Systematic Correlation is Priced as Risk Factor

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Abstract

In this study, we first measure the systematic correlation level risk ***MC*** and systematic correlation shock risk ***MCS*** based on mixed vine copula method and investigate their relationship with stock return. The empirical result shows that correlation is significantly and negatively priced as risk factor in China which is dynamic through different regimes. Then we find out that transformation mechanism between idiosyncratic correlation and systematic correlation is supported at stock-level and index-level.
Key words: Systematic correlation risk;Idiosyncratic correlation risk ;MacBeth price;Regime-switching

1. Introduction

Correlation is critical for asset allocation of investment portfolio as it reflects the level of diversification. The systematic correlation between asset and market is also important for several applications. For example, low market-correlated portfolio is better immune to dramatic fall of net asset value in the downside market, nevertheless, portfolio consisting of those assets with high correlation to market perform better in the upside trending market. Previous researches on correlation have revealed the significant impact of correlation risk in financial market. Literatures such as Bollerslev (1988)[?] and Longin and Solnic (1995)[?] have shown that correlation in financial market is time variant and there is considerable evidence on the negative relation between correlation and market return. Researchers like Gravelle (2006)[?] and Acharya(2008)[?] studied the influence of correlation risk event to market, and in their studies, correlation risk event is indicated by market shock such as financial crisis. The former concluded on the abnormally high correlation in currency and bonds during financial crisis while the latter found out correlation increases in bearish market. It is natural to ask whether the correlation is priced in asset returns and whether the price varies in bearish market and bullish market.

Based on the intertemporal capital asset pricing model, in the frictionless market with transparent information, the price change follows Itô's lemma and the price of asset is irrelevant to the utility preference, which apparently is not practical in real financial world. The Intertemporal Capital Asset Pricing Model, proposed by Robert Merton(1973)[?], forecasts changes in the distribution of future returns or income when investors are faced with more than one uncertainty. Within the framework of ICAPM model, the asset with return which is co-varying with correlation provides a hedge against correlation. The demand of assets that pay off where in highly-correlated condition would drive up the asset price and it leads to narrowing down of correlation premium, which is one of two competing theories about correlation price. The other theory regards correlation risk as one component of systematic risk. When the market consists of large number of assets, correlation risk partly contributes to integrate risk. Other related studies by Pollet and Wilson(2010)[?] explained the deterioration on return by correlation increase as the result of increased volatility and decreased benefits of diversification. Consequently, investors prefer securities with positively correlated return with market trend as a protection for welfare loss.

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Driessen (2005)[?] investigated S&P 100 and the options on component stocks and concluded on the negative risk price of market correlation. Krishnan (2008)[?] applied cross-sectional regression approach to test price of security in United States and found out negative price as well. ZHANG Zhenglong (2007)[?] identified the conditional correlation in Chinese stock market is a negative risk price. But the research on correlation risk is represented by simple average of pair-wise linear correlation coefficients without distinguishing different market conditions.

In this paper, we first use mixed vine copula and general Pareto distribution to measure systematic correlation in the first section. The mixed vine copula method considers the asymmetry of correlation in downside and upside trends. In section 2 we examine price of correlation level risk and correlation shock risk of Chinese A share market for short term and long term respectively. Using both daily return and monthly return of listing stocks, the empirical results reflect that short-term systematic correlation level risk is more significantly priced than long term, and the correlation shock risk is negatively priced in spite of examination window. By including markov switching regimes in the model, the significance of negative price of short-term correlation is well supported and further shows the asymmetry of correlation risk in different regimes.

Finally, we propose a transformation mechanism between systematic correlation and idiosyncratic correlation. We examine this transformation procedure at individual stock level and index level, which both produce sufficient evidence that during the market thrill the increasing systematic correlation risk would release idiosyncratic correlation risk with the constant market-wide volatility.

The following sections are organized as Section 2 introduces the mixed vine copula-based measurement of correlation level risk and correlation shock risk. Section 3 demonstrates the significance of negative price of correlation risk and shows the result in markov regime-switching copula model. We investigate how idiosyncratic correlation transfers into systematic correlation in Section 4 and conclude in Section 5.

2. Measurements of Correlation Risk

In this section, we demonstrate the measurement and estimation of correlation risk using mixed-vine copula and extreme theory.

2.1. Mixed Vine Copula and GPD

The copula method is gathering more attention among academics and practitioners in the field of finance as it is sensitive to features in tails, which is an effective answer to fat-tail problem since most financial data do not follow normal distribution. Sklar(1959)[?] firstly defined copula as a connection function illustrating the dependence relationship. Copula functions in Archimedean class are often used as the correlation measurement, Kendall or Pearson correlation are computed based on consistent copula parameter. Although copula-based correlation can illustrate other kinds of correlation changes other than linear changes,it is difficult to estimate the parameters when the number of assets increased due to ”dimension explosion”.Kjersti and Claudia(2009)[?] used pair-copula decomposition to exhibit complex pattern of dependence in the tails, which is named Canonical Vine Copula. This method is a flexible methodology to construct higher-dimensional copulas when approximating pair-wise copula to be connected by vines. In this paper, we use C-vine copula to model the dependence for n assets as follows:

$$c(x_1, ..., x_n) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,...,j-1}(F(x_j|x_1, ..., x_{j-1}), F(x_{j+i}|x_1, ..., x_{j-1})) \tag{1}$$

Equation ?? is the C-vine copula function and its likelihood function is Equation ??

$$L(c) = \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^{T} \log \left[c_{j,j+i|1,...,j-1,t}(F(x_j|x_1, ..., x_{j-1}), F(x_{j+i}|x_1, ..., x_{j-1})) \right] \tag{2}$$

In Equation ??,$c_{j,j+i|1,...,j-1}$ is the copula function of x_i and x_j, and $F(x_j|\cdot)$ is the conditional marginal function of x_j. Normal distribution is biased when sample data is skewed. In this article, we use Generalized Pareto distribution(GPD) to estimate marginal distribution. $R = (R_1, R_2, ..., R_n)$ is the set of asset returns and $\theta = (\theta_1, \theta_2, ..., \theta_n)$ is the threshold set that models tail data with marginal distribution G_R^θ. In spite of location θ, scale $\sigma > 0$ and shape $k \in \mathbb{R}$ of GPD, dependence function $D(u_1, u_2, ..., u_n)$ is also needed to approximate multi-variant joint distribution of tails.

