Estimating and Forecasting West Africa Stock Market Volatility Using Asymmetric GARCH Models

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Abstract

Known as one of the key risk measures, volatility has attracted the interest of many researchers. These aim, in particular, to estimate and explain its evolution over time. Several results reveal that volatility is characterized, among other things, by its asymmetric variations (Chordia and Goyal 2006, Mele 2007, Shamila et al 2009, etc.). In this article, we seek to analyze and predict the volatility of the BRVM through these two indices. The data used are daily and start from the period from 04 January 2010 to 25 May 2016. We use three models of the GARCH family with asymmetric volatilities with different density functions. The results show a presence of asymmetry in the market yields. Also testifying to the presence of leverage in this market. The EGARCH model presents the best results in the analysis of the dynamics of market volatility behavior.

Keywords: Stock Market Volatility-Brvm-Garch models-Asymmetric Variation - Leverage-Forecasting.

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1. INTRODUCTION

Charreaux (2001) argues that any financial phenomenon can be understood as a temporal transfer of wealth, which is fundamentally risky. He thus comes to the conclusion that there are two basic dimensions of financial reasoning. Which are on the one hand, time and on the other hand, the risk.

Traditionally, therefore, financial uncertainty is associated with statistical uncertainty about the change in the price of assets, and its canonical measure is volatility. That's when volatility sparked the interest of many researchers. The latter aim, in particular, to estimate and explain its evolution over time, Bezat and Nikeghbali (2000). For these authors, stock market volatility plays a central role in modern finance because it evokes the typical observed (or expected) magnitude of stock price movements over a given period of time. In addition, modern financial theory shows that the volatility of financial assets must be measured to build efficient portfolios.

In the area of emerging markets[2], the issues of market volatility are much greater than elsewhere. It should also be noted that reducing the uncertainty associated with the knowledge of the future, improves the quality of the information and the resulting decisions remain the main objectives of the forecast. Bezat and Nikeghbali (2000). The prediction of the volatility of financial time series has been widely examined over the last three decades. The theory predicts that an estimate and especially an accurate forecast of the volatility of asset prices would have important implications for investment, valuation security, risk management and monetary policy decision-making, N'dri (2015).

Market volatility therefore becomes a measure of risk that has a significant contribution to investment decisions and efficient[3] portfolio selection. Finally, policymakers rely on the results of estimates and forecasts of market volatility as a barometer for containing the vulnerability of financial markets and the economy in the treatment of monetary policy, N'dri (2015).

[2] These markets are known to have much higher volatility than developed markets, according to the International Finance Corporation (IFC). Thus the high volatility to which is added the absence of a compromise between the risk and the future profitability makes necessary the studies dealing with this phenomenon.

[3] For a good forecast of the volatility of asset prices over the holding period of the investment is the starting point for assessing investment risk.

However, it should be noted that volatility has long been and continues to be of concern to researchers in economics, primarily in the financial sector. One of the main issues that volatility raises is the estimation method used.

Indeed, the Brownian movement that conditions the normality of stock prices and the hypothesis of efficiency supported by Fama (1965, 1970) are hypotheses very often accepted in financial theory, but which struggle to respond to the actual dynamics of time series. Mandelbrot (2000). First, the assumption of normality is almost rejected in most studies conducted on financial assets (exchange rates, stock market indices, macroeconomic aggregates, etc.). Some researchers, such as Walter and Véhel (2002), have empirically argued that the introduction of normal Brownian motion generates an underestimation of risk[4]. For these authors, this is due to the shape of the normal law (which characterizes the Brownian motion), extremely flattened at the ends and whose tails are very thin, largely ignoring the extreme values. Thus the use of Gaussian processes in the estimates of financial series proves to be incapable in the prevention of the occurrence of crises and the advent of extreme risks.

Another hypothesis that is empirically refuted is that of homoscedasticity. Which states that volatility is a constant variable over time. However, the fluctuations and upheavals that the financial landscape is incessantly experiencing point to the existence of a conditional volatility autoregressive effect (ARCH effect) present in the stochastic component of financial series. Indeed, Alberg et al. (2008), think that it is the observation of certain phenomena such as Mandelbrot's excess of kurtosis (1963) and the leverage effect by Black (1976), which occurs when stock prices are negatively correlated fluctuations in volatility in financial time series, which has led to the use of a wide range of different variance models to estimate and predict volatility.

In his seminal paper, Engle (1982) proposed a conditional variance model that varies over time and uses delayed perturbations (ARCH). This is due to the inability of ARMA models to estimate financial series due to the consistency of their conditional variance. The ARCH model in turn has two major drawbacks: the first, raised by Bollerslev (1986), which results from the large number of necessary parameters used in modeling. This may lead to the violation of the positivity constraint of the conditional variance. For this purpose he proposes

[4] Several stock market shocks have occurred since the beginning of the 20th century to the present day knowing that their probability of occurrence was practically zero

to generalize the ARCH model to obtain the GARCH. The second problem is the inability of the ARCH model to account for the asymmetry of volatility (Nelson 1991, Glosten Jagannathan and Runkle 1993, Zakoian 1994, etc.).

To try to solve these imperfections, an increasing volume of extensions of the ARCH model has been developed. We distinguish two main families: ARCH type models with symmetric volatility; these are linear models where the magnitude and not the sign of shocks influences the conditional variance. Thus, positive and negative shocks of the same magnitude have the same effect on volatility. The most innovative: (GARCH, IGARCH and GARCH-M). Then, ARCH models have asymmetric volatility. In these models, the authors introduce an explicit modeling of the conditional variance that responds asymmetrically to shock according to its sign. Thus, a negative shock will be followed by a more pronounced increase in the conditional variance than that caused by a positive shock of the same magnitude. The most innovative ones are the exponential GARCH (EGARCH), the APARCH model and the GARCH dual speed model (GJR-GARCH).

