The 1-good-neighbor diagnosability of the Cayley graph generated by Pyramid graph

Chunlei Xu[a,b]

[a]Mongolian State University of Education, Ulaanbaatar Mongolia

[b]College of Mathematics, Inner Mongolia University for Nationalities, Tongliao, P.R.China

Abstract

Diagnosability of a multiprocessor system is one important study topic. A new measure for fault diagnosis of the system is called g-good-neighbor diagnosability that restrains every fault-free node containing at least g fault-free neighbors. CJ_n is the Cayley graph generated by pyramid graph. In this paper, we prove that the 1-good-neighbor diagnosability of CJ_n is $4n-8$ for $n \geq 4$.

Key words: Pyramid graph; Cayley graph; Diagnosability

1. Introduction

Many multiprocessor systems take interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph where nodes represent processors and links represent communication links between processors. We use graphs and networks interchangeably. For the system, study on the topological properties of its network is important. Furthermore, some processors may fail in the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability of a system G is the maximum value of t such that G is t-diagnosable [1].

In this paper, we prove that the 1-good-neighbor diagnosability of CJ_n is $4n-8$.

2. Preliminaries

2.1. *Definitions and Notations*

A multiprocessor system is modeled as an undirected simple graph $G=(V,E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Given a nonempty vertex subset V' of V, the induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v is the number of edges incident with v. We denote by $\delta(G)$ the minimum degrees of vertices of G. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor or a neighbor vertex of v for $u\in N_G(v)$. Let $S\subseteq V$. We use $N_G(S)$ to denote the set $\cup_{v\in S}N_G(v)\backslash S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if for any vertex v, $d_G(v)=k$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. A fault set $F\subseteq V$ is called a g-good-neighbor faulty set if $|N(v)\cap(V\backslash F)|\geq g$ for every vertex v in $V\backslash F$. A g-good-neighbor cut of a graph G is a g-good-neighbor faulty set F such that $G-F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is said to be the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. For graph-theoretical terminology and notation not defined here we follow [2].

A system $G=(V,E)$ is g-good-neighbor t-diagnosable if F_1 and F_2 are distinguishable for each distinct pair of g-good-neighbor faulty subsets F_1 and F_2 of V with $|F_1|\leq t$ and $|F_2|\leq t$. The g-good-neighbor diagnosability $t_g(G)$ of the system G is the maximum value of t such that G is g-good-neighbor t-diagnosable.

Proposition 2.1. *([4]) For any given system G, $t_g(G)\leq t_{g'}(G)$ if $g\leq g'$.*

In a system $G=(V,E)$, a faulty set $F\subseteq V$ is called a conditional faulty set if it does not contain all of neighbors of any vertex in G. A system G is conditional t-diagnosable if every two distinct conditional faulty subsets $F_1, F_2 \in V$ with $|F_1| \leq t, |F_2| \leq t$, are distinguishable. The conditional diagnosability $t_c(G)$ of G is the maximum number of t such that G is conditional t-diagnosable.

Theorem 2.2. *([5]) For a system $G = (V, E)$, $t(G) = t_0(G) \leq t_1(G) \leq t_c(G)$.*

2.2. *Pyramid graph*

In this section, its definition and some useful properties are introduced.

In the permutation $\binom{1\ \ 2\ \ \cdots\ \ n}{p_1\ \ p_2\ \ \cdots\ \ p_n}$, $i \longrightarrow p_i$. For the convenience, we denote the permutation $\binom{1\ \ 2\ \ \cdots\ \ n}{p_1\ \ p_2\ \ \cdots\ \ p_n}$ by $p_1\ p_2\ \ldots\ p_n$. Every permutation can be denoted by a product of cycles [6]. For example, $\binom{1\ \ 2\ \ 3}{3\ \ 1\ \ 2} = (132)$. Specially, $\binom{1\ \ 2\ \ \cdots\ \ n}{1\ \ 2\ \ \cdots\ \ n} = (1)$. The product $\sigma\tau$ of two permutations is the composition function τ followed by σ, that is, $(12)(13) = (132)$. For terminology and notation not defined here we follow [6].