According to the maximum likelihood method to estimate the joint tail distribution by Ledford(1997)[?], we firstly hypothesize that time-series data of asset returns R_1 and R_2 with thresholds θ_1 and θ_2 are time-independent. $\left\{A_{jk} : j = I(R_1 > \theta_1), k = I(R_2 > \theta_2))\right\}$ differentiates sample data into four zones . The dependence function D_R^θ of asset return R beyond threshold θ represents the asymmetry of upside correlation and downside correlation, where comprising Gumbel Copula Frank Copula and Clayton Copula. Gumbel Copula is sensitive to positive co-movements and Clayton Copula is better explaining the downside correlation. Correlation derived from Frank Copula is symmetrical and we include Frank Copula in mixed-copula aiming at calibrating the relative upside-sensitive weight and downside-sensitive weight.

Suppose bivariate asymmetric dependence relationship between asset return as:

$$D_R^\theta = \sum_{i=1}^{3} w_i C_i(F_{R_1}^{\theta_1}(x_1), F_{R_2}^{\theta_2}(x_2)) \tag{3}$$

Denoting $F_{R_i}^{\theta_i}(x_i)$ as the joint tail distribution of asset i return beyond threshold θ_i:

$$F_{R_i}^{\theta_i}(x_i) = 1 - p_i(1 + k_i \frac{x_i - \theta_i}{\sigma_i})^{-\frac{1}{k_i}} \tag{4}$$

where p_i is the probability of R_i beyond θ_i and generalized perato distribution $G_{R_i}^{\theta_i}(x_i)$ is the approximation of tail distribution of over-threshold R_i. For bivariate dependence at time t,$L_{jk}(R_{1,t}, R_{2,t})$ is the likelihood contribution of R_1 and R_2 in A_{jk} zone.

The likelihood function of asset return series R_1 and R_2 within time window T is:

$$L(\{R_{1,t}, R_{2,t}\}_{t\in[1,T]}, \phi) = \prod_{t=1}^{T} L(R_{1,t}, R_{2,t}, \phi) \tag{5}$$

where:

$$L(R_{1,t}, R_{2,t}, \phi) = \sum_{j,k\in\{0,1\}} L_{jk}(R_{1,t}, R_{2,t}) \cdot I_{jk}(R_{1,t}, R_{2,t})$$
$$\phi = (p_1, p_2, \sigma_1, \sigma_2, k_1, k_2, w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3)$$

2.2. Systematic Level Risk and Shock Risk

Systematic correlation risk measures the co-movement between asset and aggregate market, and its asymmetry is revealed by previous empirical evidence. Asset's different responses to good news and bad news on market is due to the uncertainty of overall market state. Usually the asset is more sensitive to bad news which causes the assets to fall together. On the other hand, when the market condition is promising and investors are confident about expected returns, further good news have little impact on increasing asset price. Thus, in this section we investigate the impact of correlation shock risk as well as correlation level risk.

Consistent with the joint distribution function in ??, we define the asset-market joint distribution is as:

$$F_{i,m}^{\theta_i,\theta_m} = \exp\left(-V(-1/\log\left\{F_i^{\theta_i}(R_i)\right\}, -1/\log\left\{F_m^{\theta_m}(R_m)\right\})\right) \tag{6}$$

In Equation ?? $F_m^{\theta_m}(R_m)$ and $F_i^{\theta_i}(R_i)$ is the GPD distribution of market return R_m and asset return R_i respectively. V is the dependence function between asset i and market. The threshold here is $\theta_{m,T} = \overline{R}_{m,T} \pm n \times \sigma_{m,T}$ where $n \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$.

During our sample period T, we assume the market contains N assets, thus there are $N+1$ assets including market return as the aggregate market asset return. The vine copula function beyond threshold of $N+1$ assets is with R_m as the critical vine:

$$c(R_{i,T}^{\theta_{i,T}}, ..., R_{m,T}^{\theta_{m,T}}) = \prod_{i=1}^{N} c_{i,m}(F^{\theta_{i,T}}(R_{i,T}|R_{1,T}, ..., R_{N-1,T}, R_{m,T}), F^{\theta_{m,T}}(R_{m,T}|R_{1,T}, ..., R_{N,T}))) \quad (7)$$

Parameters $\phi = (p_1, p_2, \sigma_1, \sigma_2, k_1, k_2, w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3)$ are calculated using EM algorithm. The equal weighted Kendall correlation τ of different thresholds is the indicator of systematic correlation. For instance, we have five joint downside distribution for $\theta_{m,T} = \overline{R}_{m,T} - n \times \sigma_{m,T}, \theta_{i,T} = \overline{R}_{i,T} - n \times \sigma_{i,T}$ where $n \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$ and the weight of Clayton copula is decisive for the relevant significance of asymmetry. We then standardize τ_{down} as the downside systematic correlation level risk. The calculation of upside and middle systematic correlation are calculated similarly. As investors care most about their asset price decreasing with the whole market, the systematic correlation level risk MC in this paper specifically refers to downside correlation τ_{down}.

