**Convective Heat and Mass Transfer Effects over a Vertical Plate with Uniform Prandtl Number**

Oyem Onyekachukwu Anselm**1**, Omowaye Adeola John2 and Koriko Olubode Kolade3

**1**Department of Mathematical Sciences, Faculty of Science, Federal University Lokoja, Nigeria

Email: anselmoyemfulokoja@gmail.com, reallityo@yahoo.co.uk

2,3Department of Mathematical Sciences, Faculty of Science, Federal University of Technology Akure, Nigeria

**Abstract**

The paper examines a steady two-dimensional incompressible viscous free convective heat and mass transfer fluid flow over a vertical plate with variable thermal conductivity in the presence of heat generation, thermal radiation and species concentration. The governing nonlinear partial differential equations describing the flow and heat transfer problem are transferred into coupled nonlinear ordinary differential equations using similarity transformation and the resulting problem is solved numerically using Runge-Kutta fourth order shooting technique. The problem is studied for a uniform Prandtl number . And the analysis of results obtained showed that the magnetic field parameter have significant influence on the flow and heat transfer.

**Keywords:** Uniform Prandtl number; heat and mass transfer; vertical plate; convective flow

**1. Introduction**

Free convection flow driven by temperature differences is of great interest in a number of industrial applications and there has been increasing need for the continuous study of the behaviour of free convective flow under several phenomena due to its wide range of applications in the field of Science and Technology. Prandtl number establishes the similarity between velocity and temperature fields in free convection flow. This has been found to be the parameter that relates the relative thicknesses of the hydrodynamic and thermal boundary layers. During the last few years, great efforts were made in materials science on experiments in a microgravity environment. The influence of Prandtl number on free convection in a rectangular cavity was investigated by Graebel [1]. He observed that the approach would be applicable to more general cavity shapes. However, would not be suitable for non-horizontal walls which are adiabatic. Schneider and Straub [2] studied influence of the Prandtl number on laminar natural convection in a cylinder caused by g-Jitter. They observed that in terms of Fourier number, a fluid is more sensitive to a gravity pulse for high Prandtl numbers. Also, for the higher and (Rayleigh number of pulse amplitude), there is more tendency for the transient velocity to overshoot the steady-state velocity.

Ali [3] studied the buoyancy effects on the boundary layers induced by continuous surfaces stretched with rapidly decreasing velocities. Similarity solutions of free convection boundary layer flow on a horizontal plate with variable wall temperature were investigated by Deswita et al. [4]. They observed that temperature gradient at the surface increases as Prandtl number decreases and the higher Prandtl number fluid has, a lower thermal conductivity results in thinner thermal boundary layer. Similarly, Olajuwon [5] investigated flow and natural convection heat transfer in a power law fluid past a vertical plate with heat generation. Rawi et al.[6] investigated g-Jitter induced free convection of heat and mass transfer flow near a two-dimensional stagnation point in micropolar fluid. They revealed that both velocity and angular velocity decrease near the plate while they increase away from the plate which satisfies the boundary conditions for the increasing of micropolar parameter.

Recently, Oyem et al. [7] studied some thermo-physical properties on free convective heat and mass transfer over a vertical plate with variable thermal conductivity. In this paper, we extend the work of Oyem et al. [7] by examining the flow and convective heat and mass transfer effect of a uniform Prandtl number with variable thermal conductivity over a vertical plate.

**2. Problem Formulation**

This study considers a steady two-dimensional free convective heat and mass transfer fluid flow over a vertical plate with variable thermal conductivity, heat generation, thermal radiation and species concentration. The flow in the -axis is taken along the vertical plate in the upward direction and the -axis is normal to the plate. Under the Boussinesq’s approximation, the boundary layer is governed by the mass, momentum, energy and concentration equations along the velocity component as shown [7]:

with boundary conditions:

where are the velocity components in directions respectively, is density of the fluid, is the specific heat capacity at constant pressure, is the kinematic viscosity, is the acceleration due to gravity, is electrical conductivity, is temperature of the fluid, and are coefficient of volumetric expansion and magnetic field intensity, is variable thermal conductivity of the fluid, is the coefficient of mass diffusivity, is heat absorption coefficient and is the fluid species concentration.

