**Key Words:** Fit Indices; Structural Equation Modeling; Bernoulli Digits; Latent Constructs; Educational Performance

**1. Introduction**

Fit refers to the ability of a model to reproduce the data (i.e., usually the variance-covariance matrix).  A good fitting model is one that is reasonably consistent with the data and so does not require respecification and also its measurement model is required before estimating paths in a structural model [2].

[3], [4], and others distinguish between several types of fit indices: *absolute fit indices*, *relative fit indices*, *parsimony fit indices*, and those based on the *noncentrality* parameter.

There are several fit indices that fall into the category of *absolute indices*, including the Goodness-of-fit index (GFI), the adjusted goodness of fit index (AGFI), ratio, Hoelter’s CN (“critical N”), Akaike’s Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Expected Cross-validation Index (ECVI), the root mean square residual (RMR), and the standardized root mean square residual (SRMR).

*Relative fit indices* compare a chi-square for the model tested to one from a so-called *null model* (also called a “baseline” model or “independence” model). There are several *relative fit indices*, including Bollen’s Incremental Fit Index (IFI), the Tucker-Lewis Index (TLI), Bentler-Bonett Nonnormed Fit Index (BBNFI), and the Bentler-Bonett Normed Fit Index (NFI).

A number of *parsimonious fit indices* was developed (which are adjustments of most of the relative fit indices) include PGFI (based on the GFI), PNFI (based on the NFI), PNFI2 (based on Bollen’s IFI), PCFI (based on the CFI mentioned below).

*Noncentrality*-based indices include the Root Mean Square Error of Approximation (RMSEA), Bentler’s Comparative Fit Index (CFI), McDonald and Marsh’s Relative Noncentrality Index (RNI), and McDonald’s Centrality Index (CI).

Considerable controversy has flared up concerning fit indices recently.  Some researchers do not believe that fit indices add anything to the analysis (e.g., [5]) and only the chi square should be interpreted.  The worry is that fit indices allow researchers to claim that a mis-specified model is not a bad model.  Others (e.g., [6]) argue that cutoffs for a fit index can be misleading and subject to misuse.  Most analysts believe in the value of fit indices, but caution against strict reliance on cutoffs.

Also problematic is the “cherry picking” a fit index.  That is, computing a many fit indices and picking the one index that allows you to make the point that you want to make.  If you decide not to report a popular index (e.g., the TLI or the RMSEA), you need to give a good reason why you are not.

[7] has also argued that fit indices should not even be computed for small degrees of freedom models.  Rather for these models, the researcher should locate the source of specification error.

SEM scholars distinguish two classes of fit indices: those that reflect “absolute” fit, and those that reflect a model's “incremental” fit, or the fit of one model relative to another. Absolute indicators of model fit include **** and SRMR, among others. Incremental fit statistics include CFI, among others. However, [8] distinguish two classes of fit indices into large fit indices (*NFI, NNFI, CFI, GFI, PGFI,* *AGFI, PNFI* and *IFI*) and in this paper, we shall consider the fit indices such as *AIC, CAIC, RMR, SRMR, RMSEA* and****/dfwith small values considered indicators of good fit to educational performance model with adult mathematics learners as our subjects. Here are their definitions and basic behavioral properties.

Table 1: Equations of some fit indices and their authors

|  |  |  |
| --- | --- | --- |
| Fit Indices | Equations | Authors |
| Root Mean Square Residual (RMR) | $$RMR=\sqrt{\frac{\sum\_{i=1}^{p}\sum\_{j=1}^{i}\left(s\_{ij}-\hat{σ}\_{ij}\right)^{2}}{{k\left(k+1\right)}/{2}}}$$ | Browne *et al.,* 2001 |
| Standardized Root Mean Square Residual (SRMR) | $$SRMR=\sqrt{\frac{\sum\_{i=1}^{p}\sum\_{j=1}^{i}\left[{\left(s\_{ij}-\hat{σ}\_{ij}\right)}/{\left(s\_{ii}s\_{jj}\right)}\right]^{2}}{{k\left(k+1\right)}/{2}}}$$ | Hu and Bentler, 1999 |
| Chi Square () | $$χ^{2}=\left\{N\left[tr\left(SΣ^{-1}\right)+log\left|Σ\right|-log\left|S\right|-\left(p+q\right)\right]\right\}$$ | Gerbing and Anderson, 1992 |
| Root Mean Square Error of Approximation (RMSEA) | $$RMSEA=\sqrt{{\left(χ^{2}-df\right)}/{df\left(N-1\right)}}$$ | Kenny and McCoach, 2003 |

**2. Models**

Let \*pqrs denotes a baseline model of four constructs together with a combination of none, one, two, three or four additional constructs; where \* indicates the latent variables: educational performance, socio-economic label, self concept and parental authority. The variables p, q, r, s denote Bernoulli or dichotomous digits 0 (if excluded) or 1 (if included) for each additional construct, that is

;

;

; and

.