Systematic correlation level risk MC reveals the absolute level of correlation risk of overall market. MC sustains high when the correlation between assets and market tend to be high. As we examine the relationship between asset return and MC respectively for short-term and long-term, the value of MC mirrors the average correlation during rolling period instead of unexpected correlation change. In order to recognize the correlation shock risk, we also examines the temporal correlation change using autoregressive model.

The simple representation of AR model of MC with lag 1 has the form as:

$$MC_{down,t} = c + \varphi MC_{down,t-1} + \epsilon_{MC,t} \quad (8)$$

$\epsilon_{MC,t}$ in Equation ?? is defined as the correlation shock risk MCS. It is necessary to study correlation shock risk in the market decline in order to protect asset price from further falling.

3. Pricing of Correlation as a Risk Factor

When the high-correlated assets are added into portfolio, the benefit of diversification is weakened, thus causing negative impact on portfolio wealth. Under the assumption from studies of Merton(1973)[?], the asset return is related to observable risk exposures.In certain circumstances, the correlation between assets better reveal the aggregate systematic risk rather than market variance. If some assets provide higher returns as a hedge tool for higher correlation, it can avoid the portfolio loss from correlation event. In this section, we start by examining the price of MC and MCS and consequently model the pricing of correlation using regime-switching models.

3.1. MacBeth Pricing Model

To abstract the effect of correlation risk on asset returns from impacts of other risk factors, we include SMB,HML,Mom,Rev,Vol,Liq,$Skew$,$Kurt$,$Co-Skew$,$Sentiment$ and PIM as control variables. SMB and HML are typical risk factors from Fama-French model and Mom,Rev,Vol,Liq represent momentum, reversal, volatility and liquidity. Since real financial data is not normal-distributed and usually leptokurtosis and fat-tail, higher-momentum risk factors like $Skew$,$Kurt$ and $Co-Skew$ are denoted as well. The pricing process of risk factor is corresponding to price-related information flow,Wang (1993) [?] presented a dynamic asset-pricing model under asymmetric information.Furthermore, recent works by [?] [?] have shown that the relationship between market return and market correlation is more significant when investor confidence is shrinking, because bad news would be magnified by negative investor's sentiment leading to sell pressure. Thus we derive that extent of correlation risk affects asset return via influencing investor sentiment.

Fama and MacBeth expanded capital asset pricing model noted as Fama-MacBeth method in 1973 [?] for multi-factor pricing. Given n risk factors, $R_{i,t}$ the asset i return from time $t-1$ to time t is:

$$R_{i,t} = \gamma_{1,t} + \gamma_{2,t}\beta_{1,i,t} + \gamma_{3,t}\beta^2_{1,i,t} + \gamma_{2,t}\beta_{2,i,t} + \gamma_{3,t}\beta^2_{2,i,t} + ... + \gamma_{2,t}\beta_{n,i,t} + \gamma_{3,t}\beta^2_{n,i,t} + \gamma_{4,t}s_i + \eta_{i,t} \tag{9}$$

where s_i is unsystematic risk of asset i while $\beta_{i,t}$ is the systematic risk. Formally, Fama-MacBeth stands for three assumptions:(1)$E(\gamma_{3,t}) = 0$;(2)$E(\gamma_{4,t}) = 0$;(3)$E(\gamma_{2,t}) = E(R_{m,t}) - E(r_f) > 0$. The second step of Mac-Beth method is cross-sectional regression when we use the estimation $\hat{\beta}_i$ rather than the real value, which result in the estimation error. To address EIV problem, we construct portfolio following rank of $\hat{\beta}_i$ as base asset as well as examining individual stock. For portfolio with N stocks, the portfolio beta is $\hat{\beta}_p = \sum_{i=1}^{N} w_i\hat{\beta}_i$.

The decrement of error-in-variables is at the cost of information loss. The portfolio with top β overestimates $\hat{\beta}_p$ and vice versa. We then first construct portfolio according to $beta$ in period T_1 and estimate $\hat{\beta}_p$ in following T_2. We denote MC as systematic correlation risk and F as other risk factors mentioned above. The final regression is as Equation ??:

$$R_i = \gamma_{MC}\beta_{i,MC} + \gamma_F\beta_{i,F} + \epsilon_{i,t} \tag{10}$$

3.2. Data and Statistics

We use daily log return data of stocks listed on A share market in China from January 1996 to June 2017. We first remove de-listed stocks and those special traded stocks during sample period for their abnormal volatility and high speculation. Then we exclude stocks with less than 15 trading days per month. Due to lack of trading, their stock price used calculation correlation may cause biased result. Finally, we adjust the observations per month for outliers. After the data cleaning process, our sample include 2571 stocks and the sample rolling month for computation of correlation risk is 60 months. The measurement window for short-term correlation risk indicator is 6 months and 36 months for long-term correlation risk.

Table 1. Statistics of Extreme Daily Return from 1996 to 2017

Date	Negative Return %	Date	Positive Return %
2016/9/1	-0.769	2007/3/30	1.908
2017/4/7	-0.766	2010/10/14	0.302
2016/4/29	-0.763	2009/6/10	0.279
2016/5/27	-0.758	2006/3/13	0.226
2016/6/8	-0.748	2009/3/5	0.157
2015/9/24	-0.740	2006/12/15	0.134
2017/5/23	-0.721	2009/4/17	0.120
2016/3/8	-0.719	2006/5/8	0.105
2015/11/11	-0.719	2007/5/21	0.104
2016/7/26	-0.715	2005/7/19	0.102

3.3. Empirical Result

The empirical result of regression result of MacBeth Pricing Model for both short-term systematic correlation level risk MC_{short} and long-term systematic correlation level risk MC_{long} are listed in Panel A and Panel B of table ??. The first column Model (1) contains risk factors Fama-French three factor model:R_m,SMB and HML other than MC. The price of MC_{short} is -1.017, significant at the 1% level while the insignificant price of MC_{long} -9.602. The column 2 reports results when including Mom,Rev and Liq as control factors for momentum, reverse and liquidity. MC_{short} remains significant with t-value of -2.01.

