Abstract Some widely-accepted practices on banking ATM networks may be negatively affecting an efficient liquidity management. This paper analyses ATM cash management in light of empirical evidence which suggests that banking ATMs tend to be overloaded beyond the customers' needs. This, in turn, results in high opportunity costs. Besides banking ATMs, it might have a damaging impact on other business which revolve exclusively around ATM networks, such as *cashback sites.*

Dormant money may be overcome by an appropriate tool matching the ATMs' cash to the users' needs. Supported by a database of banking records, this paper provides model validation for a set of theorems previously developed by the author, resulting here in a cutting-edge, reliable forecasting system, suitable for anticipating ATMs cash demand. These characteristics allow this methodology to co-exist with other technologies as a complement which supports branch managers' decisions.

Keywords ATMs cash management, Stochastic processes, Bank data processing, New methodology tested, Cashback sites

Mathematics Subject Classification (2000) C61, C63, G17, G21

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Improving cash-management systems (banking ones and others) at low cost

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1 Introduction

The management of liquidity is amongst the most important activities conducted at firms and banks since traditionally it provides control in key areas such as treasury management, working capital financing and business valuation. Due to the financial crisis which started in 2007, bank liquidity management has become one of the main concerns for the banking sector since the recent crunch has made visible the financial system' fragility if banks do not have sufficient safety liquidity levels. Besides, liquidity management is of paramount importance because a liquidity shortfall at a single institution can have system-wide repercussions. Thus, an efficient cash management is aimed at helping banks to provide a cushion of capital, available to cover losses of any kind, in order to comply with those regulatory reforms which set the safety liquidity levels (Basel III rules amongst others).

As many authors claim, the *branch* efficiency study could significantly help improve the *global* bank institution performance, [6], [17], Cash management in ATMs is amongst the activities involving liquidity management in banking *at the branch level*. Since the notion of cash management comprises all short- and medium-term cash flows no matter whether they are accounting-based or cash-based, in this paper we focus on improvements in optimization of cash inventories *at branch ATM level* under the premise that any improvement at the aggregate level has a positive impact on the global institution' performance.

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For what reasons is it recommendable to undertake a new revision of the costs of ATMs as cash manipulation channels? Mainly due to the spectacular increase in the number of ATM machines. Indeed, forty years after the first ATM (called DAC, De La Rue Automatic Cash System) a total of 3 million ATMs have been distributed across the length and breadth of the whole world[1]. Then, although the introduction of ATMs along with other technological innovations has reduced the management costs of bank liquid assets [7], this impressive usage of ATMs recommends upgrading this cash supply channel. It should not be forgotten that, in the current situation of fierce competition, bank entities are under unprecedented pressure to keep costs under control. Bank managers, however, may argue that current low interest rates mitigate the impact of these potential losses. Even in that case, banks would still incur in opportunity costs of not generating profits if cash is invested in appropriate financial products.

But, apart from the banking case, there are other examples in which business revolve exclusively around ATMs and, in consequence, such liquidity management is of capital importance for keeping the company afloat. Exchange currency companies constitute an example of this. This is also the case of **cashback sites**. Cashback sites -physical and websites- are currently a highly topical subject since they are being thought to be implemented in Spain, although they have already been operating for a long time in some countries like UK throughout supermarkets, post offices etc. Cashback sites offer services to retail customers whereby an amount is added to the total purchase price of a transaction paid by debit card and the customer receives that amount in cash along with the purchase[2]. As cashback sites provide cash to the customers if required, they act as ATMs: thus, they should anticipate uncertain demand without generating dormant money to avoid opportunity costs. While opportunity costs are not perceived by banks as particularly harmful, it might have a damaging impact on cashback sites.

The main purpose of this paper is to show that the cash management of an ATM network has significant room for improvement particularly related to some practices which may be generating losses and opportunity costs. We mainly refer to overloading ATMs beyond the real cash necessities. Actually, this paper analyses branch ATM cash management in light of empirical evidence (database formed by real ATM-level records) showing a mismatch between quantities of case placed in the ATMs and real cash needs of ATM' consumers. Along with the problem, this paper attempts to provide a potential solution to overcome dormant money: a new methodology, as a monitoring program to guide short-term corrective cash management actions of the branch's

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[1] Date research was conducted: March 29, 2017. Sources: ATMIA, National ATM Council, [19]

[2] For instance, a customer purchasing 8.99€worth of goods at a supermarket might ask for thirty euros cashback. He would pay a total of 38.99€(8.99 + 30.00) with their debit card and receive 30€in cash along with their goods.

staff. The theoretical fundamentals of the proposed methodology, developed by the author of this paper in [12], were conceived only as a set of theorems based on stochastic jump processes together with a dynamic mathematical setting in order to model the ATMs cash flow. In this paper, by means of the corresponding model validation, it will be proved that this set of theorems, pointed in the right direction, become an effective forecasting system for ATMs. As a matter of fact, the aim of this paper is twofold. First, we warn of the pressing need to improve the ATM cash management by specifically being aware of some widely-accepted practices which may result in inefficiencies. The second aim of this paper is to promote this new forecasting system as long as it is showing an immediate practical relevance for management practitioners.

