# Applied Mathematical Theory of Governance

—Impact of Group Size To Governance—

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This paper analyzes governance and enforcement mechanisms $F(X, Y, T)$ for different group sizes $T$. The European sovereign debt crisis has demonstrated the need of efficient governance for different group sizes. I find that self-governance only works for sufficiently homogenous and small neighbourhoods $T \leq T^*$. Second, as long as the union expands $T > T^*$, the effect of credible self-governance decreases. Third, spill-over effects $\zeta$ amplify the size effect. Fourth, I show that sufficiently large monetary unions, $T > T^{**}$, are better off with costly but external governance or a free market mechanism. Finally, intermediate-size unions $T^* \leq T < T^{**}$ are most difficult to govern efficiently.

*JEL classification*: E61, E62, E66

*Keywords:* Economic Governance, Group Size, European Monetary Union

## 1 Introduction

Since the onset of the European Monetary Union (EMU) in 1999, there is no doubt about the need of efficient fiscal and economic governance. Already in 1997, European policy-makers legislated the European 'Stability and Growth Pact' (SGP) to eliminate unsustainable public finances. The SGP is a 'rule-based' approach and can be seen as an internal governance mechanism. However, the increasing public deficit levels in the eurozone, without sufficient correction and rule enforcement, demonstrates the failure of the European economic governance framework. In fact, the SGP went through cycles of ups and downs. In 2005, the pact was modified towards more flexibility. In 2012, after the financial and European crisis, the pact was strengthened and supplemented with new elements such as the Euro-Plus-Pact and the fiscal compact. These changes demonstrate that policy is still looking for the optimal degree of fiscal and economic governance in the EMU.

Why is governance so difficult in the eurozone? How does governance change with group size? What happens at the border between self- and external governance? In this paper, I approach this topic with a tractable analytical model in line with Dixit [1, 2]. However, my model is different in two respects and studies the impact of governance for the eurozone. In fact, governance is a mixed blessing and it works differently in different environments. Hence, I first study the determinants and limits of self-governance. In doing so, I find unprecedented answers to current debates in the eurozone. The paper extends the literature in two respects: First, I include area-spill-over effects in a mathematical model. Second, I extend the standard model with

an external enforcement or governance mechanism. This model captures the specific constellation in the EMU.
The paper is structured as follows. Section 2 is a brief literature review. In section 3, I develop the basic model and discuss benchmark results. Thereafter, I consider external governance in Section 4. Finally, section 5 provides concluding remarks.

## 2 Literature Review

A reference point in this paper is the analysis of regulatory systems [3, 4]. However, I approach the problem with a different analytical model. In my model, I describe the evolution of cooperation in groups where pairs of countries/agents meet at random without direct reciprocity. Then, I examine whether and how the information networks can work and sustain a cooperative outcome. I find, in line with Nobel Laureate Ostrom (1990), that a small size, reciprocity, repeated interactions, and embeddedness are important conditions for a successful resolution of collective problems [5]. This finding is of great importance for the eurozone, too. However, interestingly, I also find that large country groups can overcome these problems with special arrangements, such as hierarchies across smaller groups or external governance schemes.
There is substantial literature about self-governance by Kandori [6] and Ellison [7]. Both authors developed theoretical models. Similarly, Ostrom [8] and Ellickson [9] present empirical evidence via case studies in line with theory. Later, Greif [10, 11] confirmed the first research findings. Today, the economic literature agrees that efficient governance requires the dissemination of information and the credibility of punishment. However, both is only possible in small and rather close groups.
Greif [12] added a new point to this literature by showing that in case of two groups, there are different internal and external enforcement schemes. And the choice of the system relies on the size of the group. Li [13] argues that self-enforcing governance is dependent on group size because more group members create rising marginal costs. This is in contrast to the opposite view that governance has only fixed costs of setting up the legal scheme, but once these costs have been incurred, the marginal costs are low. Consequently, relation-based governance becomes more efficient for large groups. But it is still unclear what happens when there are strong spill-over effects, such as in the eurozone.

