**A PROPOSED SECOND–ORDER FOLDOVER RESOLUTION DESIGN**

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**ABSTRACT**

Interest in the type of effect to be estimated accounts for the creation of several resolution designs. A resolution design can be created using the sign table or the defining contrast and establishing the aliases structure. This is time consuming and may be prone to error especially if a large number of factors is involved. Foldover designs are expected to provide a platform where a higher-order resolution can be obtained from a lower-order resolution. However the existing method of constructing fold over designs only permits the creation of a foldover from one resolution to the immediate next higher resolution. In this paper, a proposed foldover design that projects a lower-order resolution to the next is provided and should be called 1st-order foldover design. Also proposed is a 2nd-order fold over design where a higher-order Resolution design, say RessolutionT+2 is obtained from a ResolutionT (lower-order resolution) design.

**Keywords:** Resolution design, fractional factorial design, generalized interaction, aliases, generator.

**1.0 INTRODUCTION**

A foldover design is a resolution design of a higher-order obtained from a lower-order resolution design. A resolution design is a $2^{K-P}$ Fractional Factorial Design. These are fractions of $2^{K}$ Factorials designs created by specifying appropriate generators or defining relations. Fractional Factorial Designs were created to solve the problem of handling volumes of experimental runs often associated with $2^{K}$ factorials designs. Using a full factorial design involves difficulty in interpreting high-order interactions. The principle of parsity of effects (Hamada & Wu, 1992; Kutner, Nachtsheim, Neter & Li, 2004; Wu & Hamada, 2000) is another reason for the adoption of Fractional Factorial Designs. The theory of fractional factorial designs was developed originally by Finney (1945). Montgomery (1978) provided a thorough X-ray of the Fractional Factorial Designs together with Resolution Dessigns. Other contributors are Box, Hunter and Hunter (2005), Mukerje and Wu (2006). Kessels, Goos and Vandebroek (2006) have developed statistical packages like Minitab, Statistica, SYSTAT for the creation of Fractional Factorial Designs. Resolutions I and II designs are not useful because no effect can be independently estimated. At resolution III, main effects can be estimated but they aliase with 2-factor interactions. Resolution IV designs are useful because main effects can be estimated and are not confounded with any two-factor interactions even though 2-factor interactions are confounded with each other. A consequence of fractional factorial design is the introduction of aliases for factorial effects. Complete Foldover designs were created designed to de- aliase factorial effects in two-level designs by reversing the signs of one or two factors. Many authors have extensively discussed this. These include: Box, G.E.P., Hunter, W.G., Hunter, J.S. (1978), Montgomery (2001), Montgomery and Runger (1996), Neter, J; Kutner, M.H; Nachitsheim, C.J; Wasserman, W. (1996), Wu and Hamada (2000) developed a general decomposition structure of the foldover plan where they used general regular s-level fractional factorial designs instead of two-level designs. Li and Mee (2002) proposed an alternative foldover design from Resolution III to resolution IV such that the inability of estimating sufficient number of two-factor interactions due to a reduction of degree of freedom is overcome. Foldover designs obtained by reversing all factors provide fewer than half the degrees of freedom used for estimating two-factor interactions. Jacroux, M and Kealy-Dichone, B. (2013) developed optimal foldover plans for regular fractional factorial split-plot designs as an improvement of the minimum aberration criterion for both the initial design and the combined design. Defining a foldover plan as a collection of columns whose signs are to be reversed in the foldover design, William and Dennis (2003) developed optimal foldover plans for commonly used fractional factorial designs by using the aberration of the combined design where a combined design refers to the combination of the initial design and its foldover. To determine whether a particular fraction of a factorial design is of any resolution, use the aliases structure. For a resolution III design, main effects are not aliased with each other. They are rather aliased with 2-factor interactions. For instance, a $2^{3-1} A×B×C $ design has the following aliases:

$$A=A.ABC=A^{2}BC=BC$$

$$B=B.ABC=AB^{2}C=AC$$

$$A=C.ABC=ABC^{2}=AB$$

If 2-factor interactions are of interest, this design cannot be used. This is because the main effect cannot be (explicitly) estimated without considering the 2-factor interactions negligible. For a Resolution IV design, main effects are not aliased with each other or 2-factor interaction effects, two – factor interactions are aliased with each other, main effects are aliased with 3-factor interactions. For example, a $2^{4-1} A×B×C×D$ design has the following 2-factor confounds:

$$AB=AB.ABCD=A^{2}B^{2}CD=CD$$

$$AC=AC.ABCD=A^{2}BC^{2}D=BD$$

etc. In this design, main effects can be estimated by assuming that 3-factor interactions are negligible. Thus 2-factor interactions can also be estimated. A Resolution V design allows for the estimation of main effects and two-factor interactions such that particular interactions are not confounded with each other. $2^{K-P}$ fractional factorial designs are known as Box-Hunter designs. Designs for which only main effects are estimated are called saturated main effects designs or placket-Burman designs (Ledolter and Swersey, 2007).

A $2^{6-2}$ design with generators

$I\_{1}=ABCE, and I\_{2}=ACDF$ has the generalized interaction $I\_{3}=I\_{1}×I\_{2}$ mod 2

$$=A^{2}BC^{2}DEF=BDEF$$

The aliases structure is as follows:

$A=A.ABCE$ mod 2 $=A^{2}BCE=BCE$

$A=A.ACDF$ mod 2 $=A^{2}CDF=CDF$

$A=A.BDEF=ABDEF$

Thus $A$ is confounded with $BCE, CDF, ABDEF.$ Estimating $A$ effect means estimating $(A+BCE+CDF+ABDEF)$ effect.

Similarly, $B$ is confounded with $ACE, DEF$ and $ABCDF$, $C$ is confounded with $ABE, ADF$ and $BCDEF$. The two-factor interaction $BD$ for instance is confounded with $EF, ACDE$ and $ABCE$ while $ABF$ is confounded with $CEF, BCD$ and $ADE$

**2.0 RESOLUTION IV DESIGNS**

A $2^{K-P}$ fractional factorial Design is of resolution IV if main effects are clear of two-factor interactions and some two factor interactions are aliased (confounded) with each other. (Main effects are also confounded with 3-factor interactions). These designs are of interest if when an experimenter seeks to determine if 2-factor interactions are important at all. For instance the design $2\_{IV}^{4-1}$ with defining relation $I=ABCD$ is a Resolution IV design. By using the defining contrast

$L=α\_{1}X\_{1}+α\_{2}X\_{2}+α\_{3}X\_{3}+α\_{4}X\_{4}$

with $α\_{1}=α\_{2}=α\_{3}=α\_{4}=1$

we obtain $2^{4-1}$ runs as $\left[\left(1\right), ad, bd, ab, cd, ac, bc, abcd\right]$

The other fraction (with $I=-ABCD)$ is

$\left[a,b,c,d,abc,abd,acd,bcd\right]$

The aliases structure for ABCD is obtained as follows:

$A=A.I=A.ABCD=A^{2}BCD=BCD$

So that estimating $A$ effects means estimating $A+BCD$ effect.

Similarly, $B=ACD, C=ABD, D=ABC$ and $AB=CD, AC=BD, BC=AD$

Considering a $2\_{IV}^{6-2}$ design with generators $I\_{1}=ABCE$, $I\_{2}=ACDF.$ The generator $I\_{1}=ABCE$ applied on $2^{6}A×B×C×D×E×F$ yields $2^{6-1}$ fractions as follows:

$\left[\begin{array}{c}\left(1\right), d,f,ab, ac,bc,df,ae,be,ce,abd,acd,bcd,aef,bef,cef,ade,bde,cde,abf,acf,bcf,abce,adef,\\cdef,bdef, abdf, acde, bcdf, abcdef,abade, abcdef \end{array}\right]$

and

$\left[\begin{array}{c}a,b,c,e,ad,bd,cd,de,af,bf,cf,ef,abc,def,bce,bdf,cdf,bcd,abe,adf,abcd,bcef,abde,acde,bcde,\\abcf,abcdf,abef,acef,abdef,acdef,bcdef\end{array}\right]$.You can take three (3) lines in each bracket.

