Are Global Factors Useful for Forecasting the Exchange Rate?

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ABSTRACT

This paper applies a common factors analysis framework to an exchange rate data panel, in order to better understand the forces behind exchange rate dynamics and provide a set of variables for exchange rate forecasting. Results demonstrate the model's ability to extract key information to understand the driving forces of exchange rate movements using the exchange rate Brazilian Real to U.S. dollar; in addition, the model also shows statistically significant improvement in terms of forecasting performance in relation to the random walk benchmark as well as to traditional macroeconomic models found in economics literature.

Keywords: exchange rate, common factors, forecasting
JEL Classification: F31, F37, F47

1 Introduction

The collapse of the Bretton Woods system in the early 1970s meant the end of the direct convertibility of U.S. dollars into gold, the standard of parity for international currencies. Replacing it, floating exchange rates were gradually adopted worldwide; Brazil in particular adopted this new regime in 1999, as a sustaining pillar of a new macroeconomic policy model.

Meese and Rogoff (1983) verified the difficulty in forecasting the exchange rate using macroeconomic models. Basing themselves in structural models suggested by economic theory, their tests demonstrated that conventional models had fared no better at the task as compared to a random walk. This result is particularly striking when taking into account that the structural models used for forecasting took realized values of the explanatory variables, thus constituting what became known as the Meese and Rogoff Puzzle.

Recently several papers tried to exploit different factors besides macroeconomic variables to explain exchange rate movements. These papers advocate that global risk aversion, liquidity, and financial cycles are important drivers of the exchange rates, causing a significant level of comovement among exchange rates. Examples of this literature are Cayen et al. (2010), Lustig, Roussanov, and Verdelhan (2011), Greenaway-McGrevy et al. (2018), Aloosh and Bekaert (2019), Baku (2019) and Estradas and Romero (2022).

The paper analyzes whether common movements of the exchange rate of a panel data of world currencies is able to beat the random walk in forecasting the exchange rate Brazilian Real to the U.S. dollar. The paper goes one-step ahead and studies the main determinants of the comovements in the exchange rates. The paper extends the results of Felício and Rossi (2014) verifying the robustness of the results, including more explanatory variables and analyzing the behavior of the exchange rate during the Covid-19 pandemic, a period when the global economy underwent a significant shock.

In addition, the Brazilian Real brings an exceptional case study. Besides adopting a free floating exchange rate regime according the IMF classification and it is financially and economically related to the global economy. During the period of the study between 2002 and 2021, the country has gone through a prolonged recession from 2014 to 2017, with mounting fiscal problems pressed further with the pandemic expenditures, bringing sufficient volatility to the domestic macroeconomic variables.

The results show a statistically significant improvement in predictive performance of the factor model compared to traditional macroeconomic models suggested by Economics literature as well as to the random walk, the benchmark posed by the Meese & Rogoff Puzzle. Moreover, The paper is able to link the extracted factors to observable variables acting as proxies to global economic shocks, thereby offering an economic interpretation to the factors which can be useful for the study of exchange rate dynamics.

The paper is structured as follows: the second section covers some of the relevant literature pertaining to the exchange rate dynamics' determinants and to its forecasting. Section three presents the data used in my work, while section four explains the factor model and the specifications I used for forecasting. In the fifth section the paper analyzes the extracted common factors; forecasting results, both for in-sample and out-of-sample tests, are presented in section six. Finally, the paper concludes pointing for potential future research.

2 Literature Overview

Since Meese and Rogoff's (1983) paper, further studies in the line of finding a relation between the exchange rate and other macroeconomic variables, such as monetary aggregates, interest rates, inflation, output gaps, productivity, among others, has attempted to find a model capable with better forecasting power than the random walk benchmark proposed by the authors.

Monetary models of exchange rate determination highlight this variable's concept as a relative price between two different currencies, a price which is derived from relative supply and demand of monetary stock, as well as relative interest rate and output differentials. In his 1976 paper on the subject, Dornbusch (1976) constructed a sticky price model of exchange rate adjustment, drawing a connection between monetary expansions and the depreciation of the exchange rate, postulating the occurrence of an overshooting of the exchange rate from its equilibrium level as a reaction to short term monetary variations.

Though the theoretical framework of monetary models has found prominence in the exchange rate determination literature, the validity of its findings has found mixed reception. The model had a revival in Mark (1995), who found a significant relation between exchange rate movements and monetary fundamentals; despite this, the robustness of his findings have been questioned in later papers (Berkowitz and Giorgianni, 2001; Kilian, 1999; Groen, 1999; Faust et al., 2003). Ever since, other attempts to find a link between this class of variables and the exchange rate has gone through different iterations, with varying levels of success, as in Mark and Sul (2001) and Groen (2005).

In their 2005 paper, Cheung et al. (2005) seek to test the performance of several exchange rate determination models suggested by 1990s economic literature. Their results echo those of Meese and Rogoff (1983), not finding a model that could consistently outperform the random walk in the short run.

An alternative way to analyze the relation between the exchange rate and monetary policy would be to see the latter as being determined as a specific reaction function, taking interest rate differentials as primary tools for policy enforcement, instead of the level of macroeconomic variables (Mark, 2009). Engel and West (2004), Mark (2009) and Moldtsova and Papell (2009) model an exchange rate determination function through Taylor Rule variables, the performances of which have been found to be promising in the forecasting of change of direction of the exchange rate in the long run.

The core question about the relative failure in coming up with models with good forecasting power for the exchange rate, however, might be in this variable's own determination. As highlighted by Felício and Rossi Júnior (2014), if we take the exchange rate to be a discounted value of present and future macroeconomic variables, the dynamic of non-observable variables, such as noise trading and risk premiums, could exert some influence over exchange rate movement, along with well-known observable variables from past literature—price levels, the output gap, interest rates, among others. Moreover, as pointed out in Mark (2009), if the correlation between observable and non-observable variables is small, it is possible that a sizeable chunk of exchange rate variation is not being appropriately captured by models based on the usual parameters, thus resulting in a poor forecasting performance.

From this, we can imagine that the inconsistent relation between macroeconomic fundamentals and exchange rate movements be such that, essentially, the models are not correctly capturing the influence of non-observable factors, or expectation formation by market agents.