It should also be noted that the estimation of volatility by the classical GARCH model, i.e. under the assumption of the normality of the errors, gives a positive excess of the flattening coefficient (kurtosis) of the non-linear conditional distribution. The major disadvantage of this model is that, in general, it fails to fully account for the leptokurtosis character of the modeled series, especially for the high frequency series according to Giot and Laurent (2003 and 2004). To overcome these problems, several authors have introduced the concept of conditional density to obtain thicker tails. [Bollerslev (1987), Baillie and Bollerslev (1989), and Beine et al. (2002)] who used the Student's distribution in the use of GARCH models. In the same way to capture the skewness (asymmetry coefficient), Liu and Brorsen (1995) use a stable asymmetric density. Fernandez and Steel (1998) use the asymmetric Student distribution to model both the asymmetry coefficient (skewness) and the flattening coefficient (kurtosis). Then the asymmetric Student distribution was extended to the GARCH framework by Lambert and Laurent (2000 and 2001).

Empirical studies have been conducted on developed and emerging stock markets by [Sandoval (2006); Chuang et al. (2007); Komain (2007); Kovacic (2008); Curto et al. (2009); Lee (2009); Shamiri and Isa (2009); Liu and Hung (2010); Su (2010); etc.]. The few studies that have attempted to analyze African stock markets, however, are limited to [Appiah and

Menyah (2003), Ogun et al. (2005), Eskandar (2005), Alagidede and Panagiotidis (2009) and especially, N'dri (2015), Coffie (2015)].

This article aims to complement and contribute to the existing empirical literature by analyzing the BRVM volatility forecast using the different asymmetric GARCH models by applying three density functions.

The rest of the study is organized as follows: Section 2 deals with the description of the market with the presentation of the data. Section 3 presents the econometric methodology used. In section 4, the empirical results are highlighted and discussed. Finally Section 5 concludes this study.

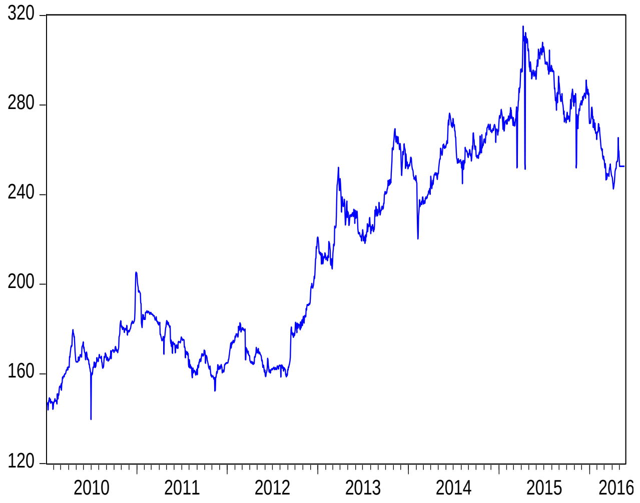
2. DESCRIPTION OF THE MARKET AND PRESENTATION OF DATA.

This study focuses on the BRVM, an integrated market common to the 8 UEMOA countries (Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal and Togo.). Created on September 16, 1998, the capital of the BRVM is subscribed by regional economic actors of West Africa. The two stock market indexes (BRVM) represent the activity of stock market securities. The BRVM Composite which consists of all listed securities. The BRVM 10 is composed of the ten most active companies on the market. The formulation and selection criteria of the BRVM COMPOSITE and the BRVM 10 are based on the main stock market indices of the world, especially the FCG index of the International Financial Corporation, a World Bank affiliate.

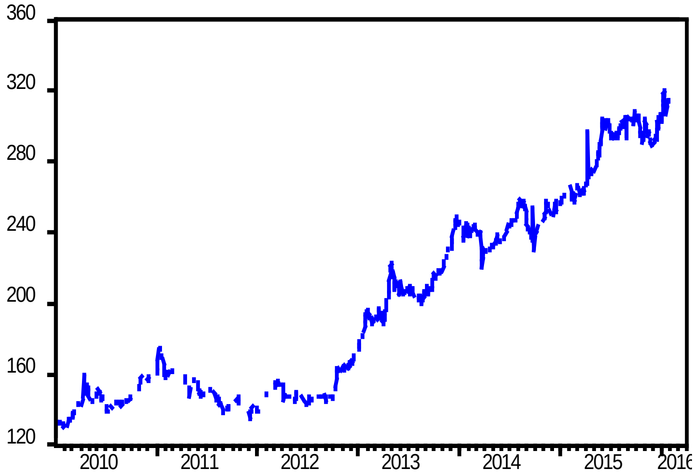
The index formula takes into account market capitalization, trading volume per trading session and trading frequency. We use daily data from the BRVM 10 and BRVM Composite indices during the period from 04 January 2010 to 25 May 2016, i.e. 1667 observations for the BRVM 10 and from 04 January 2010 to 31 March 2016, i.e. 1583 observations for the BRVM composite. They are extracted from the Official Bulletin of the Cote (BOC) which summarizes at the end of each trading session, statistics relating to BRVM 10, BRVM Composite, sectorial indices, and transaction volumes among others.