Let $[n] = \{1, 2, \cdots, n\}$, and let S_n be the symmetric group on $[n]$ containing all permutations $p = p_1 p_2 \cdots p_n$ of $[n]$. It is well known that $\{(1i) : 2 \leq i \leq n\}$ is a generating set for S_n. So $\{(1, i) : 2 \leq i \leq n\} \cup \{(i, i+1) : 2 \leq i \leq n-1\}$ is also a generating set for S_n. Pyramid graph is the graph (V, E), $V = \{1, 2, \cdots, n\}$, $E = \{(1i), (2j) | i = 2, 3, \cdots, n, j = 3, 4, \cdots, n\}$. The n-dimensional Cayley graph generated by Pyramid graph $CJ_n$is the graph with vertex set $V(CJ_n) = S_n$ in which two vertices u, v are adjacent if and only if $u = v(1, i)$, $2 \leq i \leq n$, or $u = v(2, j+1)$, $2 \leq j \leq n-1$. It is easy to see from the definition that CJ_n is a $(2n-3)$-regular graph on $n!$ vertices.

Note that CJ_n is a special Cayley graph. CJ_n has the following useful properties.

Proposition 2.3. *For any integer $n \geq 3$, CJ_n is $(2n-3)$-regular, vertex transitive.*

Proposition 2.4. *For any integer $n \geq 3$, CJ_n is bipartite.*

Proof. Suppose $n \geq 3$, CJ_n is not bipartite. There must exist a cycle $v_1 v_2 \cdots v_t v_1$, where t is odd. According to the definition of CJ_n, There exist $s_1, s_2, \cdots, s_t \in Q$, where Q is the generating set of CJ_n, such that $v_2 = v_1 s_1, v_3 = v_2 s_2, v_4 = v_3 s_3, \cdots, v_1 = v_t s_t$. Therefore, $s_1 s_2 \cdots s_t = (1) \in S_n$. (1) is the even permutation, which contradict to t is odd. So we have CJ_n is bipartite. □

Proposition 2.5. *For any integer $n \geq 3$, the girth of CJ_n is 4.*

Proof. According to Proposition 2.4, CJ_n is bipartite, there is no 3-cycle in CJ_n, but $(1), (ab), (ab)(cd), (cd), (1)$ is a 4-cycle in CJ_4, where $(ab), (cd) \in Q$. □

Theorem 2.6. *([7]) Every nonidentity permutation in the symmetric group is uniquely (up to the order of the factors) a product of disjoint cycles, each of which has length at least 2.*

Theorem 2.7. *([8]) Let H be a simple connected graph with $n = |V(H)| \geq 3$. If H^1 and H^2 are two different labelled graphs obtained by labelling H with $\{1, 2, \cdots, n\}$, then $Cay(H^1, S_n)$ is isomorphic to $Cay(H^2, S_n)$.*

We can partition CJ_n into n subgraphs CJ_n^1, $CJ_n^2, \ldots, CJ_n^n$, where every vertex $u = x_1x_2\ldots x_n \in V(CJ_n)$ has a fixed integer i in the last position x_n for $i \in [n]$. It is obvious that CJ_n^i is isomorphic to CJ_{n-1} for $i \in [n]$. Let $v \in V(CJ_n^i)$, then $v(1n)$ and $v(2n)$ are called outside neighbors of v. And we can easily verified for any $v \in V(H_i)$ the outside neighbors must be in two different $V(H_j)(j \neq i, j \in \{1, 2, \cdots, n\})$.

Proposition 2.8. *Let CJ_n^i be defined as above. Then there are $2(n-2)!$ independent cross-edges between two different H_i's.*

Proof. The number of edges between any two $H_i, H_j (i \neq j)$ is $\frac{n!}{\binom{n}{2}} = 2(n-2)!$, where $n!$ is the number of edges among all $H_1, H_2, \cdots, H_n$, $\binom{n}{2}$ is the total number of pair $(H_i, H_j)(i \neq j)$. □

Proposition 2.9. *Let CJ_n be the cayley graph generated by pyramid graph. If two vertices u, v are adjacent, there is no common neighbor vertex of these two vertices, i.e., $|N(u) \cap N(v)| = 0$. If two vertices u, v are not adjacent, there are at most three common neighbor vertices of these two vertices, i.e., $|N(u) \cap N(v)| \leq 3$.*

Proof. If u, v are adjacent, then the one is odd permutation, another is even permutation in S_n. Without loss of generality, let u is odd permutation, v is even permutation, then the neighbour of u must be the even permutation in S_n, and the the neighbour of v must be the odd permutation in S_n. Therefore, $|N(u) \cap N(v)| = 0$.

If u, v are not adjacent, it is sufficent to prove the impossibility of $|N(u) \cap N(v)| \geq 4$. Suppose $\{(ai), (bj), (ck), (dl)\} \subseteq N(u) \cap N(v)$, $|\{(ai), (bj), (ck), (dl)\}| = 4$. Without loss of generality, let $u = (1)$, since $(1),(ai),v,(bj),(1)$ is a 4-cycle, $v = (ai)(bj)$. $(1),(ck),v,(xy),(1)$ is also a 4-cycle, where is $(xy) \in E(J_n)$, then $v = (ck)(xy)$. We consider the following cases.