## 3 Basic Model

Consider a continuum of agents, each of them acting in a different country. I assume that they are uniformly distributed along a circle of circumference $2T$, where $T$ is labelled as a critical threshold. The mass of agents per unit length of arc is normalized to one. The distance between two agents is measured as the shorter of two arc length

from the initial position (Figure 1). The circle stands for the spectrum of any relevant difference. Thus, the difference among agents and countries is defined for several dimensions, such as resources, technology, geography, culture, economy and policy.
Each agent or country is randomly matched with another in two separate time periods. The first period representing the present and the second the future. In period 1, the agents have to choose whether they are complying (be honest) or whether they behave incompliant, i.e. cheat or breach the existing rules. The decision in period 1 either leads to rewards or punishments in period 2. The matches are independent across time. This creates the need for a compliance/governance mechanism in order to detect the behaviour of others in the community. We assume that it is more likely to cooperate with a closer neighbour than with a more distant one, $X$, with decreasing exponential probability $p$ as

$$\frac{e^{-\alpha X}}{\frac{2}{\alpha}[1-e^{-\alpha T}]}, \tag{1}$$

where $\alpha > 0$ indicates that the probability declines with distance. This idea is in line with empirical studies about the 'gravity law' in trade theory. The mathematical intuition behind the level of $\alpha$ is that with more distant agents, cooperation is less likely. But if $\alpha$ tends to 0, I obtain a uniform distribution over the whole circle. Therefore, $\alpha$ captures the matching technology. The $\alpha$ can also be viewed as a time constraint because in limited time an agent selects others that have similarities with himself. The denominator is a normalizing factor to ensure that the probabilities of matches at all distances are between 0 and $T$ on either side and sum up to 1.
The payoffs from cooperation at distance $x$ are proportional to $e^{\theta X}$. Notice that I will define $\lambda := \alpha - \theta > 0$. The parameter $\theta > 0$, captures the benefits from expanding the scope of (honest) cooperation across countries. The intuition of that assumption becomes clear if you consider the benefits of the provision of public goods. For instance, the Euro currency is a pubic good for all member countries in the EMU. However, member countries are different in respect of GDP growth rates, debt and deficit levels or the size of government [14]. The current debate about closer economic and fiscal coordination in the EMU indicates that people would expect benefits through internalizing external effects.
If an agent/country $A$ cheats another agent/country $B$ or breaches the common rules, the probability that a third person, located at distance $Y$, receives information of this behaviour is given by $e^{-\beta Y}$ (Figure 1). The parameter $\beta$ is positive and reflects the communication technology, i.e. $\beta$ can be seen as the decay of information at a constant rate with distance. The intuition is that the longer it takes, the weaker the exchange of information. This mechanism can be viewed as a 'peer-pressure' system of governance.
The payoffs of an agent/country are defined in Table (1). It is a simultaneous game with two strategies and two periods. Notice that the payoffs in Table (1) must be multiplied by $e^{\theta X}$.

Figure 1: Model Description

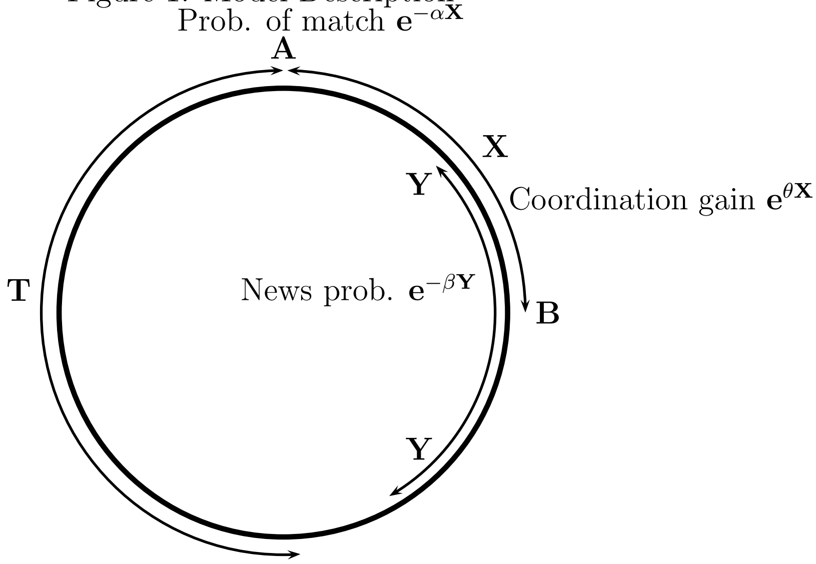



Table 1: Payoff Matrix

| | | Agent B | |
|---|---|---|---|
| | | Comply | Breach |
| Agent A | Comply | $C_t$ / $C_t$ | $L_t$ / $W_t$ |
| | Breach | $W_t$ / $L_t$ | $B_t$ / $B_t$ |

