Measurement of the monetary poverty in Cameroon using fuzzy measure theory

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Abstract. In this paper, we construct a unidimensional fuzzy Foster-Greer-Thorbecke (FGT) index which enables us to measure the monetary poverty. The approach adopted for that consists to build in a measurable space, a fuzzy measurement. This fuzzy measurement allows us afterwards to define our unidimensional fuzzy FGT index as measurement of the fuzzy set of the poor. This fuzzy FGT index is afterwards used to measure the monetary poverty in Cameroon in the year 2014. The obtained results are compared with those obtained when the classical (non-fuzzy) FGT index is used.

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1. Introduction

Poverty has been in existence for many years and continues to exist in a large number of countries in the World. Therefore, targeting of poverty alleviation remains an important policy issue in many countries [1]. During the past few decades, many attempts have been made to find a suitable way of measuring poverty. The first step is obviously to define poverty. This leads to the poor being identified. The next step is to aggregate the information on each individual or household, leading to an index number that summarises the extent of poverty for the whole population [9].

The monetary poverty is an approach used by the world bank to apprehend the poverty from the angle of the consumption or income. This approach is founded on the threshold which can fluctuate from one context to another or from a season to another [18]. Consequently, this approach is thus defined by reference to a threshold : below we are poor, beyond we are non-poor. The fixing of such a threshold poses however several problems. To have recourse to the fuzzy measure theory, enabled us to avoid major part of this difficulty. The use of a gradual scale gives us then a better account of the situation of individuals in relation to the poverty [23].

In a pioneering contribution to the measurement of poverty using fuzzy set theory, Cerioli and Zani (see [7]) identified the poor as the individuals excluded from the dominant way of live, because they are deprived of wide-spread goods and have a way of live inferior to the current standards of the population. The objective of the Cerioli and Zani works (see [7]) has consisted to reveal handicaps which express oneself from the more expand manner than no let believe the insufficiency of incomes. However, the Cerioli and Zani approach is a multidimensional approach, that is, poverty is measured by usage of several well-being indicators.

The goal of this paper is to propose a unidimensional fuzzy index of the measurement of poverty. In the literature such an approach has already been used (see for example [4], [5]). This paper is then organized as follows. In section 2, we summarize some basic results of fuzzy measure theory and of measurement of the monetary poverty allowing us to construct our unidimensional fuzzy index in section 3. In section 4, we proceed to the measurement of the monetary poverty in Cameroon in the year 2014. The last section is intended to be the conclusion.

2. Preliminaries

In this section, we introduce certain terminologies, notations and definitions that will be used in the sequel.

2.1. Basic results of the fuzzy measure theory

Definition 2.1. Let X be a universal set and A a subset of X. The fuzzy set A is a set of ordered pairs

$$A = \{(x, \mu_A(x)) | x \in X\} \tag{2.1}$$

where

$$\mu_A : X \to [0, 1] \tag{2.2}$$

is a mapping where the range $\mu_A(x)$ of $x \in X$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in A.

Remark 2.2. Let us notice that if A is a fuzzy set then we have $\mu_A(x) = 0$ if x does not belong to A, $0 < \mu_A(x) < 1$ if x belongs partially to A and $\mu_A(x) = 1$ if x is all in A.

Definition 2.3. Let A be a fuzzy set in X. The support of A, denoted by S(A), is the crisp set of all $x \in X$ such that $\mu_A(x) > 0$.

Definition 2.4. The α - level set (or α - cut) of a fuzzy set A of X is a classical set (or crisp interval) denoted by A_α and defined as:

$$A_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}. \tag{2.3}$$

Remark 2.5. A_α is a classical set with the characteristic function
$$\chi_{A_\alpha}(x) = \begin{cases} 1 & if \quad \mu_A(x) \geq \alpha; \\ 0 & otherwise. \end{cases}$$