The key decision for the bank as far as its ATMs are concerned is what amount to maintain daily in that account given the overall sum to be loaded. It should be noticed that, despite the fact that IT technologies are present at branches (commonly used from centralized IT planning centers), the procedure to compute the initial amount of cash to be loaded into the ATM strongly relies on managers' expertise, who further fine-tune the centralized predictions by taking into account the specific branch features derived from local demographics. And managers' expertise relies on historical data handling as part of the branch' routines. That means that the branch registers the cash quantity on particular weeks (workable, holidays, beginning/end-of month, etc) and the result obtained (exceed or shortage of case) and copies the successful amounts. However, in the decision-making process, the staff in charge often reaches a decision with partial information. As a result, the branches tend to overload the ATMs to avoid refilling the ATMs more than once a day.

In truth, the banking information processed in this paper suggests that this could be happening. This banking information is database of real banking records *based on ATMs data transactions.* As a matter of fact, more than 250,000 excel multicolumn cells have been processed. As far as the author knows, this is one of the few times that such dataset has been employed in the literature, due to the existing difficulties when accessing real sufficiently detailed data at branch level. Database of banking records has been also employed in order to validate the new methodology proposed in this paper, showing it as matching solution which adjusts ATMs' cash to users' needs. Actually, the model validation process has been carried out in parallel with the attempt to find out that ATMs tend to be overloaded, proceeding by comparing the following three items: a)real banking data on cash loaded into ATMs, b)consumers' real cash necessities and c)forecasted cash amounts obtained from the method which predicts the right quantities of money that must be loaded into the ATM according to a future unknown demand.

It should be noticed that this new methodology has been mainly intended and designed for forecasting *future ATM cash needs* from the analysis of *past needs.* Hence, as its forecasting mechanism is based on past branch data which implicitly include specific ATM features inside, this methodology does take into account such specific branch characteristics for each case. On one hand, this forecasting system is very precise with minimum human intervention. On the other hand, it is very simple to be implemented at daily branch practices, assuring costs reductions either in personnel training or in the implementation of the program itself. These characteristics allow this methodology to co-exist with other technologies as a complement which supports branch managers' decisions and helps notably ameliorating the ATM cash management. Moreover, it has the potential to be applied to other contexts apart from the banking environment (or cashback sites) providing thus sustainable competitive advantage since, in general terms, demand forecasting is an important issue in any supply chain planning process, [18]. Actually, the use of technology to anticipate demand establishes an accurate management to provide up to-date data to inform demand forecasting and supply management, particularly to provide early warning of potential oversupply or stock-outs. As a matter of fact, the generality of the employed methodology greatly extends the range of its possible industrial uses[3].

This paper is structured as follows: section 2 consists of a literature review. Section 3 is devoted to outlining the general formulation of the theoretical methodology stating its main features as well as running throughout a small sample intended for illustrative purposes. Section 4 is devoted to the computational experiments. Finally section 5 concludes the paper.

2 Literature review

A branch of the banking literature points at liquidity management as an essential function of banks. In particular, the seminal references on the role of financial intermediaries [10] already paid attention to deposits as the main input for banks, as well as to the consequences of random deposit withdrawals and the role of deposit insurance to reduce the risk of bank runs. These models have evolved over time, as in [3], [2] and have been largely revisited after the financial crisis that started in 2007, given that liquidity tensions have been a major concern in the banking industry and for financial stability in general, see, for example, [4], [11].

There has been also a strand of the literature dealing with the evolution of cash in the economy, as well as on the impact that electronic payments and ATMs have on the demand for currency. Authors in [15] and [7] estimate that this shift from cash to electronic payments could generate a total cost saving close to 1% of GDP for a sample of 12 EU countries (and similar gains for Spain respectively). Authors in [1] analyze the impact of ATM transactions on the demand for currency and estimate the elasticities of the demand for currency to changes in factors such as ATM transactions. They find that these

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[3] In subsequent papers, applications for computer components suppliers and electric utility industry will be studied.

elasticities are close to the theoretical values implied by standard inventory models, although they are found to be lager for consumers holding an ATM card. Additionally, other studies have shown the importance of ATMs and cash in reducing the penetration of debit and credit cards in some countries, as in [7] for Spain.

From a bank management-level perspective there is, to the author's knowledge, a more limited number of studies dealing with efficiency improvements in liquidity management. In particular, there is a paucity of research analyzing branch-level cash management. In [21] author undertakes simulations on how to optimize an ATM network and finds that up to 28% cost saving can be achieved by improving the inventory policies and cash transportation decisions.

As the problem stated and solved in this paper may be viewed as the optimal management of an inventory of cash holdings within the banks' ATM under uncertainty, models of supply chain planning and inventory models should be mentioned. Some of them have relied on supply management optimization techniques, A good summary of these models can be found in [16] who, in turn, show themselves that there are many similarities between cash supply chains and the typical chains for physical products. [8] follows an operational research perspective and build a solution to optimize the management of ATM cash based on algorithms which administer the cash in ATMs and banks. Finally, a more recent perspective has made use of clustering and neural networks to forecast cash demand at ATMs. In particular, [20] show that the cluster-wise cash demand forecast helps the bank's top management to design similar cash replenishment plans for all the ATMs in the same cluster. This cluster-level replenishment plans could result in saving huge operational costs for ATMs operating in a similar geographical region. Other papers of the existing literature focused on ATMs are[5], who optimize the ATM cash replenishment or develop different systems for predicting the daily amounts withdrawn from ATM´s.