I assume the prisoners dilemma payoff structure that is common in public goods games:

$$W_t > C_t > B_t > L_t \qquad \text{for} \qquad t = 1, 2. \tag{2}$$

There are two types of agents/countries, so-called 'Type-N' and 'Type-M'. Both types are uniformly distributed along the circle. The proportion in the population is $(1-\epsilon)$ for the N-Type and $\epsilon$ for the M-Type. I assume $\epsilon \lll 1$. If an N-Type chooses to play against a random opponent, rather than taking the outside opportunity, the following payoff inequality is obtained $(1-\epsilon)B_t + \epsilon L_t > 0$. If rule breaching is detected with certainty, then there exists an equilibrium in which all N-Types choose 'comply' in period 1. Hence, I obtain

$$W_1 - C_1 < B_2. \tag{3}$$

This inequality denotes that in a full transparent world only 'comply' is efficient in period 1. To simplify the algebra, I define the last relationship as $\xi := (W_1 - C_1)/B_2 < 1$. Next, let me introduce the effect of externalities in this model. If an agent or country breaches the rule in $t = 1$, it would create a spill-over effect $\zeta$ to the

neighbours. I assume that the spill-over spreads as the information, $\beta$, and thus decay over time, i.e. $\zeta e^{-\beta Y}$.
For the solution of this game, I use the Bayesian equilibrium concept. The equilibrium is characterized by an optimal number $X$, such that all N-Types behave honestly in period 1 when meeting someone located within distance $X$. Otherwise, they cheat or breach the rules. The equilibrium strategy for N-Types is as follows: (a) In period 1, they choose to comply if the partner's location is $X$ or less. They choose to breach if it is between $X$ and $T$. (b) In period 2, if you receive information from period 1 that your payoff match received $L_1$, then you choose not to play; else choose to breach. Now, it remains to check under what conditions it is optimal to comply in period 1, if the current match is located at distance $X$ or less.

**Proposition 1** *Suppose that an N-Type finds that his period 1 partner is distant $X$. If he believes that this partner will play 'breach' even if it is an N-Type, then his best response is 'breach', i.e. $F(X,Y,T) < 0$. If he beliefs that this partner, if an N-Type, will play 'comply', then his best response is comply, i.e. $F(X,Y,T) \geq 0$, where the function $F(X,Y,T)$ is defined by*

$$F(X,Y,T) = \frac{\beta e^{-\lambda X} - \lambda e^{-\beta X}}{\beta^2 - \lambda^2} + \frac{\lambda e^{\beta X} - \beta e^{\lambda X}}{\beta^2 - \lambda^2} * e^{-(\beta+\lambda)T} - \xi e^{\theta X}(1 - \zeta e^{-\beta Y})\frac{1 - e^{-\alpha T}}{\alpha}. \quad (4)$$

**Proof.** I compute the expected future costs of breaching. If agent or country $A$ believes that his partner plays breach in period 1, if an N-Type, then agent $A$ expects the payoff of breaching, too. By definition of equations (2) and (3), breaching is the best response independently of the type. The expected cost of this decision arises from the possibility that the partner in period 2 is N-Type and has heard that $A$ has been cheating. In this case, the partner will refuse to play with $A$ in period 2. However, if agent $A$ believes that his partner is located at distance $X$ and does play comply, if an M-Type, then agent $A$ has an immediate gain reduced by the spill-over effect:

$$(1 - \epsilon)(W_1 - C_1)e^{\theta X}(1 - \zeta e^{-\beta Y}). \quad (5)$$

To calculate the expected costs, I calculate the probability of each possible match and the probability that the partner's having heard of agent $A$'s behaviour for each possible location $Z$:

$$\begin{aligned} &\frac{(1-\epsilon)B_2}{2(1-e^{-\alpha T})\alpha^{-1}}\Bigg[\int_0^X e^{-\alpha Z_1} e^{-\beta(X-Z_1)} e^{\theta Z_1} dZ_1 + \int_X^T e^{-\alpha Z_2} e^{-\beta(Z_2 - X)} e^{\theta Z_2} dZ_2 \\ &+ \int_0^{T-X} e^{-\alpha Z_3} e^{-\beta(X+Z_3)} e^{\theta Z_3} dZ_3 + \int_{T-X}^T e^{-\alpha Z_4} e^{-\beta[2T-(X+Z_4)]} e^{\theta Z_4} dZ_4\Bigg]. \end{aligned} \quad (6)$$