Proposition 2.6. *If A is a fuzzy set in X then* $\forall x \in X$

$$\mu_A(x) = \sup_{\alpha \in]0,1]} \alpha.\chi_{A_\alpha}(x). \tag{2.4}$$

Definition 2.7. Let A be a fuzzy set in X. The height h(A) of A is defined as:

$$h(A) = \sup_{x \in X} \mu_A(x). \tag{2.5}$$

If $h(A) = 1$ then the fuzzy set A is called a *normal fuzzy set.*

Definition 2.8. Let A be a fuzzy set in X. The kernel $ker(A)$ of A is defined as:

$$ker(A) = \{x \in X, \mu_A(x) = 1\}. \tag{2.6}$$

Definition 2.9. Let A be a fuzzy set in X. The cardinality $card(A)$ of A is defined as:

$$card(A) = \sum_{x \in X} \mu_A(x). \tag{2.7}$$

Definition 2.10. Given a universal set X and a non-empty family $\mathcal{C}$ of subsets of X (usually with an appropriate algebraic structure), a *fuzzy measure*, g, on $(X, \mathcal{C})$ is a function

$$g : \mathcal{C} \to [0, \infty]$$

that satisfies the following requirements :

(g1) $g(\emptyset) = 0$ when $\emptyset \in \mathcal{C}$ (vanishing at the empty set);

(g2) for all A, $B \in \mathcal{C}$, if $A \subseteq B$, then $g(A) \leq g(B)$ (monotonicity);

(g3) for any increasing sequence $A_1 \subseteq A_2 \subseteq \ldots$ of sets in $\mathcal{C}$, if $\bigcup_{i=1}^{\infty} A_i \in \mathcal{C}$, then $\lim_{i\to\infty} g(A_i) = g(\bigcup_{i=1}^{\infty} A_i)$ (continuity from below);

(g4) for any decreasing sequence $A_1 \supseteq A_2 \supseteq \ldots$ of sets in $\mathcal{C}$, if $\bigcap_{i=1}^{\infty} A_i \in \mathcal{C}$, then $\lim_{i\to\infty} g(A_i) = g(\bigcap_{i=1}^{\infty} A_i)$ (continuity from above).

Remark 2.11. A fuzzy measure g is regular iff $X \in \mathcal{C}$ and $g(X) = 1$ (see [24]).

Remark 2.12. We call $(X, \mathcal{C})$ a *measurable space*, $(X, \mathcal{C}, g)$ a *fuzzy measure space* and any element in $\mathcal{C}$ is called a *measurable set*. The number $g(A)$ assigned to a measurable set A indicates the measurement of A (see [24]).

2.2. Methodology of measurement of the monetary poverty

The analysis of the monetary poverty requires three factors (see [18]) : A well-being indicator, a poverty line and measurement poverty indicators.

Definition 2.13. The well-being indicator is a cardinal measurement (i.e. a real number) which allows us to assign to a household a certain standard of well-being.

Remark 2.14. In the case of the measurement of the monetary poverty, the well-being indicator is either income or consumption. For the developing countries, the use of consumption as well-being indicator is better than the use of income (see [11]).

Definition 2.15. The poverty line is a standard of the well-being indicator which leads to defining if a household is poor (in the case where its well-being indicator is less than the threshold) or non-poor (in the contrary case).

Remark 2.16. There are two standards of poverty : the absolute poverty and the relative poverty. In the standpoint of the absolute poverty, an individual is poor if he cannot satisfy basic elementary needs such as to eat, to dress oneself or to dispose of an appropriate roof. Whereas from the standpoint of the relative poverty, an individual is poor not because he has a given standard of living, but because his standard of living is very low if it is compared with that of the other members of the society. For the developing countries, the appropriate definition of the threshold poverty is given in the understanding of the absolute poverty (see [19]).

Remark 2.17. A measurement poverty indicator can be interpreted as being the social loss due to the fact that a group of the population has a standard income less than the poverty line (see [3]).

3. Fuzzy measure of the monetary poverty

In this section, we make use of results of the section 2 to propose a methodology allowing us to measure the monetary poverty.

We consider a sample of a human population X subdivided in n discrete entities ($n = 1, 2, \ldots$) such that the size of X is n. The discrete entities can be the individuals, the households or any demographic strata. We assume that all the discrete entities are identified by a single common well-being indicator. Let $\mathcal{P}(X)$ be the power set of X (i.e. all subsets of X). We consider the function

$$\beta : \mathcal{P}(X) \to [0, \infty]$$

defined for all $A \in \mathcal{P}(X)$ by

$$\beta(A) = \frac{CardA}{n}. \tag{3.1}$$

Theorem 3.1. *$(X, \mathcal{P}(X), \beta)$ is a regular fuzzy measure space.*

Proof. It suffices to verify the conditions of the definition 2.10 and of the remark 2.11.