Playing the role of barometers of economic activity in a market economy, the financial market indices reflect the evolution of the values that are quoted as shown in the following graphs:

Graphique 3a: Daily evolution of the BRVM10 index from January 2010 to October 2016



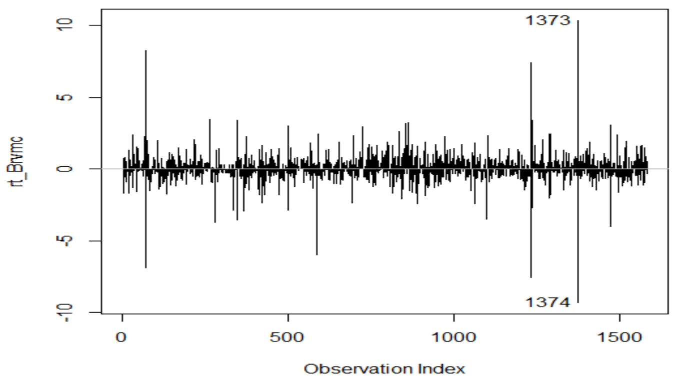
Graphique 3b: Daily evolution of the composite BRVM from January 2010 to March 2016



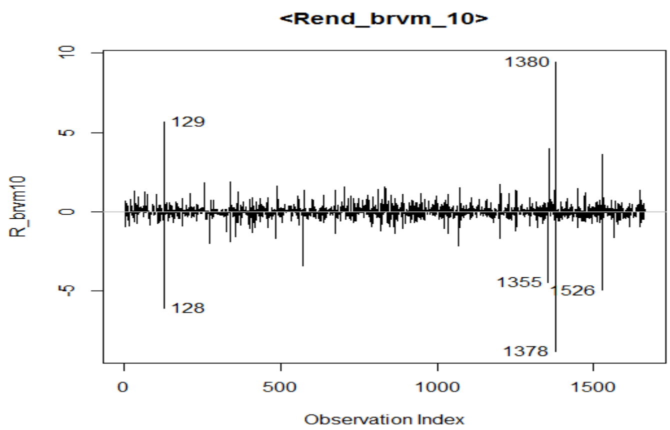
Source: Author from the data of the official bulletin of the BRVM rating.

For the calculation of yields, it should be noted that we use the first differences of the logarithms of the raw series. $R_t\_Brvm = \ln(P_t) - \ln(P_{t-1}) \times 100$, Where P_t being the price of the BRVM index at the date t

Graphique 4a: Daily evolution of the performance of composite BRVM index from January 2010 to October 2016.



Graphique 4b: Daily evolution of the performance of the BRVM10 index from January 2010 to October 2016.



The descriptive statistics of our data are presented in Table 1 below. This table clearly indicates the nature and type of data we have available for our analysis.

Table 1: Descriptive statistics for logarithm differences $100\,[\ln(P_t) - \ln(P_{t-1})]$ of BRVM 10 and BRVMC

	Obs.	Average	Max	Min	SD	Skewness	Kurtosis	Jarque-Bera Stat.
r_t**_brvm10**	1667	0.0324	21.697	-20.237	1.370	-0.0615	86.67	486213.4 (0.000)

	Obs.	Average	Max	Min	SD	Skewness	Kurtosis	Jarque-Bera Stat.
r_t**_brvmC**	1583	0.0537	10.354	-9.3017	0.911	0.255	33.804	62606.40 (0.000)

Table 1, above, gives kurtosis coefficients of 86.67 and 33.804 which are well above 3 for a normal distribution. This indicates a high probability of extreme points that is to say that the tails of the distribution are therefore thicker than those of the normal distribution which is consistent with one of the characteristics (leptokurtic distribution) of the financial series. There is also a skewness (asymmetry coefficient) of -0.061 for the BRVM10 and 0.255 for the composite BRVM compared to zero (0) for the normal distribution, this shows that the distribution of the series is asymmetric and bent respectively towards the left and right according to the index. This asymmetry may be a sign of the presence of non-linearity in the process of evolution of returns. This possible non-linearity can testify to the presence of an ARCH effect (autoregressive conditionally heteroscedastic), frequently encountered in the financial series. Finally, the Jarque-Bera statistic confirms the non-normality of the studied series through the probability associated with this statistic. We will test the ARCH effect which could be the cause of the non-linearity in the process of evolution of the profitability through 2 different methods. The one proposed by Engle (1982) which consists of estimating ε_t par $\hat{\varepsilon}_t$ (les residues). That is, regression of the model $\hat{\varepsilon}_t^2 = \partial_0 + \partial_1 \hat{\varepsilon}_{t-1}^2 + ... + \partial_p \hat{\varepsilon}_{t-p}^2 + \upsilon_t$ and calculate TR^2 with T, with T, the sample size and $T \times R \sim \chi^2(m)$. McLeod and Li (1983), which is a test similar to the Ljung-Box test, but here it is the squared residuals that are evaluated. That is to say $Q^2(m) = T(T+2)\sum_{j-1}^{m} \frac{\hat{e}_j}{T-j}$

The results obtained with the McLeod test for the two indices shows that with an optimal delay of 1 day for the BRVM10 and 5 days for the composite BRVM, we have statistical values of 244.63 for the BRVM10 and 212.77 for the BRVM composite with p-value less than 5%. This allows us to reject the null hypothesis of the absence of heteroscedasticity. The Engle test confirms in turn a strong presence of the ARCH effect through its F-statistics of 21.18 for the BRVM10 and 15.75 for the composite index with p-values lower than 5% (Table 2). Following).

Table 2: Test of the ARCH effect according to the McLeod test and that of the Engle Lagrange Multiplier

ARCH effect test	Ljung-Box test according to Macleod	Engle Lagrange Multiplier Test
Rbrvm10	Q^2(m)=244.63, m=1, p-value=0.000	F-stat=21.18, m=1,p-value=0.000
RbrvmC	Q^2(m)=212.77, m=5,p-value=0.000	F-stat=15.75, m=5,p-value=0.000

H0: $\alpha_0 = \alpha_1 = ... = 0$ presence of unit root (non-stationarity).