After integrating and using the definition for $\lambda := \alpha - \theta$, I obtain

$$\frac{(1-\epsilon)B_2}{(1-e^{-\alpha T})/\alpha}\left[\frac{\beta e^{-\lambda X}-\lambda e^{-\beta X}}{\beta^2-\lambda^2}+\frac{\lambda e^{\beta X}-\beta e^{\lambda X}}{\beta^2-\lambda^2}*e^{-(\beta+\lambda)T}\right]. \tag{7}$$

Finally, subtracting equation (5), yields the function $F(X,Y,T)$. □

**Proposition 2** *For all $T$ and $Y$, there exists a function $X(Y,T)$, such that an equilibrium exists according to Table (1) for any $X$ under the condition $0 \leq X \leq X(Y,T)$.*

**Proof.** Using proposition 1, I differentiate the function $F(X,Y,T)$. I obtain

$$\left.\frac{\partial F}{\partial X}\right|_{Y,T} = \frac{-\beta\lambda}{\beta^2-\lambda^2}\left[(e^{-\lambda X}-e^{-\beta X})-e^{-(\beta+\lambda)T}(e^{\beta X}-e^{\lambda X})\right]-\xi\theta e^{\theta X}(1-\zeta e^{-\beta Y})\frac{1-e^{-\alpha T}}{\alpha}$$
$$< 0 \qquad \text{for all} \qquad 0 \leq X \leq T.$$

There are three possible cases:

1. There exists one $Y,T$ so that $F(0,Y,T) < 0$. Then $F(X,Y,T) < 0$, for all $X \iff X(Y,T) = 0$.
2. There exists one $Y,T$ so that $F(T,Y,T) > 0$. Then for all $X\epsilon[0,T]\times[0,Y]$, I obtain $X(Y,T) = T$.
3. However, if $F(0,Y,T) < 0 < F(T,Y,T)$, then by monotonicity there is a unique $X(Y,T)\epsilon[0,T]\times[0,Y]$ such that $F(X,Y,T) \geq 0$ for $0 \leq X \leq X(Y,T)$ and $F(X,Y,T) < 0$ for $X(Y,T) < X \leq T$. □

Without loss of generality, I study the best $X$ for each $T$. In other words, the function $X(T)$. This function can be called the *extent of spill-overs*. In the next section, I examine its properties. But before doing so, I demonstrate the impact of spill-overs.

**Proposition 3** *For all $T$, a function $X(Y,T)$ exists with an equilibrium defined in Table (1) for any $Y$ satisfying $0 \leq Y \leq X(Y,T)$.*

**Proof.** Differentiating the expression in equation (4), yields

$$\left.\frac{\partial F}{\partial Y}\right|_{T,X} = -\xi\beta\zeta e^{\theta X}e^{-\beta Y}\frac{1-e^{-\alpha T}}{\alpha} < 0 \qquad for \qquad 0 \leq Y \leq T. \qquad \square$$

A greater *extend of spill-over* indicates that agent $A$ chooses more likely to breach. This behaviour is even leveraged with greater group size $T$. Consequently, a larger monetary union reduces the internalization of external effects and the incentive to play comply. This is in an important insight for an efficient design of European economic and fiscal governance. At the moment, supranational rules in Europe, such as the Stability and Growth Pact or the Fiscal Compact, do not appreciate that issue. The reason for that can be explained by the typical free-rider behaviour in public goods environments. In the next proposition, I show whether the extend of spill-over is complementary to the extend of breach.