In Model (3), Model (4) and Model (5), short-term systematic correlation risk MC_{short} are all significant and negative at the 10% confidence level, the pricing of MC_{short} are respectively -1.185, -1.127 and -1.133. However, long-term systematic correlation risk MC_{long} are negative but insignificant.

As the increasing control risk factors, the price of MC_{short} presented in Panel A is significantly negative indicating that faced with the surge of systematic correlation, those under-diversified assets may suffer from price decline due to their high downside correlation with market return, nevertheless, assets with relatively low correlation with market return would provide higher return as hedging benefit. For the long term period, the changes of MC_{long} is well-adopted and revealed by asset price, consequently there is no significant relationship between long-term systematic correlation risk and asset return.

We also examine the relation between correlation shock risk MCS and asset return by dividing MCS into short-term correlation shock MCS_{short} and long-term correlation shock MCS_{long}. In Model (1), the short-term correlation shock risk price is -0.110 and the long-term correlation shock is price as -0.025, both of which are significant at 1% level. We add Mom Rev and Liq in Model (2), the result shows that after controls of other three factors, MCS_{short} and MCS_{long} are negatively priced(-0.156,-0.034) with significance. From column 3 to column 5, we add more risk factors in the capital asset pricing regression model, MCS_{short} and MCS_{long} remain significant indicating that unexpected correlation change have negative impact on asset return due to their unpredictability. Stocks that are able to defend themselves from correlation shock have higher implied value. That is, when the asset has negative exposure to MCS, the negative price leads to higher asset return and vice versa.

Table 2. MacBeth Pricing of MC for Individual Stocks

This table shows the result of MacBeth pricing regression for both short-term systematic correlation level risk MC_{short} and long-term systematic correlation level risk MC_{long}. The first column Model (1) contains risk factors Fama-French three factor model:R_m,SMB and HML other than MC. Model (2) added Mom,Rev and Liq to control momentum effect, reverse effect and liquidity factor. High-moment factors $Skew$, $Kurt$, $CoSkew$ are included in Model (3) and Vol, $idioVol$ are included in column 4 of Model (4). In the last column, Model (5) shows the result of pricing of all risk factors. γ is the pricing of risk factor and $t-value$ is their corresponding significance.

	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)	
Panel A. Short term MC_{short}										
Risk Factor	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$
R_m	0.472	5.21	0.370	5.87	0.279	4.78	0.338	4.60	0.331	4.59
SMB	0.027	3.13	0.152	1.71	0.001	1.72	0.001	1.21	0.001	1.26
HML	-0.010	-1.07	0.007	9.91	0.003	9.49	0.007	9.53	0.007	9.58
MC_{short}	-1.017***	-2.59	-1.137**	-2.01	-1.185*	-1.78	-1.127*	-1.87	-1.133*	-1.69
Mom			0.250	8.09	0.250	7.82	0.244	8.09	0.238	8.20
Rev			-0.628	-0.28	-0.650	-0.88	-0.650	-0.85	-0.643	-0.65
Liq			-1.149	-10.46	-1.278	-10.44	-1.244	-9.92	-1.262	-9.97
Vol							-1.027	-11.38	-1.031	-11.46
$idioVol$							-1.723	-10.83	-1.732	-10.90
$Skew$					2.579	7.24	2.369	6.67	2.329	6.60
$Kurt$					-6.199	-10.46	-5.856	-9.67	-5.958	-9.80
$CoSkew$					1.876	8.19	1.819	8.01	1.848	8.12
$Sentiment$									-2.353	-10.67
PIM									-5.290	-17.21
Panel B. Long Term MC_{long}										
	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$
R_m	0.449	4.86	0.453	6.09	0.357	5.04	0.393	4.84	0.337	4.81
SMB	0.002	3.20	0.245	1.76	0.002	1.67	0.001	1.14	0.002	1.18
HML	-0.102	-1.09	0.009	10.03	0.006	9.64	0.009	9.64	0.008	9.55
MC_{long}	-9.602	-1.250	-1.047	-1.05	-1.093	-1.54	-1.044	-1.0	-1.051	-1.06
Mom			0.244	8.29	0.247	7.85	0.242	8.07	0.24	8.10
Rev			-0.618	-0.08	-0.64	-0.79	-0.645	-0.96	-0.645	-0.82
Liq			-1.138	-10.45	-1.269	-10.44	-1.235	-9.92	-1.263	-10.01
Vol							-1.032	-11.34	-1.038	-11.49
$idioVol$							-1.723	-10.84	-1.738	-10.95
$Skew$					2.724	7.45	2.476	6.79	2.354	6.64
$Kurt$					-6.183	-10.44	-5.862	-9.69	-5.964	-9.82
$CoSkew$					1.911	8.33	1.833	8.05	1.855	8.15
$Sentiment$									-2.340	-10.70
PIM									-5.338	-17.49

∗significant at 10 % level; ∗∗significant at 5 % level; ∗ ∗ ∗significant at 1 % level

Table 3. MacBeth Pricing of MCS for Individual Stocks

This table shows the result of MacBeth pricing regression for both short-term systematic correlation shock risk MCS_{short} and long-term systematic correlation shock risk MCS_{long}. The first column Model (1) contains risk factors Fama-French three factor model:R_m,SMB and HML other than MC. Model (2) added Mom,Rev and Liq to control momentum effect, reverse effect and liquidity factor. High-moment factors $Skew$, $Kurt$, $Coskew$ are included in Model (3) and Vol, $idioVol$ are included in column 4 of Model (4). In the last column, Model (5) shows the result of pricing of all risk factors. γ is the pricing of risk factor and $t-value$ is their corresponding significance.