Note that: CIRCUM represents circumstances;

 TRAINENV represents training environment;

 HEALT represents health characteristic; and

 SEC represents socio-economic characteristic.

We shall consider some 16 progressively nested models using the data from model sample as enumerated in Table 2. It varies from the baseline model \*0000 to the ultimate model \*1111.

 Table 2: Coding for Models by included Latent Constructs

|  |  |
| --- | --- |
| **Code Name** | **Latent Constructs** |
| **\*0000** | educational performance, socio-economic label, self concept and parental authority |
| **\*1000** | educational performance, socio-economic label, self concept, parental authority and circumstances |
| **\*0100** | educational performance, socio-economic label, self concept, parental authority and training environment |
| **\*0010** | educational performance, socio-economic label, self concept, parental authority and health characteristic. |
| **\*0001** | educational performance, socio-economic label, self concept, parental authority and socio-economic characteristic. |
| **\*1100** | educational performance, socio-economic label, self concept, parental authority, circumstances and training environment |
| **\*1010** | educational performance, socio-economic label, self concept, parental authority, circumstances and health characteristic. |
| **\*1001** | educational performance, socio-economic label, self concept, parental authority, circumstances and socio-economic characteristic. |
| **\*0110** | educational performance, socio-economic label, self concept, parental authority, training environment and health characteristic. |
| **\*0101** | educational performance, socio-economic label, self concept, parental authority, training environment and socio-economic characteristic. |
| **\*0011** | educational performance, socio-economic label, self concept, parental authority, health characteristic and socio-economic characteristic. |
| **\*1110** | educational performance, socio-economic label, self concept, parental authority, circumstances, training environment and health characteristics. |
| **\*1101** | educational performance, socio-economic label, self concept, parental authority, circumstances, training environment, and socio-economic characteristic. |
| **\*1011** | educational performance, socio-economic label, self concept, parental authority circumstances, health characteristic and socio-economic characteristic. |
| **\*0111** | educational performance, socio-economic label, self concept, parental authority, training environment, health characteristic and socio-economic characteristic. |
| **\*1111** | educational performance, socio-economic label, self concept, parental authority, circumstances, training environment, health characteristic and socio-economic characteristic. |

 **3. Goodness-of-Fit Statistics on Modeling Sample**

Having considered some 16 progressively nested models starting with model \*0000 using the data from the modeling sample, we shall now employ some fit indexes which are commonly used in the literature (such as , *GFI*, *AGFI, NNFI, CFI, RMSR, RMSEA*, among others) to test the fitness of the model.

As the values in Table 3 reveal, the fit indexes of the models are included in the values which are acknowledged in the literature [1]. The commonly used measures of model fit, based on results from analysis of the structural model, are summarized in Table 3. In practice, model AIC, sat. AIC, model CAIC, RMR and ****are indicative of small values, and SRMR has less than 0.1, RMSEA has less than 0.06 or 0.08 and Chi-square/degree of freedom has less than 3.00 for good fit.

From Table 3, models \*0100 (with value 262.89), \*0110 (with value 376.02) and \*0111 (with value 633.63) have smaller values compared with other competing models for model AIC. Models \*0100 (with value 156), \*0010 (with value 156), \*0110 (with value 210) and \*0111 (with value 342) have smaller values compared with other competing models for saturated AIC. Moreso, models \*0100 (with value 458.98), \*0110 (with value 607.76) and \*0111 (with value 924.80) have smaller values compared with other competing models for model CAIC. Models \*0100 (with value 619.49), \*0010 (with value 619.49), \*0110 (with value 833.93) and \*0111 (with value 1358.11) have smaller values compared with other competing models for saturated CAIC. Furthermore, models \*1000 (with value 1.82), \*1001 (with value 1.47), \*1101 (with value 1.34) and \*1011 (with value 1.34) have smaller values compared with other competing models for RMR. Models \*0100 (with value 0.045), \*1100 (with value 0.046) and \*1101 (with value 0.050) have smaller values less 0.1 compared with other competing models for SRMR. In addition, models \*1000 (with value 0.055), \*1100 (with value 0.050) and \*1101 (with value 0.049) have smaller values less than 0.06 compared with other competing models for RMSEA. Models \*0100 (with value 196.89), \*0110 (with value 298.02) and \*0111 (with value 535.63) have smaller values for **** compared with other competing models. Finally, models \*1000 (with value 4.16), \*1100 (with value 3.59) and \*1101 (with value 3.51) have smaller values compared with other competing models for **/** Df.

