# EFFECTS OF SOME THERMO-PHYSICAL PROPERTIES ON FREE CONVECTIVE HEAT AND MASS TRANSFER OF REACTING FLOW OVER A VERTICAL PLATE

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**Abstract**
The problem of some thermo-physical properties on free convective heat and mass transfer of reacting flow over a vertical plate in the presence of viscous dissipation is investigated. The governing partial differential equation is transformed to a coupled nonlinear ordinary differential equation with the help of similarity variables. Two special cases are analysed under some assumptions and the resulting coupled nonlinear ordinary differential equations for both cases is solved numerically by shooting method along with Runge-Kutta fourth order technique. The effects of thermo-physical parameters on velocity and temperature are shown graphically while numerical data for the local skin friction coefficient and Nusselt number have been tabulated for various values of certain parameters.

**Keywords:** MHD; reacting flow; heat and mass transfer; Eckert number; vertical Plate.

**MSC:** 80A20

## 1 Introduction

Free convective flow driven by temperature differences is of great interest in a number of industrial applications. There has been increasing need for the continuous study of the behaviour of free convective flow under several phenomena due to its wide range of applications in the field of Science and Technology. This is a flow which plays an important role in agriculture, engineering and petroleum industries. The problem of free convection under the influence of magnetic field has attracted many researchers in view of its application in geophysics, astrophysics, geological formations, and thermal recovery of oil, and in assessment of aquifers, geothermal reservoirs and underground nuclear waste storage site, etc. Researchers are motivated by the fact that free convection appears to be increasingly important due to its various applications in applied sciences, engineering, industries and technology as nuclear reactors, heat exchangers, solar powers, oceanography, cooling applications, fossil fuel combustion energy processes, astrophysical flows, satellites, solar power technology, space vehicle re-entry, etc. In most of the existing works in literature, many investigations dealing with heat flow and mass transfer have been reported by a considerable number of researchers. Heat and mass transfer characteristics and flow behaviour on magneto-hydrodynamics (MHD) flow near the lower stagnation point of a porous isothermal horizontal circular cylinder was studied by Ziya and Manoj [1]. Their result showed that velocity increases with viscosity parameter, while temperature decreases with the same parameter. Also, both velocity and temperature decreases with increase in radiation and increases with increase in thermal conductivity. Basant [2], considered the effects of applied magnetic field on transient free-convective flow in a vertical channel. His result showed that as magnetic parameter increases, velocity decreases, while it increases with increase in time $(t)$. Mansuor et al. [3] considered a steady two dimensional nonlinear MHD boundary layer flow of an incompressible, viscous and electrically conducting fluid in the presence of a uniform magnetic field with heat, mass transfer and chemical reaction in a porous medium. The fluid properties were assumed to be constant. The results showed that the flow field was influenced appreciably by the presence of chemical reaction, viscous dissipation and suction or injection flow. Kishore et al. [4] investigated the unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate, by taking into account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. Their investigation showed that velocity increases with increase in thermal Grashof number, accelerated parameter, Eckert number and time. While temperature increases with increasing values of Eckert number and time. Gideon and Eletta [5] discussed viscous dissipation effect on the flow through a horizontal porous channel with temperature dependent viscosity. It was observed that high Darcy number leads to a higher velocity and that velocity is parabolic while reversal flow takes place at low Darcy number. The use of Arrhenius equation in kinetics for many fluids, have been reported for countless industrial and engineering processes like geological materials, liquid foams, polymeric fluids, slurries, hydrocarbon oils and grease. Also, its applications in a number of technological processes include the production of polymer films or thin sheets, wire drawing, fiberglass and paper production. Steady Arrhenius laminar free convective MHD flow and heat transfer past a vertical stretching sheet with viscous dissipation was studied by Omowaye and Koriko [6] and their results indicated that velocity and temperature profile increases with increase in local Grashof number and Eckert number. Omowaye and Ayeni [7], studied unsteady MHD free convection flow and heat transfer along an infinite vertical porous plate under Arrhenius kinetics. Their studied showed that velocity of the fluid decreases with the increase in Prandtl number and Hartmann number but increases with increase in Grashof number. While, temperature decreases with increase in Prandtl number. Thermal criticality for a reactive gravity driven thin film flow of a third-grade fluid with adiabatic free surface down an inclined plane was studied by Makinde [8]. His investigation revealed that an increase in the material parameter enhances the thermal stability of the liquid and his series summation procedure can be used as an effective tool to investigate several other parameter-dependent nonlinear boundary-value problems in science and engineering. Galwey and Brown [9] studied the application of the Arrhenius equation to the kinetics of solid state reactions. Galwey [10] further looked into a general and critical analysis of theories used to interpret those thermochemical rate measurements that are directed towards investigations of the mechanism of chemical changes that result from the heating of initially solid reactants. Analytical solutions for the problem of heat and mass transfer by steady flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field of first order chemical reaction was studied by Chamkha [11]. His result showed that fluid velocity decreased as Prandtl number, the Schmidt number and the strength of the magnetic field was increased but increased as thermal and concentration buoyancy effects were increased.

Takhar et al. [12] studied the effect of thermophysical quantities on the natural convection flow of gases over a vertical cone and effects of some thermo-physical properties on force convective stagnation point on a stretching sheet with convective boundary conditions in the presence of thermal radiation and magnetic field. Khaleque and Samad [13] studied the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Mohyud-Din et al. [14] investigated modified Variational Iteration Method (MVIM) for free-convective boundary-layer equation using Padé approximation. In the literature, the problem of variable thermal conductivity, magnetic field effect and the reacting fluid flow with laminar free convection flow over a vertical plate has not been adequately dealt with to our best of knowledge. Hence, the need to study the effects of the physical features for the free convective flow problem and give the numerical solution of the problem, which allows us to critically analyse the physical features of our problem to providing better and efficient results in the model.

## 2.0 Governing Equation

Consider a steady two-dimensional laminar free convective flow of a viscous, incompressible fluid over a vertical plate with variable thermal conductivity and magnetic field effects. The $x$-axis is taken along the vertical plate in the upward direction and the $y$-axis is normal to the plate. The schematic representation of the problem under consideration is shown in figure 1. The fluid is reacting and a uniform magnetic field is applied normal to the flow field.

Under the Boussinesq's approximation [14], the modified partial differential governing mass, momentum and energy equation is given as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma\beta_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right)+\frac{AQexp^{-\frac{E}{RT}}}{\rho c_p}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

where $u, v$ are the velocity components in $x, y$ directions respectively, $\rho$ is density of the fluid, $c_p$ is the specific heat capacity at constant pressure, $\upsilon$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\sigma$ is electrical conductivity, $T$ is the temperature of the fluid, $\beta$ and $\beta_0$ are coefficient of volumetric expansion and magnetic field intensity, $\kappa(T)$ is variable thermal conductivity of the fluid, $A$ is the pre-exponential (frequency) factor, $Q$ is heat release, $E$ is the activation energy, $R$ is the universal gas constant and $\mu$ is the fluid viscosity coefficient. The boundary conditions for the velocity and temperature fields are:

$$u=0 \quad v=0 \quad T=T_w \quad at \quad y=0 \quad (4)$$
$$u\to 0 \quad T\to T_\infty \quad as \quad y\to\infty \quad (5)$$

where $T_w$ is the wall dimensional temperature and $T_\infty$ is free stream dimensional temperature.