	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)	
Panel A. Short term MCS_{short}										
Risk Factor	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$	γ	$t-value$
R_m	0.058	2.59	0.380	4.97	0.315	4.64	0.339	4.40	0.388	4.78
SMB	0.002	2.19	0.005	1.47	0.001	1.20	0.001	0.78	0.002	1.01
HML	-0.095	-10.35	0.003	9.47	0.006	9.95	0.008	10.00	0.004	9.51
MCS_{short}	-0.110***	-3.41	-0.156**	-2.13	-0.168***	-3.53	-0.113**	-2.45	-0.128***	-2.74
Mom			0.229	8.90	0.232	8.51	0.234	8.53	0.24	8.12
Rev			-0.593	-0.65	-0.617	-0.07	-0.635	-0.45	-0.646	-0.86
Liq			-1.096	-10.12	-1.200	-10.15	-1.139	-9.44	-1.253	-9.99
Vol							-0.968	-10.83	-1.037	-11.52
$idioVol$							-1.62	-10.27	-1.742	-10.93
$Skew$					2.959	7.76	2.591	6.94	2.394	6.64
$Kurt$					-5.952	-10.11	-5.484	-9.22	-6.037	-9.87
$Coskew$					1.959	8.54	1.779	7.84	1.871	8.26
$Sentiment$									-2.337	-10.68
PIM									5.275	17.28

	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)	
Panel B. Long term MCS_{long}										
R_m	0.123	0.07	0.368	4.42	0.318	4.24	0.297	4.20	0.327	4.57
SMB	0.254	2.18	0.247	1.29	0.001	1.24	0.001	0.95	0.002	1.23
HML	-0.087	-10.31	0.002	9.67	0.005	9.95	0.008	10.02	0.005	9.53
MCS_{long}	-0.025***	-3.75	-0.034***	-5.21	-0.046***	-5.05	-0.053***	-4.88	-0.06***	-5.06
Mom			0.216	9.43	0.221	8.87	0.229	8.56	0.235	8.12
Rev			-0.583	-0.98	-0.603	0.45	-0.626	-0.21	-0.638	-0.68
Liq			-1.035	-9.68	-1.166	-9.88	-1.151	-9.54	-1.280	-10.14
Vol							-0.963	-10.84	-1.038	-11.57
$idioVol$							-1.618	-10.32	-1.756	-11.02
$Skew$					2.911	7.67	2.470	6.83	2.230	6.45
$Kurt$					-5.671	-9.71	-5.481	-9.26	-6.106	-9.96
$Coskew$					1.881	8.01	1.763	7.65	1.853	8.04
$Sentiment$									-2.399	-10.85
PIM									5.271	17.20

*significant at 10 % level; **significant at 5 % level; ***significant at 1 % level

3.4. Pricing of Correlation in Regime-Switching Market

The empirical results so far show that systematic correlation risk generally has a negative price. In order to find out the price of correlation risk in different regimes, we follow Rodriguez (2007)[?] to model dependence with switching-parameter copulas and expand to RS-copula with three regimes. Let ($R_{1,t}$ and $R_{2,t}$) denote asset return at t in regime $s_t = j$

$$f(R_{1,t}, R_{2,t}|I_{t-1}, s_t = j) = c_j(u_t, v_t|\psi_c^j)\prod_{i=1}^{2} f_i(R_{i,t}|I_{t-1};\psi_i) \quad , j = 0, 1, 2 \tag{11}$$

Then we use two-step max likelihood method EM for estimation. As the marginal distribution of asset return does not switch between regimes, so the log likelihood function is:$L(R\psi, \alpha) = \sum_{t=1}^{T} \log f(R_t|I_{t-1};\psi, \alpha)$ where $L(R\psi, \alpha)$ can be split into log likelihood of marginal distribution L_m and log likelihood of dependence function L_c.

$$\begin{aligned} L(R\psi, \alpha) &= L_m(R;\psi_m) + L_c(R;\psi_m, \psi_c) \\ L_m(R;\psi_m) &= \sum_{t=1}^{T}\sum_{i=1}^{2} \log f_i(R_{i,t}|I_{i,t-1};\psi_{m,i}) \\ L_c(R;\psi_m, \psi_c) &= \sum_{t=1}^{T} \log c(u_{t,1}|\psi_{m,1}, u_{t_2}|\psi_{m,2};\psi_c) \end{aligned} \tag{12}$$

We first estimate ψ_m in $\hat{\psi}_m = \arg_{\psi_m} \max L_m(R;\psi_m)$ using maximum likelihood estimation. Then we substitute $\hat{\psi_m}$ for ψ_m in $L_c(R;\psi_m, \psi_c)$ to calculate ψ_c as $\hat{\psi}_c = \arg_{\psi_c} \max L_c(R;\hat{\psi_m}, \psi_c)$

In Equation ??,we include regime-switching parameters in mixed vine copula expression, the calculated τ is used to construct RS correlation level risk and RS correlation shock risk. Table ?? lists the smoothing switching probability and the corresponding risk factor price. As the switching regimes only have influence on dependence relation which is reflected by the weights and Kendall correlation correlation of Gumbel Copula Frank Copula and Clayton Copula, the measurements of MC and MCS based on mixed copula estimation can presents prices of correlation risk in different regimes. We summarize the 6 month rolling averaged MC and MCS for short-term and long-term in Table ??.