The radiative heat flux is describes by the Rosseland approximation given by

where is the Stefan-Boltzmann constant and is the Rosseland mean absorption coefficient. Following Chamkha (1997), the temperature difference within the flow is assumed to be sufficiently small so that may be expressed as a linear function of temperature . This is accomplished by expanding in a Taylor’s series about the free stream temperature and truncating after the second term and neglecting higher order terms, it becomes

From the physical parameters of the coupled nonlinear partial differential equations (1) – (4), subject to the boundary conditions (5), the following similarity and dimensionless variables are introduced

Introducing stream functions, equation (1) is satisfied and applying equations (6) – (8) in equations (1) – (5), taking the take the following assumptions:

1. (ii) (iii) (iv)

The coupled nonlinear ordinary differential equations with boundary conditions are obtained and given as:

subject to the boundary conditions

where prime denotes differentiation with respect to the variable . The coupled nonlinear differential equations (9) – (12) is solved numerically and their results are displayed to depict the effects of the parameters on velocity, temperature and concentration profiles. Here

(local thermal Grashof number),

(local modified thermal Grashof number),

(local magnetic field parameter),

(Prandtl number),

(heat generation parameter),

(Schmidt number),

(variable thermal conductivity parameter),

(thermal radiation parameter of the flow).

**3. Numerical Computation**

The nonlinear governing boundary layer equations (9) – (11) together with the boundary conditions (12) are solved numerically by using Runge-Kutta fourth order technique along with the shooting method. First of all, higher order nonlinear differential equations are converted into simultaneous linear differential equations of first order and further transformed into initial value problem by applying the shooting technique. The step-size of is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin friction coefficient, Nusselt number and Sherwood number are obtained and are presented in tables.

**4. Results and Discussions**

The parameters of the flow , , , , , and are taken from [7]. The effects of magnetic field parameter, thermal Grashof number, thermal conductivity variation, modified thermal Grashof number, thermal radiation parameter, heat generation parameter and Schmidt number on the velocity are shown in figures 1 – 7. It is observed that velocity increases with increasing values of variable thermal conductivity parameter . It is evident that velocity profiles increases gradually along the plate towards the free stream velocity (figure 1). Moreover, the rise in the magnitude of the velocity is quite significant in the present case, showing that the volume rate of flow perpendicular to the plate increases with an increase in .



**Figure 4.46:** Velocity profiles for different values of with , , , , and

In figure 2, the effect of thermal Grashof number on velocity is presented. It is observed that velocity initially increases in the velocity along the plate with increasing values of and later experienced a twist in the flow and gradually decreases away from the plate with increasing values of . This signifies the relative effect of the thermal boundary force to the viscous hydromagnetic force in the boundary layer.



**Figure 4.47:** Velocity profiles for different values of with , , , , and

The effect of radiation on velocity is shown in figure 3. It is observed that velocity increases as thermal radiation parameter increases but the reverse is depicted in figure 4 with a decrease in velocity as magnetic field increases in value. It is because that the application of magnetic field will result in a resistive type of force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity.



**Figure 4.48:** Velocity profiles for different values of with , , , , and



**Figure 4.49:** Velocity profiles for different values of with , , , , and



**Figure 4.50:** Velocity profiles for different values of with , , , , and

In figures 5 – 7, the effects of modified thermal Grashof number , mass diffusivity in terms of Schmidt number and thermal radiation parameter on the velocity are presented. It is evident to note from figures 5 and 7 that velocity profiles increases with increasing values of and but decreases towards the free stream velocity with increasing values of Schmidt number as shown in figure 6.



**Figure 4.51:** Velocity profiles for different values of with , , , , and



**Figure 4.52:** Velocity profiles for different values of with , , , , and

The numerical values of the governing parameters on shear stress are shown in table 1. It is revealed that skin friction coefficient decreases with decreasing values of , and . Similarly, as and increases in values, the skin friction coefficient increases likewise. But skin friction decreases with increasing values of magnetic parameter and thermal radiation parameter .

The effects of variable thermal conductivity , thermal Grashof number , modified thermal Grashof number , radiation parameter , Schmidt number , magnetic parameter and heat source on the temperature are shown in figures 8 – 14. Figure 8 reveals that temperature in the fluid increases as increases, making the rise in the magnitude of the temperature quite significant. While an increase in the thermal Grashof number results in a decrease in the temperature (figure 9).