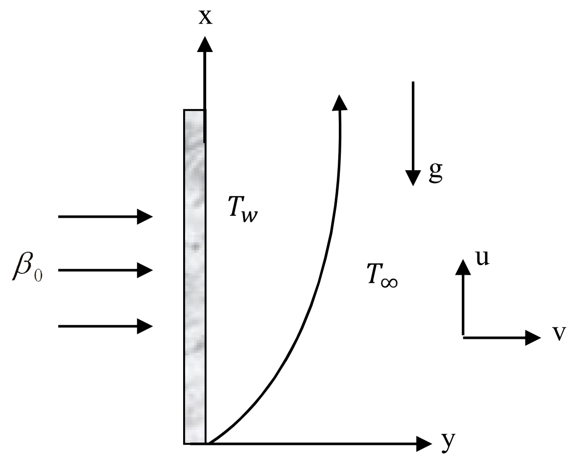


**Figure 1:** Schematic representation of the problem

Introducing the stream function $\psi(x, y)$, equation (1) is satisfied. Equations (2) and (3) with boundary conditions (4) and (5) reduces to

$$\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial^2\psi}{\partial y^2}\right)=\upsilon\left(\frac{\partial^3\psi}{\partial y^3}\right)+g\beta(T-T_w)-\frac{\sigma\beta_0^2}{\rho}\left(\frac{\partial\psi}{\partial y}\right) \quad (6)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial\kappa(T)}{\partial T}\left(\frac{\partial T}{\partial y}\right)^2+\frac{\kappa(T)}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{AQexp^{-\frac{E}{RT}}}{\rho c_p}+\frac{\mu}{\rho C_P}\left(\frac{\partial}{\partial y}\frac{\partial\psi}{\partial y}\right)^2 \quad (7)$$

$$\frac{\partial\psi}{\partial y}=0 \quad -\frac{\partial\psi}{\partial x}=0 \quad T=T_w \quad at \quad y=0$$
$$(8)$$
$$\frac{\partial\psi}{\partial y}=0 \quad T\to T_\infty \quad as \quad y\to\infty$$

In this research, the fluid thermal conductivity $\kappa$ is assumed to vary as a linear function of temperature is assumed. Variation of the normalized thermal conductivity is written in the form (Elbashbeshy and Ibrahim [15], Seddeek and Abdelmeguid [16]) as:

$$\kappa(T)=\kappa^*[1+\gamma\theta] \quad (9)$$

where $\kappa^*$ is the ambient fluid thermal conductivity and $\gamma$ is a constant depending on the nature of the fluid. In order to resolve the governing partial differential equations (6) and (7) along with the boundary conditions (8), the following dimensionless quantities are introduced

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}}\ ;\ f'(\eta)=\frac{u}{U_0}\ ;\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ ;\ \psi=\sqrt{2x\upsilon U_0}\,f(\eta) \quad (10)$$

using (10), equations (6) and (7) subject to the boundary conditions (8) reduces to

$$\frac{\eta}{y}\sqrt{2xvU_0}\ f'(\eta)\left(-\frac{\eta^2}{2y}\sqrt{\frac{2vU_0}{x}}\ f''(\eta)\right)-\left(-\frac{1}{2}\sqrt{\frac{2vU_0}{x}}\ (\eta f'-f)\right)\left(\frac{\eta^2}{y^2}\sqrt{2xvU_0}\ f''(\eta)\right)$$
$$=v\left(\frac{\eta^3}{y^3}\sqrt{2xvU_0}\ f'''(\eta)\right)+g\beta(T_w-T_\infty)\theta(\eta)$$
$$-\frac{\sigma\beta_0^2}{\rho}\left(\frac{\eta}{y}\sqrt{2xvU_0}\right)\ f'(\eta) \qquad (11)$$

$$-\frac{\eta^2}{2y}\sqrt{\frac{2vU_0}{x}}\ f'(\eta)\,\theta'(\eta)-\left(-\frac{\eta^2}{2y}\sqrt{\frac{2vU_0}{x}}\ f'(\eta)\,\theta'(\eta)+\frac{\eta}{2y}\sqrt{\frac{2vU_0}{x}}\ f(\eta)\,\theta'(\eta)\right)$$
$$=\frac{\gamma\kappa^*}{\rho c_p}\left(\frac{\eta^2}{y^2}\theta'^2(\eta)\right)+\frac{\kappa^*[1+\gamma\theta]}{\rho c_p}\left(\frac{\eta^2}{y^2}\ \theta''(\eta)\right)+\frac{AQexp^{-\frac{E}{RT\infty}}}{\rho c_p(T_w-T_\infty)}exp^{\left(\frac{\theta}{\varepsilon\theta+1}\right)}$$
$$+\frac{\mu}{\rho C_P}\left(\frac{U_0^3}{2xv(T_w-T_\infty)}\right)\left(f''(\eta)\right)^2 \qquad (12)$$

on simplification of equations (11) and (12), the dimensionless governing equations for momentum, energy and their boundary conditions are obtained. Hence, the coupled nonlinear ordinary differential equations:

$$f'''+ff''+Gr\theta-Mf'=0 \qquad (13)$$
$$[1+\gamma\theta]\theta''+\gamma\theta'^2+Prf\theta'+\delta exp^{\left(\frac{\theta}{\varepsilon\theta+1}\right)}+PrEcf''^2=0 \qquad (14)$$

The corresponding boundary conditions are

$$\left.\begin{array}{llll} f=0, & f'=0, & \theta=1 & at\quad \eta=0 \\ f'=0, & \theta=0 & & as\quad \eta\to\infty \end{array}\right\} \qquad (15)$$

where prime denotes the differentiation with respect to $\eta$, $f$ is the dimensionless velocity and $\theta$ the dimensionless temperature, $\gamma=\delta(T_w-T_\infty)$ is the thermal conductivity variation parameter of the flow, $Gr=\frac{2xg\beta(T_w-T_\infty)}{U_0^2}$ is the local Grashof number, $Pr=\frac{v}{\alpha}$ is the Prandtl number, $\varepsilon=\frac{RT_\infty}{E}$ is the activation energy parameter, $\delta=\frac{v}{\alpha}\frac{2xAQexp^{-\frac{E}{RT\infty}}}{\rho C_pU_0(T_w-T_\infty)}$ is the modified Frank-Kamenetskii parameter, $Ec=\frac{U_0^2}{C_p(T_w-T_\infty)}$ is the Eckert number and $M=\frac{2x\sigma\beta_0^2}{\rho U_0}$ is the local magnetic field parameter of the flow. the physical quantities of principal interest are the skin friction coefficient $C_f$ and Nusselt number $Nu$ given as:

$$\left(\frac{y}{\eta}\right)\frac{\rho U_0}{2\mu}C_f=f''(\eta)$$
$$\frac{q_wy}{\kappa(T)(T_w-T_\infty)\eta}=-\theta'(\eta)$$

## 3 Special Forms of the Governing Equation

Special forms of some of the governing equations are investigated when dealing with certain types of fluid flow.