Before beginning the econometric estimations, we proceeded to several tests of stationarity, to reassure us or to eliminate any presence of unit root in the series studied. The t-statistic values are compared to the different critical values in brackets. The statistical values of all 4 tests are lower than the different critical values. Hence the rejection of the null hypothesis of non-stationarity (presence of unit root). The 4 tests carried out all confirm the stationarity of the yield level of our two indices, namely the BRVM10 and the BRVM composite. (See table 3 next).

Table 3: Unit Root Tests

Indices	Stat.ERS	Stat.ADF	Stat.pp	Stat.KPSS
BRVM10	-22.616 **(-1.94)	-26.125**(-2.56)	-28,100**(-2.56)	0.063**(0.463)
BRVMC	-8.320**(-1.94)	-13.524**(-2.56)	-27,134*(-2.56)	0.137(0.463)

Notes: Stat. ADF is the value of the Augmented Dickey-Fuller statistic to be compared with the critical value of -2.56 at the 5% threshold. Asterisks indicate significant values. Stat.pp is the value of the Philips and Perron statistic. Stat. ERS is the value of the Elliott-Rothenberg-Stock statistic to compare with the critical value of -1.94 at the 5% threshold.

3. ECONOMETRIC METHODOLOGY

We draw inspiration from the work of Alberg et al. (2008). Who presented a model for estimating volatility by its ability to predict and capture stylized facts received on conditional volatility. Such as the persistence of volatility and the impact of shocks according to their different signs, by studying the prediction performance of models GARCH, EGARCH, GJR and APARCH through their different density functions.

To do this, we start from the fact that Engle (1982) proposes the first ARCH model in two equations. The first describes the relationship that exists, at a given date t, between the Y yield and the vector of the variables that explain X.

$$Y_t = X_t\beta + \varepsilon_t \quad (1)$$

With $\varepsilon_t = \sigma_t z_t$, such as $\varepsilon_t | I_{t-1} \sim N(0,\sigma^2)$. Where β represents the vector of the real, ε is the shock, σ the conditional variance, Z , is i.i.d. random variable with mean zero and variance one. I_{t-1} is the information available at time t-1.

The second equation links, through an autoregressive process, the conditional variance σ^2, shock ε to the squares of the past values of this shock, that is:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (2)$$

Where $\varepsilon_t = \sigma_t z_t$, such as $Z_t \sim N(0,1)$. z_t follows a Gaussian distribution law and is independently and identically distributed (i.i.d). As restrictions, we have:

$$\alpha_0 > 0,\ \alpha_i \geq 0 \ \ for\ i > 0$$

Reducing the high number of parameters required in the modeling will lead us to the use of a GARCH (p, q) presented in the following form:

$$r_t = \mu + \varepsilon_t \quad (3)$$

With $\varepsilon_t = \sigma_t z_t$, such as $Z_t \sim N(0,1)$.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (4)$$

α_i, β_j , *and* ω, are the parameters to estimate. r_t , μ and ε_t are respectively the return on the asset at the date t, the average yield and the term of the innovation.

Equation (4) also shows that the variance is:

$$\sigma^2 \equiv E(\varepsilon_t^2) = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j} \tag{5}$$

Like ARCH, some restrictions are needed to ensure that σ_t^2 is positive for all t. Bollerslev (1986) shows that imposing $\omega > 0,\ \alpha_i \geq 0,\ i = 1...q\ \ and\ \ \beta_j \geq 0,\ j = 1...p$ is sufficient for the conditional variance to be positive.

To capture the asymmetry observed in the data, a new class of ARCH models was introduced: the GJR-GARCH by Glosten and al. (1993), the exponential GARCH (EGARCH) by Nelson (1991) and the APARCH model by Ding, and al. (1993). This last model that has the feature to generate many ARCH models by varying the parameters is expressed as: APARCH (1, 1):

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-1})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \tag{6}$$

With $\alpha_0 > 0,\ \delta \geq 0,\ \beta_j \geq 0,\ (\mathrm{j} = 1,...,\mathrm{p}),\ \alpha_i \geq 0\ \ and\ \ -1 < \gamma_i < 1\ \ (\mathrm{i} = 1,...,\mathrm{q})$.

Where $\delta\ and\ \gamma$ are the parameters allow us to capture the asymmetric effects. The presence of a leverage effect can be investigated by testing the hypothesis that $\gamma_i < 0$. With a number of variations of the parameters of the APACH model, we obtain the following models:

- ✓ ARCH of Engle (1982), when $\delta = 2,\ \gamma_i = 0\ \ (\mathrm{i} = 1,...,\mathrm{p})\ and\ \ \beta_j = 0\ (j = 1,...,\mathrm{p})$
- ✓ GJR-GARCH, Glosten and al. (1993) when $\delta = 2$
- ✓ TGARCH of Zakoian (1994), when $\delta = 1$
- ✓ TS-GARCH of Taylor (1986) and Schwert (1990), for $\delta = 1,\ \ and\ \ \gamma_i = 0\ \ (i = 1,...,q)$
- ✓ N-ARCH of Higgins and Bera, when $\gamma_i = 0\ (i = 1,...,q)\ and\ \ \beta_j = 0(j = 1,...,\mathrm{p})$ Etc.

Nelson (1991) investigated asymmetric variance trends using the EGARCH models, highlighting that rising and falling movements give different effects on volatility dynamics by using logarithm of the conditional variance.

EGARCH(1,1):

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right] - \gamma (\frac{\varepsilon_{t-1}}{\sigma_{t-1}}) \tag{7}$$

Where γ is the asymmetry parameter and is supposed to be positive so that a negative shock increases future volatility ie has more impact on volatility while the opposite effect is observed for a positive shock. This model is all the more interesting for the simple reason that it imposes no restriction on the estimated parameters.

