\lambda}_{Bay}, \hat{\mu}_{Bay} \hat{\sigma}_{Bay})$.

4.2.3.1. Presentation of simulation by the inverse cumulative distribution function.

The Monte Carlo Method consists of simulating an important sample of realizations p_j of size $J = 100000$ in the following manner: for $1 \leq j \leq J$

(1) Simulate a realization n_j of the frequency N from the law of frequency chosen $(P(\lambda)\ or\ BN(a, b))$

(2) Simulate n_j realizations x_i, $1 \leq i \leq n_j$, of the severity X, from the law of severity chosen ($LN(\alpha,\beta)$ or $Wei(\alpha,\beta)$)

(3) Calculate $p_j = \sum_{i=1}^{n_j} x_i$ which will constitutes a realization of the loss $P_N = \sum_{i=1}^{N} X_i$.

In the following paragraph, we present the used algorithms to simulate the law $P(\lambda)$ of the frequency and the law $LN(\alpha,\beta)$ of the severity.

Before presenting the simulation by Monte Carlo method, we cite firstly the theorem of the inverse cumulative function that allows the simulation of the continuous random variables.

Theorem: If U is uniform random variable on the interval $[0,1]$ and F a cumulative distribution function continuous and strictly increasing. Let Y be the random variable defined from the inverse cumulative distribution function F^{-1} by $Y = F^{-1}(U)$. Then the cumulative distribution function of Y is F.

Consequently, for simulating a realization y_i of the random variable Y which has F as a cumulative distribution function, it suffices to:

−Simulate a realization u_i of the Uniform distribution $U[0,1]$.

−Calculate the inverse cumulative distribution function $y_i = F^{-1}(u_i)$. Then y_i is considered as a realization of Y.

4.2.3.2. Simulation of the realizations n_j for $1 \leq j \leq 100000$:

To simulate the realizations of the frequency N, we use the Poisson distribution $P(\lambda)$ or the gamma distribution $\Gamma(a,b)$.

4.2.3.2.1. Simulation Poisson distribution.

***Propriety*:** Let $(V_i)_{i\geq 1}$ be a sequence of exponential random variables of parameter λ. Then the random variable defined by:

$$M = Sup\left\{k \in \mathbb{N}^* / \sum_{i=1}^{k} V_i \leq 1\right\} \text{ and } M = 0 \ if \ V_1 > 1$$

is a Poisson random variable of parameter λ.

To simulate the realizations of the Poisson's law of the parameter λ we use the following algorithm.

Step 1 : Simulation of n_1

To simulate the realization n_1 of the frequency we proceed as follows :

(1) Simulate a realization v_1 of the law $Exp(\lambda)$ by the inverse cumulative distribution function. For that we must :

- Simulate a realization u_1 of the Uniform law $U[0,1]$.
- The cumulative distribution function of the exponential law $Exp(\lambda)$ is defined by $F^{-1}(u) = -\frac{ln(1-u)}{\lambda}$. We deduct $v_1 = F^{-1}(u_1) = -\frac{ln(1-u_1)}{\lambda}$

(2) If $v_1 > 1$ then $n_1 = 0$.

If not, simulate a second realization v_2 of the exponential law $Exp(\lambda)$ according to the procedure 1. If $v_1 + v_2 > 1$ then $n_1 = 1$ is a realization of Poisson of the parameter λ, otherwise, simulate the k realizations v_i, $1 \leq i \leq k$ until that $\sum_{i=1}^{k} v_i \leq 1$ and $\sum_{i=1}^{k+1} V_i > 1$. The value k that verify the last two inequalities is the realization $n_1 = k$ of the frequency.

Step *j* : Simulation of $n_j, 2 \leq j \leq 100000$

We redo 100000 times the step 1. We obtain thus 100000 realization n_j.

4.2.3.3. Simulation of the laws $LN(\alpha,\beta)$.

To simulate the laws $LN(\alpha,\beta)$we use the inverse cumulative distribution function method as follows:

(1)Simulate a realization u_i of the Uniform law $U[0,1]$.

(2)Calculate $x_i = F_{(\alpha,\beta)}^{-1}(u_i)$ where $F_{(\alpha,\beta)}$ is a cumulative distribution function of the law $LN(\alpha,\beta)$. As $F_{(\alpha,\beta)}^{-1}(u_i)$ has not analytical expression we simulate numerically x_i.

4.2.3.4. Determination of operating losses.

For each n_j realization of the law of frequency, n_j realizations of the law of severity must be simulated. The simulated loss p_j is the sum of the simulated realizations:

$$p_j = \sum_{i=1}^{n_j} x_i$$

4.2.4. Calculation of the Capital at operational risk (VaR).

The capital at operational risk is calculated by the determination of the percentile 99.9% of the empirical distribution of the losses $p_j = \sum_{i=1}^{n_j} x_i$, for $1 \leq j \leq 100000$, simulated by Monte Carlo.