Baccheta and van Wincoop (2004 and 2013) corroborate this line of thinking, through an apparatus called The Scapegoat Theory. The authors explore the possibility that excessive importance is attributed to some macroeconomic variables in the determination of the exchange rate, due to heterogeneous market information; the true force behind exchange rate movements would come in the form of a set of variables which are non-observable to the agents.

The microstructure approach sees exchange rate determination as derived from sequences of order flow, that is, net demand pressures from the several agents involved in FX operations (Evans and Lyons, 2002; Chinn and Moore, 2011). Order flow aggregates informational aspects processed in regards to macroeconomic fundamentals that will be accordingly priced in the exchange rate. Exchange rate variations, thus, would be consequential to this process, anticipating future changes in level of macroeconomic components (Vitale, 2007).

This paper follows a different line from the aforementioned models and propositions, using principal factors in order to extract common components from exchange rate fluctuations of 18 countries, starting from a time after Brazil's adoption of the free floating exchange rate regime. In order to obtain relevant information with regard to exchange rate forecasting, I expect that a parsimonious number of factors be related to non-observable variables whose movements influence exchange rate dynamics in a meaningful way (Groen 2006; Felício and Rossi Júnior, 2014).

Vector Autoregression (VAR) methods may not be appropriate when using sizeable panel data. This is due to the technique's requirement of the estimation of several parameters (Forni and Reichlin, 1998). Therefore, the factor model appears as a more viable alternative to estimation and application in this paper, in the sense that it simplifies the model, lessens the amount of parameters to be estimated, and can extract and compute the main non-observable drivers of exchange rate movement; additionally, data is more readily obtained, compared to the microstructure approach. I hope to establish, by applying this model, a link between factor characteristics and proxy variables to global shocks, so as to better understand exchange rate dynamics.

The hypothesis is that the exchange rates contain information hidden in common trends that can be extracted in order to get more accurate models for forecasting. This withheld information can come from variables whose measurement is not exactly representative of their true levels. In Groen (2006), Engel et al. (2015) and Greenway-McGrevy et al. (2018), exchange rate forecasting with factor models has obtained promising results relative to the performance of the random walk.

3 Data

The data used are of a monthly frequency, beginning in January 2002 and ending in September 2021. Save indication of the contrary, the data is in log-difference format; thus, the total sample includes 236 observations. Exchange rate data refers to end-of-month rates, in national currency units relative to one U.S. dollar, among a selection of 19 countries and one monetary union. These were selected for adopting a free-floating exchange rate regime as well as independent monetary policy, according to a report from the International Monetary Fund (2021): Australia, Brazil, Canada, Chile, Colombia, Iceland, Israel, Japan, Mexico, New Zealand, Norway, Peru, Poland, South Africa, South Korea, Sweden, Switzerland, the Philippines, the United Kingdom and the Euro Zone. These data were collected for the most part from historical series available from the respective central banks of the selected countries, or from the Federal Reserve St. Louis (FED) database.

With relation to macroeconomic variables, most traditional models from literature have used price levels, output gap and monetary aggregates differentials to forecast the exchange rate. For the seasonally adjusted monthly inflation indices accumulated in twelve periods It is used the *Índice Nacional de Preços ao Consumidor* (IPCA), published by the *Instituto Brasileiro de Geografia e Estatística* (IBGE) and the American Consumer Price Index (CPI), obtained from the U.S. Bureau of Labor Statistics. For monetary aggregate it is used the M2 series for Brazil and the U.S., which It is obtained from the seasonally adjusted historical series published by the Brazilian Central Bank and FED St. Louis, respectively. It is used the seasonally adjusted industrial production of Brazil and the U.S. as a proxy for national output, as the latter is generally only available on a quarterly basis; the output gap was estimated from a Hodrick-Prescott filter.

Recent focus on the risk aversion against Brazilian assets, as the debate on the government's fiscal constraints becomes ever more heated, brings to mind that macroeconomic variables relating to the public debt and debt-risk perceptions may influence the exchange rate. The variables included in this vein are the SELIC rate, the yield rate on ten-year Brazilian government bonds, and the Brazilian nominal deficit. The SELIC rate and the Brazilian nominal deficit as a percentage of GDP were obtained from the historical series published by the Brazilian Central Bank, while the yield rate for Brazilian government's 10 year bonds (NTN-Fs) was obtained from the Brazilian National Treasury.

Figure 1 - BRL/USD nominal exhange rate

The Brazilian Real/U.S. dollar nominal rate during the considered period is plotted in figure 1, with the variable in level form. After a depreciation throughout 2002 with the uncertainties associated to the presidential election, the exchange rate showed a steady movement of appreciation up until 2008, when the global financial crisis hit; the exchange rate appreciated in 2009 and presented relative stability from 2010 to 2012. From 2012 to 2015 the exchange rate depreciated; with the start of an economic crisis in Brazil from the last quarter of 2014, the exchange rate began a continuous and strong movement of depreciation, which was partially subsided in the second semester of 2016, after which a relative stability period was maintained for two years. A new depreciation movement followed starting from the second half of 2018, once more associated to uncertainties related to the electoral period. From the last quarter of 2019, a new cycle of depreciation began, influenced by international trade tensions between the United States and China; this movement gained significant momentum in 2020 with the Covid-19 pandemic, as the adoption of restrictive measures had adverse effects on the economy at the global and national levels.