Other approaches on ATM forecasting techniques are in [9], where a brief summary of the existing methods for cash forecasting are presented. As a matter of fact, following [9], techniques used for cash demand forecasting can be broadly classified into four groups: (1) *Time series methods* that predicts future cash need based on the past values of variable and/or past errors. (2) *Factor analysis method*, based on the cash demand determinants in order to model their correlation with actual cash withdrawal. (3) *Fuzzy expert systems*, which try to imitate the reasoning of a human operator by reducing the analogical thinking behind the intuitive forecasting to formal steps of logic (rule-based methods).

As we shall see later (see section 5), the proposed forecasting method has the possibility of learning without being explicitly programmed, like a methodology with recognized learning abilities.

3 Methodology

This section, devoted to explaining the methodology used, is divided into two parts: the first one (3.1) gives an overview of those theoretical foundations which were conceived as a set of theorems, developed in [12]. In next section, through the corresponding model validation, it shall be proved that these theorems, pointed in the right direction, result into an effective forecasting method for ATMs.

It should be also mentioned that the theoretical foundations upon which the forecasting method tested in this paper is based, do not specify *how* to compute the expected cash amount. This leaves the door open to executing the method in several ways[4]. Hence, in the second part of this section (3.2), one of the ways by which this method may work in practice beyond its fundamentals shall be detailed. This will be carried out through a small sample (one week) intended for illustrative purposes.

3.1 Fundamentals

In this section a brief summary on the theoretical setting developed in [12], will be exposed. These are based on stochastic jump processes (compound Poisson processes).

With regard to the process of withdrawing cash from ATMs, there are two stochastic unknowns: the number of ATM customers and the amounts of cash withdrawn each time. The first one is described by means of the arrival process known as counting process: if N_t is the number of ATM users in the time interval $(0, t)$, the main properties of this arrival process considered as Poisson process with parameter λ, are the following:

- the number of arrivals to the ATM in an interval of length t has a Poisson distribution with parameter $\lambda \cdot t$; that is, $P[N_t = n] = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$ measures the probability of n ATM customers in the time t.
- The mean and variance of N_t are $E[N_t] = \lambda \cdot t, var[N_t] = \lambda \cdot t$. Particularly, the rate of the Poisson process N, λ, is the average of withdrawals made by ATM customers in a day.

As for the second stochastic unknown, the amounts of cash withdrawn each time A_i^*, these are controlled by means of a compound Poisson process:

$$X_t = \sum_{i=1}^{N_t} A_i^*.$$

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[4] The best option to execute the method would depend on the (economic, social, demographic) circumstances of each ATM bank branch. See the section 5 for complete explanations about possible methods of computing.

In [12] was proved that X_1 is the total amount that have been withdrawn by the N_1 ATM users throughout the day. By the compound Poisson process' properties, this is equal to $\lambda \cdot E[A_i^*]$ where λ is the average of the number of withdrawals per day whereas $E[A_i^*]$ represents the average of quantity withdrawn from the ATM per day. Hence, if x_0 represents the quantity to be loaded into the ATM at the beginning of the day, this may be computed as follows:

$$\begin{aligned} x_0 &= \lambda \cdot E[A_i^*] = \\ &= \begin{array}{l}\text{Average of number of}\\ \text{withdrawals per day}\end{array} \cdot \begin{array}{l}\text{Average of quantity withdrawn}\\ \text{from ATM per day.}\end{array} \end{aligned} \tag{1}$$

This equation shall be at the heart of the subsequent model validation. Let it be noticed that the proposed forecasting method -equation (1)- does not predict ATM cash demand depending on total number of ATM arrivals but only on ATM arrivals in which money has been withdrawn. That is, other arrivals without cash withdrawn at the ATM[5], including an eventual ATM failure, are not considered in equation (1).

The theoretical setting developed *for ATMs* in [12], on which the present model validation relies, was enlarged in subsequent works *for branches*. Specifically, in [13] a theoretical programme of cash efficiency for banks' branches is proposed thereby providing a significant reduction of cash holdings at branches. In [14], a sound and low-cost algorithm to optimize branch cash holdings is designed as an accurate tool to improve the performance of branches with respect to its cash management. Following on from forecasting methods in algorithmic form, it should be said that the nearest future research project of the author of this paper is to implement the proposed methodology into an algorithm or and expert system which may act may cat as set of rules or monitoring program to guide short-term corrective cash management actions of the branch's staff (see Conclusion section for further details).

3.2 Running the method through a short sample

As mentioned before, the set of theorems developed by the author in [12] may be applied in several ways in order to produce adequate forecasted amounts of cash. This subsection is devoted to detailing one of these: in few words, database processing will be made in such a way that each iteration uses as inputs the mean of cash withdrawn in *all* previous stages. Other ways of executing the method would select only a group of inputs instead of using all of them, simulating some widely-accepted managers' practices of clustering the time into groups (weeks/months/years) of similar features[6].

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[5] Like paying routine bills, fees, taxes, printing bank statements, updating passbooks, transferring money between linked accounts, purchasing tickets -concert tickets, lottery tickets, movie tickets, train tickets etc- and many other functions.