**Figure 4.53:** Temperature profiles for different values of with , , , , and



**Figure 4.54:** Temperature profiles for different values of with

, , , , and

Similarly, it is revealed in figure 10 that as thermal radiation of the fluid increases, temperature also increases rapidly away from the plate towards the free stream region.



**Figure 4.55:** Temperature profiles for different values of with

, , , , and



**Figure 4.56:** Temperature profiles for different values of with

, , , , ,

In figure 11, it is observed that greater magnetic parameter causes a rise in the temperature as increase in the Schmidt number also results in the increase in temperature of the fluid (figure 13). But as increases, temperature of the fluid decreases along the plate towards the free stream, satisfying boundary conditions (figure 12). In figure 14, increase in the heat source indicates a rise in temperature also. The effects of the governing parameters , , , , , and on rate of heat transfer in terms of Nusselt number are shown in table 2. It is seen that as , , and increases, the Nusselt number increases. Also, Nusselt number decreases with increase of and .



**Figure 4.57:** Temperature profiles for different values of with

, , , , and



**Figure 4.58:** Temperature profiles for different values of with

, , , , and



**Figure 4.59:** Temperature profiles for different values of with , , , , and

The effects of variable thermal conductivity , thermal Grashof number , modified thermal Grashof number , radiation parameter , Schmidt number , magnetic parameter and heat source on concentration profiles are shown in figures 15 – 21. Figure 15 shows that concentration in the fluid decreases as increases as a decrease in the thermal Grashof number also results in a decrease in the concentration (figure 16). It is revealed in figure 17 that as thermal radiation decreases, concentration equally decreases gradually along the plate towards the free stream region.



**Figure 4.60:** Concentration profiles for different values of



**Figure 4.61:** Concentration profiles for different values of



**Figure 4.62:** Concentration profiles for different values of



**Figure 4.63:** Concentration profiles for different values of



**Figure 4.64:** Concentration profiles for different values of



**Figure 4.65:** Concentration profiles for different values of



**Figure 4.66:** Concentration profiles for different values of

From figures 18 – 21, it is observed that concentration of the fluid decreases with increasing values of , and but increases as magnetic parameter increases. The effects of the governing parameters , , , , , and on rate of mass transfer in terms of Sherwood number are shown in table 3. It is seen that with increasing values of and , the rate of mass transfer increases. Also, as thermal radiation parameter and modified thermal Grashof number decreases, mass transfer rate increases but decreases with decreasing values of magnetic parameter . Meanwhile, both heat source and Schmidt number increases with increase in the concentration of the fluid.

The physical quantities of principal interest are the wall shearing stress in terms of skin-friction coefficient and the rate of heat transfer in terms of Nusselt number respectively. The shear stress in terms of the skin-friction coefficient is defined as;

were

And the skin friction coefficient of the fluid flow is given by:

The rate of heat transfer in terms of the Nusselt number (given by Fourier law) at the plate is obtained by

where is the temperature gradient at the surface measured normal to the surface. Hence, the Nusselt number is given by

expressed in terms of the wall heat flux and temperature difference.

**Table 1:** Numerical values of skin friction coefficient for

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 0.5 | 1.0 | 1.2 | 1.0 | 1.5 | 0.22 | 0.01 | 1.78787 |
| 1.0 |  |  |  |  |  |  | 1.80092 |
| 1.5 |  |  |  |  |  |  | 1.81230 |
| 2.0 |  |  |  |  |  |  | 1.82236 |
|  | 3 |  |  |  |  |  | 2.91872 |
|  | 5 |  |  |  |  |  | 3.94856 |
|  | 7 |  |  |  |  |  | 4.91270 |
|  |  | 1.0 |  |  |  |  | 1.77901 |
|  |  | 2.0 |  |  |  |  | 1.81601 |
|  |  | 3.0 |  |  |  |  | 1.84046 |
|  |  |  | 4 |  |  |  | 1.12364 |
|  |  |  | 6 |  |  |  | 0.94614 |
|  |  |  | 8 |  |  |  | 0.83274 |
|  |  |  |  | 1.5 |  |  | 1.78787 |
|  |  |  |  | 2.0 |  |  | 2.10115 |
|  |  |  |  | 2.5 |  |  | 2.40543 |
|  |  |  |  |  | 0.32 |  | 1.73880 |
|  |  |  |  |  | 0.42 |  | 1.70121 |
|  |  |  |  |  | 0.52 |  | 1.67103 |
|  |  |  |  |  |  | 0.04 | 1.79230 |
|  |  |  |  |  |  | 0.10 | 1.80166 |
|  |  |  |  |  |  | 0.15 | 1.81005 |