### 3.1 Case I

In this case, the following assumptions are introduced into the dimensionless governing equations (13) and (14):

(i) $\delta\neq 0$ (ii) $Ec=0$

The governing equations become

$$f'''+ff''+Gr\theta-Mf'=0 \qquad (16)$$

$$[1+\gamma\theta]\theta'' + \gamma\theta'^2 + Prf\theta' + \delta exp^{\left(\frac{\theta}{\varepsilon\theta+1}\right)} = 0 \quad (17)$$

the corresponding boundary conditions (15).

### 3.1.1 Numerical Computation

In this section, the set of equations (16) and (17) under the boundary conditions (15) are solved numerically by applying the Runge-Kutta fourth order scheme along with shooting method.

Let $f = y_1,\ f' = y_2,\ f'' = y_3,\ \theta = y_4,\ \theta' = y_5$. Hence, equations (16) and (17) are transformed into a system of first order differential equations as follows:

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= -y_1y_3 - Gry_4 + My_2 \\ y_4' &= y_5 \\ y_5' &= \frac{1}{1+\gamma\theta}\left(-Pry_1y_5 - \gamma y_5^2 - \delta exp^{\left(\frac{y_4}{\varepsilon y_4 + 1}\right)}\right) \end{aligned} \quad (18)$$

subject to the following initial conditions:

$$y_1(0) = 0, y_2(0) = 0, y_3(0) = s_1, y_4(0) = 1, y_5(0) = s_2 \quad (19)$$

The unspecified initial conditions $s_1$ and $s_2$ are guessed. Numerical computation on the behaviour of the physical parameters $Gr, Pr, M, \varepsilon$ and $\delta$ are calculated including the skin-friction $f''(0)$ and the Nusselt number $-\theta'(0)$.

### 3.1.2 Discussion of Results

The problem of magnetic field and variable thermal conductivity on laminar free convective heat flow and mass transfer over a reacting vertical plate is considered. The governing parameters are the local Grashof number $Gr$, Prandtl number $Pr$, activation energy $\varepsilon$, magnetic field parameter $M$, modified Frank-Kamenetskii parameter $\delta$ and thermal conductivity variation parameter $\gamma$. To illustrate the behaviour of these physical quantities on the velocity and temperature profile, numerical values were computed with respect to the variations in the governing parameters and the analysis are presented graphically.

**Table 1:** Numerical values of $f''(0)$ and $\theta'(0)$ for various values of the governing parameters

| $\gamma$ | $Pr$ | $Gr$ | $M$ | $\varepsilon$ | $\delta$ | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.72 | 1 | 1.5 | 0.1 | 0.01 | 0.6700 | 0.2149 |
| 2.0 | 0.72 | 1 | 1.5 | 0.1 | 0.01 | 0.7078 | 0.1547 |
| 0.5 | 4 | 1 | 1.5 | 0.1 | 0.01 | 0.5740 | 0.3649 |
| 0.5 | 0.72 | 7 | 1.5 | 0.1 | 0.01 | 3.6731 | 0.4502 |
| 0.5 | 0.72 | 1 | 2.5 | 0.1 | 0.01 | 0.5588 | 0.1807 |
| 0.5 | 0.72 | 1 | 1.5 | 1.5 | 0.01 | 0.6682 | 0.2201 |
| 0.5 | 0.72 | 1 | 1.5 | 0.1 | 0.08 | 0.7709 | 0.0147 |

From table 1 above, the numerical values of $f''(0)$ and $\theta'(0)$ for different values of dimensionless parameters $Pr$, $Gr$, $M$, $\varepsilon$, $\gamma$ and $\delta$ indicates that skin friction coefficient $f''(0)$ increases with increasing values in $\gamma$, $Gr$ and $\delta$ but decreases with increasing values in $Pr$, $M$ and $\varepsilon$. More so, the Nusselt number coefficient $-\theta'(0)$ decreases as $Pr$, $\varepsilon$ and $Gr$ increases but increases with increasing values in $M$, $\gamma$ and $\delta$. The parameters of the flow $\gamma$, $Pr$, $M$ and $Gr$ can be taken as follows (Loganathan et al. [17], Elbashbeshy [18], Reddy and Reddy [19], Kishore et al. [4]): $0.7 \le Pr < 7.0$, $0.5 \le \gamma < 6$, $0.5 < M \le 2.5$, $1 \le Gr \le 7$.

The effect of magnetic field parameter $M$ on the velocity $f'$ and temperature $\theta$ of the flow with variable magnetic field parameter $M(1, 1.5, 2, 2.5)$, $Gr = 1$, $Pr = 0.72$, $\varepsilon = 0.1$, $\gamma = 0.5$ and $\delta = 0.01$ is illustrated in figures 2 – 3. From figure 2, it is observed that velocity decreases as the magnetic parameter increases, but in figure 3, temperature $\theta$ increases as $M$ increases in the vicinity of the plate.