The negative correlation between shocks and returns is a salient feature of the stock market. The sign and magnitude of shocks have asymmetrical effects on returns. Therefore, Glosten, Jagannathan and Runkle in (1993), introduced a GARCH model with the diverging effects of negative and positive shocks taking into account the phenomenon of leverage. Due to asymmetric effects, asymmetric distributions are used in the modeling of market returns. This model assumes a specific parametric form for conditional heteroscedasticity. Called GJR-GARCH and is as follows:

GJR-GARCH (1,1):

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}) \ \ \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{8}$$

$$with \ \ \varepsilon_t = \sigma_t z_t \quad and \quad I_{t-1} = \begin{cases} 1 \ if \ \ \varepsilon_{t-1} < 0 \\ 0 \ if \ \ \varepsilon_{t-1} \geq 0 \end{cases}$$

As restrictions, we have: $\alpha + \beta + \frac{1}{2}\gamma \leq 1, \ \omega \geq 0, \ \beta \geq 0, \ \alpha + \gamma \geq 0$

3.1. Estimation methods

If the prediction of volatility using the GARCH model is simple, the one using the asymmetric models must take into account the law of innovations. When the distribution is Gaussian, the probability of having a negative shock is 50%. When the distribution is of the asymmetric Student type, the probability will depend on the asymmetry and flattening parameters.

Since GARCH models are parametric, the maximum likelihood and quasi-maximum likelihood methods proposed by Bollerslev and Wooldridge (1992) are usually used for estimation. For this, it is necessary to impose a law on innovations. Because in practice, the use of a Gaussian law does not correspond to the behavior of shocks, which favors non-normal distributions with additional parameters for asymmetry and kurtosis.

Gaussian Conditional Likelihood is derived from equation (2):

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_m \varepsilon_{t-m}^2 \quad (9)$$

By posing that $\eta_t = \varepsilon_t^2 - \sigma_t^2$, we will have: $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_m \varepsilon_{t-m}^2 + \eta_t$. So the likelihood function will be of the following form:

$$\begin{aligned} f(\varepsilon_1,...,\varepsilon_T \left|\alpha\right.) &= f(\varepsilon_T \left|F_{T-1}\right.) f(\varepsilon_{T-1} \left|F_{T-2}\right.)... f(\varepsilon_{m+1} \left|F_m\right.) f(\varepsilon_1,...,\varepsilon_m \left|\alpha\right.) \\ &= \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp(-\frac{\varepsilon_t^2}{2\sigma_t^2}) f(\varepsilon_1,...,\varepsilon_m \left|\alpha\right.) \end{aligned} \quad (10)$$

With $\alpha = (\alpha_0, \alpha_1, ..., \alpha_m)'$ *and* $f(\varepsilon_1,...,\varepsilon_m \left|\alpha\right.)$, being the density function of the joint probability of $\varepsilon_1, ..., \varepsilon_m$. This likelihood function can also be written as follows:

$$L_T(\theta) = \mathrm{L}_T(\theta, \varepsilon_1, ..., \varepsilon_T) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp(-\frac{\varepsilon_t^2}{2\sigma_t^2}) \quad (11)$$

Where the σ_t^2 are defined recursively, for $t > 1$ by equation (4). For a given value of θ, under the assumption of second-order stationarity, the unconditional variance (corresponding to this value of θ) is a reasonable choice for unknown initial values $\varepsilon_0^2 = \varepsilon_{1-q}^2 = \sigma_0^2 = ... = \sigma_{1-p}^2 = \omega$ or $\varepsilon_0^2 = \varepsilon_{1-q}^2 = \sigma_0^2 = ... = \sigma_{1-p}^2 = \varepsilon^2$. Maximizing the conditional likelihood function is like maximizing its logarithm, which is easier to manage. The conditional log likelihood function is:

$$l(\varepsilon_{m+1},...,\varepsilon_T \left|\alpha, \varepsilon_1, .., \varepsilon_m\right.) = \sum_{t=m+1}^{T} \left[-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{\varepsilon_t^2}{2\sigma_t^2} \right] \quad (12)$$

Since the first term ln (2π) does not involve any parameters, and then the log likelihood function transforms and becomes:

$$l(\varepsilon_{m+1},...,\varepsilon_T \left|\alpha, \varepsilon_1, ..., \varepsilon_m\right.) = -\sum_{t=m+1}^{T} \left[\frac{1}{2}\ln(\sigma_t^2) + \frac{1}{2}\frac{\varepsilon_t^2}{2\sigma_t^2} \right] \quad (13)$$

Where $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_m \varepsilon_{t-m}^2$, can be evaluated recursively. In general, and in some applications, it is more appropriate to assume that Z_t follows a thick-tailed distribution such as a standardized Student distribution. Let x_v, the Student's distribution with v the degree of freedom, the density function of Student's asymmetric distribution is as follows:

$$D(z_t;v)=\frac{\Gamma((v+1)/2)}{\Gamma(v/2\sqrt{\pi(v-2)}}(1+\frac{z_t^2}{v-2})^{-(v+1)/2} \tag{14}$$

Where $v>2$, is the degree of freedom. With $z_t=\varepsilon_t/\sigma_t$, *and* $\Gamma(v)=\int_0^\infty e^{-x}x^{v-1}dx$ is the gamma function and v is the parameter that measures the tail thickness.

3.2. Normal distribution.

The normal distribution is by far the most used distribution in the estimation and prediction of GARCH models. If we express the equation of the mean, that is equation (1), with $\varepsilon_t = z_t\sigma_t$, the log-likelihood function of the normal distribution is given by:

$$L_T=-\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi)+\ln(\sigma_t^2)+z_t^2\right], \tag{15}$$

With T, the number of observations.