Case 1 If (ai) is disjoint to (bj), then $v = (ck)(xy) = (ai)(bj)$, x is equal to one element of $\{a,b,i,j\}$, without loss of generality, let $x = a$, then $y = i$, we have $(ck) = (bj)$, which contradict to $|\{(ai),(bj),(ck),(dl)\}| = 4$. The rest of subcases can be proved similiarly.

case 2 If (ai) is joint to (bj), without loss of generality, let $a = b$, $(ai)(bj) = (ai)(aj) = (aji)$. Then $(ck)(xy) = v = (aji)$. By Theorem 2.6, x is equal to one element of $\{c,k\}$, or y is equal to one element of $\{c,k\}$. Let $x = c$, then $(ck)(xy) = (ck)(cy) = (cyk) = v = (aji)$. So c is equal to one element $\{a,i,j\}$. Without loss of generality, let $c = a$, then $y = j$ and $k = i$. Therefore, we have $(ck) = (ai)$, it contradict to $|\{(ai),(bj),(ck),(dl)\}| = 4$. The rest of subcases can be proved similiarly. The rest of subcases can be proved similiarly.

Therefore, $|N(u) \cap N(v)| \leq 3$. □

Proposition 2.10. *Let CJ_n be the Cayley graph generated by $n-$dimension Pyramid graph, then $\kappa(CJ_n) = 2n-3$.*

Proof. Let $F = N_{CJ_n}((1))$, since (1) is an isolated vertex in $CJ_n - F$, then F is a vertex cut, $|F| = 2n-3$. Therefore, $\kappa(CJ_n) \leq 2n-3$.

Then we will prove that for any $F \subseteq V(CJ_n)$, $|F| \leq 2n-4$, F is not vertex cut of CJ_n.

If $n = 3$, CJ_3 is just complete bipartite graph $K_{3,3}$. Obviously, for any $F \subseteq V(CJ_3)$, $|F| \leq 2$, F is not vertex cut. But F consist of all even permutations, $|F| = 3$, F is a vertex cut, so $\kappa(CJ_3) = 3$.

If $n = 4$, $F = \{(12),(13),(14),(23),(24)\}$, $CJ_4 - F$ is not connected, so we have $\kappa(CJ_4) \leq 5$.

We divide into four cases to prove, for any $F \subseteq V(CJ_4)$, $|F| \leq 4$, F is not vertex cut. For the simplicity of proof, let $|F_1| \geq |F_2| \geq |F_3| \geq |F_4|$.

Case 1 If $|F_1| = 4$, then $|F_2| = |F_3| = |F_4| = 0$. Since for any two $H_i, H_j (i,j = 2,3,4, i \neq j)$, there are 4 cross-edges between them. Therefore, $CJ_4[H_2 \cup H_3 \cup H_4]$ is connected. Furthermore, for any $u \in V(H_1 - F_1)$, there are 2 outside neighbours in $V(H_2) \cup V(H_3) \cup V(H_4)$. So we have $CJ_4 - F$ is connected.

Case 2 If $|F_1| = 3$, then $1 \geq |F_2| \geq |F_3| \geq |F_4|$. There are 4 cross-edges between H_3, H_4, so $CJ_4[H_3, H_4]$ is connected. For any $u \in V(H_1 - F_1)$, there are two neighbours in $V(H_2) \cup V(H_3) \cup V(H_4)$, and one of them must be in $V(H_3)$ or $V(H_4)$, so $CJ_4[(H_1 - F_1) \cup H_3 \cup H_4]$ is connected. And for any $v \in V(H_2 - F_2)$, there are 2 neighbours in the two of $V(H_1) \cup V(H_3) \cup V(H_4)$, and one of them must be in $V(H_3)$ or $V(H_4)$, so $CJ_4[(H_2 - F_2) \cup H_3 \cup H_4]$ is connected. Therefore, $CJ_4 - F$ is connected.

Case 3 If $|F_1| = 2$, then $2 \geq |F_2| \geq |F_3| \geq |F_4|$.