Let F_P be the empirical cumulative distribution function of the loss P determined from the simulated realizations p_j. The function F_P is given by :

$$F_P(y) = \frac{numbre\ of\ p_j \leq y}{number\ of\ p_j} \qquad (24)$$

The value at risk VaR is expressed by the formula:

$$VaR = Inf\{y/F_P(y) \geq 99{,}9\%\} = Inf\left\{y/\frac{number\ of\ p_j \leq y}{number\ of\ p_j} \geq 99{,}9\%\right\}$$

In this paper, the modelling of the frequency is made for a horizon of one year $T = 12\ month$ or by dividing the year T into k sub-horizons $T_k = \frac{T}{k}$ for k an integer $2 \leq k \leq 12$.

4.2.5. The annual VaR with segmentation of the database by risk category

The operational loss P_c of risk category RT_c is a random variable defined by $P_c = \sum_{\mathrm{i}=1}^{N_c} X_{ci}$. With :

- N_c : the random variable that represents the frequency of losses of the risk category RT_c
- X_{ci} : the random variable, for $1 \leq i \leq N_c$, that represents the severity of the losses of the risk category RT_c

Let n_{jc}, $1 \leq j \leq 100000$, the annual frequency of the losses collected for the risk category RT_c and let x_{ci} be the simulated realizations of the losses of the risk category RT_c.

The realizations $p_{jc} = \sum_{i=1}^{n_{jc}} x_{ci}$, $1 \leq j \leq 100000$ permit to calculate the capital at risk VaR_c for each risk category RT_c. The annual VaR is the sum of the VaR_c because it is supposed that the risk categories are independent.The modelling of the frequency of the loss is made for a horizon of one year $T = 12\ month$ or by dividing the year T into k sub-horizons $T_k = \frac{T}{k}$ for k an integer $2 \leq k \leq 12$.

- **The modelling of the Loss frequency for an annual horizon**

The horizon chosen is a year $T = 12\ month$ and the level of confidence is $1 - \alpha = 99.9\%$.

The empirical cumulative distribution function F_{P_c} of the losses for the risk category RT_c is defined by:

$$F_{P_c}(y) = \frac{number\ of\ p_{jc} \leq y}{number\ of\ p_{jc}} \qquad (25)$$

The capital of operational risk for the category risk RT_c is:

$$VaR_c = Inf\{y/F_{P_c}(y) \geq 99.9\%\} \qquad (26)$$

The capital at risk on the annual horizon is the sum of the VaR_c:

$$VaR = \sum_{c=1}^{7} VaR_c \qquad (27)$$

- **The modelling of the Losses frequency for the sub-horizon $T_k = \frac{T}{k}$, $2 \leq k \leq 12$:**

Let F_{PT_c} be the empirical cumulative distribution function of the operational risk of a given risk category RT_c for the horizon $T = 1\ year$ defined by the formula (44) and determined from the simulate realizations p_{jc} with n_{jc} is a realization of the frequency of losses on the horizon T.

The cumulative distribution function F_{PT_c} is simulated k times on the horizon T. Let $F_{PT_{ci}}$ be the i[th] simulation. The capital at operational risk VaR_c on an annual horizon is the sum of the $VaR_{ci}, 1 \leq i \leq k,\ VaR_{ci}$ is the i[th] capital at operational risk determined from the i[th] simulation of the losses.

The capital at risk on the annual horizon is the sum of the $VaR_{ci},\ 1 \leq i \leq k$:

$$VaR_c = \sum_{i=1}^{k} VaR_{ci} \qquad (28)$$

The capital at risk on the annual horizon is the sum of the VaR_c :

$$VaR = \sum_{c=1}^{7} \sum_{i=1}^{k} VaR_{ci} \qquad (29)$$

5. Bayesian modelling of expert opinion

5.1. Collecting and modelling expert opinion.

5.1.1. Organization of the process for collecting expert opinion.

Obtaining expert opinion can be defined as a process of collecting information and data, or answering questions about problems to be solved. In this study, we must define the parameters of the frequency and severity of operational risk events. Therefore, the approach adopted must ensure a high level of accuracy and reliability of the expert opinion in order to reduce the impact of this data on the bank's risk profile.

The modelling of expert opinion has been the subject of various studies that have used various techniques for collecting expert opinions, such as the Delphi technique defined by Helmer (1968) and the practical guides proposed by Ayyub (2001).

In our study, we used the Delphi technique after adapting it to the specificities of collecting information from experts in the field of operational risk.