[6] That means periods where spending tends either to increase such as pre-holidays (beginning of July and December) or to decrease, such as the so-called 'hard January' with

With illustrative purposes, a small sample (one week) will be processed. The banking information comes from partial extracts of daily ATM cash count sheets corresponding to a representative office in demographic and sociological terms of an emblematic Spanish bank firm. Due to confidentiality arrangements we provide some general descriptive statistics. Throughout this section, the banking partial extracts of daily ATM cash count sheets' specific terminology has been kept: particularly, the term *return* means *withdrawals* while the label *Total Delivered* coincides with the *real needs of cash* delivered by the ATM at the end of the day.

In order to explain how the method may be executed in practice, we will carry out the contrast amongst a)banking data on quantities of cash charged into ATMs, b)users' real cash needs and c)ATM forecasts. The final result shall be displayed at Table 3. Previous proceedings in order to get final computations are shown in following Table 1.

	Total Delivered TD	Total Returns TR	Average of quantity withdrawn from ATM $= \frac{\text{TD per day } i}{\text{TR per day } i}$.
Day 1	10.090 €	104	97,01 €
Day 2	3.160 €	17	185,8 €
Day 3	3.980 €	34	117,05 €
Day 4	3.090 €	24	128,75 €
Day 5	5.050 €	51	99,01 €
Day 6	6.540 €	79	82,78 €
Day 7	1.320 €	17	77,64 €
		326	788,04 €
		Total Returns per week	Total average quantities withdrawn per week

Table 1: Previous proceedings on the ATM forecasting method' execution

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personal habits of moderation and austerity, see section 5, where a seasonality coefficient is presented in order to correct possible peaks and troughs.

Now, by applying equation 1, this comes into the following amount:

$$
\begin{aligned}
x_0 &= \begin{array}{l}\text{Average of number of}\\ \text{withdrawals per day}\end{array} \cdot \begin{array}{l}\text{Average of quantity withdrawn}\\ \text{from ATM per day}\end{array}\\
&= \frac{\text{Total Returns per week}}{7} \cdot \frac{\begin{array}{l}\text{Total average quantities}\\ \text{withdrawn per week}\end{array}}{7}\\
&= \frac{326}{7} \cdot \frac{788,04}{7} = 5.242,54€.
\end{aligned}
$$

That is to say, by processing the small sample of banking records, the ATM forecasting methodology produces an output of $\boxed{5.242,54€}$ which should be enough to satisfy the demand of cash of ATM users every day.

	Total Delivered (€)	Total Intro (€)
Day 1	10.090	25.770
Day 2	3.160	47.100
Day 3	3.980	43.940
Day 4	3.090	39.960
Day 5	5.050	36.870
Day 6	6.540	31. 820
Day 7	1.320	23. 680
	33.230 €	249.140 €
	Total Delivered	Total Intro

Table 2: Mismatch between real needs and cash loaded

Before contrasting the aforementioned forecasted amount with real necessities of cash, let us pay attention to the data displayed in following Table 2. From Table 2 the big difference between real daily needs of cash (labeled Total Delivered) and amounts of case loaded into the ATM (labeled Total Intro) are prominently displayed. Later on, when processing database of real banking records (more than 250,000 excel multicolumn cells with information about city and rural ATMs), the mentioned hypothesis of overloading ATMs will be reinforced. This alone should be reason enough to revisit the current ATM cash management procedures.

In order to finally reach a global comparison amongst a) provision of funds for the ATMs on the banking firms (i.e., average of the Total Intro by the branch staff), b)ATM users real cash needs (i.e., average of the ATM Total Delivered) and c) predictions of cash on the proposed methodology, as the two first quantities are both averages *per week*, the corresponding quantities should be now

$$\frac{249.140}{7} = 35.591{,}43 \text{ €} \quad \text{and} \quad \frac{33.230}{7} = 4.741{,}14 \text{ €}.$$

Thus, the mentioned global comparison is shown in Table 3:

Average Total Delivered	Average Total Intro	Forecasted amount
4.741,14 €	35.591,4 €	5.242,54 €

Table 3: Global comparison

which displays a mismatch between provision of funds and real cash needs for banking ATMs (see the two items circled around).

This mismatch is shown as well in Figure 1, where the area between green and blue/red lines (blue and red lines practically coincide) represents the ATM *surplus* cash. Incidentally, both Table 3 and Figure 1 also shows the high level of precision and reliability of the proposed ATM forecasting methodology:

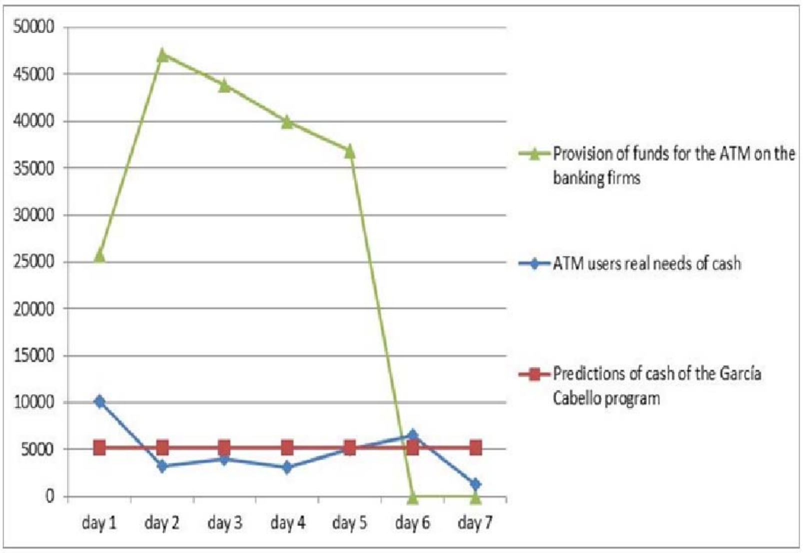

Fig. 1: Comparative graph

4 Data description and data-processing of real banking records.

In this section, some numerical experiments in order to back test our hypothesis are performed. These experiments are based upon two excel files that contain all daily ATM branch operations from June 2013 to March 2014 of two representative Spanish branches of a well known Spanish bank. For this, more than 250,000 excel multicolumn cells have been processed. The numerical experiments have been carried out as a sensitivity test on ATMs withdrawals for two kind of branches: metropolitan' and rural'. Despite our initial data set was originally written using the entity's specific code, significant external operations have been extracted/separated from those internal organizational orders (accounting entries) as part of the database processing. To comply with legislation, the name of the bank must be kept private.

For both kind of ATMs, at an urban/rural location, two graphics have been developed: the first one (bars chart) sets the comparison between quantities of cash charged into ATMs and real cash needs in order to state trends in ATMs practices, as the possible overloading. The second graphic (diagram of functions) displays jointly the three functions corresponding to a)banking data on quantities of cash loaded into ATMs, b)real cash needs and c)ATM forecasts, aimed at establishing the degree of accuracy of the proposed forecasting methodology for ATMs. For all these graphics, the x axis shows months of the years 2013/2014 starting in June, while cash amounts in Euros appear on the y axis. As mentioned before, there is more than one way to execute the methodology, which would be more or less suitable depending on the context. In order to be consistent with former sections (3.2), database processing will be still made in such a way that each iteration uses as inputs the mean of cash withdrawn in *all* previous stages.

Let it be noticed that the distinction between city and rural branch is the usual categorization of branches amongst branch managers although it does not correspond only to demographic parameters. On the contrary, it includes other factors like branch size.

Branch size is not a closed concept: on the contrary, it may be measured by means of several parameters. Actually, *there exists a strong relationship between branch size and local demographics*: branch size depends on branch cash transactions -number and amounts- while branch cash transactions depend on branch customers' needs for cash, which are strongly related to local demographics (a heavy retail stores area will require much more cash than a heavy industrial area where firms do not deal with much cash).

Hence, “city center/rural site branch” shall be implicitly considered for model validation when using the above categorization of branches because grouping the branches into city centre', rural' or business centre' by practitioners implicitly include their demographic features inside. Evidence of this is the fact that branch managers, in practice, categorize branches on city centre', rural' or business centre' depending, not (only) on their branch geographical location but on their number and amount of transactions, *on a not clearly de-*

fined basis: i.e., although a branch is geographically located at a rural area, it could be considered by practitioners as city centre' if its number and amount of transactions exceed the internal benchmarks for rural branches.

4.1 A city center branch

According to an expert bank manager, the main feature of metropolitan bank branches ATMs is a constant and high client flow, where over 50% of them are not habitual ones. Branch ATM consumer habits are not therefore fully known.

In Figure 2, blue bars represent real cash needs and red ones the quantities loaded into ATMs. A huge difference can be seen between both items. The same conclusion is reached as concerns Figure 3, where, additionally, a high degree of coincidence may be observed between forecasted amounts of cash and real needs.

4.2 A rural site branch

The main features of ATM bank branches located at rural areas are a constant and medium/low client flow, where less than 20% are not regular ones. Branch consumer habits are well known by branch staff in consequence. The withdrawals flow is homogeneous with medium/low level on cash quantities.

Average Total Delivered	Average Total Intro	Forecasted amount
608,000,00€	10.800,000,00€	685,000,00 €

Table 4: Global comparison for city center' ATM

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Average Total Delivered	Average Total Intro	Forecasted amount
55.750,00 €	79.700,00 €	57.000,00 €