4 A model for exchange rate forecasting

An exchange rate determination model can be defined as such:

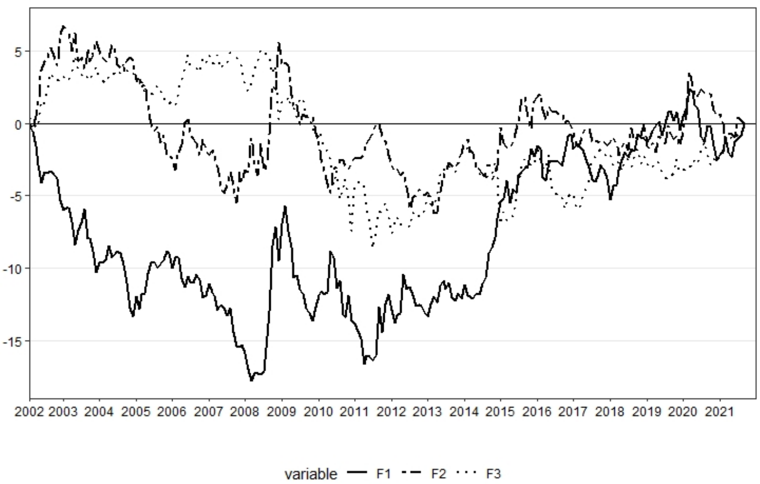
$$\Delta S_t = \alpha + \beta F_t + \gamma Z_t + u_t \tag{1}$$

Where ΔS_t is the change in the exchange rate (in logarithmic form), F_t is the set of common global factors and Z_t the set of macroeconomic variables — this latter set following all specifications detailed in the previous section. The sole variable which is not in logarithmic form is that of the Brazilian government's nominal deficit as a percentage of GDP.

4.1 Common factors

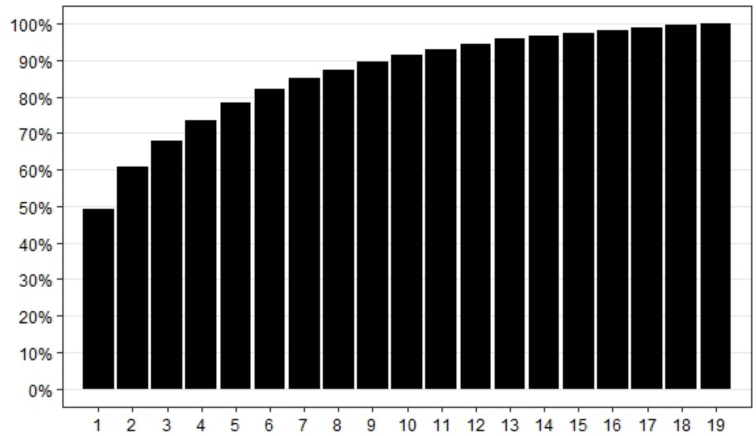
The idea behind using common factor analysis is to investigate a multivariate space, explaining the observable variables' variance in terms of a small number of non-observable variables called factors. The analysis follows with the estimation of factor loadings, which are the link between the factors and the observable variables — the goal is to establish economic meaning to the extracted factors through an interpretation of the loadings.

Figure 2: Evolution of the common factors



It is used an exploratory factor analysis to extract common factors from a panel of the exchange rates from 19 countries, estimating an orthogonal model from this data. The Kaiser-Guttman criteria was used, together with a scree plot, in order to determine an appropriate number of factors for the model; the criteria pointed to the extraction of three factors as the most adequate to analyze the dynamics of exchange rate movements. Figure 2 plots the evolution of the three estimated factors, accumulated by period. Figure 3 shows the percentage of the exchange rate's variance that is cumulatively explained by the first n factors estimated in the sample from 2002 to 2021. Note that the first three factors to be used in the analysis explain almost 70% of the variance, 49%, 11.7% and 7.2% of data variance, respectively.

Figure 3: Accumulated variance explained by the first 19 extracted factors



5 Common Factor identification

Attributing an economic interpretation to the estimated factors may help in establishing a connection between the driving forces behind exchange rate fluctuation and the observable variables, providing a better understanding of exchange rate determination. This identification and attribution of meaning to the factors can be done, primarily, by analysing the estimated factor loadings.

Table 1 shows, to the left, the estimated factor loadings associated to each of the 19 currencies from the sample — with the exception of Brazil's, as its inclusion in the estimation model would contaminate the sample for future forecasting purposes. The first loading has positive weight for all currencies considered; as a weighted mean of all exchange rates, following Felício and Rossi Júnior's (2014) approach that this factor reflects common movements of all currencies with relation to one reference currency — namely, the U.S. dollar. Put it another way, this factor would be representative of the Dollar Effect, that is, the strength of the U.S. dollar in relation to the the mean of the other currencies.

Table 1: Factor loadings

	Factor Loadings			Regressions			
	F1	**F2**	**F3**	**F1**	**F2**	**F3**	**Adj. R^2**
AUS	0.8284	0.2934	0.1121	0.029 (31.048)*	0.012 (11.947)*	0.006 (5.172)*	0.828
CAN	0.6635	0.4129	-0.0734	0.018 (17.797)*	0.013 (12.033)*	-0.003 (−2.423)**	0.664
CHE	0.8143	-0.4331	0.2067	0.024 (82.075)*	-0.015 (−47.435)*	0.009 (25.646)*	0.976
CHL	0.5511	0.3263	0.2282	0.020 (12.324)*	0.014 (7.928)*	0.012 (6.281)*	0.517
COL	0.5553	0.4522	0.2058	0.022 (14.130)*	0.021 (12.503)*	0.012 (6.446)*	0.627
EUR	0.9105	-0.1927	-0.1432	0.025 (53.105)*	-0.006 (−12.210)*	-0.006 (−10.283)*	0.929
GBR	0.6654	0.0727	-0.2019	0.017 (14.922)*	0.002 (1.772)***	-0.008 (−5.573)*	0.519
ISL	0.5121	0.1181	0.0007	0.021 (9.368)*	0.005 (2.348)**	0.00004 (0.016)	0.278
ISR	0.5042	0.0965	0.0207	0.011 (9.139)*	0.002 (1.900)***	0.007 (0.461)	0.264
JPN	0.2477	-0.3206	0.2455	0.006 (4.517)*	-0.009 (−6.351)*	0.009 (5.509)*	0.273
KOR	0.6827	0.1075	0.2006	0.021 (15.833)*	0.003 (2.709)*	0.009 (5.726)*	0.550
MEX	0.5553	0.4365	0.0519	0.018 (12.829)*	0.017 10.957)*	0.002 (1.476)	0.547
NOR	0.8400	0.0957	-0.1806	0.029 (28.16)3*	0.004 (3.487)*	0.009 (−7.452)*	0.785
NZL	0.5095	0.1056	0.2702	0.032 (10.113)*	0.008 (2.277)**	0.026 (6.602)*	0.386
PER	0.4274	0.3023	0.1214	0.006 (8.018)*	0.005 (6.162)*	0.002 (2.803)*	0.313
PHL	0.4411	0.2369	0.2131	0.007 (8.383)*	0.004 (4.892)*	0.005 (4.985)*	0.331
POL	0.8559	0.1020	-0.0879	0.034 (28.080)*	0.004 (3.636)*	-0.005 (−3.551)*	0.775
SWE	0.8964	-0.0181	-0.2001	0.029 (41.638)*	-0.0007 (−0.912)	-0.010 (−11.440)*	0.888
ZAF	0.5876	0.3119	0.2085	0.028 (13.398)*	0.018 (7.726)*	0.015 (5.851)*	0.535