It is clear that $\emptyset$ and X belong to $\mathcal{P}(X)$. Moreover since $Card\emptyset = 0$ and $CardX = n$, we have according to (3.1), $\beta(\emptyset) = 0$ and $\beta(X) = 1$.

Let A, B belong to $\mathcal{P}(X)$ such that $A \subseteq B$. We have $A \subseteq B$ implies $CardA \leq CardB$, consequently, it is clear that $\beta(A) \leq \beta(B)$. This proves the monotonicity.

Let $A_i \in \mathcal{P}(X)$ $(i = 1, 2, \ldots)$ be an increasing sequence of sets. It is well known that $\mathcal{P}(X)$ is the largest σ-algebra over X (see [24]). Thus, we have $\bigcup_{i=1}^{\infty} A_i \in \mathcal{P}(X)$. Moreover since $A_i \subseteq \bigcup_{i=1}^{\infty} A_i$ for each i, it follows from monotonicity that $\beta(A_i) \leq \beta(\bigcup_{i=1}^{\infty} A_i)$ for each i. Thus, since X is finite , $\mathcal{P}(X)$ is finite and $A_i \in \mathcal{P}(X)$ $(i = 1, 2, \ldots)$ is an increasing sequence of sets, $\exists j_0$ such that $\lim_{i\to\infty} A_i = A_{j_0} = \bigcup_{i=1}^{\infty} A_i$. That is, $\lim_{i\to\infty} \beta(A_i) = \beta(\bigcup_{i=1}^{\infty} A_i)$. This proves continuity from below.

In the same way, Let $A_i \in \mathcal{P}(X)$ $(i = 1, 2, \ldots)$ be an decreasing sequence of sets. Since $\mathcal{P}(X)$ is the largest σ-algebra over X (see [24]), we have $\bigcap_{i=1}^{\infty} A_i \in \mathcal{P}(X)$. Moreover since $\bigcap_{i=1}^{\infty} A_i \subseteq A_i$ for each i, it follows from monotonicity that $\beta(\bigcap_{i=1}^{\infty} A_i) \leq \beta(A_i)$ for each i. Thus, since X is finite , $\mathcal{P}(X)$ is finite and $A_i \in \mathcal{P}(X)$ $(i = 1, 2, \ldots)$ is a decreasing sequence of sets, $\exists j_0$ such that $\lim_{i\to\infty} A_i = A_{j_0} = \bigcap_{i=1}^{\infty} A_i$. That is, $\lim_{i\to\infty} \beta(A_i) = \beta(\bigcap_{i=1}^{\infty} A_i)$. This proves continuity from above. $\square$

Proposition 3.2. *The fuzzy measure β satisfies the following requirements:*

1. $0 \leq \beta(A) \leq 1$ *for any* $A \in \mathcal{P}(X)$.
2. β *is self-dual, i.e.,* $\beta(A) + \beta(A^c) = 1$ *for any* $A \in \mathcal{P}(X)$ *with* $A^c = X \backslash A$.

Proof. 1. It is clear that $CardA \geq 0$ for any $A \in \mathcal{P}(X)$. Moreover, since $A \subseteq X$ for any $A \in \mathcal{P}(X)$, it follows from monotonicity that $\beta(A) \leq \beta(X) = 1$, from where $0 \leq \beta(A) \leq 1$.

2. It is due to the fact that $CardA + CardA^c = CardX$. $\square$

We are now able to build our fuzzy model of monetary poverty and proceed to its measurement.

The fuzzy set of the poor of X denoted by P is defined by:

$$P = \{(x, \mu_P(x)) | x \in X\}. \tag{3.2}$$

Let us recall that (see remark 2.2): $\mu_P(x) = 0$ if $x \in X$ is non-poor certainly, $0 < \mu_P(x) < 1$ if $x \in X$ is poor partially and $\mu_P(x) = 1$ if $x \in X$ is poor completely.