5.1.2. Presentation of the Delphi method.

The Delphi method includes eight steps according to Ayyub (2001) defined as follows:

(1) Selection of issues or questions and development of questionnaires.
(2) Selection of experts who are most knowledgeable about issues or questions of concern.
(3) Issue familiarization of experts by providing sufficient details on the issues on the questionnaires.
(4) Elicitation of experts about the issues. The experts might not know who the other respondents are.
(5) Aggregation and presentation of results in the form of median values and an inter-quartile range (i.e., 25% and 75% percentile values).
(6) Review of results by the experts and revision of initial answers by experts. This iterative re-examination of issues would sometimes increase the accuracy of results. Respondents who provide answers outside the inter-quartile range need to provide written justifications or arguments on the second cycle of completing the questionnaires.
(7) Revision of results and re-review for another cycle. The process should be repeated until a complete consensus is achieved. Typically, the Delphi method requires two to four cycles or iterations.
(8) A summary of the results is prepared with argument summary for out of inter-quartile range values.

5.1.3. Summary presentation of the process for collecting expert opinions.

The approach for collecting expert opinion is based on that defined by Ayyub (2001) with readjustments to better adapt the process to the area of operational risk:

(1) Definition of the information requested.
(2) Definition of interveners in data collecting process.
(3) Identification of problems, information sources and insufficiency.

(4) Analysis and collecting of pertinent information.
(5) Choice of interveners in data collecting process.
(6) Knowledge of the objective of the operation by the experts and formation of experts.
(7) Soliciting and collecting opinions.
(8) Simulation, revision of assumptions and estimates, if the expert manifests his consent, we pass to the next step otherwise we repeat steps 6, 7 and 8.
(9) Aggregation of estimates and overall validation.
(10) Preparation of reporting and determination of results.

5.1.3.1. Definition of the information requested.

The collecting of information from experts has two objectives:

(1) The first consists in modelling the law a priori of the frequency and severity of data by risk category. Indeed, the expert must provide the forms of the laws a priori of frequency and severity and the estimation of their parameters $(\lambda_e, \mu_e, \sigma_e)$.
(2) The second objective is estimation of the expert weighting with the control functions (internal audit and permanent control).

5.1.3.1.1. Modelling the a priori law.

In this case the expert must provide:

(1) The estimation of the parameter μ_{e} of the lognormal law $LN(\mu, \sigma)$ which models the severity X_i by risk category knowing that σ is a constant and $\mu \sim N(\mu_{\mathrm{e}}, \sigma_0)$.
(2) The estimation of the parameter λ_e the Poisson's law $P(\lambda)$ which models the frequency N by risk category over a horizon (T) knowing that $\lambda \sim$gamma (a_0, b_0).

5.1.3.1.2. Weighting of the expert opinion.

The objective of weighting the expert opinion is to determine the parameters of a posteriori law. Indeed, for frequency, the weighting permits to determine the parameter $\hat{\lambda}_{Bay}$ of the Poisson's law relating to the frequency of losses by risk category RT_c. Whereas, for severity, it allows to determine the parameter $\hat{\mu}_{Bay}$ of the LogNormal law relating to the severity of losses by risk category.

5.1.3.2. Definition of interveners in data collecting process.

The evaluation of the parameters of a priori law involves all operational entities concerned as well as the risk management function:

− **The Risk managers**.

They have the status of evaluator because they must conduct the evaluation process with the various experts.

− **The responsible for declaring incidents (Risk correspondents) and their managers.**

This is an essential population of great added value given their experience in collecting incidents and their contributions to correct collecting bias.

− **Expert from the operating entities and des business lines.**

The operational losses are dependent on the business line and the activity exercised. Indeed, the severity and frequency generally reflect the risk profile of each activity and business line because they depend on the size of the transactions concluded by the business line (or activity exercised) and on their frequencies.

Consequently, the use of experienced and well-qualified experts is the first step in the evaluation process, which will be followed by a phase of estimation and aggregation of the data collected, which takes into account the specificities of the activity targeted by the evaluation.

− **Internal auditors and permanent controllers.**

The internal audit and permanent control functions have a right of supervision over all activities and execute, on a permanent or periodic basis, audit and control missions for the various business lines and operational entities. Their verification approaches are based on a risk identification approach using risk mapping and the database of events collected. Therefore, the recourse to the service of this category for the weighting of the experts' opinions is necessary.

5.1.3.3. Identification of problems, information sources and insufficiency.

The main reason for using expert opinion modelling is to reduce the uncertainty due to the change in the bank's risk profile caused by changes in the organization level and in the process of control and risk management, given that the distributions of observed historical losses in frequency and severity follow the Poisson law and the LogNormal law respectively .Indeed, uncertainty is linked to the change in the parameters of the two laws because the use of historical data alone can bias the estimation of risk capital.