**Proposition 4** *For all $T$, the impact of spill-over diffusion $Y$ is complementary for any $X$ with $0 \leq X \leq X(Y,T)$.*

**Proof** Differentiating the function $F(X,Y,T)$ in equation (4) in respect to $X$, and later $Y$ respectively, yields

$$\left.\frac{\partial \frac{\partial F}{\partial X}}{\partial Y}\right|_T = -\xi\zeta\theta^2\beta e^{\theta X}e^{-\beta Y}\frac{1-e^{-\alpha T}}{\alpha} < 0 \qquad for \qquad 0 \leq X \leq T. \qquad \square$$

Consequently, I find that the extend of spill-over is complementary to the extend of breach. Put it in other words: the greater the spill-over effect as well as the distance — defined as socioeconomic differences— the lower the probability that agents/countries choose comply. On the one hand this seems counter-intuitive but notice that on the other hand the spill-over diffusion is in line with lower idiosyncratic benefits within a group of countries.

## 4 Model Extension

The matches between two N-Types who are located in distance $X$, have benefits in equilibrium due to cooperation. Without the multiplicative factor $(1-\epsilon)^2(C_1 - B_1)$, the gain function, $V(X,T)$, is given by

$$V(X,T) = \frac{\alpha}{2(1-e^{-\alpha T})} * 2\int_0^X e^{-\alpha Z}e^{\theta Z}dZ = \frac{\alpha}{\alpha-\theta}\frac{1-e^{-(\alpha-\theta)X}}{1-e^{-\alpha T}}. \tag{8}$$

The function $V(X,T)$ represents the payoff in case of play comply in period $t=1$. This occurs if and only if two N-Types meet. This function is increasing in $X$, i.e. $\partial V/\partial X > 0$, and decreasing in $T$, i.e. $\partial V/\partial T < 0$.[1] In case of to comply, stable cooperation is reached over the whole cycle in period 1, if

$$V(T,T) = \frac{\alpha}{\alpha-\theta}\frac{1-e^{-(\alpha-\theta)T}}{1-e^{-\alpha T}}. \tag{9}$$

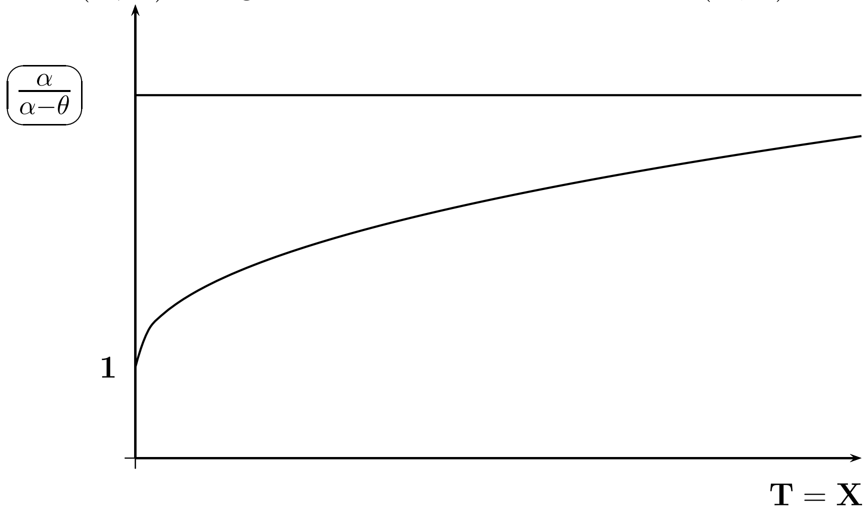
For $T \to \infty$ it results:

[1] Simple prove by computing the first-order derivatives of the function $V(X,T)$.

$$V(T,T) = \frac{\alpha}{\alpha - \theta}. \tag{10}$$

Figure (2) demonstrates the properties of the function $V(X,T)$ for $T = X$ graphically. Note that both preconditions, $\theta > 0$ and $\alpha > \theta$ need to fulfilled to require the convergence.

Figure 2: Limit of Gain Function $V(X,T)$




**Proposition 5** *There exist a unique $T^*$ such that $X(Y,T) = T$ for $0 \leq T \leq T^*$ and $X(Y,T) < T$ for $T > T^*$.*

**Proof.** I examine $F(T,Y,T) \geq 0$ for all $X$. If $X = T$, the function $F(T,Y,T)$ in equation (4) simplifies to

$$\begin{aligned} F(T,Y,T) &= \frac{e^{-\lambda T} - e^{-\beta T}}{\beta - \lambda} - \xi e^{-\theta T}(1 - \zeta e^{-\beta Y})\frac{1 - e^{-\alpha T}}{\alpha} \\ &= e^{-\lambda T}\left[\frac{1 - e^{-(\beta-\lambda)T}}{\beta - \lambda} - \xi(1 - \zeta e^{-\beta Y})\frac{1 - e^{-\alpha T}}{\alpha}\right]. \end{aligned} \tag{11}$$