Table 1 – Presents the loadings of each estimated factors and the results from estimating the equation $\Delta S_{it} = \alpha_i + \sum_{j=1}^{3} \delta_{ij} + v_{it}$ for each country i, where $\sum_{j=1}^{3} \delta_{ij} = F_t$ the set of three common factors (the values pertaining to the estimation of the α constant have been omitted here). The set of 19 countries and monetary union is as follows: Australia (AUS), Canada (CAN), Switzerland (CHE), Chile (CHL), Colombia (COL) Eurozone (EUR), United Kingdom (GBR), Iceland (ISL), Israel (ISR), Japan (JPN), Mexico (MEX), Norway (NOR), New Zealand (NZL), Peru, (PER) Philippines (PHL), Poland (POL), Sweden (SWE), and South Africa (ZAF). (*), (**) and (***) represent p-values below the 1%, 5%, and 10% significance values, respectively, while the t-value is found between parentheses.

An analysis of figure 2 lends support to this interpretation. From 2002 to 2008, the U.S. dollar depreciated in relation to other currencies, coinciding with the factor's downwards movement along the period. Soon follows a rapid increase in the factor; the flight-to-quality trend in the fallout of the 2008 financial crisis strengthened the U.S. dollar. The factor's fall in 2009 up to 2011 — with the exception of 2010's first half — also coincides with the U.S. dollar's appreciation trend compared to its peers in the period. After a relative stabilisation until the end of 2014, the dollar gains strength in 2015, as a reflection of the American economy's strength, and once again after 2018, with a new flight-to-quality movement as a result of an increase in international trade tensions and the Covid-19 pandemic; in figure 8, the factor followed these developments, increasing with the strenghtening of the U.S. dollar and decreasing when it faltered.

To table 1's right are the results of equation 1's estimation, taking in consideration only the common factors. The first factor had a positive impact in all considered currencies, on top of being statistically significant for all as well; moreover, as seen on figure 3, this factor explains over 50% of the exchange rate sample variance.

As to the second factor, an analysis of the factor loadings seem to indicate that the factor appears to indicate a division between countries concerning a perception of national currency quality by part of investors. Currencies with negative second factor loadings — above all the Japanese Yen, the Euro and the Swiss franc — can be taken as desirable value reserves in moments of upheaval of the global economy. Meanwhile, currencies such as the Mexican Peso and the Colombian Peso are the ones with the highest positive factor loadings and estimated coefficients. Thus, this second factor would represent the movement of capital flight to develop countries with more robust currencies.

The third factor, however, poses a conundrum in that there is no clear pattern between the factor loadings and listed countries that incites a natural association or economic interpretation, contrary to the previous two.

If the factors are an amalgamation of information hidden in several exchange rate drivers, another way to identify the factors is to analyse their correlation with other, visible proxy variables, representative of global shocks. McGrevy et al. (2017) determine that the common movements in exchange rates originate in factors related to shocks to the U.S. dollar and the Euro — the most traded currencies in the global foreign exchange market. The authors suggest their results could give a risk-based interpretation of the factors, with the currencies in question acting as measures of global risk in a macro level — conclusion such based on the concept of the exchange rate as a representation of the future discounted value of economic activity. Similar risk-based interpretation is highlighted by Lustig et al. (2011), who found a relation between extracted factors from a currency portfolio and share market volatility. In addition, the role of liquidity risk is found as significant in exchange rate determination in Menkhoff et al. (2012) and Banti et al. (2012).

The proxy variables included are the CRB commodity index — Cayen et al. (2010) identify commodity prices as correlated to global factors in their own estimation of a common factor model —; the spot price of gold, due to its role as a value reserve in times of turbulence of the U.S. economy; Bank of America's High Yield Spread, as a proxy variable to risk aversion; the VIX index of implicit options volatility in the S&P 500; as well as the TED spread as a proxy of liquidity shocks. Table 2 shows the correlation matrix between the factors and the chosen proxy variables.

An analysis of table 2 shows that the factors have significant correlation with the

Table 2: Correlations

	F1	F2	F3	CRB	Gold	VIX	TED Spread	High Yield Spread
F1	1							
F2	0	1						
F3	0	0	1					
CRB	−0.583*	−0.297*	0.110	1				
Gold	−0.439*	0.051	−0.151*	0.309*	1			
VIX	0.320*	0.428*	0.013	−0.309*	0.011	1		
TED Spread	0.106	0.124	0.069	−0.194*	-0.061	0.136*	1	
High Yield Spread	0.422*	0.505*	-0.001	−0.534*	-0.059	0.586*	0.314*	1

Table 2 – Correlation between estimated common factors and some proxy variables to obervable global shocks. All variables are in log-difference. CRB is the Commodity Research Bureau's commodity price index; Gold refers to the spot price of gold; VIX is the Chicago Board Options Exchange Market Volatility Index; TED Spread is the difference between rates of inter-bank lending and the short run rate of the U.S. government's debt bonds; High Yield Spread is Bank of America's High Yield Spread. (*) represents a 5% level of significance.

chosen proxy variables; the results also help in the attribution of economic meaning to the factors.

The first factor is more strongly correlated with the CRB index, corroborating with the interpretation that this factor represents the relative strength of the U.S. dollar in comparison to the other currencies, as is standard practice for commodity products worldwide to be priced in U.S. dollars.