Consequently, the expert opinion makes it possible to define the a priori law on the one hand and to weight the experts' estimate on the other hand. To do this, we will estimate with the business experts the average loss defined by formula (4), which will permit to determine the parameters λ_e and μ_e respectively.

5.1.3.4. Analysis and collecting of pertinent information.

In order to carry out the evaluation mission and ensure an acceptable reliability of the expert opinion, we have collected a series of relevant information such as:

(1) The evolution of the bank's size in terms of net banking income, the number of transactions, the number of incidents, the size of the banking network and the number of customer claims.
(2) The organizational and business changes such as the introduction of new products, industrialization of sales, control and treatment processes, external audits, control activities, outsourcing of activities.
(3) The major losses suffered and the action plans implemented and their impact on the control and risk management device.
(4) The formation programmers of operational risk and their frequency.

5.1.3.5. Choice of interveners in data collecting process.

5.1.3.5.1. Choice of Expert from the operating entities and responsible for declaring events of risk.

In our study, we weighted the expert opinion at 25%. However, the approach used is valid for any desired weighting.

Therefore, we have carried out the estimation with experts who can be weighted at 25%. To choose them, we drew a list of experts from the operating entities and responsible for declaring events of risk at the level of each business line and we scaled the estimate that each responsible and each expert can provide with a scoring system that we constructed, then we selected only those whose estimate can be weighted at 25%.

The determination of the score is made with the hierarchical managers and validated with the internal audit and permanent control functions on the basis of the following elements:

(1) Relevant expertise, academic and professional formation as well as the professional experience.
(2) The number of risk incidents declared and treated.
(3) Knowledge and mastery of the control device.
(4) The level of formation and knowledge in operational risk.
(5) The level of knowledge of descriptive and inferential statistics.
(6) Excellent communication abilities, flexibility, impartiality and capacity to generalize and simplify.

The score must give a value that corresponds to a grid of (10%, 25%, 40%, 50%, 75%). Indeed, each criterion must have a qualification between low, medium and high. To calculate the score, a rating was assigned to each qualification as follows:

Table 1 : Rating of the criteria for scoring experts

Qualification	Low	Medium	High
Rating	1	2	3

The expert score function that we retained for our study is equal to the sum of the ratings assigned to all criteria and the weighting is defined according to the score obtained, as follows:

Table 2 : Expert weighting according to the score function

Score	(6 à 7)	(8 à 9)	(10 à 11)	(12 à 14)	(15 à 18)
Weighting	10%	25%	40%	50%	75%

5.1.3.5.2. Choice of evaluators for internal audit and permanent control.

For the choice of evaluators for permanent control and internal audit, we based our choice on the following elements:

(1) Relevant expertise, academic and professional formation as well as the professional experience.
(2) The number of control and audit missions conducted annually.
(3) The level of formation and knowledge in operational Risk.
(4) The level of knowledge of descriptive and inferential statistics.
(5) Excellent communication abilities, flexibility, impartiality and capacity to generalize and simplify.

The designation of evaluators is made by consensus with the audit function and the permanent control function.

5.1.3.6. Knowledge of the objective of the operation by the experts and formation of experts.

Once, we selected the experts and evaluators, we organized an introductory session of the evaluation mission by presenting the main lines of the mission, the objectives, the speakers and the realization planning. Then, the following elements are sent to the participants before launching the evaluation meetings and workshops:

(1) The description of the objective of the operation.
(2) The list of experts from the operating entities, responsible for declaring events of risk, hierarchical managers and the evaluators for internal audit and permanent control.
(3) A summary description of risks, tools and operating system, organization and controls.
(4) Basic terminology, definitions that should include probability density, arithmetic and weighted mean, standard deviation, mode, median…etc.
(5) A detailed description of the process by which meetings and workshops to collect expert opinion are conducted and the average duration of their conduct.
(6) Methods for aggregating expert opinions.

5.1.3.7. Simulation, revision of assumptions and estimates.

To have the expert's consent to the estimates obtained, we proceeded as follows:

(1) The expert estimates the average loss per risk category that will be used to determine the parameters of frequency law $P(\hat{\lambda}_{expert})$ and severity law $LN(\hat{\mu}_{expert}, \hat{\sigma}_{expert})$ knowing that $\hat{\sigma}_{expert}$ is equal to σ determined by the likelihood. These parameters will be used to simulate, by Monte Carlo, three samples of the realizations concerning respectively the individual loss X_i, the frequency N and the annual loss $P(\sum_{i=1}^{n} X_i)$ Then we analyze the characteristics of these samples with the expert, in particular the average, median, maximum, minimum and maximum values… etc.
(2) If the expert accepts the simulations and their characteristics, the estimation of the parameters $\hat{\lambda}_{expert}$, $\hat{\mu}_{expert}$ and $\hat{\sigma}_{expert}$ will be validated.
(3) If the expert rejects the simulations, we will eliminate the outliers rejected by the expert and we will revise the expert's initial estimates and reputed the simulations in an iterative manner until the expert's consent is obtained.