Since the poverty line Z can not be estimated with absolute certitude (see [4] and reference therein), we assume that $Z \in [Z_{\min}, Z_{\max}]$. We then define the membership function $\mu_P(x)$ $(x \in X)$ by:

$$\mu_P(x) = \begin{cases} 1 & if \quad 0 \leq x < Z_{\min} \\ (\frac{Z_{\max} - x}{Z_{\max}})^{\alpha} & if \quad Z_{\min} \leq x < Z_{\max} \\ 0 & if \quad x \geq Z_{\max}, \end{cases} \tag{3.3}$$

where the nonnegative real α is the *poverty aversion degree.*

The main result of this part is the following.

Theorem 3.3. *The poverty measurement index is given by:*

$$I_\alpha = \frac{cardP}{n} \tag{3.4}$$

Proof. The poverty measurement index is the measurement $\beta(P)$ of the set $P \in \mathcal{P}(X)$. □

Remark 3.4. I_α is the unidimensional fuzzy Foster-Greer-Thorbecke (FGT) index (see [1], [8], [6]).

Remark 3.5. I_0 is the fuzzy rate (or index) of poverty, I_1 is the fuzzy poverty depth and I_2 is the fuzzy poverty harshness.

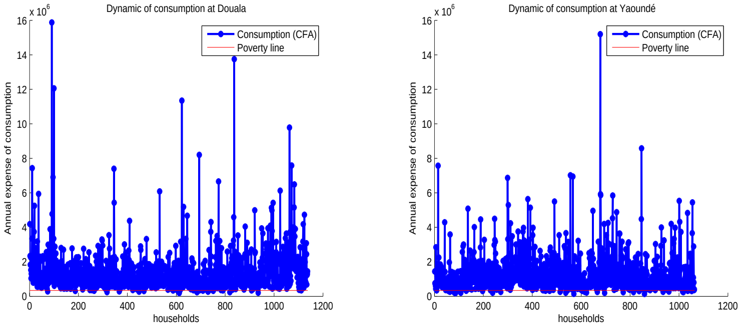
4. Measurement of the monetary poverty in Cameroon

In this section, we proceed to the measurement of the monetary poverty in Cameroon in the year 2014 by using the fuzzy FGT index (3.4).

In 2014, the National Institute of Statistics (NIS) of Cameroon proceeded to the measurement of the monetary poverty in this country by using the classical (non fuzzy) FGT index (see [18]). In their study (see [18]), the well-being indictor is measured by the annual expenditure of an aggregate of equivalent-adult consumption . The annual poverty line given is 339715FCFA with 1 dollar $\simeq$ 550FCFA. This annual poverty line is measured using absolute cost of basic needs method (see for example [1] and [18] for the knowledge on this method). In [18], the standards by kilocalorie used to calculate the annual poverty line fluctuate from 1800 to 3000 kilocalories. But the value of 2900 kilocalories is the one which has been used to obtain the value of 339715FCFA as annual poverty line. A simple rule of three allows us to have for 1800 kilocalories $Z_{\min} = 210860$ FCFA and for 3000 kilocalories $Z_{\max} = 351430$ FCFA. In this section we will focus our study on the measurement of the monetary poverty in the two main towns of Cameroon, that is, the political capital Yaoundé and the economic capital Douala. To measure the monetary poverty in this two towns, a 1063 households sample survey in Yaoundé and a 1137 households sample survey in Douala were realized by the National Institute of Statistics (NIS), a household of the sample representing on the average 625.2719815 households in the global population, the average size of a household being on the average of 4.5 individuals. The National Institute of Statistics (NIS) of Cameroon (see [18]) is the source of all data used for calculations and simulations realized in this work. The dynamics of the annual expense of consumption in these two towns are given in the figure 1.

4.1. Measurement of the monetary poverty using classical FGT index

In this subsection, we measure the monetary poverty in Douala and in Yaoundé using the classical (non fuzzy) FGT index. Let us recall that (see [12], [22])



(a) (b)

FIGURE 1. Dynamics of the annual expense of consumption in Douala (a) and in Yaoundé (b).

the classical FGT index is given by:

$$I_\alpha = \frac{1}{n}\sum_{i=1}^{q}(\frac{\widetilde{Z}-x_i}{\widetilde{Z}})^\alpha, \quad \alpha \geq 0 \tag{4.1}$$

where n is the total number of households, q the number of poor households, $\widetilde{Z} = 339715$FCFA the annual poverty line, x_i the annual expenditure by equivalent-adult of the household i and α the poverty aversion degree. Moreover, let us recall that a household is poor if $x_i < \widetilde{Z}$ and non-poor otherwise.