Table 5: Global comparison for rural' ATM

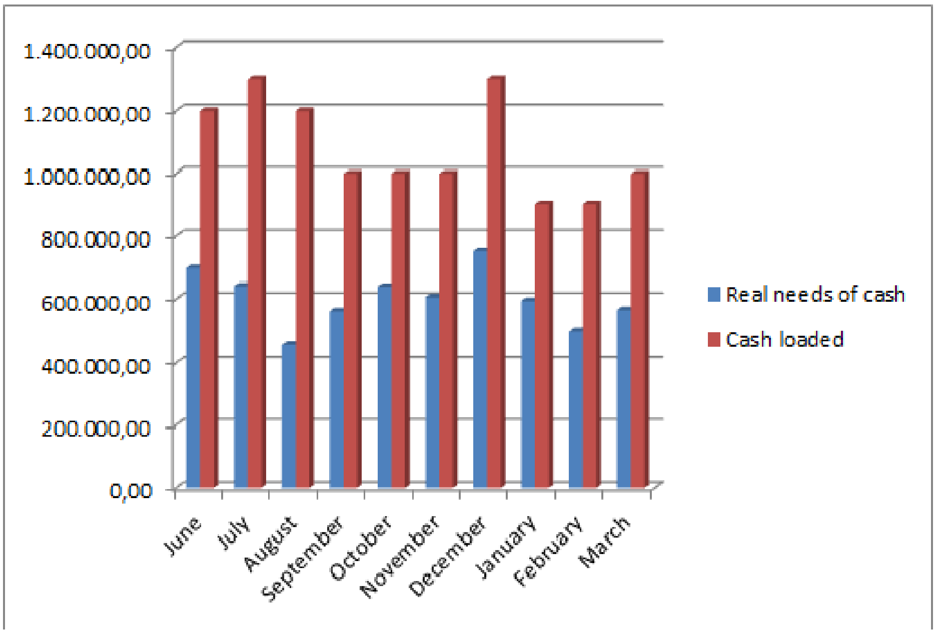

Fig. 2: City center' ATM overload

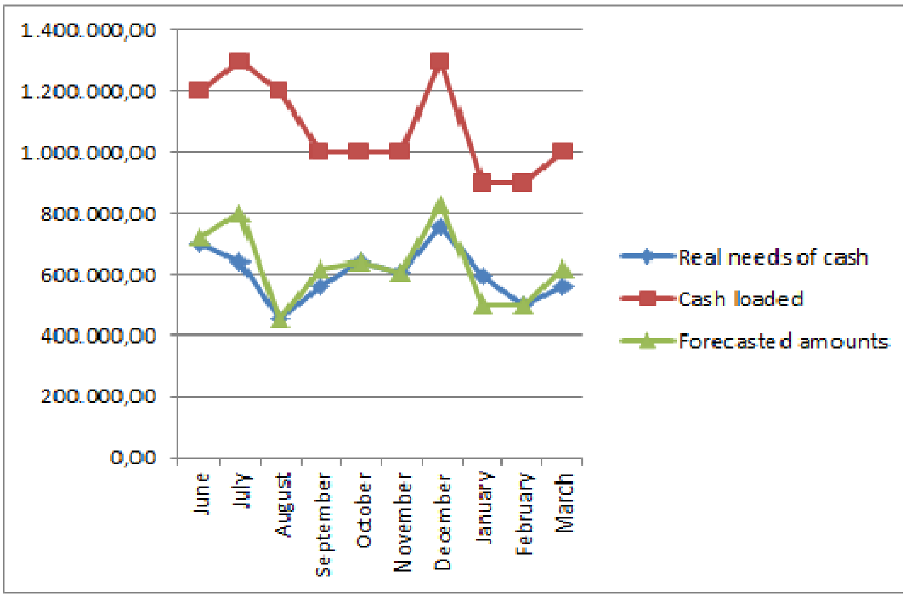

Fig. 3: Global comparison for city center' ATM

Figure 4 and 5 provide similar findings to those reached by the urban case: firstly, the mismatch between quantities of case placed in the ATMs and