5.1.3.8. Aggregation of estimates and validation.

In our study, the expert's estimate concerns the parameters $\hat{\lambda}_{expert}$, $\hat{\mu}_{expert}$ and $\hat{\sigma}_{expert}$. Therefore, we need to aggregate historical and expert estimates to determine the Bayesian estimator.

5.2. Determination of the Bayesian Estimator.

In the theoretical study, we showed that the Bayesian Estimators of the parameters of the severity and frequency distributions of losses are defined as follows:

(1) For frequency, formula (20) defines the Bayesian estimator of λ by:

$$\hat{\lambda}_{Bay} = \varepsilon_1 \times \lambda_{\text{expert}} + (1 - \varepsilon_1) \times \lambda_{16\text{bserve}}$$

(2) For severity, formula (21) defines the Bayesian estimator of μ by:

$$\hat{\mu}_{Bay} = \varepsilon_2 \times \mu_{\text{expert}} + (1 - \varepsilon_2) \times \mu_{\text{observé}}$$

In our study, the weights ε_2 and ε_2 are fixed at 25 % which corresponds to the scores of the selected experts.

6. Empirical study.

6.1. Data description.

In this study we used a database of loss incidents concerning the retail banking business line of a Moroccan banking institution, the database were constituted from the losses registered by the bank since the 1990s as well as the reports and missions of the audit.

The database is composed of 3581 losses, i.e. 2069 distinct amounts, the statistical characteristics of distinct losses are summarized as follows:

Table 3 : Descriptive statistics of losses (in amounts)

Mean	Standard Deviation	Skewness	Kurtosis
468 730	8 719 755,32	36,28	1 430,99

The distribution of the database by risk category RT_c shows that the losses of the category « RT_3 » represent 45%, followed by « RT_6 » that represent 19%, in third position the category « RT_1 » that represent 12%, followed by « RT_7 » with 10% and the other categories represent 15%. The statistical characteristics of the loss amounts by risk category are summarized in the following table:

Table 4 : Statistical characteristics of losses by risk category (in amounts)

RT_c	Distribution of losses in number	Mean	Distribution of losses by amount	Standard Deviation	Skewness	Kurtosis
RT_1	12%	781 175,04	16,8%	9 355 985,39	15,33	234,22
RT_2	9%	13 213,10	0,3%	63 647,81	7,123	52,090
RT_3	45%	34 573,71	0,7%	363 852,27	20,47	457,38
RT_4	4%	183 689,13	3,9%	781 324,16	5,81	32,690
RT_5	2%	199 146,42	4,3%	84 075,40	1,853	4,475
RT_6	19%	190 746,84	4,1%	1 379 733,78	10,248	113,639
RT_7	10%	3 249 849,84	69,9%	25 490 889,73	13,408	184,914

To determine the frequency of losses we will segment the database by a semi-annual horizon. The choice of horizon is made on the basis of the data available for modelling, which must be greater than 30 observations.

The statistical characteristics of the frequency by risk category are as follows:

Table 5 : Statistical characteristic of frequency by risk category

RT_c	Mean	Standard Deviation
RT_1	10,57	11
RT_2	11,87	14,54

RT_3	52,96	54,92
RT_4	3,17	2,71
RT_5	2,92	3,15
RT_6	38,72	52,82
RT_7	7,12	10,08

6.2. The LDA approach.

6.2.1. Statistical estimation of parameters.

The estimation of the parameters of the laws of severity and frequency based on observed data by risk category is as follows:

6.2.1.1. The parameters of the severity.

The adjustment test of the data with the lognormal law $LN(\mu_h, \sigma_h)$ s based on the Kolmogorov- Smirnov test. As a result, the estimation of the parameters and the results of the adjustment tests by risk category are as follows:

Table 6 : Estimation and adjustment test of the $LN(\mu_h, \sigma_h)$ by risk category

RT_c	$LN(\mu_h, \sigma_h)$		Kolmogorov- Smirnov test
	μ_h	σ_h	p-value
RT_1	10,60	1,67	0,084
RT_2	7,51	1,58	0,258
RT_3	8,59	1,49	0,419
RT_4	9,84	2,09	0,831
RT_5	12,14	0,35	0,649
RT_6	8,08	2,49	< 0,0001
RT_7	11,52	2,49	0,723

The Kolmogorov-Smirnov fit test shows that data from all categories adjust with the lognormal law except the category RT_6.