To compute the classical FGT index (4.1), we can use the following algorithm:

Step 1: Enter the data x_i.

Step 2: For $i = 1$ to $i = n$ do

Step 21: if $x_i < \widetilde{Z}$ then compute $S_i = \left(\frac{\widetilde{Z}-x_i}{\widetilde{Z}}\right)^\alpha$

Step 22: if $x_i \geq \widetilde{Z}$ then $S_i = 0$.

Step 3: Compute $I_\alpha = \frac{1}{n}\sum_i^n S_i$.

The table 1 shows the measurement of the monetary poverty in Douala and in Yaoundé using the classical (non fuzzy) FGT index (4.1).

TABLE 1. Measurement of the monetary poverty using classical FGT index.

Town	I_0	I_1	I_2	**number of poor**
Douala	0.0211	0.0029	7.3285×10^{-4}	24
Yaoundé	0.0386	0.0085	0.0027	41

4.2. Measurement of the monetary poverty using fuzzy FGT index

In this subsection, we make use of data given in annexe of this paper to measure the monetary poverty at Douala and at Yaoundé using the fuzzy FGT index (3.4) with $Zmin = 210860$FCFA and $Zmax = 351430$FCFA in the membership (3.3).

To compute the fuzzy FGT index (3.4), we can use the following algorithm:

Step 1: Enter the data x_i.

Step 2: (Compute the membership (3.3)) For $i = 1$ to $i = n$ do

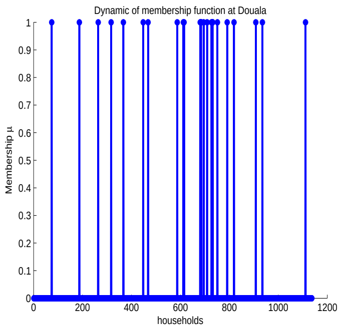
Step 21: if $x_i < Zmin$ then $\mu_i = 1$

Step 22: if $Zmin \leq x_i < Zmax$ then compute $\mu_i = \left(\frac{Zmax - x_i}{Zmax}\right)^\alpha$

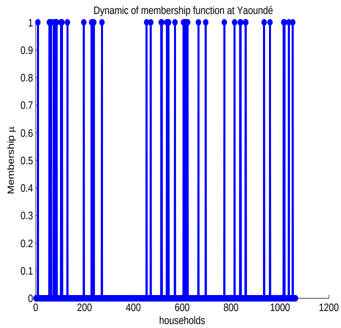
Step 23: if $x_i \geq Zmax$ then $\mu_i = 0$.

Step 3: Compute $I_\alpha = \frac{1}{n} \sum_i^n \mu_i$.

The dynamics of the membership function in Douala and in Yaoundé are given in figures 2 ($\alpha = 0$), 3 ($\alpha = 1$) and 4 ($\alpha = 2$) .



(a)



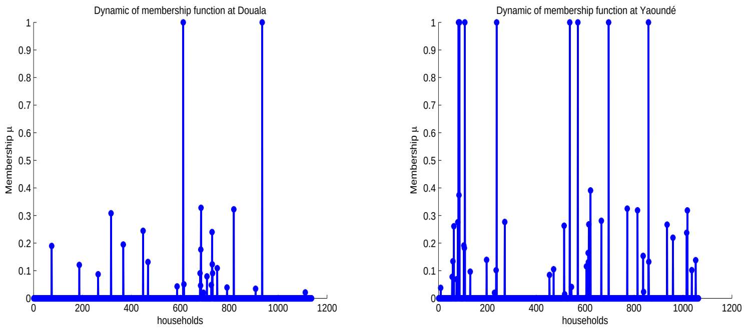
(b)

FIGURE 2. Dynamics of the membership function in Douala (a) and in Yaoundé (b) when $\alpha = 0$.

The table 2 shows, using the membership function (3.3) the distribution of the poor in Douala and in Yaoundé. While the table 3 shows the measurement of the monetary poverty in Douala and in Yaoundé using the fuzzy FGT index (3.4).