I define $\Omega(Y,T)$ to be the expression in the bracket. Note, that the sign of $F(T,Y,T)$ is the same as the sign of $\Omega(Y,T)$. On the one hand $\Omega(Y,0) = 0$, and on the other hand

$$\Omega_T := \frac{\partial\Omega(Y,T)}{\partial T} = e^{-(\beta-\lambda)T} - \xi(1 - \zeta e^{-\beta Y})e^{-\alpha T}$$

so $\Omega_T(Y,0) = 1 - \xi(1 - \zeta e^{-\beta Y}) > 0$. Therefore, $\Omega(Y,T) > 0$ for $T$ sufficiently small. Next, I show the sign of $\Omega(Y,T)$ if $T$ is sufficiently large.

**(A)** If $\beta - \lambda > 0$, as $T \to \infty$ the first term of $\Omega(Y,T)$ goes to $1/(\beta - \lambda)$ and the second term to infinity, for all $Y$. Consequently, $\Omega_T(Y,T)$ is negative.

**(B)** If $\beta - \lambda < 0$, as $T \to \infty$, I obtain

$$\Omega(Y,T) = e^{\alpha T}\Big[-\frac{e^{-\alpha T} - e^{-(\beta+\theta)T}}{\lambda - \beta} - \xi(1 - \zeta e^{-\beta Y})\frac{1 - e^{-\alpha T}}{\alpha}\Big].$$

The first term in the bracket limits to zero and the second to $-\xi(1-\zeta e^{-\beta Y})/\alpha$. Again $\Omega(Y,T)$ becomes negative for all $Y$. □

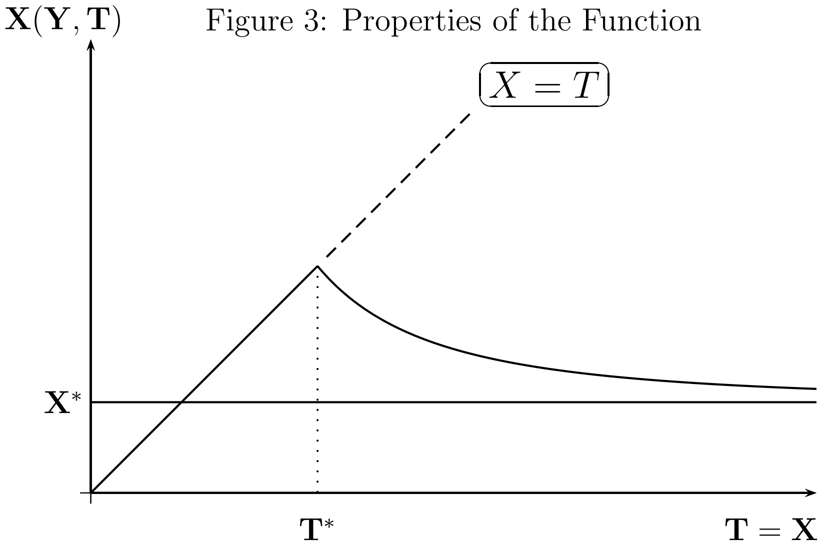
**Proposition 6** *As $T \to \infty$, $X(Y,T)$ tends to a positive limit $X^*$ if $\alpha > \xi(\beta+\lambda)(1-\zeta e^{-\beta Y})$, else $X(Y,T)$ reaches zero.*

**Proof.** When $T \to \infty$, then equation (4) obtains a limit of $X^*$ which is the solution to

$$F(X,Y,\infty) = \frac{\beta e^{-\lambda X} - \lambda e^{-\beta X}}{\beta^2 - \lambda^2} - \xi e^{\theta X}(1 - \zeta e^{-\beta Y})\frac{1}{\alpha} > 0.$$

This is positive if and only if $\alpha > \xi(\beta + \lambda)(1 - \zeta e^{-\beta Y})$. Then the $X^*$ defined by $F(X^*, \infty) = 0$ is positive. Otherwise, by continuity of the function $F(X,Y,T)$, the function at $X = 0$ becomes negative and remains beyond a finite $T$. □