Applying $I\_{2}=ACDF$ on the first fraction (first block) yields the two fractions of $2^{6-2}$ runs each as follows:

$\left[\left(1\right),ac,df,be,abd,bcd,ade,cde,abf,bcf,aef,cef,abce,bdef,acdf,abcdef\right]$

$\left[d,f,ab,bc,ae,ce,acd,bde,acf,bef,adef,cdef,abdf,acde,bcdf,abcde\right] $

**3.0 FOLDOVER DESIGNS**

A resolution IV design can be obtained from a resolution III design through the process of fold over. When this happens, the number of factors increases by one and the number of runs is doubled. For example $2\_{IV}^{K+1-P}$ (resolution IV) design can be obtained from $2\_{III}^{K-P}$ (resolution III) design.

$2\_{IV}^{K+1-P}$ is a resolution IV design with (K+1) factors, each at two levels and $2^{P}$ fractions. A$ 2\_{IV}^{4-1}$ design with $I=ABCD$ can be obtained from a $2\_{III}^{3-1}$ with $I=ABC$ as follows:

**Table 1:** $2\_{IV}^{4-1} $Design Obtained by Fold Over

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | I | A | B | C |
|  | I | D | A | B | C |
| Cd | + | + | - | - | + |
| Ad | + | + | + | - | - |
|  Bd | + | + | - | + | - |
| Abcd | + | + | + | + | + |
| Ab | + | - | + | + | - |
| Bc | + | - | - | + | + |
| Ac | + | - | + | - | + |
| (1) | + | - | - | - | - |

Principal block (fraction) = $\left[\begin{array}{c}(1)\\ab\\bc\\ac\\cd\\ad\\bd\\abcd\end{array}\right]$

To free main effects from 2-factor interactions, we foldover from Resolution III to Resolution IV

**Proposed Method**

1. Take any fraction of the $2\_{III}^{3-1}$ design
2. Multiply each of the effects by the extra main effect of the $2^{4}$ design
3. Obtain the complements of the combinations (runs) arising from step 2.
4. Merge the combinations (runs) of steps 2 & 3.

The result is a resolution IV ($2\_{IV}^{4-1})$ design.

**Example**

Starting with $2\_{III}^{3-1} A×B×C$ design, we can obtain a resolution IV design ($2\_{IV}^{4-1} A×B×C×D)$ as follows:

1. One fraction of $2\_{III}^{3-1} A×B×C$ is

Block 1 = $\left[\begin{array}{c}a\\b\\c\\abc\end{array}\right]$

1. Multiply each element of block 1 by $d$ (the remaining main effect of $2^{4} A×B×C×D)$ to obtain

Block 2 = $\left[\begin{array}{c}ad\\bd\\cd\\abcd\end{array}\right]$

1. Compliments of block 2 yield block 3 as

Block 3 = $\left[\begin{array}{c}bc\\ac\\ab\\(1)\end{array}\right]$

1. Add blocks 2 and 3 to obtain the $\frac{1}{2}$ fraction of a $2\_{IV}^{4-1}$ design.

**4.0 A PROPOSED 2ND-ORDER FOLDOVER DESIGN OF RESOLUTION V**

In this paper, a 2nd-order fold over resolution design is proposed by post multiplying the extra main effects of the fire word generator with the treatment combinations of the original resolution with respect to its generator. This is repeated with the original resolution complement.

For instance, a resolution IV $(2\_{IV}^{K+2-P})$ design can be obtained from $2\_{III}^{K-P}$ by the proposed 1st-order fold over design, and a resolution V $(2\_{V}^{K+2-P})$ design can be obtained from $2\_{III}^{K-P}$ by the proposed 2nd-order fold over design.