Table 4. Averaged Pricing of Correlation Risk based on RS Copula

.

Factorγ	Averaged Pricing			Average Regime Duration(month)		
	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
MC_{short}	-0.9134***	-0.1546***	0.0651	5.64	16.57	1.45
	(-4.59)	(-3.30)	(0.69)			
MC_{long}	-0.1208	-0.1001	0.0942	1.49	1.50	1.52
	(-0.13)	(-0.11)	(0.97)			
MCS_{short}	-0.0258***	-0.0093*	-0.0070	16.17	1.09	2.03
	(-6.43)	(1.71)	(-0.89)			
MCS_{long}	-0.0081	-0.0153	0.0063***	1.57	1.55	13.42
	(-0.03)	(-0.38)	(2.98)			

In regime 1,the average price for MC_{short} is -0.9134 and in regime 2 the average price is -0.1546. Both of price for MC_{short} in regime 1 and regime 2 are significant at 1% level. Rather than negative price, the average price for MC_{short} in regime 3 is positive and insignificant(0.0651) which is obviously distinguished from other two regime. Regime 2 covers the post-financial crisis period after 2008 and 2015 in China and the mean duration of regime 2 in 16.57 months. The average price for MC_{long} in three regimes are -0.1208,-0.1001 and 0.0942 respectively and all of them are not significant statistically.

The transition probabilities across regimes are similar which cannot reject that the prices for MC_{long} in different regimes are indifferent. Figure ?? plots the smoothing transition probability for short-term and long-term correlation risk factor.

Table ?? also shows the RS pricing for correlation shock risk MCS. There are 16.17 months in average that among regime 1 when the short-term averaged price of -0.0256. In regime 2, the average price for MCS_{short} is -0.0093 while the significance drops from 1% level to 10%s level. The duration in regime 1 is longest among three regimes for short-term correlation shock risk factor, however, for long-term, MCS_{long} stays in regime 3 for longest time with 13.42 months in average and the price in that period is significant positive (0.0063). The price for MCS_{long} in regime 1 and regime 2 are -0.0081 and -0.0153, both of which are negative and insignificant.

3.5. Robustness Check

For robustness, we further use portfolio return as base asset returns to examine the price of correlation risk factor. We first construct 25 portfolios using beta to MC rank and market value rank; then we construct 25 portfolios by beta to MC rank and book-to-market ratio rank; we also group stocks by market value rank and book-to-market ratio rank and finally we use 29 SW first-level industry classification for 29 industry portfolio. We name the above four portfolio as correlation-value portfolio, correlation-style portfolio, value-style portfolio and industry portfolio. The portfolio return and risk factors are the value-weighted return and risk factor values of member stocks.

Controlling for all the mentioned risk factors as in Model (5) in table ?? and table ??, we investigate the MacBeth pricing of MC: for correlation-value portfolios, the price of MC_{short} is -1.62 which is significant at 5% level with t-value -2.54; for value-style portfolios, the MC_{short} price is 1% significant with γ of -9.52. The price of MC_{short} for value-style portfolios and industry portfolios are also negative but not as significant as for the other groups, the price γ are -1.778 and -3.600 and t-value are -1.32 and -1.25. For the long-term correlation level risk factor MC_{long}, its price is statistically significant and negative for four portfolios.

In order to check the robustness of correlation shock risk MCS price, we use the same way to form portfolios, the empirical result represents that price of MCS_{short} is -0.15 for correlation-value portfolios, -0.133 for correlation-style portfolios and -0.983 for industry portfolios. The prices are all significant at 5% level. The result for long-term correlation shock risk is similar to that for short-term. The MCS_{long} price is -0.02 for correlation-value portfolios and -0.11 for value-style portfolios, both of which are of 1% level of significance, while the prices of MCS_{long} for industry portfolios and correlation-style portfolios are significant at 5% level with -0.11 and -0.047.

In order to further check the robustness of MacBeth regression result, we reconstruct MC and MCS using factor mimicking portfolios. Theoretically, portfolio return can be interpreted as the linear regression of risk factor return as in Equation ?? where risk factor weights are $w_P = [w_1, w_2, ..., w_n]^T$ and risk factor exposure $x_P^m = w_P^T x_m = \sum w_i x_{i,m}$.

$$\begin{aligned} r_P &= x_P^1 f_1 + x_P^2 f_2 + ... + x_P^m f_m + w_P^T \mu \\ &= w_P^T x_1 f_1 + w_P^T x_2 f_2 + ... + w_P^T x_m f_m + w_P^T \mu \end{aligned} \tag{13}$$

When $x_P^T \mu = 0$, the portfolio is only related to one risk factor, with exposure 1 to this factor and exposure 0 to any other risk factor. This is the definition of pure risk factor portfolio $r_P = 1 \cdot f_m + w_P^T \mu$. We first specify the base assets return R_B and compute their exposure to target risk factor. Taking MC for example, we use the above construction procedure and get $MC_t = c_{B,t}(R_{B,t} - r_f) + e_t$ where $(R_{B,t} - r_f)$ is the excess return of base asset and $c_{B,t}(R_{B,t} - r_f)$ is MC in return formality. By summing up $\sum c_{B,t}(R_{B,t} - r_f)$, we can get factor-mimicking return $MimickMC$, so as $MimickMCS$.

The short-term correlation level risk $MimickMC_{short}$ has negative price when investigating portfolio return as sample data. The prices γ for correlation-value portfolios, correlation-style portfolios and value-style portfolios are -0.027,-0.017 and -0.024 and all of three are 1% level significant. Consistent with results of $MimickMC_{short}$, the prices of $MimickMC_{long}$ are negative and significant for all portfolios except value-style portfolios. The price of $MimickMCS_{short}$ and $MimickMCS_{long}$ are all negative and significant. To sum up, the MacBeth pricing result for both individual asset and portfolios and result for both original correlation measurement and factor mimicking portfolio show that the negative price of correlation risk in China is robust.