3.3. Student's t-distribution.

For $D(z_t;v)$, the log-likelihood function of $\{y_t(\theta)\}$ for the Student's t-distribution is given by:

$$L_T(\{y_t\};\theta)=T\left(\ln\Gamma\left(\frac{v+1}{2}\right)-\ln\left(\frac{v}{2}\right)-\frac{1}{2}\ln\left(\pi(v-2)\right)\right)-\frac{1}{2}\sum_{t=1}^{T}\left(\ln\left(\sigma_t^2\right)+(1+v)\ln\left(1+\frac{z_t^2}{v-2}\right)\right) \tag{16}$$

Where θ is the vector of parameters to be estimated for the conditional mean, the conditional variance and the density function. When $v\sim\infty$ we have a normal distribution, so that the lower v is, the fatter are the tails.

3.4. skewed Student's t-distribution.

Asymmetry and flattening are important phenomena in financial applications in many respects (in asset valuation models, portfolio selection, option price theory or Value-at-Risk among others). Therefore, a distribution that can model these two moments seems appropriate.

Recently, Lambert and Laurent (2000, 2001) extended the skewed Student's t-distribution proposed by Fernandez and Steel (1998) to the GARCH framework. Using $D(Z_t;v)$, the log-likelihood function of $\{y_t(\theta)\}$ for the skewed Student's t distribution is given by:

$$L_T(\{y_t\};\theta) = \mathrm{T}(\ln\Gamma(\frac{v+1}{2}) - \ln(\frac{v}{2}) - \frac{1}{2}\ln(\pi(v-2)) + \ln(\frac{2}{\xi + (1/\xi)}) + \ln(s))$$

$$-\frac{1}{2}\sum_{t=1}^{T}(\ln(\sigma^2) + (1+v)\ln(1+\frac{(sz_t+m)^2}{v-2}\xi^{-2I_t})) \quad (17)$$

Where ξ is the asymmetry parameter, and v the degree of freedom of the distribution and:

$$I = \begin{cases} 1 \quad si \quad z_t \geq -\frac{m}{s} \\ 0 \quad si \quad z_t < -\frac{m}{s} \end{cases} \quad with \quad m = \frac{\Gamma((v+1/2))\sqrt{v-2}}{\sqrt{\pi}\Gamma(v/2)}(\xi - 1/\xi) \quad and \quad s = \sqrt{(\xi^2 + \frac{1}{\xi} - 1) - m^2}$$

Different distributions such as the normal, Student and asymmetric student distribution are used in this article for estimating ARCH / GARCH models. Although we use various distributions, we will present the results of the best fit only, ignoring the rest. For the estimation, we use the software R according to Tsay (2014).

3.5 Prediction

For forecasting, it should be noted that the predictive ability of GARCH models has been widely discussed by Poon and Granger (2003). We evaluate 20 forecasts in one step using a 1667 window and 1587 observations for the BRVM10 and BRVMC. This is done for both the mean equation and the variance equation. The forecasts we will obtain will be evaluated using four different measures[5].

3.6 Measures of the quality of the forecast

The advantage of using many predictive measures lies in the robustness in choosing an optimal predictor model. We consider the following measures:

- ✓ Mean squared error (MSE)

$$MSE = \frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{\sigma}_t^2 - \sigma_t^2)^2 \quad \mathbf{(18)}$$

- ✓ Median squared error (MedSE)

$$MedSE = Inv(f_{Med}(e_t)), avec\ e_t = (\hat{\sigma}_t^2 - \sigma_t^2) \quad (19)$$

- ✓ Mean absolute error (MAE)

[5] Indeed, the daily squared returns may not be the appropriate measure to evaluate the forecast performance of the different GARCH models for the conditional variance according to Andersen and Bollerslev (1998).

$$MAE = \frac{1}{h+1}\sum_{t=S}^{S+h}\left|\hat{\sigma}_t^2 - \hat{\sigma}_t^2\right| \quad (20)$$

- ✓ Adjusted mean absolute percentage error (AMAPE)

$$AMAPE = \frac{1}{h+1}\sum_{t=S}^{S+h}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 - \sigma_t^2}\right| \quad (21)$$

Where h is the number of step, S is the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual variance.

MSE and MAE are generally affected by larger errors, as in the case of outliers. But the MedSE and AMAPE have the advantage of reducing the effect of outliers.

4. RESULTS AND DISCUSSIONS

We present in Tables 4 and 5 below the results obtained from our estimates. The basic estimation model consists of two equations. One for the mean which is a simple autoregressive AR model and the other for the variance that is identified by a particular ARCH specification. Such as GARCH (1, 1), EGARCH (1, 1), GJR (1, 1) and, APARCH (1, 1) for the two indices of the BRVM. The models are estimated using the approximate quasi-maximum likelihood estimator assuming Student, Normal, or Student asymmetric errors.

Note that it is obvious that the recursive evaluation of the maximum likelihood depends on the unobserved values and that, therefore, the estimate can not be considered perfectly accurate. To compare the different models, we apply several standard criteria: The Q (.) And Q2 (.) Which are the statistics of Box-Pierce with the delay of the standardized standardized residuals and squares, the AIC which is the criterion of Akaike information and the Log-Lik value of log-likelihood.

Tables 4 and 5 present the results of the estimates. Indeed, the Akaike Information Criteria (AIC) and log-likelihood values reveal that the EGARCH, APARCH and GJR models estimate the series better than the traditional GARCH.