6.2.1.2. The parameters of the frequency.

The test for adjusting the frequency data with the Poisson law and the negative binomial law is based on the chi-square test. As a result, the estimation of the parameters and the results of the adjustment tests by risk category are as follows:

Table 7 : Estimation and adjustment test of the $P(\lambda_h)$ and $BN(a_h, b_h)$ by risk category

RT_c	Poisson $P(\lambda_h)$	p-value chi-square test	Negative-Binomial $BN(a_h, b_h)$		p-value chi-square test
	λ_h		a_h	b_h	
RT_1	10,57	< 0,0001	0,83	12,87	0,01
RT_2	11,87	< 0,0001	0,74	16,12	0,040
RT_3	52,96	< 0,0001	0,87	60,84	0,385
RT_4	3,17	< 0,0001	2 ,93	1 ,08	0,017
RT_5	2,92	< 0,0001	2,38	1,23	0,030
RT_6	38,72	< 0,0001	0,42	92,66	0,054
RT_7	7,12	< 0,0001	1,25	5,68	< 0,0001

The fit test shows that for a 5% threshold, the data does not adjust with Poisson law and Negative-Binomial law except the category RT_7 which adjusts with the Negative-Binomial law while for a 1% threshold, all categories does not adjust with the Poisson law but adjusts with the Negative-Binomial law except the category RT_7 which does not adjust with the Negative-Binomial law.

6.2.2. Experts' estimates.

The mean annual loss is determined by risk category by maintaining the same allocation structure for the mean annual losses. The calculation of the mean loss for the business line is defined as a percentage of the activity level of the business line (N_a). Indeed, the level of activity is deducted from the activity indicator presented above. In our research, the activity level N_a is defined as follows :

$$N_a = Min\left[\left(\frac{1}{3}\sum_{i=1}^{3}|PI_i - CI_i|\right) ; 2{,}25\% \times \left(\frac{1}{3}\sum_{i=1}^{3} API_i\right)\right] + Max\left[\left(\frac{1}{3}\sum_{i=1}^{3} PHC_i\right) ; \left(\frac{1}{3}\sum_{i=1}^{3} CHC_i\right)\right]$$

For the bank studied, the level of activity of the Retail Banking line is equal to 4 500 million MAD.

The experts' estimate of the mean loss for the business line is set at 1.5%, i.e. an mean loss of 6,75 million MAD, allocated as follows:

Table 8 : Expert estimate of mean losses (P_M) by risk category RT_C in millions of MAD

RT_c	mean losses (P_M) by category	Mean losses structure by category	Expert estimate of mean losses (P_M) by category
RT_1	18 966	31,6%	21 313
RT_2	314	0,5%	352
RT_3	2 349	3,9%	2 640
RT_4	2 266	3,8%	2 546
RT_5	1 704	2,8%	1 915
RT_6	4 706	7,8%	5 289
RT_7	29 762	49,5%	33 445

The estimation by the experts is made in two steps. Indeed, we will first estimate the mean semi-annual frequency (λ_e) then we will estimate the parameter μ_e of $LN(\mu_e, \sigma_h)$ from the formula (11) using the mean loss per risk category RT_c.

6.2.2.1. Expert estimation λ_e of the parameter λ.

The experts' estimate of the parameter λ_e is based on the approach defined above. in fact, the expert gives a first estimate based on historical data. This estimate is used to simulate the realization of the Poisson law according to the algorithm presented above, then the values judged outliers by the expert are deleted. We determine the new mean of the simulated sample that will be confirmed with the expert. The simulation is repeated until the expert's validation of the mean frequency by risk category is obtained.

The results of this approach are as follows:

Table 9 : The experts' estimate of the parameter λ_e by risk category

RT_c	λ_e
RT_1	11,5
RT_2	14,3
RT_3	54,6
RT_4	4,07
RT_5	3,40
RT_6	5,8
RT_7	3,5

6.2.2.2. Estimation of the parameter μ_e

The expert estimate of the parameter μ_e from the estimation of the mean loss and the mean frequency is made by the following formulas determined from the formulas (8) et (11) :

$$\begin{cases} P_M = \lambda_e E(X) \\ \mu_e = \ln\big(E(X)\big) - \dfrac{{\sigma_h}^2}{2} \end{cases}$$

As a result, the estimate of the parameter μ_e by risk category is presented as follows:

Table 10 : The expert estimate of the parameter μ_e by risk category

RT_c	P_M	λ_e	μ_e
RT_1	21 313	11,5	6,13
RT_2	352	14,3	1,95
RT_3	2 640	54,6	2,77
RT_4	2 546	4,07	4,25
RT_5	1 915	3,40	6,27
RT_6	5 289	5,8	3,72
RT_7	33 445	3,5	6,06