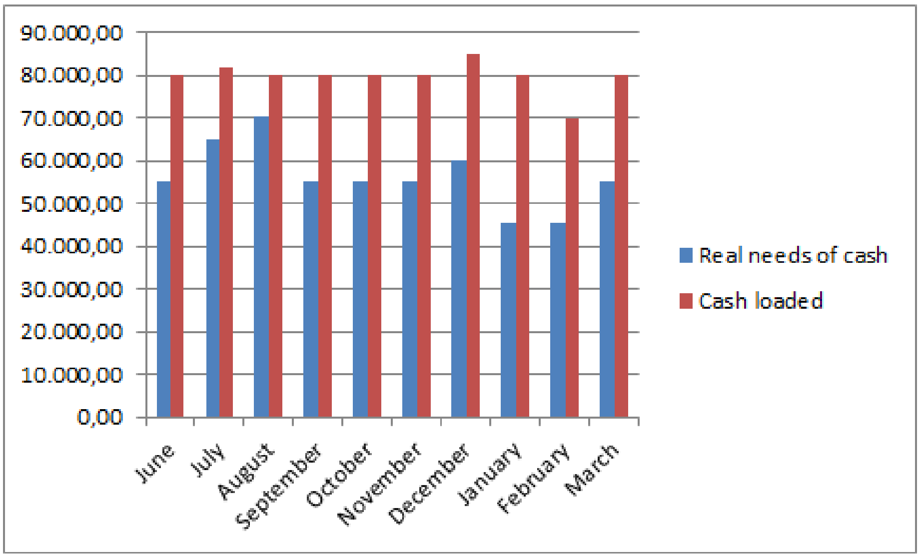

Fig. 4: Rural' ATM overload

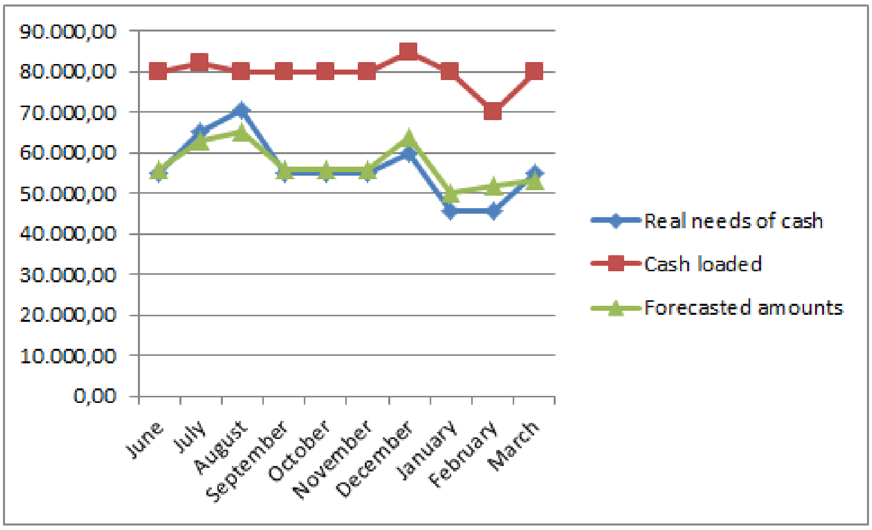

Fig. 5: Global comparison for rural' ATM

real cash needs of ATM' consumers. And secondly, a high level of accuracy of the proposed methodology in forecasting the quantities of money that must be loaded into the ATM according to a future unknown demand. One further conclusion may be drawn: in rural locations, the ATM consumer' habits tend to be more homogeneous than in metropolitan areas. This is the probable reason as to why such branch managers do not overload ATMs so disproportionately than in the urban locations.

5 Exploring the forecasting method possibilities

The precise way of computing used in the model validation is detailed when running the method throughout a small sample (see section 3.2). However, *the existence of alternative ways of computing is suggested* (see footnote 5). This is because, as mentioned earlier, the set of seminal theorems at the proposed methodology' heart does not specify how to compute the forecasted amounts for ATMs leaving the door open to several possibilities, whose effectiveness may be tested in different scenarios.

To this regard, recall that the theoretical model (summarized in the central equation (1)) sets up how to compute the expected cash amount x_0 by means of two unknowns: i) withdrawals and ii) quantities withdrawn *per day.* Of course, the days from which these data are extracted (i.e., the days that serve as reference) must be prior to the forecasting-day. Since these reference-days may be chosen following several methods, this feature left possibilities open depending on needs. Some ways of computation x_0 are listed below as well as some intrinsic characteristic of them (such as learning capabilities, i.e., progressively improving performance) which would help to identify those contexts where such way of computing would fit better:

- Last day: Use for the current day the data extracted from the last one. This method assumes that all days are similar. This way of computation would be suitable for those branches with not too peaks-and-troughs.
- Last *similar* day: Use for the current day the data extracted from the last similar one where “similar” means “with similar specific features”. In such case, days are grouped (clustering) depending on these specific features: work days, holidays, beginning/end-of-month days, etc in order to take samples from the corresponding cluster. This way of computation would be suitable for those branches with more extreme swings.
- Accumulated average: Use the historical average of ATM cash needs. This method assumes that all days in the year are equal and will never account for extreme values. It will however, slowly adapt to rising or decreasing needs since temporal sequence, with every step becoming broader. The corresponding **outputs** of the proposed forecasting method, apart from x_0, are new data on withdrawals and quantities withdrawn which perform as inputs for next iteration. That way, the mean values of these two unknowns meet a temporal sequence, with every step becoming broader. Hence, the successive application of method minimizes the cumulative error while computing the forecasting ATM cash amounts as well as stabilizing the method.

- Accumulated average with initial learning period: A modification of the former because during the initial period, the average is very sensible to extreme variations.

Both accumulated-average and accumulated-average-with-initial learning period methods would be suitable versions for branches with large volumes of ATMs transactions. These are general guidelines while exploring potential fine-tunings of the proposed method should be carried out by testing the procedure with real data of each kind of branch (see Conclusions section for further details). Thus, in addition to the versatility in executing the method, other further fine-tunings could be implemented in order to best-fit the method to the needs of each ATM location.

As a matter of fact, each ATM location constitutes itself a *particular scenario* whose set of features ranges from the market conditions to the special conditions of the site where the ATM is located (special conditions pertaining to the socioeconomic and cultural scope of each country/region/stat). In this line, let us explain a didactic example of fine-tuning of the method depending on location, intended for illustrative purposes. When the special conditions of the location would include the possibility of being a tourist destination, thus number and amounts of ATM withdrawals would increase at certain times of the year. In such case, a seasonality coefficient may be designed, specially when the proposed method has been implemented in algorithmic form This could be carried out as follows: in the first place, it should be noticed that the seasonality coefficient is time-dependent. Let T be the chosen time unit, although a day is the usual time unit for ATMs since the ATM often needs to be replenished on a daily basis. Let $s_c(T)$ be the seasonality coefficient. Thus, if x_0 denotes as usual the quantity to be loaded into the ATM at the beginning of the day, $s_c(T)x_0$ represents such quantity corrected in accordance with the seasonality coefficient, where the corrections could be made depending on peaks-and-troughs. That is,

Quantity which must be loaded into the ATM

without coefficient	with coefficient		
x_0	$s_c(T) \cdot x_0$	$s_c(T) > 1$	when an increment of cash is presumed
		$s_c(T) < 1$	when a decrement of cash is presumed

Table 6: An example of seasonality corrector

Apart from the different versions of the method, let it be noticed that the data corresponding to the first day (the data to start the computation) are unknown. This set of possible initial values may be also changed depending on each particular scenario. Thus, these wide range of adjustments left open the possibility of designing a different version of the proposed method which works particularly well for each different scenario.