When we analyze the densities, we find that the two Student distributions (symmetric and asymmetric) far exceed the normal distribution. Indeed, the log likelihood function increases when using the asymmetric Student distribution.
This leads to AIC criteria of 2,413 and 2,909 for normal density versus 2,007 and 2,487 for non-normal densities for BRVMC and BRVM10, respectively. For all models, the dynamics of the first two moments of the series are tested with the Box-Pierce statistics for residues and square residues that reject no serial correlation at the 5% level.
In addition, stationarity is satisfied for each model selected and for each density.

With one exception, all results are in the 95% predictive intervals.
All estimates are significantly different from zero at the 5% level. The model control statistics show that these models used are adequate for the BRVM series of returns. The EGARCH and APARCH models are selected as the best estimate results.
It should be noted that for both indices, the beta parameter β is positive and significant in most cases. This reflects a strong presence of persistence that can be interpreted as the persistence of the price differential with respect to the fundamental value.
In terms of portfolio management, we can then assume that this persistence could be explained by the prolongation of a climate of pessimistic uncertainty fueled by bad news. Or by persistent valuation errors on the part of investors.

There is also asymmetry of the impact of negative and positive shocks on volatility since the gamma coefficient is significant and since it is negative in the GJR-GARCH model, we can deduce that there is a negative relationship between the stock market returns and their volatilities and therefore that there is a good and a leverage effect on the BRVM market like most financial markets. Our results corroborate those found by Alberg et al. (2008), Loudon and al. (2000), Mele (2007), and Shamila and al. (2009). Etc.

See Tables 4 and 5 below for the results of the estimates.

Table 4: Results of the BRVM 10 estimates by the different GARCH models with the three density functions.

BRVM10	DISTRIBUTION											
	Normal				Student's t				Asymmetric student's t (Skewed t)			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
μ	0.046** (0.031)	0.030** (0.029)	0.022** (0.029)	0.029** (0.029)	0.022** (0.019)	0.014** (0.018)	0.013*** (0.001)	0.011** (0.018)	0.036** (0.024)	0.033** (0.025)	0.063** (0.027)	0.025 (0.025)
ω	0.0008 (0.000)	0.074 (0.029)	0.652 (0.094)	0.485 (0.090)	0.002 (0.001)	0.089 (0.088)	0.478 (0.158)	0.388 (0.113)	0.423 (0.149)	0.086 (0.087)	0.005 (0.001)	0.381 (0.112)
α	0.000 (0.000)	-0.135 (0.031)	0.044 (0.025)	0.109 (0.024)	0.000 (0.001)	-0.071 (0.057)	0.295 (0.137)	0.423 (0.117)	0.507 (0.175)	-0.064 (0.057)	0.001 (0.000)	0.410 (0.113)
β	0.999*** (0.000)	0.525* (0.085)	0.294* (0.087)	0.467* (0.089)	0.999*** (0.000)	0.684* (0.072)	0.403* (0.095)	0.478* (0.093)	0.445 (0.102)	0.687* (0.072)	0.999*** (0.000)	0.485* (0.094)
γ	-	0.255** (0.046)	-0.200* (0.061)	1.00*** (0.000)	-	0.542 (0.101)	-0.457 (0.034)	0.235 (0.105)	-	0.536 (0.101)	-0.006*** (0.000)	0.225 (0.107)
δ	-	-	-	1.126 (0.239)	-	-	-	1.302 (0.366)	-	-	-	1.288 (0.351)
ϑ	-	-	-	-	2.421 (0.152)	2.613 (0.235)	2.599 (0.235)	2.582 (0.228)	2.586 (0.231)	2.613 (0.233)	2.504 (0.118)	2.586 (0.226)
Log-Lik	-1882.8	-1792.78	-1797.049	-1791.102	-1567.522	-1531.064	-1533.212	-1532.064	-1535.855	-1530.589	-1563.39	-1531.726
AIC	3.053	**2.909**	2.916	2.919	2.545	**2.487**	2.490	2.490	2.494	2.488	2.541	2.491
BIC	3.0697	2.929	2.936	2.904	2.565	2.512	2.515	2.519	2.519	2.517	2.570	2.524
Q(1)	10.84** (0.006)	1.760 (0.676)	1.612 (0.712)	1.876 (0.648)	20.7*** (0.000)	2.246 (0.056)	2.544 (0.496)	2.505 (0.504)	2.418 (0.523)	2.158 (0.581)	31.01*** (0.000)	2.444 (0.517)
$Q^2(1)$	233.6 (0.000)	0.274 (0.999)	0.390 (0.999)	2.857 (0.782)	260.8 (0.000)	0.478 (0.999)	0.386 (0.999)	0.409 (0.999)	0.304 (0.999)	0.477 (0.999)	164.1 (0.000)	0.000 (0.999)

Table 5: Results of the BRVM Composite Index estimates by the different GARCH models with the three density functions.