6.2.3. The Bayesian estimators of parameters.

The Bayesian estimators of frequency and severity are determined by the following relationships:

$$\begin{cases} \hat{\lambda}_{Bay} = \varepsilon_1 \times \lambda_e + (1-\varepsilon_1) \times \lambda_h \\ \hat{\mu}_{Bay} = \varepsilon_2 \times \mu_e + (1-\varepsilon_2) \times \mu_h \end{cases} \quad \text{with } \varepsilon_1 = \varepsilon_2 = 25\%$$

As a result, Bayesian estimators of severity and frequency by risk category, knowing that variance is a constant determined by likelihood, are presented as follows:

Table 11 : The Bayesian estimators of parameters by risk category

RT_c	$P(\lambda)$	$LN(\mu,\sigma)$	
	$\hat{\lambda}_{Bay}$	$\hat{\mu}_{Bay}$	σ_h
RT_1	10,80	9,48	1,67
RT_2	12,48	6,12	1,58
RT_3	53,37	7,14	1,49
RT_4	3,40	8,44	2,09
RT_5	3,04	10,67	0,35
RT_6	30,49	6,99	2,49
RT_7	6,22	10,16	2,49

6.2.4. Determination of VaR by risk category.

The determination of the VaR is made according to the approach presented above. Indeed, the breakdown of VaR based on historical LDA and the Bayesian LDA by risk category is as follows:

Table 12 : The VaR under Historical and Bayesian LDA by risk category (in $kMAD$)

RT_c	$VaR_{C,h}\ (\lambda_h, \mu_h, \sigma_h)$	$VaR_{C,bay}\ (\hat{\lambda}_{Bay}, \hat{\mu}_{Bay}, \sigma_h)$
RT_1	45 526	14 449
RT_2	1 560	367
RT_3	6 926	1 767
RT_4	45 526	11 764
RT_5	4 026	955
RT_6	164 222	45 554
RT_7	1 641 160	384 680

The use of expert opinion has permitted to minimize VaR by risk category. Indeed, the experts readjusted the parameters of the distributions of severity and frequency for all categories, in order to take into account organizational changes and the strengthening of the control device.

6.3. Capital allocation.

The capital is allocated in accordance with formula (1). Indeed, each category benefiting from a percentage of the capital allocated to the retail banking business line equivalent to the ratio of its VaR_C and the sum of the $VaRs$ of all categories ($\sum_{c=1}^{7} VaR_C$). he capital share of each class is as follows:

Table 13 : Capital allocation through historical and Bayesian approaches

RT_c	Percentage of capital allocated under the classical LDA	Percentage of capital allocated under the bayesian LDA
RT_1	2,38%	3,14%
RT_2	0,08%	0,08%
RT_3	0,36%	0,38%
RT_4	2,38%	2,56%
RT_5	0,21%	0,21%
RT_6	8,60%	9,91%
RT_7	85,97%	83,71%

The use of VaR through the use of the traditional approach or by incorporating expert opinion shows that the capital allocated to retail banking will be divided mainly into the risk categories RT_6 and RT_7. Indeed, the category RT_7 benefited respectively from 85,97% and 83,71% of the allocated capital, followed by the category RT_6 respectively from 8,60 et 9,91% while the other categories benefited respectively from 5,43% and 6,38%.

7. Conclusion

Internal models permit to determine the economic capital independently of the regulatory capital and to determine the impact of the occurrence of risk events at the level of the different entities and at the aggregate level under the Bottom-up approach or the Top-down approach.

for the risk identification process, banks are free to use their own models to achieve the objective of risk supervision in accordance with the second pillar relating to prudential risk management. This situation encourages the use of internal models that can be based either on historical data, expert opinion or a combination of historical data and expert opinion.

The use of expert opinion is essential in risk management given the recurrent changes in organization, business size and control device. Indeed, the expert opinion permits to readjust the estimates and assumptions based on historical data taking into account the changes operated.

The reliability of models incorporating expert opinions depends on the approach used to collect the requested information. Indeed, it is necessary to adopt rigorous procedures and approaches at the theoretical and practical levels in order to avoid the risk of a model.

In this context, we have presented in this paper a process for collecting information from experts specific to operational risk, based on the Delphi method, which we believe will give the relevant results for risk measurement if it is correctly administered.

For the prospects of internal models for quantifying operational risk, banks must separate regulatory capital requirements from internal requirements for managing the return/risk trade-off. Indeed, they must develop internal risk measures allowing them to manage their activities through risks and allocate the necessary equity capital for their business plans.

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Annexe1 :
Démonstration :

1. Montrons que pour $n \geq 1$ l'évènement $(N \geq n)$ est équivalent à $(S_n = \sum_{i=1}^{n} V_i \leq 1)$.

En effet, l'évènement $(N \geq n)$ signifié que le plus grand des k tels que $\left(S_k = \sum_{i=1}^{k} V_i \leq 1\right)$ est au moins égal à n, ce qui se résume par $(S_n \leq 1)$.