The second factor, meanwhile, is more strongly correlated with the risk aversion and volatility indices, represented here by the VIX and BofA's High Yield Spread; this also falls in line with the previous interpretation given to this factor, that it would be a measure of risk perception by part of global investors in the dynamics of foreign exchange. The third factor doesn't appear to have strong correlation with any of the chosen proxy variables, though it might by noted that it found its strongest correlation with the spot price of gold.

Table 3 was built as an attempt to establish a formal connection between the common factors and the proxy variables chosen. To the right of the table, an ordinary least squares regression for each factor was performed, with all variables in log-difference; to the left side the regression exercise is that of a dynamic ordinary least squares regression (DOLS) in order to capture a possible long-run relationship between the variables. As the possibility of cointegration between the variables is being tested, in this latter exercise the proxy variables are formatted in levels.

Just as in table 2, results from table 3 show that the first factor is more strongly linked to commodity prices. The second factor, meanwhile, shows significant relations with the volatility index (the VIX), risk aversion (BofA's High Yield Spread), as well as gold and commodity prices. The third factor doesn't appear to have a consistent link with

Table 3: Results of factor identification

	F1	F2	F3	F1	F2	F3
CRB	-7.128	-1.305	3.681	-0.077	-0.018	0.003
	$(-6.241)^{*}$	(-1.088)	$(3.024)^{*}$	$(-15.054)^{*}$	$(-3.688)^{*}$	(0.475)
Ouro	-6.318	1.693	-3.500	-0.0001	-0.004	-0.006
	$(-6.148)^{*}$	(1.568)	$(-3.196)^{*}$	(0.120)	$(-3.792)^{*}$	$(-6.654)^{*}$
VIX	0.611	0.826	0.140	-0.019	0.164	0.219
	$(2.185)^{**}$	$(2.814)^{*}$	(0.470)	(-0.461)	$(2.790)^{*}$	$(2.475)^{**}$
TED Spread	-0.159	-0.077	0.199	2.160	-1.573	2.920
	(-0.956)	(-0.442)	(1.120)	$(2.485)^{**}$	(-1.367)	$(4.015)^{*}$
High Yield Spread	1.280	3.098	0.538	-0.355	-0.087	-0.687
	$(1.988)^{**}$	$(4.578)^{*}$	(0.784)	(-0.415)	(0.531)	$(-3.129)^{*}$
Constant	0.067	0.001	0.022	12.835	6.150	3.398
	(1.378)	(0.020)	(0.434)	$(8.339)^{*}$	$(3.296)^{*}$	(1.558)
R^2	0.456	0.291	0.065	0.902	0.644	0.762
Adjusted R^2	0.445	0.276	0.045	0.897	0.628	0.751
Cointegration				Yes	Yes	Yes

Table 3 – The table shows results from the identification of common factors. To the left of the table, variables are in log-difference. To the right are the results of an Engle-Granger cointegration test between the variables through a dynamic OLS (DOLS) framework with the inclusion of one lag; variables are in level. CRB is the Commodity Research Bureau's commodity price index; Gold refers to the spot price of gold; VIX is the Chicago Board Options Exchange Market Volatility Index; TED Spread is the difference between rates of inter-bank lending and the short run rate of the U.S. government's debt bonds; High Yield Spread is Bank of America's High Yield Spread. (*), (**) e (*) represent 1%, 5% and 10% significance levels, respectively.

any variable in either regression specification, save for gold prices; in the OLS framework the relationship with both commodity and gold prices is significant, while in the DOLS framework, the gold price, TED Spread and High Yield Spread variables have the most significant coefficients.

An analysis of the R^2 and adjusted R^2 of each specification shows the chosen variables have good enough predictive power for the first couple of factors, while showing disparate results for the third factor — while the OLS regression points to the chosen variables being poor predictors, the same set of variables points to the other direction under the DOLS framework. The cointegration tests did not reject the hypothesis of common movement in the long-run for the set of variables at a 10% significance level; while this corroborates with the previous results concerning the first two factors, it mounts to the confusion surrounding the third factor's identification in the context of exchange rate dynamics.

6 Exchange rate forecasting

In this section It is performed in-sample and out-of-sample tests so as to verify equation 1's performance in forecasting the exchange rate. It is tested whether the factor model is able to extract key information to the understanding of the drivers behind exchange rate fluctuations and if it is possible to outperform the forecasting benchmark set by the random walk.

6.1 Unit root tests

Testing for a unit root in each model's variables is necessary in order to check the stationarity of the series. Were the stationarity of the series not to be assessed, estimation and results obtained from equation 1 may not be valid. Tables 4 shows the results for the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root tests.

Table 4: Results of the unit root tests

Variable/Unit root test	ADF	KPSS
Exchange rate (BRL/USD)	-9.27^{*}	0.255^{+}
F1	-10.36^{*}	0.330^{+}
F2	-10.30^{*}	0.044^{+}
F3	-11.37^{*}	0.118^{+}
Brazil		
Price Level	-3.14^{*}	0.279^{+}
Industrial Output	-11.57^{*}	0.156^{+}
Monetary Supply (M2)	-4.13^{*}	0.463^{++}
U.S.		
Price Level	-7.61^{*}	0.141^{+}
Industrial Output	-11.56^{*}	0.062^{+}
Monetary Supply (M2)	-5.04^{*}	0.544^{++}

Table 4 – Unit root tests results, with all variables in log-difference, taking the sample from January 2002 to September 2021. ADF is the Augmented Dickey-Fuller unit root test, whose null hypothesis is that the variable in question has a unit root. KPSS refers to the Kwiatkowski-Phillips-Schmidt-Shin unit root test, whose null hypothesis is that the variable is stationary. (*) represents a 1% significance level of rejection of the null hypothesis for the ADF test; (++) and (+) represent the 1% and 10% significance levels for the KPSS test at which the null hypothesis cannot be rejected.

In table 4 the ADF test for all ten variables rejects the null hypothesis that the series are not stationary with a 1% level of significance. Meanwhile the KPSS test results show the null hypothesis which that the series are stationary cannot be rejected for all variables with a 10% level of significance, with the exception of the monetary supply (M2) series for both Brazil and the U.S., for which the null cannot be rejected at a 1% level of significance. Thus, the tests provide evidence favorable to accepting the stationarity hypothesis for all series used.