BRVMC	DISTRIBUTION											
	Normal				Student't				Asymmetric student't (Skewed t)			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
μ	0.036** (0.022)	0.049** (0.017)	0.039** (0.024)	0.039** (0.022)	0.029** (0.014)	0.026** (0.014)	0.026** (0.014)	0.026** (0.014)	0.044** (0.019)	0.039** (0.020)	0.038** (0.020)	0.038** (0.020)
ω	0.287** (0.041)	-0.129** (0.040)	0.280** (0.041)	0.188** (0.044)	0.018** (0.071)	-0.035** (0.078)	0.218** (0.085)	0.212** (0.077)	0.176 (0.069)	-0.039* (0.074)	0.207* (0.083)	0.205* (0.079)
α	0.171** (0.033)	0.012** (0.030)	0.187** (0.043)	0.109** (0.037)	0.415** (0.149)	-0.058** (0.054)	0.323 (0.132)	0.424 (0.141)	0.398 (0.143)	-0.048 (0.053)	0.319 (0.130)	0.416 (0.144)
β	0.432* (0.068)	0.542* (0.076)	0.444* (0.068)	0.412* (0.072)	0.583* (0.086)	0.740* (0.075)	0.538* (0.095)	0.550 (0.102)	0.599* (0.088)	0.750* (0.074)	0.553* (0.098)	0.567 (0.103)
γ	-	0.370** (0.043)	-0.039** (0.052)	-0.073** (0.059)	-	0.598 (0.113)	-0.263 (0.093)	0.149* (0.092)	-	0.585 (0.109)	-0.229 (0.085)	0.137 (0.103)
δ	-	-	-	3.500 (0.777)	-	-	-	1.847 (0.536)	-	-	-	1.029** (0.035)
ϑ	-	-	-	-	2.546 (0.217)	2.560 (0.218)	2.537 (0.217)	2.554 (0.212)	2.549 (0.215)	2.571 (0.218)	2.543 (0.216)	2.542 (0.215)
Log-Lik	-1491.47	-1493.608	-1490.982	-1485.565	-1238.995	-1234.712	-1237.598	-1237.558	-1238.354	-1234.177	-1237.241	-1237.19
AIC	2.419	2.424	2.423	**2.413**	2.012	**2.007**	2.012	2.013	2.013	2.0084	2.013	2.014
BIC	2.436	2.445	2.452	2.438	2.033	2.032	2.037	2.042	2.038	2.0374	2.042	2.048
Q(5)	3.000 (0.407)	3.447 (0.331)	2.582 (0.733)	2.449 (0.516)	2.869 (0.431)	3.492 (0.324)	3.177 (0.375)	3.258 (0.361)	2.888 (0.427)	3.435 (0.333)	3.146 (0.381)	3.243 (0.364)
$Q^2(5)$	1.300 (0.970)	1.024 (0.985)	0.945 (0.871)	0.602 (0.997)	0.709 (0.995)	0.885 (0.999)	0.921 (0.989)	0.965 (0.987)	0.731 (0.994)	0.863 (0.999)	0.923 (0.989)	0.978 (0.987)

4.2. Discussions of the results of the forecasts

The predictive ability is indicated by ranking the different models against the five measures used in the analysis. This is done in Tables 6 and 7 below which compare the distributions for the BRVM10 and BRVMC indices. For the BRVM10 index, the results confirm that the use of the EGARCH model with asymmetric student distribution is adequate to obtain the best forecast results. For most measures of the variance equation, the EGARCH model outperforms the APARCH model. The GARCH model provides much less satisfactory results and the GJR model provides the poorest forecasts.

For the BRVMC index, the EGARCH model gives better forecasts than the GARCH model while the APARCH and GJR models give the poorest forecasts. The asymmetric Student distribution is the most successful in predicting the conditional variance of the BRVM10, unlike the BRVMC which shows better results with the Student distribution.

Indeed, our results are in line with those of Lambert and Laurent (2001). Which have shown that the asymmetric Student distribution functions are the most appropriate for modeling the NASDAQ index with respect to symmetrical densities. Those of J.J. Peter (2001), Chordia and Goyal 2006; Mele 2007; Shamila et al. 2009; etc. who found results revealing that volatility is characterized, among other things, by its asymmetric variations. The EGARCH and APARCH models had the best estimation and forecasting results.

See Tables 6 and 7 below for the results of the quality of the forecast.

Table 6: BRVM10 forecast results with the 4 GARCH models according to the best decision criteria.

BRVM10	GARCH		EGARCH	GJR	APARCH	
	Dist-std	Dist-sstd	Dist-sstd	Dist-sstd	Dist-std	Dist-sstd
MSE	0.657	0.588	0.3045	0.909	0487	0.4034
RMSE	0.708	0.702	0.508	0.807	0.606	0.543
MAE	0.455	0.364	0.345	0.878	0.466	0.274
MedSE	0.304	0.252	0.245	0.955	0.354	0.300
AMAPE	0.984	0.983	0.568	0.974	0.906	0.349

Note: Dist-std, is the student distribution and Dist-sstd is the asymmetric student distribution.

Table 7: Summary of the BRVMC forecast results with the 4 GARCH models according to the best decision criteria.

BRVMC	GARCH-std	EGARCH-std	GJR-std	APARCH-std
MSE	0.376	0.116	0.404	0.400
RMSE	0.605	0.5509	0.630	0.641
MAE	0.562	0.502	0.666	0.704
MedSE	0.306	0.2001	0.664	0.656
AMAPE	0.649	0.621	0.666	0.679

7. CONCLUSION

We have shown through our results that the BRVM is a volatile market. This trend volatility has varied and persisted while taking into account the asymmetric nature of new information (shocks). Asymmetric returns and the presence of leverage clearly indicate that market volatility is negatively correlated with BRVM index returns. This would mean that bad news would tend to have more impact on volatility or generate greater volatility than good news.

We compared the prediction performance of several GARCH models using different distribution functions. This for the returns of the two stock indexes of the BRVM. We found that the exponential GARCH model (EGARCH) proposed by Nelson in 1991 used with the asymmetric Student distribution is the most promising for characterizing the dynamic behavior of these returns.

Because it reflects their underlying process in terms of serial correlation, clustering of asymmetric volatility (clustering) and leptokurtic innovation.

The results also show that asymmetric GARCHs improve prediction performance. Among the predictions tested, the EGARCH model with the asymmetric Student distribution outperformed the GARGH, GJR and APARCH models. This result implies that the EGARCH model could be more useful than the other three models when implementing risk management strategies for the returns of the two BRVM indices.

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