2. Montrons que N suit une loi de Poisson de paramètre λ. Nous effectuons les deux étapes suivantes :

a. Montrons que $p(S_{n+1} \leq 1) = 1 - \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}$ pour $n \geq 0$:

En effet, la formule de Taylor avec reste d'intégrale donne :

$$e^{\lambda} = \sum_{j=0}^{n} \frac{\lambda^j}{j!} + \int_0^1 e^{\lambda-t} \frac{t^n}{n!}\, dt \ (n \geq 0)$$

En multipliant la formule par $e^{-\lambda}$ on obtient : $1 = \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!} + \int_0^1 e^{-t} \frac{t^n}{n!}\, dt \ (n \geq 0)$

En opérant le changement de variable $u = \frac{t}{\lambda} \Leftrightarrow t = \lambda u$ dans l'intégrale on déduit :

$1 = \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!} + \int_0^1 e^{-\lambda u} \frac{\lambda^{n+1} u^n}{n!} du$

d'où

$$\int_0^1 e^{-\lambda u} \frac{\lambda^{n+1} u^n}{n!} du = 1 - \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}$$

Montrons que $e^{-\lambda u} \frac{\lambda^{n+1} u^n}{n!}$ est la densité de S_{n+1}. En effet, de la relation (41) on déduit que la fonction de densité $h(z)$ de $S_2 = V_1 + V_2$ est le produit de convolution suivant :

$$h(z) = \int_{-\infty}^{+\infty} f(z-y) g(y) dy$$
$$= \int_0^z (-\lambda e^{-\lambda(z-y)}) * (-\lambda e^{-\lambda(y)}) dy$$
$$= \int_0^z \lambda^2 e^{-\lambda(z-y)} e^{-\lambda(y)} dy = \int_o^z \lambda^2 e^{-\lambda(z-y)} e^{-\lambda(y)} dy = \int_o^z \lambda^2 e^{-\lambda(z)} dy = \lambda^2 z e^{-\lambda z}$$

On suppose, par hypothèse de récurrence, que S_n est de densité $e^{-\lambda u} \frac{\lambda^n u^{n-1}}{(n-1)!}$. Montrons que la densité de S_{n+1} est $e^{-\lambda u} \frac{\lambda^{n+1} u^n}{n!}$. On a :

$h(z) = \int_0^z \left(-\lambda e^{-\lambda(z-y)} e^{-\lambda y} \frac{\lambda^n y^{n-1}}{n-1!} \right) dy = \int_0^z -e^{-\lambda(z)} \frac{\lambda^{n+1} y^{n-1}}{n-1!} dy = e^{-\lambda z} \frac{\lambda^{n+1} z^n}{n!}$

d'où

$$p(S_{n+1} \leq 1) = \int_0^1 e^{-\lambda u} \frac{\lambda^{n+1} u^n}{n!} du = 1 - \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}.$$

b. Montrons que $p(N \leq n) = \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}$ pour $n \geq 0$.

- Pour $n = 0$; $p(N \leq 0) = p(N = 0) = p(V_1 > 1) = 1 - p(V_1 \leq 1) = 1 - (1 - e^{-\lambda}) = e^{-\lambda}$
- Pour $n \geq 1$ on sait que :

$$p(N \geq n+1) = p(S_{n+1} \leq 1) = 1 - \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}$$

Donc

$p(N \leq n) = \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!}.$

On déduit que

$p(N = n) = e^{-\lambda} \frac{\lambda^n}{n!}.$

En effet :

$$p(N = n) = p(N \leq n) - p(N \leq n-1) = \sum_{j=0}^{n} e^{-\lambda} \frac{\lambda^j}{j!} - \sum_{j=0}^{n-1} e^{-\lambda} \frac{\lambda^j}{j!}$$
$$\Leftrightarrow p(N = n) = e^{-\lambda} \frac{\lambda^n}{n!}$$

Donc N suit une loi de Poisson.

Annexe2 :

```
dim=100000; % number of scenarios
mu=11.52;sigma=2.49;   % severity (lognormal)
```

```
% parameters
lambda=7.12;    % frequency parameter
% (average frequency)
%% Monte Carlo using cellfun
N1=num2cell(poissrnd(lambda,dim,1));
Loss1 = cellfun(@(x) sum(lognrnd...
(mu,sigma,x,1)), N1,...
'UniformOutput', false);
Loss1=cell2mat(Loss1);
% aggregate loss
% distribution (empirical
T=Loss1;
VaR=prctile(T,99.9)
```

http://em.swu.bg/index.php?option=com_content&view=article&id=84&Itemid=286