**Table 2:** Numerical values of governing parameters on Nusselt number

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 0.5 | 1.0 | 1.2 | 1.0 | 1.5 | 0.22 | 0.01 | - 0.32717 |
| 1.0 |  |  |  |  |  |  | - 0.29847 |
| 1.5 |  |  |  |  |  |  | - 0.27593 |
| 2.0 |  |  |  |  |  |  | - 0.25762 |
|  | 3 |  |  |  |  |  | - 0.38851 |
|  | 5 |  |  |  |  |  | - 0.43175 |
|  | 7 |  |  |  |  |  | - 0.46588 |
|  |  | 1.0 |  |  |  |  | - 0.34047 |
|  |  | 2.0 |  |  |  |  | - 0.28559 |
|  |  | 3.0 |  |  |  |  | - 0.25051 |
|  |  |  | 4 |  |  |  | - 0.23061 |
|  |  |  | 6 |  |  |  | - 0.19865 |
|  |  |  | 8 |  |  |  | - 0.17764 |
|  |  |  |  | 1.5 |  |  | - 0.32717 |
|  |  |  |  | 2.0 |  |  | - 0.34906 |
|  |  |  |  | 2.5 |  |  | - 0.36815 |
|  |  |  |  |  | 0.32 |  | - 0.31539 |
|  |  |  |  |  | 0.42 |  | - 0.30655 |
|  |  |  |  |  | 0.52 |  | - 0.29969 |
|  |  |  |  |  |  | 0.04 | - 0.31531 |
|  |  |  |  |  |  | 0.10 | - 0.29062 |
|  |  |  |  |  |  | 0.15 | - 0.26896 |

**Table 14:** Numerical values of Sherwood number on governing parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 0.5 | 1.0 | 1.2 | 1.0 | 1.5 | 0.22 | 0.01 | - 0.27473 |
| 1.0 |  |  |  |  |  |  | - 0.27651 |
| 1.5 |  |  |  |  |  |  | - 0.27819 |
| 2.0 |  |  |  |  |  |  | - 0.27977 |
|  | 3 |  |  |  |  |  | - 0.32266 |
|  | 5 |  |  |  |  |  | - 0.35691 |
|  | 7 |  |  |  |  |  | - 0.38413 |
|  |  | 1.0 |  |  |  |  | - 0.27307 |
|  |  | 2.0 |  |  |  |  | - 0.28039 |
|  |  | 3.0 |  |  |  |  | - 0.28568 |
|  |  |  | 4 |  |  |  | - 0.19981 |
|  |  |  | 6 |  |  |  | - 0.17710 |
|  |  |  | 8 |  |  |  | - 0.16278 |
|  |  |  |  | 1.5 |  |  | - 0.27473 |
|  |  |  |  | 2.0 |  |  | - 0.29244 |
|  |  |  |  | 2.5 |  |  | - 0.30801 |
|  |  |  |  |  | 0.32 |  | - 0.32551 |
|  |  |  |  |  | 0.42 |  | - 0.36776 |
|  |  |  |  |  | 0.52 |  | - 0.40408 |
|  |  |  |  |  |  | 0.04 | - 0.27532 |
|  |  |  |  |  |  | 0.10 | - 0.27658 |
|  |  |  |  |  |  | 0.15 | - 0.27773 |

**5. Conclusion**

The effects of prescribed governing parameters on free convective heat and mass transfer over a vertical plate with uniform Prandtl number and variable thermal conductivity is considered. In conclusion, it is observed that from the discussions that above magnetic field parameter has a significant influence in controlling the flow and heat transfer. Similarly, increasing values of and , results in the increase of mass transfer as increase in the values of , , and leads to increase in the Nusselt number.

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