6.2 In-sample forecasting

Table 5 shows the in-sample test results. They point to an improvement in performance for the exchange rate determination models including common factor variables in them, comparatively to the model with macroeconomic variables only. This latter model's

R^2 is 3.03%; when enlarged in order to include the common factors as regressors, it reaches 45.67%, a substantial increase. The model including only the common factors has a R^2 of 45.23% — thus indicating that it is able to explain more of the U.S. dollar-Brazilian real exchange rate variation in this sample period than the macroeconomic model. A comparative analysis of the adjusted R^2 yields a similar insight. The adjusted R^2 for the macroeconomic model is 1.77%, while that of the model with the common factors only is 44.52%. The adjusted R^2 of the model containing both sets of variables is 44.24%.

Table 5: In-sample tests results

	Factors only	**Macro variables only**	**Factor + Macro variables**
R^2	45.23%	3.03%	45.67%
Adjusted R^2	44.52%	1.77%	44.24%
log(SSR)	-1.101	-0.529	-1.109
AIC	-869.59	-734.78	-865.49
HR (%)	82.13%	77.45%	82.13%

Table 5 – Shows in-sample test results for three specifications of equation 1, taking into consideration the full available sample for the variables included: a model including only the estimated common factors (labeled 'Factors Only'), a model including only macroeconomic variables — excluding the SELIC rate, the return of 10 year Brazilian bonds and Brazil's nominal deficit ('Macro variables only'), and a third model which combines both sets of variables ('Factors + Macro variables'). Log(SSR) refers to the logarithm of the square sum of residuals of each model, while AIC is Akaike's Information Criteria. HR (%) refers to the hit ratio test.

An assessment of the logarithm of the square sum and Akaike's Information Criteria among the model presented also point to a better performance for the specifications including the common factors in relation to the goal of exchange rate forecasting as compared to the model containing only macroeconomic variables. The hit ratio test (HR%) was also included, which shows the percentage of times the model correctly forecasted a change of direction in exchange rate variation. Results show, once more, a better performance of the models including common factor variables; quile the model including only the macroeconomic variables correctly forecasted a change in direction of the exchange rate 77.45%, both the model containing only the common factors and the model containing both sets of variables correctly forecasted the change 82.13% of the time.

6.2.1 Granger Causality

Running Granger causality tests aims at verifying the existence of a causality relation between the estimated common factors and the U.S. dollar-Brazilian real exchange rate. If the common factors hold predictive power, it is expected that test results will indicate that it is not possible to reject the null hypothesis that the common factors Granger-cause the exchange rate. Chen et al. (2010) point out that the problem when carrying out Granger causality tests with macroeconomic variables is that there may be a causality relation between the variables used and the exchange rate, which would lessen the validity of the test. It is important to note, however, that this issue is avoided in the following tests, as the estimation of the common factors did not take into consideration the BRL/USD exchange rate dynamics, thus it is not to be expected for this latter variable to have an impact of the dynamic of the factors.

Results for the Granger causality tests are shown in table 6. Taking into consideration a 10% level of significance, it is seen that the null hypothesis that the latter two factors do not Granger-cause the exchange rate is rejected, while for the first factor the tests it is not possible to reject the null. In the other direction, results points to a rejection of the null hypothesis that the exchange rate Granger-causes the common factors, as expected.

Table 6: Granger Causality Tests

	Granger causality test
F1 does not Granger cause the ER	0.3969
ER does not Granger cause F1	0.7293
F2 does not Granger cause the ER	0.0682
ER does not Granger cause F2	0.1954
F3 does not Granger cause the ER	0.0154
ER does not Granger cause F3	0.4222

Table 6 – Shows the p-values associated to the Granger causality tests between the U.S. dollar-Brazilian real (BRL/USD) exchange rate and the estimated common factors in the whole sample period. F1 stands for the first factor, F2 stands for the second factor, F3 stands for the third factor; ER stands for the BRL/USD exchange rate.

6.3 Out-of-sample forecasting

A rolling windows-style of forecasting exercise was carried out for equation 1, which, according to Ferraro et al. (2012), is a forecasting exercise more robust to temporal variation in the parameters as it adapts to structural changes more quickly, comparatively to recursive forecasting. The out-of-sample forecasts were carried out on four different forecast horizons (h=1, 2, 3, 6 and 12 months into the future).

6.3.1 Rolling windows

The rolling windows forecasting was set up as follows: first, for a window of fixed size N (in proportion to the whole sample size) and a forecast horizon h, the common factors of the exchange rate data panel were estimated. Then equation 1 was estimated in each of the specifications, using data up until time t. Then an out-of-sample forecast was made from the estimated models and for each of the desired forecast horizons h. After this, the window advances one period in the sample and the process is repeated until the end of the total sample.

An important observation is that realized values of the factors are being used in this forecasting exercise. The reason for this is explained by Ferraro et al. (2012) — if only past values of a variable were to be used in the exercise of exchange rate forecasting and they are not good predictors of their own future values, a potential rejection of this variable's predictive quality in relation to the exchange rate would not be a result of a non-existent relation between them; rather, it would come as a result of the poor forecasting performance that past values of the variable generate for themselves.