Table 3: Summary Statistics of Small Type Fit Indices on Modeling Sample

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Fit Index** | **Model****AIC** | **Sat.****AIC** | **Model****CAIC** | **Sat.****CAIC** | **RMR** | **SRMR** | **RMSEA** |  | **/DF** |
| **Ideal Value** | **Small****Value** | **Small****Value** | **Small****Value** | **Small****Value** | **Small****value** | **0.10** | **0.08** | **Small****Value** | **3.00** |
|  **Model \*0000** | 221.32 | 110 | 369.87 | 436.82 | 2.82 | 0.051 | 0.067 | 171.32 | 5.71 |
| **Model \*1000** | 477.44 | 272 | 721.07 | 1080.13 | 1.82+ | 0.048 | 0.055+ | 395.44 |  4.16+ |
| **Model \*0100** | 262.89+ | 156+ | 458.98+ | 619.49+ | 2.37 | 0.045+ | 0.057 | 196.89+ | 4.38 |
| **Model \*0010** | 299.43 | 156+ | 495.52 | 619.49+ | 2.37 | 0.051 | 0.064 | 233.43 | 5.19 |
| **Model \*0001** | 541.04 | 210 | 754.96 | 833.93 | 2.08 | 0.062 | 0.075 | 469.04 | 6.70 |
| **Model \*1100** | 537.24 | 342 | 822.47 | 1358.11 | 1.62 | 0.046+ | 0.050+ | 441.24 |  3.59+ |
| **Model \*1010** | 619.82 | 342 | 899.10 | 1358.11 | 1.63 | 0.054 | 0.056 | 525.82 | 4.24 |
| **Model \*1001** | 742.58 | 420 | 1039.69 | 1667.85 | 1.47+ | 0.054 | 0.054 | 642.58 | 4.02 |
| **Model \*0110** | 376.02+ | 210+ | 607.76+ | 833.93+ | 2.04 | 0.051 | 0.058 | 298.02+ | 4.52 |
| **Model \*0101** | 502.42 | 272 | 751.99 | 1080.13 | 1.80 | 0.056 | 0.058 | 418.42 | 4.45 |
| **Model \*0011** | 544.46 | 272 | 794.03 | 1080.13 | 1.80 | 0.058 | 0.061 | 460.46 | 4.90 |
| **Model \*1110** | 724.70 | 420 | 1033.69 | 1667.85 | 1.50 | 0.054 | 0.053 | 620.70 | 3.93 |
| **Model \*1101** | 801.81 | 506 | 1140.51 | 2009.37 | 1.34+ | 0.050+ | 0.049+ | 687.81 |  3.51+ |
| **Model \*1011** | 891.43 | 506 | 1230.14 | 2009.37 | 1.34+ | 0.055 | 0.054 | 777.43 |  3.97 |
| **Model \*0111** | 633.63+ | 342+ | 924.80+ | 1358.11+ | 1.60 | 0.056 | 0.057 | 535.63+ |  4.39 |
| **Model \*1111** | 1005.2 | 600 | 1385.53 | 2382.65 | 1.23 | 0.055 | 0.051 | 877.23 |  3.72 |

*“+” indication of good fit model with some class of models*

where

Model AIC - Model Akaike Information Criterion

Model CAIC - Model Consistent Akaike Information Criterion

Sat. AIC - Saturated Akaike Information Criterion

Sat. CAIC - Saturated Consistent Akaike Information Criterion

RMR - Root Mean Square Residual

SRMR - Standardized Root Mean Square Residual

RMSEA - Root Mean Square Error of Approximation

 ** -**  Chi-square

Chi-square / degree of freedom

**4. Conclusion**

The study considered some 16 progressively nested models for educational performance on small type fit indices of mathematics adult learners.

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