Once all these corrections have been carried out (it would be easier when the proposed methodology is implemented into an algorithm or into an expert system[7]), the proposed methodology may be thus considered as a set of rules which acts as monitoring program to guide short-term corrective cash management actions of the branch's staff.

6 Concluding remarks and potential areas for future research

The employment of ATMs network as additional alternative to cash window has spread enormously amongst bank entities' users now reaching massive proportions. This alone should be reason enough to revisit the current ATM cash management procedures in order to detect money leaks. Moreover in Spain within a foreseeable period of time, the setting-up of new companies (cashback sites) may have a high chance of occurrence. Cashback sites would act as ATMs by offering services to retail customers while providing cash added to the total purchase price of the debit card transactions, as shown in Figure 6:

Fig. 6: Cashback companies

A similar provision exists with regard to other companies which use ATMs machines to expend money, like exchange currency companies. All these settings should anticipate uncertain demand without generating dormant money to avoid opportunity costs. Opportunity costs might have a damaging impact on cashback sites or similar companies.

This paper lays on the table the pressing necessity of enhancing ATMs performance as well as learning from possible inefficient ATM branching practices such as overloading the ATMs beyond the users' needs. As a matter of fact, the dataset formed by real banking information used in this paper suggest

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[7] This is the nearest future research project of the author of this paper, see Conclusion section for further details.

that the probable common practice is for banks to overload ATMs with cash, which in turn can be generating large losses and opportunity costs.

Along with the problem, this paper attempts to provide a potential solution to improve cash management by updating the ATM cash forecasting processes trough a cutting-edge methodology matching the ATMs' cash to the users' needs. The tests performed in this paper (through ATM database in order to reduce noise as much as possible) show this methodology as sound and reliable. Actually, our findings demonstrate that the proposed method may significantly reduce the mismatch between provision of funds for the ATMs and real cash needs of the ATM users. Furthermore, as the methodology proposed may be implemented without extra cost either in personnel training or in the implementation of the program itself, from a practical point of view, it may co-exist with IT technologies as an extra decision support system for managers and practitioners, specially when the proposed methodology is converted into an algorithm directly throughout the banking institutions' own computer services. It has also the potential to be applied to other contexts apart from the banking environment providing thus sustainable competitive advantage.

The proposed method can be expanded to incorporate a cost structure in such a way that the forecasted amount which should be loaded into the ATM, also minimize a given costs function. As a result, as the cost specification is an important characteristic of inventory management problems[8], the inclusion of the cost structure allows to locate the cash management problem described in this paper within the broader context of the optimal inventory literature.

In this line of thinking, the following is a thumbnail sketch on how a complementary cost structure may be developed. The starting point should be a cost function for ATMs, which would gather all costs involved:

$$\begin{aligned} f(x_0, \text{other ATM variables}) &= \text{costs due to cash flow} \\ &+ \text{opportunity costs}+ \\ &+ \text{insurance costs} \end{aligned}$$

in such a way that "other ATM variables" should include variables such as the cash upper bound (maximun cash amount allowed) related to insurance costs as well as many others. Once the ATM cost function is stated, f could be considered as objective function in a constrained optimization programme:

$$\begin{cases} \text{Minimize } f(x_0, \text{other ATM variables}) \\ \text{subject to} \text{necessary constrains} \end{cases}$$

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[8] In short, the problem of inventory control (see (Zhiukov, 2015) for an up-to-date and complete review of such literature) is aimed at a successful management of stocks of goods in order to meet the demand by seeking for an inventory policy that will make profits as large as possible or *costs as small as possible* This is as to why a cost structure should be given.

in such a way that the feasible domain is a non-empty compact one. As a result, according to the Weiertrass's Extreme Value Theorem, for a real-valued continuous function on a non-empty compact domain, there would exist global minimum. One of the advantages of this complementary cost structure is that the cost function could be modified as needed provided only that it verifies some slight requirements (i.e., Weiertrass's Extreme Value Theorem continue to be complied).

Besides the versatility in executing the method depending on needs exposed in section 5, other further fine-tunings could be implemented into the proposed method when special conditions pertaining to the socioeconomic and cultural scope of each country/region/stat, would need to be considered (seasonality for instance). Moreover, as mentioned in section 5, in addition to the possibility of different versions of the method, the data to start the computation are unknown producing in consequence a potential set of possible initial values which may vary depending on each particular scenario. Thus, such wide range of adjustments left open the possibility of designing a different version of the proposed method which works particularly well for each different scenario.

To this regard, since the range of possible scenarios is so wide, a future research project within a foreseeable period of time is to further fine tuning the method predictions *according to any different kind of branch.* This shall be done after converting the proposed methodology into an algorithm, which is the nearest future research project while exploring the fine tuning of the proposed method through testing the procedure with real data of each kind of branch is not yet in the bag, since in the Spanish case these data are not publicly available in the short term. In this sense, the conditions of a possible agreement between the University of Granada and some Spanish banking institutions are currently in negotiation in order to perform some experiments to best-fit the ATM methodology to the needs of each banking institution.

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