Table 7: Out-of-sample test results

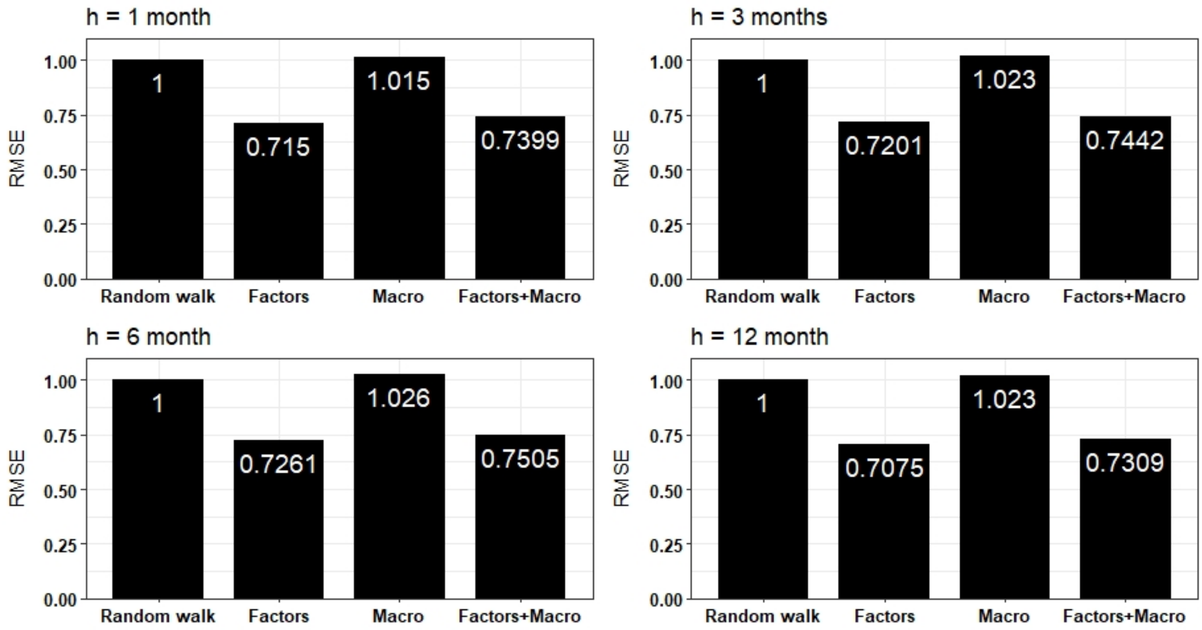
		Forecasting horizon (h)			
Model	**Benchmark model**	**1 month**	**3 months**	**6 months**	**12 months**
N = 1/2					
Only factors	Random walk without drift	0.7150*	0.7201*	0.7261*	0.7075*
Only macro variables	Random walk without drift	1.015*	1.023*	1.026*	1.023*
Factors + macro variables	Random walk without drift	0.7399*	0.7442*	0.7505*	0.7309*
N = 1/3					
Only factors	Random walk without drift	0.7372*	0.7795*	0.7055*	0.7214*
Only macro variables	Random walk without drift	1.0134*	1.0197*	0.9961*	0.9988*
Factors + macro variables	Random walk without drift	0.7611*	0.7943*	0.7233*	0.7424*
N = 1/4					
Only factors	Random walk without drift	0.6774*	0.6905*	0.6948*	0.7830*
Only macro variables	Random walk without drift	1.019*	1.025*	1.031*	1.040*
Factors + macro variables	Random walk without drift	0.7060*	0.7310*	0.7492*	0.8004*

Table 7 – Shows the Theil's U statistics associated with each model in relation to the benchmark model and a forecasting horizon, taking into consideration the sample from January 2002 to September 2021. The asterisks represent the results of the statistic elaborated in Clark and West (2006), for which (*), (**) and (***) represent test p-values below 1%, 5% and 10%, respectively.

In order to compare the forecasting performance of the models in this exercise in relation to the random walk without drift benchmark, it is used the Theil's U statistic, calculated as the square root of the ratio between the mean square prediction errors of the estimated model and the mean square prediction errors of the benchmark model. Values below one point to a more accurate forecasting performance of the estimated model, while values above one point to a better performance of the random walk. I also included results for the statistic in Clark and West (2006) as an evaluation criterion of forecasting quality.

Results from this out-of-sample forecasting exercise are shown in table 7. An analysis of the obtained statistics points to a great usefulness of the common factors in terms of predictive value for the exchange rate, as the model specifications which incorporated that set of variables had better predictive performance in relation to the

Figure 7 - Models' RMSE and Random Walk's RMSE ratio



The bar plots above show the RMSE of each BRL/USD exchange rate forecasting model relative to the random walk's RMSE (that is, the ratio is the Theil's U statistic), for the h = 1, 3, 6 and 12 month forecasting horizons. These statistics were obtained taking into consideration an initial sample for the rolling windows method of $N = 1/2$ the size of the total sample, from January 2002 to September 2021. Random Walk refers to the the random walk mode, Factors refers to the model with only the set of common factors as predictive variables, Macro refers to the model with only the set of macroeconomic variables as predictive variables, and Factors+Macro refers to the model containing both sets of common factors and macroeconomic variables as predictive variables.

benchmark model. The model including solely macroeconomic variables from economic literature had the worst relative performance in most specifications, not being able to attain better results in any specification for short run forecasting.

6.3.2 Recursive

Additionally it is performed another out-of-sample test, in the same vein as the rolling windows method, with the difference that instead of a fixed window that moves along the sample, a new data point is added to the sample at each iteration of the forecasting until the end of the sample is reached – starting with 1/4 the total sample size. Results, as shown in table 8, point that while in the short run the best performance is that of the model with only the common factors variables, in the longest horizon it is tested for, that of a year ahead, the model incorporating both sets of variables, common factors and macroeconomic variables, had the best results comparatively.

Table 8: Out-of-sample test results for the recursive method

Model	Reference Model	Forecasting horizon (h)			
		1 month	**3 months**	**6 months**	**12 months**
Only factors	Random walk without drift	0.7777*	0.8089*	0.8536*	0.9164*
Only macro variables	Random walk without drift	1.0235*	1.0259*	1.0227*	1.0140*
Factors + Macro variables	Random walk without drift	0.7948*	0.8205*	0.8591*	0.9090*

Table 8 – Shows the Theil's U statistics associated with each model in relation to the benchmark model and a forecasting horizon. The asterisks represent the results of the statistic elaborated in Clark and West (2006), for which (*), (**) and (***) represent test p-values below 1%, 5% and 10%, respectively.

7 Conclusions

The paper attempts to forecast the Brazilian real–U.S. dollar exchange rate assessing the usefulness of the inclusion of common factor variables. Results estimated from a sample of monthly data from February 2002 to September 2021, point that common factors indeed improved in-sample and out-of-sample forecasting performance of exchange rate models comparatively to traditional macroeconomic models.