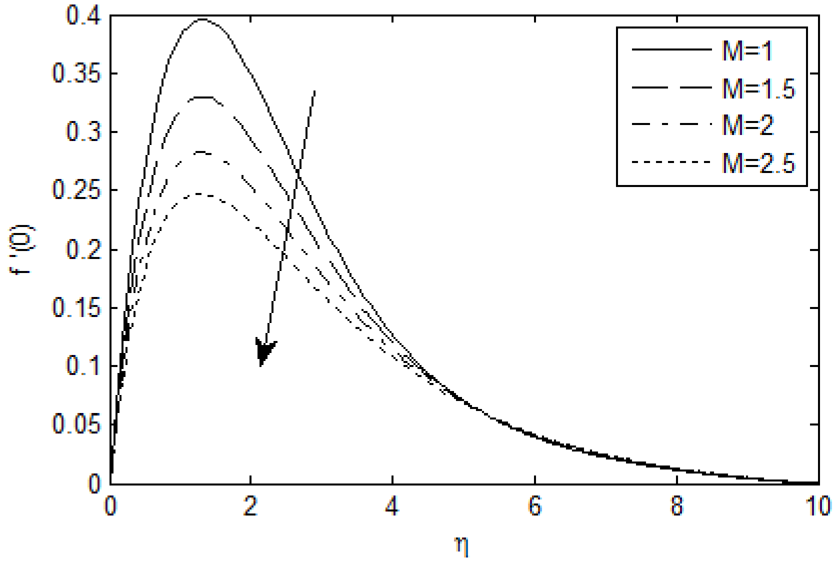


**Figure 2:** Velocity profile for different values of $M$

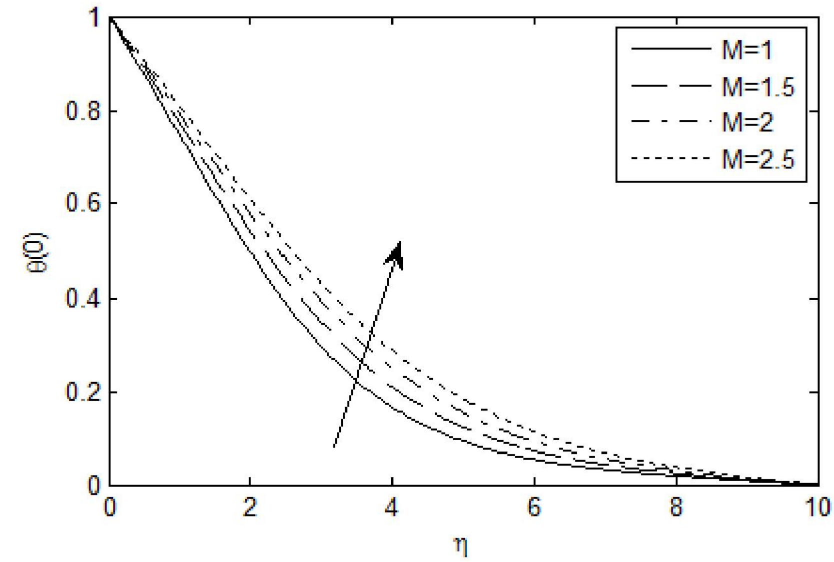


**Figure 3:** Temperature profile for various values of $M$ against $\eta$

The effect of variable Grashof number $Gr$ for heat transfer on the velocity of the flow with $Gr(1,3,5,7)$, $M = 1.5$, $Pr = 0.72$, $\varepsilon = 0.1$, $\gamma = 0.5$ and $\delta = 0.01$ is presented in figure 4. It is observed that velocity initially increases along $y$ away from the plate with increasing values of $Gr$ and later decreases towards the plate but gradually increases afterwards towards the free stream. Figure 5 show the effect of $Gr$ for heat transfer on the temperature profile. It shows that $\theta$ decreases towards the plate with increasing values of Grashof number.

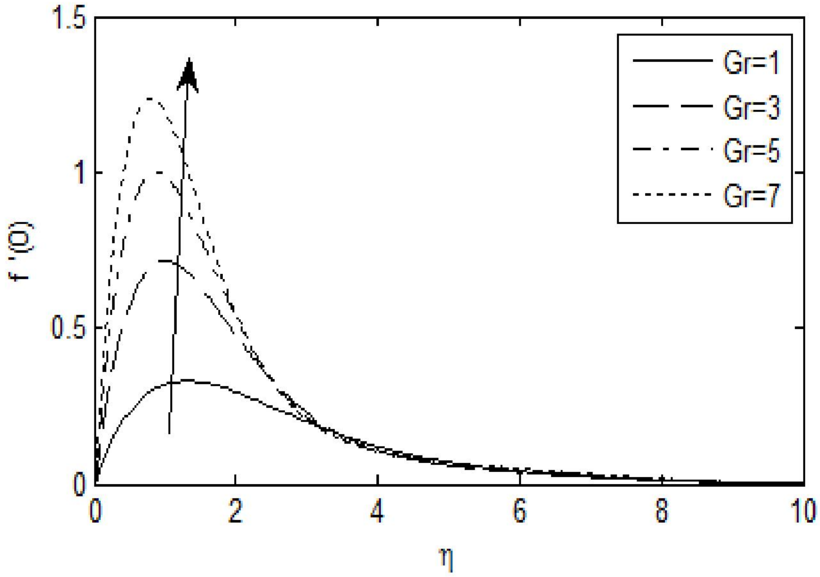


**Figure 4:** $f'$ profile against $\eta$ for various values of $Gr$

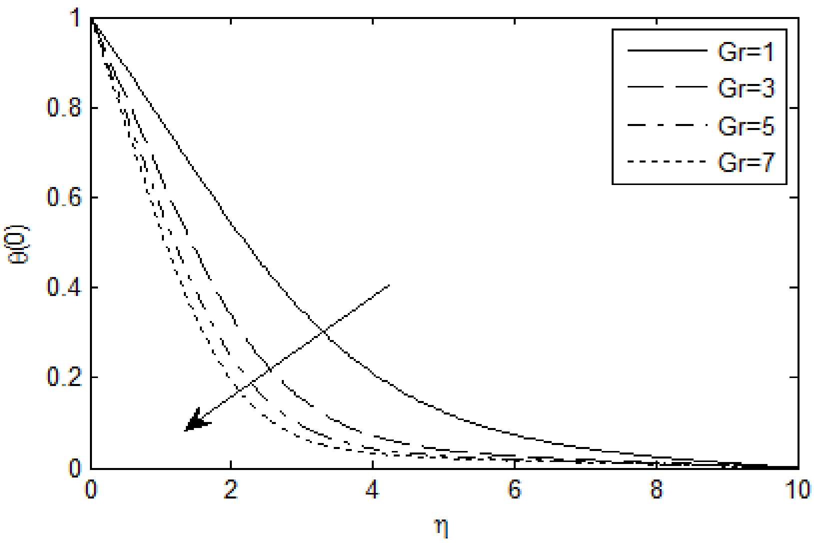


**Figure 5:** Temperature profile for different values of $Gr$

The effect of Prandtl is important in velocity and temperature profile. Figures 6 and 7 depict the effect of Prandtl number $Pr$ on velocity and temperature profiles respectively. It is observed that velocity and temperature decreases with increasing values in $Pr$ for $Pr(0.72, 2, 4)$, $M = 1.5$, $Gr = 1$, $\varepsilon = 0.1$, $\gamma = 0.5$ and $\delta = 0.01$.