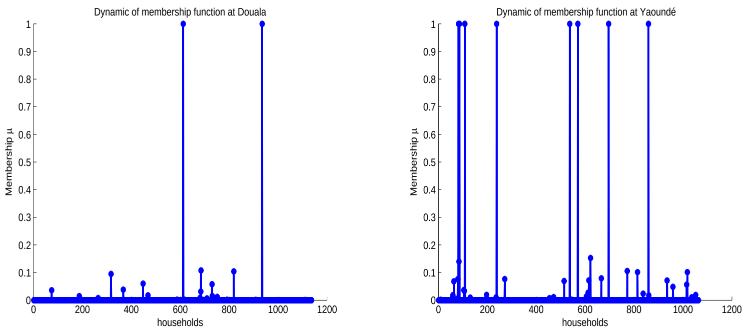
TABLE 2. Distribution of the poor using the membership function (3.3).

Town	**N. totally poor**	**N. partially poor**	**N. poor**
Douala	2	24	26
Yaoundé	8	36	44



(a) (b)

FIGURE 3. Dynamics of the membership function in Douala (a) and in Yaoundé (b) when $\alpha = 1$.



(a) (b)

FIGURE 4. Dynamics of the membership function in Douala (a) and in Yaoundé (b) when $\alpha = 2$.

TABLE 3. Measurement of the monetary poverty using the fuzzy FGT index (3.4).

Town	I_0	I_1	I_2
Douala	0.0229	0.0045	0.0023
Yaoundé	0.0414	0.0135	0.0089

4.3. Discussion

In this subsection, we proceed to the comparison of the results obtained in subsections 4.1 and 4.2.

By comparing the tables 1 and 2 at the level of the number of the poor in the two towns Douala and Yaoundé, we can notice that in the case of the use of the fuzzy measurement, the poor are subdivided into two classes:

the individuals totally poor and the individuals partially poor. While in the case of the use of the classical (non fuzzy) measurement, this classification is not enabled. Furthermore, the total number of the poor in the two towns when the fuzzy measurement is used is greater than the total number of the poor in the two towns when the classical (non fuzzy) measurement is used. This is explained by the fact that though in [18], it is clearly mentioned that the used standards per kilocalorie to calculate the annual poverty line, fluctuate from 1800 to 3000 kilocalories, the value of 2900 kilocalories has been chosen to obtain the annual poverty line used in classical measurement. However, in the case of the fuzzy measurement, the extreme values of 1800 and 3000 kilocalories have been directly used to evaluate $Zmin$ and $Zmax$. Consequently, we can notice that the fuzzy measurement brings out the fact that some individuals considered as non-poor in the classical measure are partially poor in the fuzzy measurement.

By comparing the tables 1 and 3 at the level of the measurement of the monetary poverty in the two towns Douala and Yaoundé, we can notice by comparing the index of the same nature that the fuzzy index are greater than the classical index of the measurement of the monetary poverty. In concrete terms:

The comparison of the two I_0 shows that the fuzzy measurement presents a rate of poverty greater than the rate given by the classical measurement. The explanation of this fact is due to the consideration adopted for the poverty line in the classical measurement and for the $Zmin$ and $Zmax$ in the fuzzy measurement.

The comparison of the two I_1 shows that the fuzzy measurement presents a poverty depth greater than the poverty depth given by the classical measurement. That is, the gap between the poor and the non-poor is more accented in the case of the fuzzy measurement than in the case of the classical measurement. The explanation of this fact is due to the fact that in the classical measurement, some individuals counted as non-poor are partially poor in the fuzzy measurement.

The comparison of the two I_2 shows that the fuzzy measurement presents a poverty harshness greater than the poverty harshness given by the classical measure. That is, the inequality between the poor is more accented in the case of the fuzzy measurement than in the case of the classical measurement. This data on the inequality between the poor is fundamental for the politics, since it allows us to define "the major poor".

Looking at the above comparisons and explanations, it is easy to notice that the fuzzy measurement seems more accommodated to measure the monetary poverty than the classical measurement.

5. Conclusion

In this paper, we have constructed a unidimensional fuzzy Foster-Greer-Thorbecke (FGT) index which enables us to measure the monetary poverty.