The attempt at identifying the factors estimated from an exchange rate data panel of 19 countries helped bring greater meaning to the dynamic of exchange rate determination; through tests carried out with selected proxy variables, it is attested that factors condensed information contained in a larger pool of variables corresponding to global shocks which influence the variable of interest, the exchange rate. Additionally, this analysis gives a prominent role to international demand of the U.S. dollar as a reliable asset, through its connection with the perception of market volatility, risk, liquidity and capital flows.

Results of out-of-sample exercises showed that the errors associated with exchange rate forecasting were smaller in models which included the common factors as a set of variables, comparatively to the random walk benchmark model, for all the considered forecasting horizons; results were particularly favorable to these models in short-run forecasting exercises.

Future work might want to further delve into the identification and interpretation of the common factors for exchange rates, as well as include more proxy variables so as to better understand exchange rate dynamics. Employing other model specifications, new sets of macroeconomic variables, as well as investigating the performance of machine learning techniques in the forecasting of the exchange rate might also enrich the analysis.

References

ALOOSH,G.; CHANG,K.; MANO,R.; SHAO,Y.**Currency factors.** NBER WP. 25449, 2019.

BACCHETTA, P.; VAN WINCOOP, E. **A scapegoat model of exchange-rate fluctuations.** American Economic Review, 94 (2), 114–118, 2004.

BACCHETTA, P.; VAN WINCOOP, E. **On the unstable relationship between exchange rates and macroeconomic fundamentals.** Journal of International Economics, 91 (1), 18–26, 2013.

BAKU, E.**Exchange rate predictability in emerging markets** International Economics, v. 157, p. 1-22, 2019.

BANTI, C.; PHYLAKTIS, K.; SARNO, L.. **Global liquidity risk in the foreign exchange market.** Journal of International Money and Finance, v. 31, n. 2, p. 267-291, 2012.

BERKOWITZ, J.; GIORGIANNI, L. **Long-horizon exchange rate predictability?** Review of Economics and Statistics, 83 (1), 81–91, 2001.

CAYEN, J. P.; COLETTI, D.; LALONDE,R.; MAIER, P. **What drives exchange rates? new evidence from a panel of US dollar bilateral exchange rates** Bank of canada, 2010-5, 2010.

CHEUNG, Y.-W.; CHINN, M. D.; PASCUAL, A. G. **What do we know about recent exchange rate models? in-sample fit and out-of-sample performance evaluated.** Exchange rate economics: Where do we stand, pp. 239–276, 2005.

CHINN, M. D.; MOORE, M. J. **Order flow and the monetary model of exchange rates: Evidence from a novel data set.** Journal of Money, Credit and Banking, 43 (8), 1599–1624, 2011.

DORNBUSCH, R., **Expectations and exchange rate dynamics.** Journal of political Economy, 84 (6), 1161–1176, 1976.

ENGEL, C.; WEST, K. D. **Taylor rules and the deutschmark-dollar real exchange rate.** Tech. rep., National Bureau of Economic Research, 2004.

ENGEL, C. MARK, N. C.; WEST, K. D. **Factor model forecasts of exchange rates.** Econometric Reviews, 34 (1-2), 32–55, 2015.

EVANS, M. D.; LYONS, R. K. **Order flow and exchange rate dynamics.** Journal of political economy, 110 (1), 170–180, 2002.

FAUST, J.; ROGERS, J. H.; WRIGHT, J. H. **Exchange rate forecasting: the errors we've really made.** Journal of International Economics, 60 (1), 35–59, 2003.

FELÍCIO, W. R. d. O.; ROSSI JÚNIOR, J. L. **Common factors and the exchange rate: results from the Brazilian case.** Revista Brasileira de Economia, 68 (1), 49–71, 2014.

FORNI, M.; REICHLIN, L. **Let's get real: a factor analytical approach to disaggregated business cycle dynamics.** The Review of Economic Studies, 65 (3), 453–473, 1998.

GREENAWAY-MCGREVY, R.; MARK, D. SUL, N. C.; WU, J.-L. **Identifying exchange rate common factors.** International Economic Review, 59 (4), 2193–2218, 2018.

GROEN, J. J. **Long horizon predictability of exchange rates: Is it for real?** Empirical Economics, 24 (3), 451–469, 1999.

GROEN, J. J. **Exchange rate predictability and monetary fundamentals in a small multi-country panel.** Journal of Money, Credit and Banking, pp. 495–516, 2005.

GROEN, J. J. **Fundamentals based exchange rate prediction revisited**, manuscript. Bank of England, 2006.

INTERNATIONAL MONETARY FUND. MONETARY; CAPITAL MARKETS DEPARTMENT. **Annual report on exchange arrangements and exchange restrictions 2019.** International Monetary Fund, 2020.

KILIAN, L. **Exchange rates and monetary fundamentals: what do we learn from long-horizon regressions?** Journal of applied Econometrics, 14 (5), 491–510, 1999.

LUSTIG, H.; ROUSSANOV, N.; VERDELHAN, A. **Common risk factors in currency markets.** The Review of Financial Studies, v. 24, n. 11, p. 3731-3777, 2011.

MARK, N. C. **Exchange rates and fundamentals: Evidence on long-horizon predictability.** The American Economic Review, pp. 201–218, 1995.

MARK, N. C. **Changing monetary policy rules, learning, and real exchange rate dynamics.** Journal of Money, Credit and Banking, 41 (6), 1047–1070, 2009.

MARK, N. C.; SUL, D. **Nominal exchange rates and monetary fundamentals: evidence from a small post-Bretton Woods panel.** Journal of international economics, 53 (1), 29–52, 2001.

MEESE, R. A.; ROGOFF, K. **Empirical exchange rate models of the seventies: Do they fit out of sample?** Journal of international economics, 14 (1-2), 3–24, 1983.

MENKHOFF, Lukas et al. **Carry trades and global foreign exchange volatility.**

The Journal of Finance, v. 67, n. 2, p. 681-718, 2012.

MOLODTSOVA, T.; PAPELL, D. H. **Out-of-sample exchange rate predictability with Taylor Rule fundamentals.** Journal of international economics, 77 (2), 167–180, 2009.

VITALE, P. **A guided tour of the market microstructure approach to exchange rate determination.** Journal of Economic Surveys, 21 (5), 903–934, 2007.