Figure 3: Properties of the Function



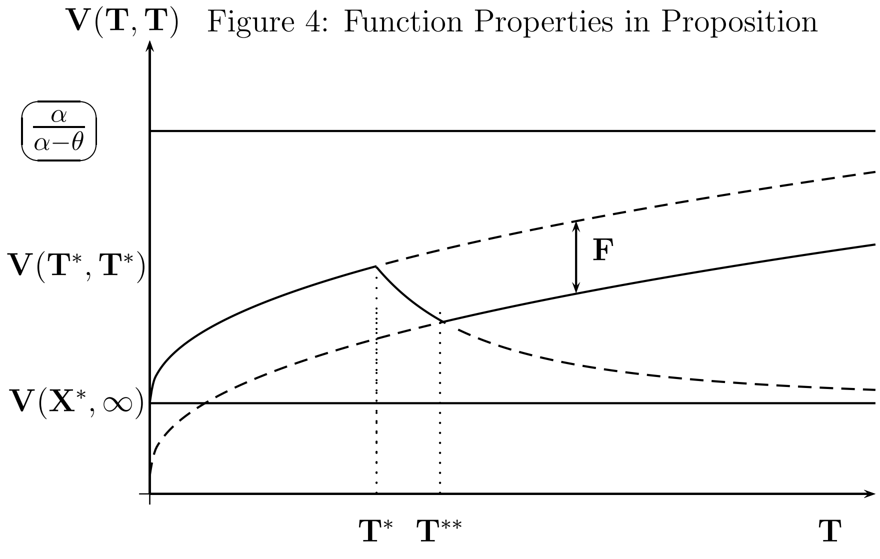

This proposition illustrates the impact of spill-over's and the size of groups on governance schemes. The spill-over creates a situation that $X^*$, the optimal extend of comply, is bigger because $F(X,Y,\infty)$ decreases with $\zeta$.

**Proposition 7** *The optimal group-size $T^*$ is a decreasing function of each $\alpha, \beta, \theta$ and $\xi$.*

**Proof.** The formal proof is relegated to the Appendix A.

In summary, this proposition demonstrates that more distant matches are less achievable and less likely with self-enforcing governance. Next, I discuss the external enforcement of governance schemes, such as in the European Monetary Union. Suppose that with costs $F$ per unit of arc length along the circle, any breach can be detected. Moreover, assume that the breaching country has to pay a fine $cX$, where $c$ is a constant. The fine increases the more distant/significant the failure. The enforcement of the governance mechanism is guided by a benevolent dictator who knows all and maximizes the representative agent's payoff. The costs are financed by a simple lump-sum tax $F$. This modeling is in line with the Stability and Growth Pact.
How does the function look like in case of an external enforcement? If $T \leq T^*$, self-governance is possible over the circle and the payoff is given by $V(T,T)$. However, if $T$ increases beyond $T^*$, then the extent of compliance $X(Y,T)$ declines. The payoff function $V(X(Y,T),T)$ is increasing in the first and decreasing in the second argument respectively. The self-enforcing payoff function $V(T,T)$ and the external enforcement function $V(T,T) - F$ is identical, however, at a lower level. But beyond a certain group size $T^*$, the self-enforcing function declines for $T \to \infty$ (Figure 4). Consequently, the function (solid curve) consists of three different segments. Each segment illustrates the optimal mode of governance for each group size, $T$.

Figure 4: Function Properties in Proposition




It is clear that for small groups, $T \leq T^*$, self-enforcement is optimal. In addition, self-enforcement has no costs. However, beyond size of $T^{**}$, self-governance becomes impossible. If $T$ is larger than $T^*$, then external enforcement is more efficient despite of the costs $F$. In case of

$$cX < V(\infty,\infty) - V(X^*,\infty) = \frac{\alpha}{\alpha-\beta} - V(X^*,\infty), \tag{12}$$

external enforcement is optimal. Thus, it is desirable to break up a large group of agents or countries in a monetary union into smaller self-enforcing communities of size $T^*$.

# 5 CONCLUSIONS

This paper explains the limits of different modes of governance while considering area-wide spill-over effects in a public goods game, such as fiscal policy spill-overs across euro area member states. I study self-enforced and externally enforced governance mechanisms.