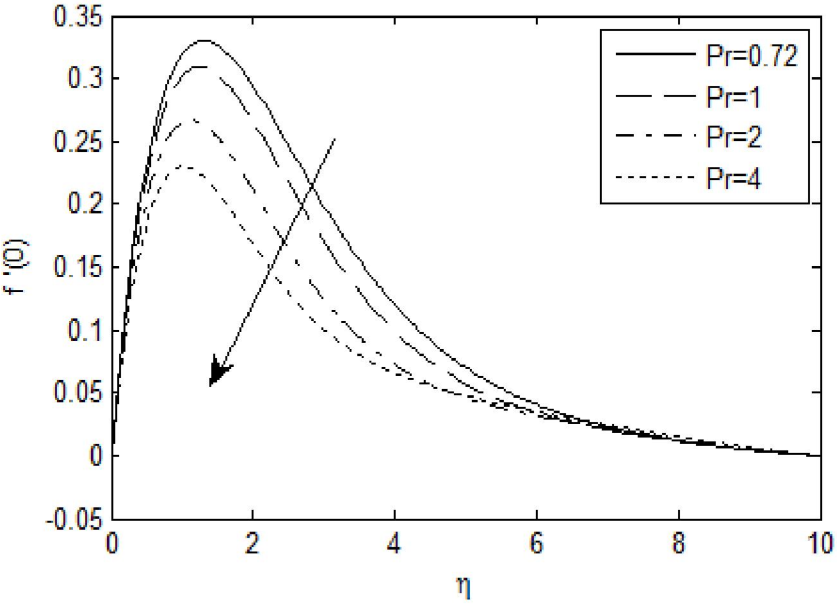


**Figure 6:** Velocity profile for different values of $Pr$

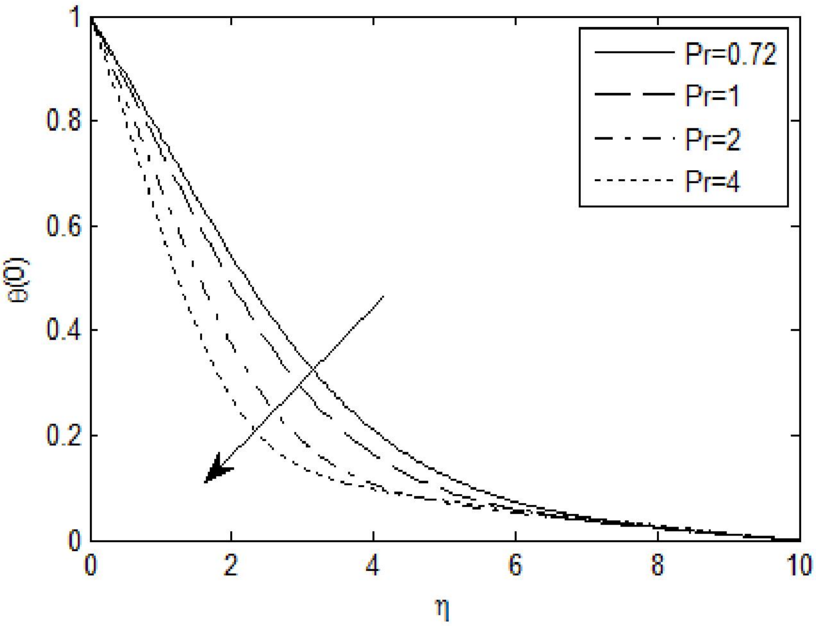


**Figure 7:** Temperature profile for various values of $Pr$

Figure 8 shows the velocity distribution for different values of activation energy $\varepsilon$. It is observed that velocity and temperature decreases with increasing values of activation energy $\varepsilon$ as shown in figure 9.

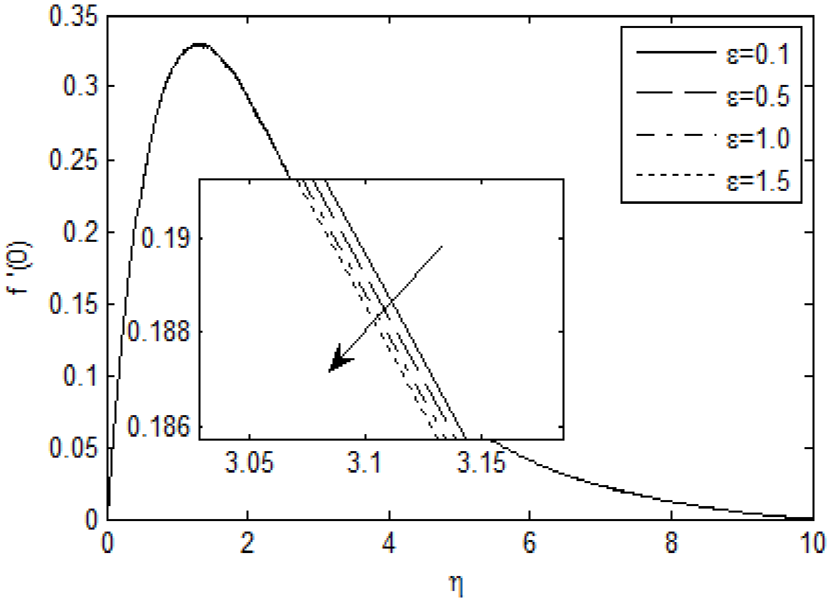


**Figure 8:** Velocity profile for different values of $\varepsilon$

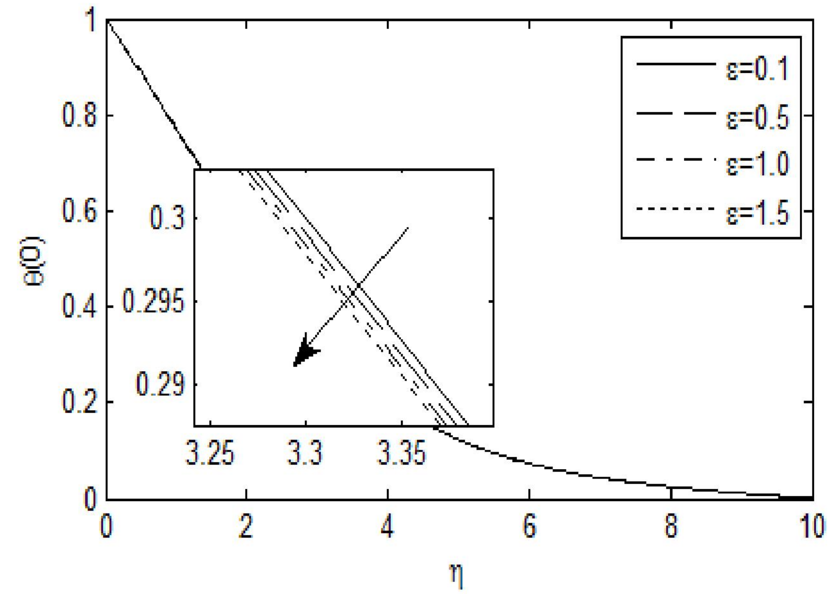


**Figure 9:** Temperature profile for different values of $\varepsilon$

The effect of velocity and temperature profile for different values of modified Frank Kamenetskii parameter is shown in figures 10 and 11. It is observed that velocity and temperature profiles increases with increasing values of modified Frank Kamenetskii parameter $\delta$. From the diagram, the rate of increase is spontaneous as it provides a convenient comparative measure of reactivity an of the temperature coefficient of reaction rate.