4. Transformation between systematic and idiosyncratic correlation

Our main objective in this section is to study the idiosyncratic correlation and try to find out the relationship between systematic correlation and idiosyncratic correlation.

4.1. Idiosyncratic Correlation Risk

We first give the definition of idiosyncratic correlation. According to capital asset pricing model, asset return $R_{i,t}$ can be separated into $R_{i,t} = \alpha_{m,i,t} + \eta_{i,t}$, where $\alpha_{m,i,t}$ denotes passive return and $\eta_{i,t}$ denotes active return. Assume an investment portfolio that constitutes of N assets $\{R_1, R_2, ..., R_N\}$ with weights $w_i i \in \{1, 2, ..., N\}$. Because $\alpha_{m,i,t}$ and $\eta_{i,t}$ are orthogonal, we have $Cov(\alpha_{m,i,t}, \eta_{i,t}) = 0$, thus the portfolio variance VAR_P is given by:

$$\begin{aligned} VAR_P &= Var(\alpha_{m,i,t}) + \sum_{i=1}^{N} w_{i,t} Var(\eta_{i,t}) \\ &= VAR_{sys} + VAR_{idio} \end{aligned} \tag{14}$$

Equation ?? breaks portfolio variance VAR_P into market-related variance risk VAR_{sys} and idiosyncratic firm-related variance risk VAR_{idio}. As for multi factor regression model, asset return R_i can be expressed by $R_i = \beta(R_m - r_f) + \epsilon_i$, and ϵ_i is the idiosyncratic return apart from systematic return. For simplicity, we assume N stocks in the market, so there are $N(N-1)/2$ stock pairs in total. we first model the dependence between idiosyncratic stock returns as:

$$c(\epsilon_{1,T}, ..., \epsilon_{N,T}) = \prod_{j=1}^{N-1} \prod_{i=1}^{N-j} c_{j,j+i|1,...,j-1}(F(\epsilon_{j,T}|\epsilon_{1,T}, ..., \epsilon_{j-1,T}), F(\epsilon_{j+i,T}|\epsilon_{1,T}, ..., \epsilon_{j-1,T})) \tag{15}$$

Similar to MC and MCS, we use GPD as the conditional tail distribution of $F^{\theta_i}(\epsilon_{i,T}|\cdot)$ and $F^{\theta_j}(\epsilon_{j,T}|\cdot)$. Then we compute their Kendall's correlation and relevant weight by estimation Equation ??:

$$F_{i,j}^{\theta_i,\theta_j} = \exp\left(-V(-1/\log\left(F_i^{\theta_i}(\epsilon_i)\right), -1/\log\left(F_j^{\theta_j}(\epsilon_j)\right)\right) \tag{16}$$

where V is the dependence function which is modelled by $c(\epsilon_{1,T}^{\theta_1}, ..., \epsilon_{N,T}^{\theta_N})$. The maximum likelihood of dependence during sample window T is:

$$L = \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^{T} \log\left[c_{j,j+i}(F(\epsilon_{j,T}^{\theta_j}), F(\epsilon_{j+i,T}^{\theta_{j+1}}))\right] \tag{17}$$

Through iteration, the averaged pair-wise Kendall's correlation derived from mixed vine copula parameters in Equation ?? is the idiosyncratic risk IC. The calculation of systematic correlation is the first-layer decomposition copula with R_m as the key vine. We first calculate $\phi = (p_1, p_2, \sigma_1, \sigma_2, k_1, k_2, w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3)$ of stock pair $\{\epsilon_i, \epsilon_j\}$ by EM algorithm and compute IC as the idiosyncratic correlation risk. The portfolio risk PC and systematic correlation SC are calculated in the same way with the raw return R_i and the market-related return $\beta(R_m - r_f)$.

4.2. Transformation Mechanism

Campbell[?] found out when overall market risk remains same, increasing averaged idiosyncratic risk would decrease the average market correlation. We referred to the approach by Kearney[?] to define the overall market risk VAR_{sys} as $VAR_{sys} = w_t' H_t w_t$, where $H_t = D_t \rho_{sys,t} D_t$ and D_t is the standard deviation matrix. The correlation coefficient matrix of $\beta(R_m - r_f)$ is $\rho_{sys,t}$. Consequently, VAR_{sys} can also be interpreted as $VAR_{sys} = D_t \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i,t} w_{j,t} \rho_{sys,ij,t} D_t$.

If we rewrite $\rho_{sys,t}$ as $\rho_{sys,t} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i,t} w_{j,t} \rho_{sys,ij,t}$, equation ?? is simplified as $VAR_{sys} = \frac{\rho_{sys,t}(i' D_t I D_t i)}{n}$. Assuming the portfolio is constructed by simple weighted average method, $\rho_{sys,t}$ is:

$$\begin{aligned}\rho_{sys,t} &= n\left(\frac{1}{n}\frac{i' D_t D_t i}{i' D_t D_t i} - \frac{VAR_{idio}}{i' D_t D_t i}\right) \\ &= 1 - \frac{VAR_{idio}}{VAR_P}\end{aligned} \tag{18}$$