Subcase 3.1 If $|F_2| = 2$, then $|F_3| = |F_4| = 0$. $H_1 - F_1$ and $H_2 - F_2$ are bipartite graph $K_{1,3}$ or $K_{2,2}$. Obviously $CJ_4[H_1 - F_1]$ and $CJ_4[H_2 - F_2]$ are connected, there are 4 cross-edges between H_3 and H_4, so $CJ_4[H_3 \cup H_4]$ are connected. For any $u \in V(H_1 - F_1)$, there are two neighbours in the two of $V(H_2) \cup V(H_3) \cup V(H_4)$, one of them must in $V(H_3)$ or $V(H_4)$, therefore, $CJ_4[(H_1 - F_1) \cup H_3 \cup H_4]$ is connected. Similiarly, $CJ_4[(H_2 - F_2) \cup H_3 \cup H_4]$ is connected too. So $CJ_4 - F$ is connected.

Subcase 3.2 If $|F_2| = 1$, according to inequality $4 > 2 > 1$, that is, the number of cross-edges between any two H_i and $H_j (i \neq j)$ is bigger than the number of $|F_i| (i = 1, 2, 3)$, so $CJ_4[(H_1 - F_1) \cup H_4]$, $CJ_4[(H_2 - F_2) \cup H_4]$, $CJ_4[(H_3 - F_3) \cup H_4]$ are connected. Therefore, $CJ_4 - F$ is connected.

Case 4 If $|F_1| = 1$, then $1 \geq |F_2| \geq |F_3| \geq |F_4|$. According to inequality $4 > 1$, that is, the number of between any two H_i and $H_j (i \neq j)$ is bigger than the number of $|F_i| (i = 1, 2, 3, 4)$. Therefore, $CJ_4 - F$ is connected.

Combin these four cases, we have $\kappa(CJ_4) = 5$.

Suppose $\kappa(CJ_{n-1}) = 2(n-1) - 3 = 2n - 5$, then for any $F \subseteq V(CJ_n)$, F is not vertex cut.

When $n \geq 5$, we divide into three cases to prove $\kappa(CJ_n) = 2n - 3$.

Case 1 If $|F_i| \leq 2n - 6 (i = 1, 2, \cdots, n)$, then F is not vertex cut of $H_i (i = 1, 2, \cdots, n)$, so $CJ_n[H_i - F_i] (i = 1, 2, \cdots, n)$ are connected, and $2(n-2)! > 2(2n-6)$, therefore, $CJ_n - F$ is connected.

Case 2 If there exist $i \in 1, 2, \cdots, n$, such that $|F_i| = 2n - 5$, without loss of generality, let $|F_1| = 2n - 5$, then $|F_i| \leq 1 (i = 2, 3, \cdots, n)$. Since $\kappa(CJ_n) = 2(n-1) - 3 = 2n - 5 > 1$, $CJ_n[H_i - F_i] (i = 2, 3, \cdots, n)$ are connected. $2(n-2)! > 1$, that is, the number of crossedges between two H_i and H_j is much bigger than $|F_i|$, so $CJ_n[(H_2 - F_2) \cup (H_3 -$

$F_3) \cap \cdots \cup (H_n - F_n)]$ is connected.

If F_1 is just the vertex cut of H_1, it produce several connected subgraphs $D_1, D_2, \cdots, D_k$ when we delete F_1 from H_1. For any $j \in \{1, 2, \cdots, n\}$, every vertex of D_j have two neighbours belonging to $V(H_i)(i = 2, 3, \cdots, n)$, and $2 > 1$, therefore, $CJ_n - F$ is connected. If F_1 is not vertex cut of H_1, similiarly with disccuse above, $CJ_n - F$ is connected.

Case 3 If there exist $i \in \{1, 2, \cdots, n\}$, such that $|F_i| = 2n - 4$. Without loss of generality, let $|F_1| = 2n - 4$, then $|F_i| = 0(i = 2, 3, \cdots, n)$. For every vertex of $H_1 - F_1$, it must exist two neighbours belonging to $H_i(i = 2, 3, \cdots, n)$. Therefore, $CJ_n - F$ is connected.

Sum up in conclusions, $\kappa(CJ_n) = 2n - 3$. □

3. The 1-good-neighbor connectivity for the Cayley graph generated by *n*-dimensional Pyramid graph

In this section, we shall show the 1-good-neighbor connectivity of the Cayley graph CJ_n generated by n-dimensional Pyramid graph .