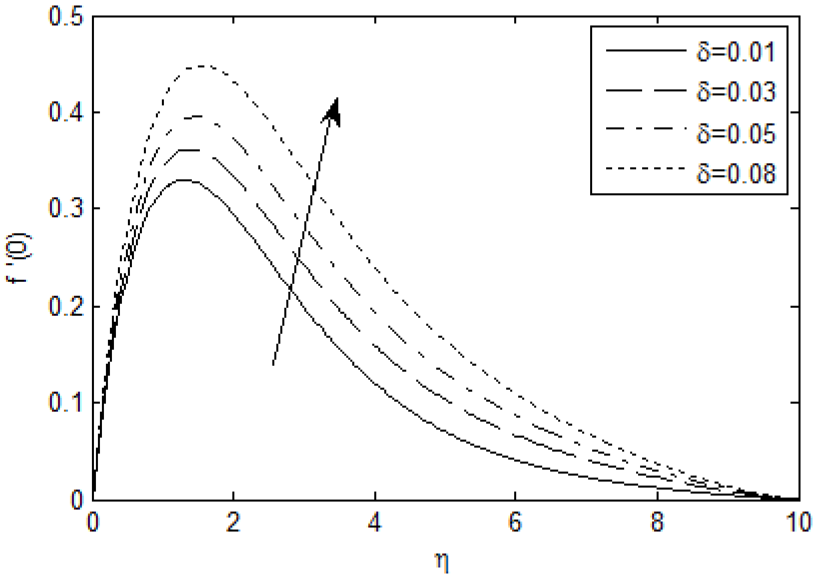


**Figure 10:** Velocity profile for various values of $\delta$

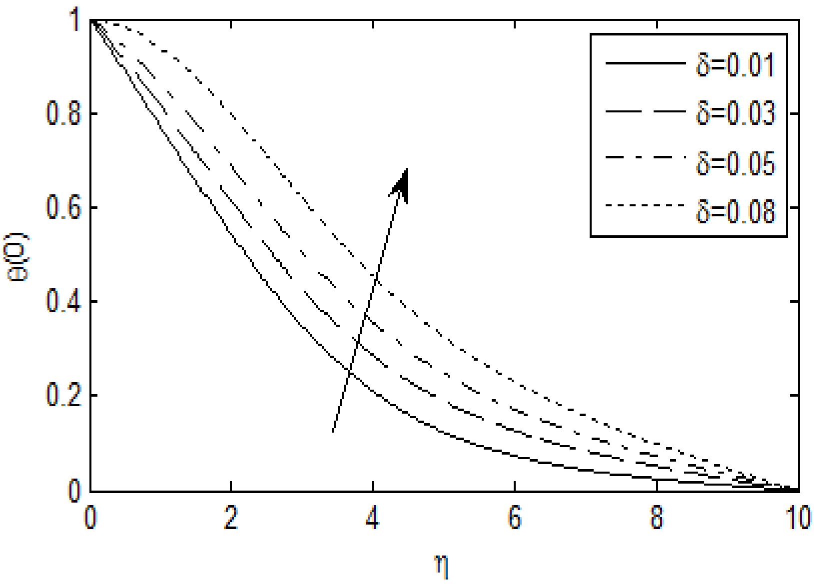


**Figure 11:** Temperature profile for various values of $\delta$

The velocity and rate of heat transfer of temperature for thermal conductivity variation parameter over the vertical plate is displayed in figures 12 and 13 respectively. From figure 12, it is observed that with increasing values in $\gamma$ parameter, the velocity profile increases away from the plate towards the free stream values. Likewise, as the variable thermal conductivity increases, temperature also increases with a decreasing boundary layer towards the free stream values as shown in figure 13.

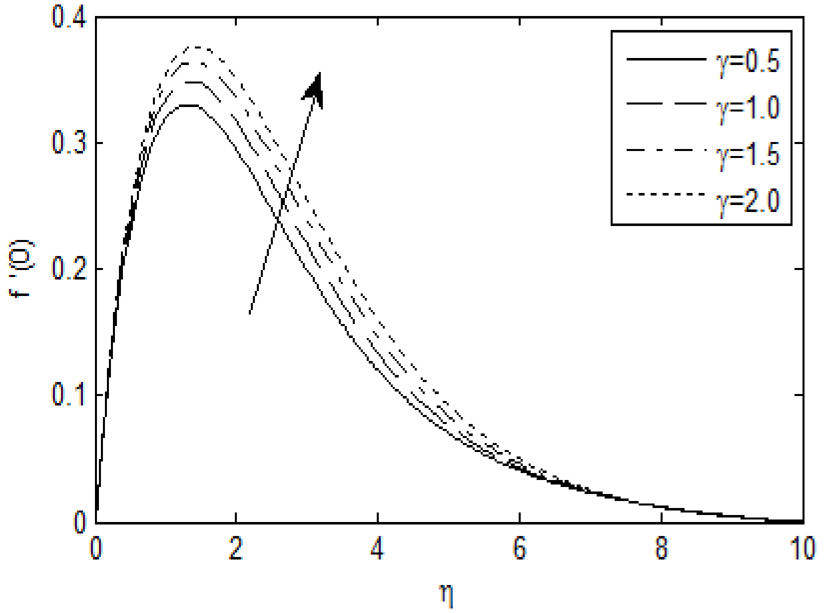

**Figure 12:** Velocity profile for variable thermal conductivity against $\eta$

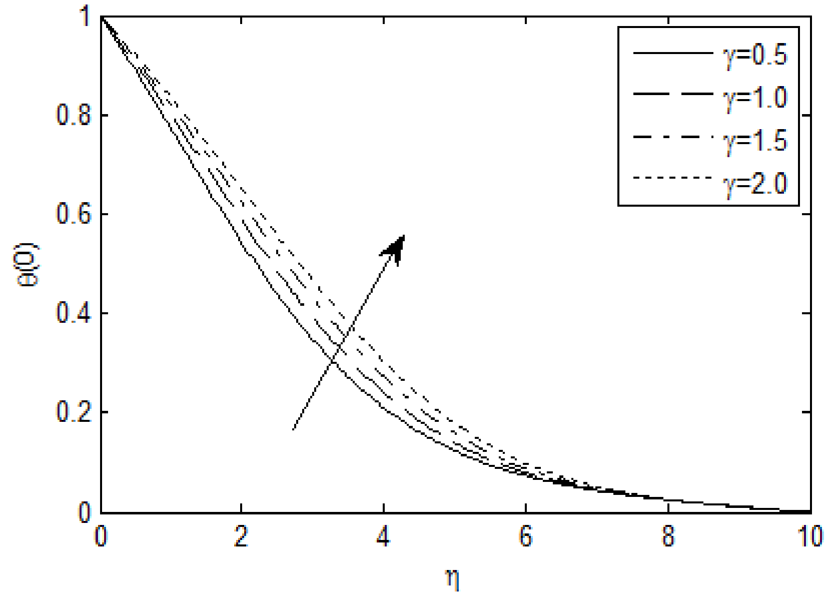

**Figure 13:** Temperature profile for variable thermal conductivity against $\eta$

### 3.2 Case II

For this case, we define some parameters in the dimensionless equations (13) and (14) by the following expressions

(i) $\delta = 0$ (ii) $Ec \neq 0$

The governing equations become

$$f''' + ff'' + Gr\theta - Mf' = 0 \tag{20}$$

$$[1+\gamma\theta]\theta'' + \gamma\theta'^2 + Prf\theta' + PrEcf''^2 = 0 \tag{21}$$

subject to the boundary conditions (15).

### 3.2.4 Numerical Computation

Here, the same numerical approach in Case I is applied to equations (20) and (21) under the boundary conditions (15).

Let $f = y_1,\ f' = y_2,\ f'' = y_3,\ \theta = y_4,\ \theta' = y_5$. The governing equations are transformed into a system of first order differential equations as follows:

$$y_1' = y_2$$

$$y_2' = y_3$$

$$y_3' = -y_1 y_3 - Gr y_4 + M y_2 \qquad (22)$$

$$y_4' = y_5$$

$$y_5' = \frac{1}{1+\gamma\theta}\left(-Pr y_1 y_5 - \gamma y_5^2 - Pr Ec y_3^2\right)$$

subject to the following initial conditions:

$$y_1(0) = 0, y_2(0) = 0, y_3(0) = s_1, y_4(0) = 1, y_5(0) = s_2 \qquad (23)$$

### 3.2.2 Discussion of Result

For the purpose of discussing the effects of various parameters on the flow profiles and the temperature distribution within the boundary layer, analysis has been carried out for various values of $Pr$, $Gr$, $M$, $\gamma$ and $Ec$. The values for the parameters are taken from Loganathan et al. [17], Elbashbeshy [18], Reddy and Reddy [19], Kishore et al. [4] respectively. The numerical results for the prescribed parameters $\gamma, Gr, M, Pr$ and $Ec$ is presented in figures 14 – 23.