Equation ?? demonstrates negative correlation between average correlation of market portfolio and the portion of idiosyncratic risk in systematic risk. When market portfolio consists of enough assets, the negative relationship still holds in spite of weighting method. In that case, we can decompose VAR_{idio} and VAR_P:

$$\begin{aligned}VAR_{idio} &= \frac{\rho_{idio,t}(i' D_{idio,t} I D_{idio,t} i)}{n} \\ VAR_P &= \frac{\rho_{P,t}(i' D_{P,t} I D_{P,t} i)}{n}\end{aligned}$$

where $\rho_{idio,t} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i,t} w_{j,t} \rho_{idio,ij,t}$ $\rho_{P,t} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i,t} w_{j,t} \rho_{ij,t}$
Let $\overline{\rho}_{idio,t} = \rho_{idio,t}/n$ $\overline{\rho}_{P,t} = \rho_{P,t}/n$ so equation ?? is:

$$\rho_{sys,t} = 1 - \frac{\overline{\rho}_{idio,t}}{\overline{\rho}_{P,t}} \times (i' D_{idio,t} I D_{idio,t} i)(i' D_{P,t} I D_{P,t} i) \tag{19}$$

From ??, there exists a transformation mechanism between portfolio aggregate correlation $\rho_{P,t}$, systematic correlation $\rho_{sys,t}$ and idiosyncratic correlation $\rho_{idio,t}$. We then relax the assumption of normal distribution and empirically analyse the relationship between the mixed vine copula based correlation risk: idiosyncratic risk IC, systematic correlation risk SC and portfolio aggregate risk PC.

4.3. Empirical Result

In this section, we firstly use hs300 index member stocks' return as sample data to construct idiosyncratic risk IC, systematic correlation risk SC and portfolio aggregate risk PC. In order to avoid over-fitting problem, we test the unit root of the above time series. The result is reported in table ?? and the ADF tests with interception and time-trend show that IC,SC and PC time series are stationary with lag 1 and lag6 for short-term and long-term. We also test unit root of indicator time series computed with base asets of hs300 industry index.

The following table ?? shows the OLS result of SC and IC/PC for hs300 index member stocks and hs300 style indexes. For hs300 index member stocks, we both measure SC and IC/PC by simple weighting method and value weighting method, and for hs300 we evaluate the result with and without standard deviation matrix between industry indexes. The empirical result confirms our assumption that the idiosyncratic correlation portion in portfolio correlation(IC/PC) is negatively related to systematic correlation SC. When the idiosyncratic correlation increases, the systematic correlation would decrease at 1% confidence no matter the indicators are for short-term and long-term and vice versa. The changes of IC/PC explains 91% changes of short-term SC and 86% changes of long-term SC. For industry index, the negative relationship is also of 1% significance.

Table 5. ADF Test for PC SC IC of hs300 Index Members and hs300 Industry Index

Variable	hs300 Index Members			hs300 Industry Index		
	DF	$ADF1$	$ADF6$	DF	$ADF1$	$ADF6$
short-term 5% tvalue=-3.145				5% tvalue=-3.448		
PC	-4.319	-4.037	-4.251	-5.878	-5.954	-6.214
SC	-3.942	-3.549	-3.703	-6.388	-6.556	-6.427
IC	-4.270	-3.724	-3.784	-5.843	-4.765	-6.094
long-term 5% tvalue=-3.451				5% tvalue=-3.036		
PC	-5.602	-6.984	-5.417	-5.131	-4.939	-5.027
SC	-6.339	-5.587	-7.138	-5.063	-4.998	-4.512
IC	-5.863	-4.877	-5.301	-5.047	-4.953	-4.709

Table 6. OLS Result of IC/PC SC IC for hs300 Index Member Stock and hs300 Industry Index

Variable	hs300 Index Member Stock		hs300 Industry Index	
	Simple-weighted	Value-weighted	Without std. matrix	with std. matrix
short-term				
$\beta_{IC/PC}$	-3.37	-3.46	-4.34	-3.16
t-value	-38.38	-15.98	-17.85	-11.59
R^2	0.91	0.834	0.724	0.79
long-term				
$\beta_{IC/PC}$	-3.69	-4.78	-6.88	-6.57
t-value	-30.55	-23.13	-39.14	-30.27
R^2	0.86	0.78	0.93	0.93

The transformation mechanism between idiosyncratic correlation and systematic correlation supported at stock-level and index-level in Chinese financial market. It is important for investors to monitor the idiosyncratic correlation changes during the market downturn for better diversification.

5. Conclusion

Correlation measures the dependence relationship between variables which is not linear in the real world. In this paper we firstly construct new correlation risk measurement considering the asymmetry of upside correlation and downside correlation using mixed vine copula and general Pareto distribution. Systematic correlation level risk and systematic correlation shock risk indicate the aggregate correlation level in the overall market and the unpredictable market downside event. We then examine the MacBeth price of these two types of correlation risk for short-term and long-term. The empirical result shows that the short-term correlation level risk is significantly and negatively priced while the long-term correlation level risk price is not of significance. This is because for long-term correlation, the negative effect is gradually digested by market participants so there is little effect for highly correlated stocks. However, the correlation shock risk has negative price in spite of the measurement window, which is result from the fact that in the market downturn with increasing systematic correlation, those stocks with high correlation with market return cannot efficiently diversify. The regime-switching result also supported the above empirical result that short-term correlation level deserves more attention for risk control through market regimes and there is no apparent difference for correlation shock risk in different regime. Then we investigate the transformation mechanism between idiosyncratic correlation and systematic correlation and find out that when idiosyncratic correlation drops, the systematic correlation is simultaneously increasing and vice versa. This empirical result implies that the acceleration of systematic correlation is the result of weakened idiosyncratic correlation.

The main contribution of this paper is measuring correlation in a more specific and realistic way, which is important for investors to maximize diversification benefit during market downturn. Furthermore, the asymmetric correlation provides another evaluation method of stocks in different market conditions so we can leverage the benefit from bull market for the control of return retracement in bear market. Finally, the idiosyncratic reflects the changes of systematic correlation and we would further look into its effect as a forward-looking indicator.

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