Example 1:

A $2\_{III}^{3-1}$ design with $I=ABC$ yields the block (fraction) $B\_{1}=\left[a,b,c,abc\right], with compliment B\_{1}^{c}=\left[bc,ac,ab,(1)\right]$

A $2\_{V}^{5-1}$ design with $I=ABCDE$ has $A$ and $E$ extra main effects.

Post multiplication of $B\_{1}$ with $de$ yields $B\_{2}=\left[ade, bde, cde,abcde\right]$ while the post multiplication of $B\_{1}^{c}$ with $e$ yields

$$B\_{3}=\left[bce, ace, abe,e\right]$$

$$B\_{2}∪B\_{3}=\left[ade, bde, cde,abcde,bce,ace,abe,e\right]$$

$$\left(B\_{2}∪B\_{3}\right)^{c}=\left[bc,ac,ab,\left(1\right),ad,bd,cd,abcd\right]$$

The resulting resolution V design, $2\_{V}^{5-1}$ with $I=ABCDE$ is

$$B\_{4}=(B\_{2}∪B\_{3})∪\left(B\_{2}+B\_{3}\right)^{c}$$

$$=\left[e,bc,ac,ab,bd,cd,ade,bde,cde,bce,ace,abe,\left(1\right),abcd,abcde\right]$$

**5.0 MEAN SQUARE ERROR CALCULATION**

A full replicate of a $2^{K}$ design has $K$ main effects, $\left(\begin{array}{c}K\\2\end{array}\right)$ 2-factor interactions, $\left(\begin{array}{c}K\\3\end{array}\right)$ 3-factor interactions… $\left(\begin{array}{c}K\\b\end{array}\right)$ b-factor interactions, and $K-$factor interactions to be estimated. Thus a $2^{4-1}$ design with $I=ABCD$ has 4 d.f. for main effects (one for each)

 3 d.f. for 2-factor interaction aliases (one for each alias)

Arising from the degrees of freedom above, ANOVA cannot be conducted for a single replicate of $2^{K-1}$ designs because there is no degree of freedom left for the error. The whole degrees of freedom (7) have all been used by the main effects and the interactions.

Several solutions have been considered as follows:

(i) Some higher-order interaction effects are assumed insignificant to pave way for the calculation of residual error variance.

(ii) The use of the normal probability plot is also advised in deciding which higher-order interactions should be neglected.

In this paper, variances of treatment effects are used to identify insignificant higher-order interactions. Interactions with high treatment variances are ignored.

**Example:** Consider the following treatment combination values for a $2^{4-1}$ design with $I=ABCD$

|  |  |  |  |
| --- | --- | --- | --- |
| Treatment Combination | Response | Treatment Combination | Response |
| (1) | 52 | A | 120 |
| Ad | 80 | B | 98 |
| Bd | 76 | C | 107 |
| Ab | 98 | D | 82 |
| Cd | 30 | Abc | 63 |
| Ac | 73 | Adc | 44 |
| Bc | 69 | Abd | 50 |
| abcd | 103 | Bcd | 83 |

The sums of squares are computed as $SS=^{\left[contrast\right]^{2}}/\_{N}$ where $N=2^{4-1}=8$

$SS\_{AB}=\frac{1}{8}\left[ab-ac-bc-ad-bd+cd+abcd+(1)\right]^{2}$

$$=28.125$$

$SS\_{BC}=91.125$, $SS\_{A}=1,081.125$, $SS\_{B}=435.125$, $SS\_{C}=351.125$, $SS\_{D}=2,080.125$

$$SS\_{ABC}=\frac{1}{8}(abc+a+b+c-d-adc-abd-bcd) $$

$$=1,540.125$$

$$SS\_{ACD}=66.125$$

Thus we can neglect 2-factor interactions and use their degrees of freedom to estimate the residual error variance (EMS).

Also negligible is the $ACD$ interaction effect. Thus we can estimate $B$ effect independently of the $ACD$ effect.

**6.0 CONCLUSION**

The proposed foldover design demonstrates clear advantages of easy transition and ease of computation over the existing method. A new design structure called second-order foldover design is also proposed in this paper. It is innovative and has potentials for projection to the 3rd-order and higher-order Resolution designs.

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