Theorem 3.1. *For $n \geq 4$, the 1-good-neighbor connectivity CJ_n is $4n-8$, i.e., $\kappa^{(1)}(CJ_n) = 4n - 8$.*

Proof. Let $A = \{(1), (12)\}$, $F_1 = N_{CJ_n}(A)$, $F_2 = F_1 \cup A$. Since $CJ_n - F_1$ is disconnected (K_2 and $CJ_n - F_2$), F_1 is a vertex cut. It is sufficient to prove for any $v \in V \setminus F_1$, $|N(v) \cap V \setminus F_1| \geq 1$. Obviously, for any $v \in V(K_2)$, $\delta(v) = 1$. By Prosition 2.10, there is at most 3 common vertices between v and (1) or v and (12), so for any $v \in V \setminus F_2$, $\delta(CJ_n - F_2) \geq 2n - 3 - 3 > 0$ when $n \geq 4$. Therefore, F_1 is a 1-good neighbour cut, $\kappa^{(1)}(CJ_n) \leq 4n - 8$.

We prove for any F, $|F| \leq 4n - 9$, F is not a 1-good neighbour cut of CJ_n.

If $n = 3$, $CJ_3 = K_{3,3}$. Obiviously, $|F| \leq 2$, F is not a vertex cut. When $|F| = 3$, there are two cases in the following. If the three vertices in one part, it produce three isolated vertices. If the three vertices in two parts, F is not a vertex cut.

If $n = 4$, CJ_4 can be partitioned into H_1, H_2, H_3, H_4, $H_i \cong K_{3,3}(i = 1, 2, 3, 4)$, there are 4 crossedges between H_i and $H_j(i \neq j, i, j = 1, 2, 3, 4)$.

If $F_1| = 6$, $0 \leq |F_i| \leq 1(i = 2, 3, 4)$. By inequality $4 > 1 + 1 = 2$, $CJ_4 - F$ is connected.

If $|F_1| = 5$, $0 \leq |F_i| \leq 2(2,3,4)$. If $u \in V(H_1 - F_1)$ is an isolated vertex, then F is not satisfied to definition of 1-good neighbour cut. If $u \in V(H_1 - F_1)$ is not an isolated vertex, by inequality $4 > 2$, $CJ_4 - F$ is not connected.

If $|F_1| = 4$, $0 \leq |F_i| \leq 3(i = 2,3,4)$. By inequality $4 > 3$, $H_2 - F_2, H_3 - F_3, H_4 - F_4$ are connected. If $u, v \in V(H_1 - F_1)$ are isolated vertices, then F is not a 1-good neighbour cut. If $u, v \in V(H_1 - F_1)$, then u, v are adjacent, by Proposition 2.9, there is no common neighbour. For both u and v, there are 4 outside neighbours, and by inequality $4 > 3$, $CJ_4[H_1 - F_1] \cup CJ_4[(H_2 - F_2) \cup (H_3 - F_3) \cup (H_4 - F_4)]$ is connected. Therefore, $CJ_4 - F$ is connected.

Suppose $|F| \leq 4n - 13$, $F \subseteq V(CJ_n)$, F is not a 1-good neighbour cut of CJ_{n-1}.

We can partition CJ_n into $H_1.H_2, \cdots, H_n$, $H_i \cong CJ_{n-1}(i = 1,2,\cdots,n)$. Let $F = F_1 \cup F_2 \cup \cdots \cup F_n$, $F_i \cap F_j = \emptyset(i,j = 1,2,\cdots,n, i \neq j)$, and $|F_1| \geq |F_2| \geq \cdots \geq |F_n|$. We divide cases in the following.

Case 1 If $|F_i| \leq 4(n-1) - 9 = 4n - 13$, then by assumption, $H_i - F_i(i = 1,2,\cdots,n)$ is connected. And by inequality $2(n-2)! > 4n - 13$, so $CJ_n - F$ is connected.

Case 2 If $4n - 11 \geq |F_1| \geq 4n - 12$, then $0 \leq |F_i| \leq 2(i = 2,3,\cdots,n)$. By inequality $2(n-2)! > 2 + 2 = 4$ if $n \geq 5$, $CJ[H_i](i = 2,3,\cdots,n$ is connected. By inequality $2(n-2)! > 4n - 11 + 2 = 4n - 9$ if $n \geq 5$. Then $CJ_n - F$ is connected.

Case 3 If $4n - 9 \geq |F_1| \geq 4n - 10$, then $0 \leq |F_i| \leq 1(i = 2,3,\cdots,n)$. For any $H_i, H_j(i,j = 2,3,\cdots,n)$, there are $2(n-2)!$ cross-edges between them, by inequality $2(n-2)! > 1+1 = 2$, $CJ_n[H_i - F_i](i = 2,3,\cdots,n)$ is connected. For any $u \in V(H_1 - F_1)$, there are two outside vertices adjacented to it. By inequality $2 > 1$, $CJ_n - F$ is connected.

Therefore, $\kappa^{(1)}(CJ_n) = 4n - 8$. □

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