The approach adopted for that has consisted in building in a measurable space, a fuzzy measurement. This fuzzy measure has allowed us afterwards to define the unidimensional fuzzy FGT index obtained as measurement of the fuzzy set of the poor. The choice of the membership of the fuzzy set of the poor considered is motivated by the fact that the poverty line belongs quasi always to an interval. The fuzzy FGT index obtained has been afterwards used to measure the monetary poverty in Cameroon in the year 2014. By comparing the results obtained in the case of the use of the classical (non-fuzzy) FGT index and those obtained in the case of the use of fuzzy FGT index, we have prove that the results obtained with fuzzy FGT index are more significant and realistic.

References

[1] Achille Lemmi and Gianni Betti *Fussy set approach to multidimensional poverty measurement*, Springer Science-FBusiness Media, LLC (2006).

[2] Atanu Sengupta and Tapan Kumar Pal *Fuzzy Preference Ordering of Interval Numbers in Decision Problems*, Springer-Verlag Berlin Heidelberg (2009).

[3] Atkinson, A-B. *On the Measurement of Poverty*, Econometrica, **55** (1987), 749–764.

[4] Besma Belhadj, Mohamed Salah Matoussi *Proposition d'un indice flou de pauvreté en utilisant une fonction d'information*, SETIT 2007 ,International conference : Sciences of electronic, technologies of information and telecommunications (2007).

[5] Besma Belhadj *A new fuzzy unidimensional poverty index from an information theory perspective* , Empir Econ, **40** (2011), 687–704.

[6] Bourguignon F, Chakravarty SR *The measurement of multidimensional poverty*, Journal of Economic Inequality, **1** (2003), 25–49.

[7] Cerioli, A. and Zani, S. *A fuzzy approach to the measurement of poverty-Studies in Contemporary Economics*, in C. Dagum et M. Zenga (eds), Income and Wealth Distribution, Inequality and Poverty, springer Verlag (1990), 272–284.

[8] Chakravarty SR, Mukherjee D, Ranade R *On the family of subgroup and factor decomposable measures of multidimensional poverty*, Research on Economic Inequality, **8** (1998), 175–194.

[9] David Miceli *Measuring poverty using fuzzy sets*, NATSEM, University of Canberra, (1998).

[10] D. Dubois, H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, (1980).

[11] Deaton, A. and Grosh, M. *Consumption In Grosh, M and Glewwe, P (eds.), Designing Household Survey Questionnaires for Developing Countries : Lessons from Ten Years of LSMS Experience, mimeo, ch.17.* (1998).

[12] Foster J., Greer J., Thorbecke E. *A class of decomposable poverty measures*, Econometrica **52** (1984), 761-766.

[13] George J. Klir *On fuzzy-set interpretation of possibility theory*, Fuzzy Sets and Systems **108** (1999), 263-273.

[14] H.J. Zimmermann, *Fuzzy set theory and its applications*, Kluwer Academic Publishers (2001).

[15] L. A. Zadeh, *Fuzzy sets*, Information and control **8** (1965), 338–353.

[16] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets Syst. **100** (1999), 9–34.

[17] L. G. Bellu, P. Liberati *Impact des politiques sur la pauvreté : mesure de l'cart de pauvreté généralisé* , www.fao.org/tc/easypool (2006).

[18] National Institute of statistics *Quatrième enquéte camerounaise auprẽs des ménages : tendances, profil et déterminants de la pauvreté au Cameroun entre* 2001-2014 , www.statistics-cameroon.org (2015).

[19] Ravallion, M. *Poverty Comparisons, Fundamentals of Pure and Applied Economics*, Chur, Switzerland: Harwood Academic Publishers **56** (1994).

[20] Rudolf Seising *The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and its Initial Applications Developments up to the 1970s*, Springer-Verlag Berlin Heidelberg (2007).

[21] Samuel Ambapour and Rufin Bidounga *Mesure de la pauvreté au Congo utilisant la logique floue* , Journal Afrika Statistika **7** (2012) 412-424.

[22] Samuel Fambon *Croissance économique, pauvreté et ingalité des revenus au Cameroun* , Revue d'économie du développement **13** (2005) 91-122. DOI 10.3917/edd.191.091

[23] Vero Josiane et Werquin Patrick *Un réexamen de la mesure de la pauvreté*, In:Economie et statistique, num. 308-310, (1998), 143–158.

[24] Zhenyuan Wang and George J. Klir *Fuzzy Measure theory*, Springer Science+Business Media New York, (1992).

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