**Table 2:** Numerical values for heat transfer rate $\theta'(\eta)$

| $\gamma$ | $Pr$ | $Gr$ | $M$ | $Ec$ | $\theta'(\eta)$ |
|---|---|---|---|---|---|
| 0.5 | 0.72 | 1 | 1.5 | 1 | - 0.1806 |
| 2.0 | 0.72 | 1 | 1.5 | 1 | - 0.1328 |
| 0.5 | 7.0 | 1 | 1.5 | 1 | - 0.3083 |
| 0.5 | 0.72 | 3 | 1.5 | 1 | 0.0774 |
| 0.5 | 0.72 | 1 | 2.5 | 1 | - 0.1735 |
| 0.5 | 0.72 | 1 | 1.5 | 3 | - 0.0450 |

Numerical values for heat transfer rate and skin friction are presented in tables 2 and 3.It is observed that increase in $Ec$, $\gamma$, $Gr$ and $M$ increases the rate of heat transfer but decreases with increasing values of $Pr$ as shown in table 2. For table 3, the numerical values for skin friction $(C_f)$ coefficient is presented.

**Table 3:** The rate of shear stress in terms of skin friction $(C_f)$

| $\gamma$ | $Pr$ | $Gr$ | $M$ | $Ec$ | $C_f$ |
|---|---|---|---|---|---|
| 0.5 | 0.72 | 1 | 1.5 | 1 | 0.6662 |
| 2.0 | 0.72 | 1 | 1.5 | 1 | 0.7058 |
| 0.5 | 7 | 1 | 1.5 | 1 | 0.5152 |
| 0.5 | 0.72 | 3 | 1.5 | 1 | 1.9140 |
| 0.5 | 0.72 | 1 | 2.5 | 1 | 0.5529 |
| 0.5 | 0.72 | 1 | 1.5 | 3 | 0.6881 |

The effects of the prescribed parameter on the skin friction $(C_f)$ coefficient shows that increase in $Pr$ and $M$ lead to a decrease in skin friction but increases with increasing values in $Ec$, $\gamma$ and $Gr$. The velocity profile for various values of Prandtl number is shown in figure 14. Also, as the velocity increases, it increases to a peak and begins to decrease exponentially to zero thereby satisfying boundary conditions. The result shows that velocity decreases with increase in $Pr$. While increase in $Gr$ leads to an increase in velocity indicating that buoyancy force assists the flow (Omowaye and Koriko [6], Chamkha [11]) as presented in figure 15. In figure 16, velocity decreases with increase in magnetic parameter thereby acting against the flow in the normal direction if applied and also, stabilizes the magnetic field. From figure 17, the result shows that as velocity increases, Eckert number increases also.