First, my argument is much more general than initially considered. The results demonstrate the complexity and differences between self-enforcement and external enforcement. The model shows that externalities —a missing parameter in other models — play an important role especially for the design of efficient governance. Second, the model captures the institutional drawbacks of the European Monetary Union. Thus, I replicate the flawed system of fiscal and economic governance in Europe. Third, an unexpected result is that too tough external enforcement mechanisms may reduce the benefits of coordination at all. An efficient alternative is an institutional rule that at least imitates the normal market mechanism. If neither an external governance mechanism nor the imitation of a market mechanism is achievable in order to govern a large group of agents/countries due to sizeable spill-over effects, a break-up remains the ultimate exit. Indeed, the design of efficient fiscal and economic governance is a delicate trade-off. Not surprisingly, this finding is in line with other related research [15, 16]. Moreover, credible and independent enforcement mechanisms mitigate domestic externalities as well and helps to govern large groups of agents/countries in general.

# APPENDIX

***Proof of Proposition 7.***
Form proposition 2, $T^*$ is defined as the unique positive T satisfying $\Omega(Y,T) = 0$ for all Y. We also saw in the proof of the proposition that at this point $\Omega(Y,T)$ goes from positive to negative values, so $\Omega'(Y,T) < 0$. Now take again the definition of $\Omega(Y,T)$ and re-write is as:

$$\Omega(Y,T) = \int_0^T e^{(\alpha-\beta-\theta)z} dz - \xi(1-\zeta e^{-\beta Y}) * \int_0^T e^{\alpha z} dz = 0 \tag{A.1}$$

To do comparative statics with respect to any parameter, say Z, differentiate totally

$$\Omega'(Y,T^*)\frac{\partial T^*}{\partial \xi} + \frac{\partial \Omega(Y,T)}{\partial \xi}\bigg|_{T=T^*} = 0 \qquad \forall Y. \tag{A.2}$$

Since $\Omega'(Y,T^*) < 0$, the sign of $\partial T^*/\partial \xi$ is the same as that of the partial derivative of $\Omega(Y,T)$ with respect to $\xi$, and

$$\frac{\partial \Omega(Y,T)}{\partial \xi} = -(1-\zeta e^{-\beta Y}) * \int_0^T e^{\alpha z} dz < 0 \qquad \forall Y. \tag{A.3}$$

Therefore, $\frac{\partial T^*}{\partial \xi} < 0$. Similarly for $\theta$:

$$\frac{\partial \Omega(Y,T)}{\partial \theta} = -\int_0^T z e^{\alpha-\beta-\theta} dz < 0, \tag{A.4}$$

thus it follows $\frac{\partial T^*}{\partial \theta} < 0$.
Also for $\beta$:

$$\frac{\partial \Omega(Y,T)}{\partial \beta} = -\int_0^T z e^{\alpha-\beta-\theta} dz - \xi(1+\zeta Y e^{-\beta Y}) < 0, \tag{A.5}$$

therefor it is $\frac{\partial T^*}{\partial \beta} < 0$.
Finally,

$$\frac{\partial \Omega(Y,T)}{\partial \alpha} = \int_0^T z e^{(\alpha-\beta-\theta)z} dz - \xi(1+\zeta e^{-\beta Y}) \int_0^T z e^{\alpha z} dz. \tag{A.6}$$

Each of the terms in $\Omega(Y,T)$ are positive and there are equal at $T = T^*$. So dividing each of them with the terms of the expression $\partial\Omega(Y,T)/\partial\alpha$, we see that the sign of $\partial\Omega(Y,T)/\partial\alpha$ at $T^*$ is the same as the sign of

$$\frac{\int_0^T z e^{(\alpha-\beta-\theta)z} dz}{\int_0^T e^{(\alpha-\beta-\theta)z} dz} - \frac{(1+\zeta e^{-\beta Y}) \int_0^T z e^{\alpha z} dz}{(1-\zeta e^{-\beta Y}) \int_0^T e^{\alpha z} dz}. \tag{A.7}$$

Both ratios are weighted averages of z over the interval [0,T]. the first has weights proportional to $e^{(\alpha-\beta-\theta)z}$, and the second has weights proportional to $e^{\alpha z}$. So the first is a relative bigger weight for small z and is therefore small. Thus $\partial\Omega(Y,T)/\partial\alpha$ is negative, and therefore, $\partial T^*/\partial\alpha < 0$. □

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