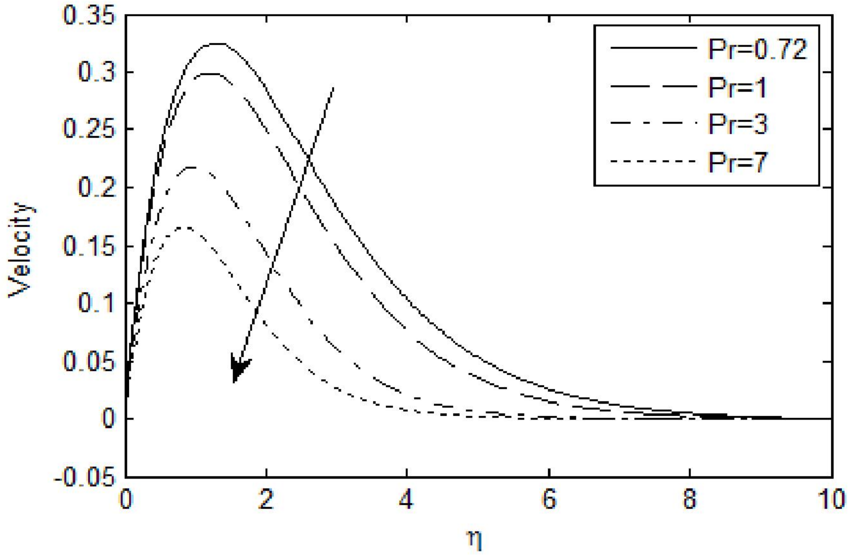


**Fig. 14:** Velocity profile for various values of $Pr$

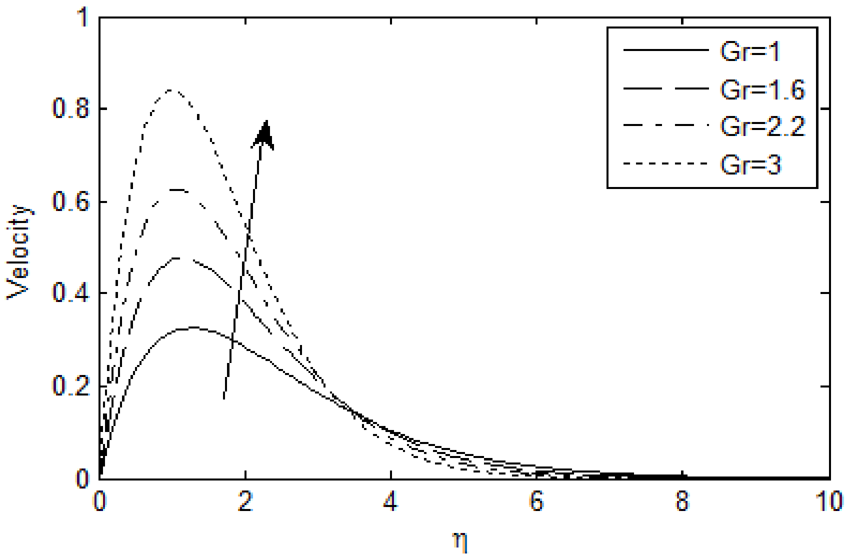


**Fig 15:** Velocity profile for different values of $Gr$

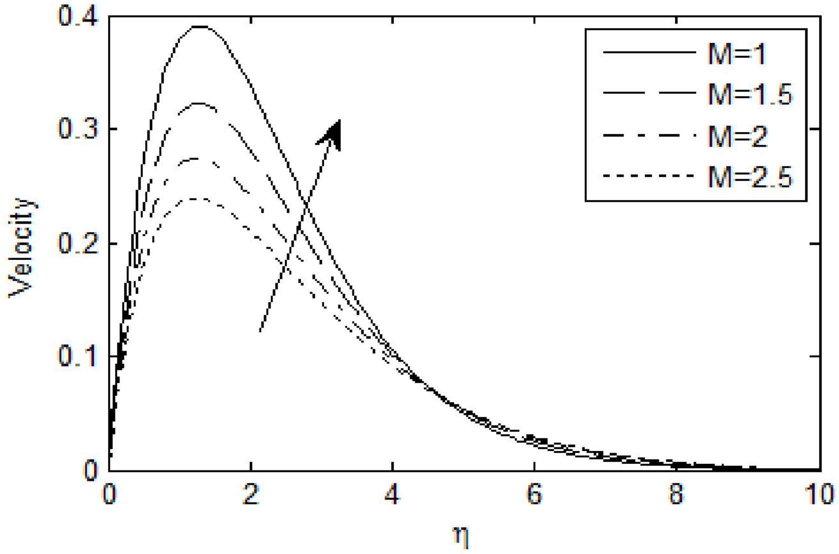


**Fig 16:** Velocity distribution against $\eta$ for various $M$

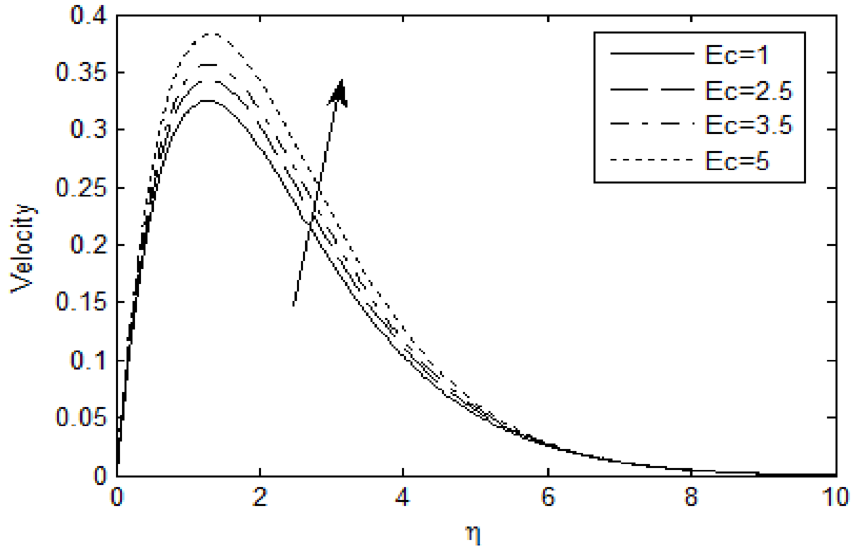


**Fig 17:** $f'(\eta)$ against $\eta$ for different values of $Ec$

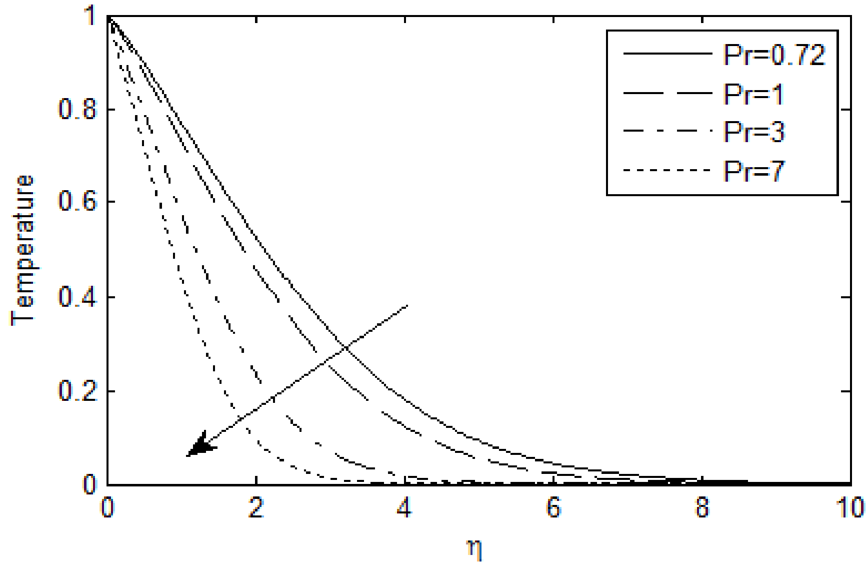


**Fig 18:** Temperature profile for various values of $Pr$

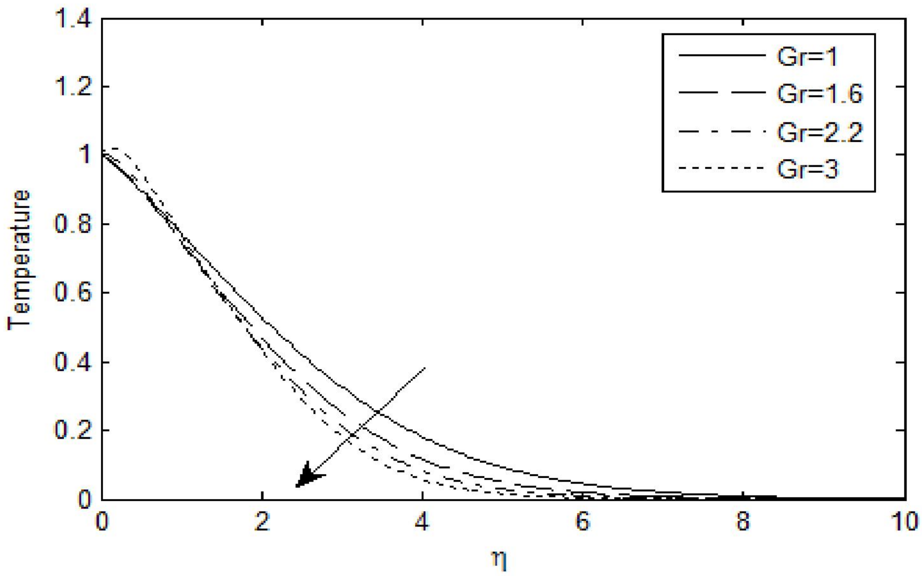


**Fig 19:** Temperature against $\eta$ for different values of $Gr$

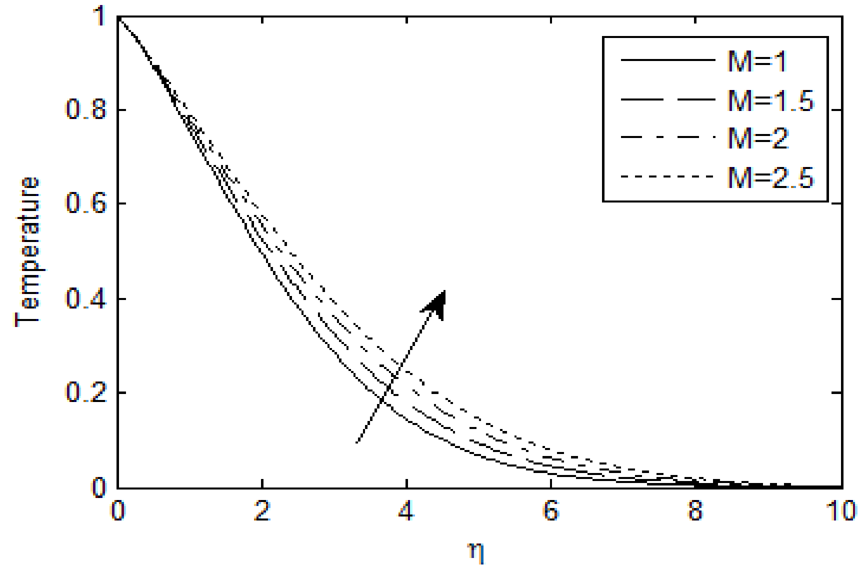


**Fig 20:** Temperature $\theta$ against $\eta$ for various values of $M$

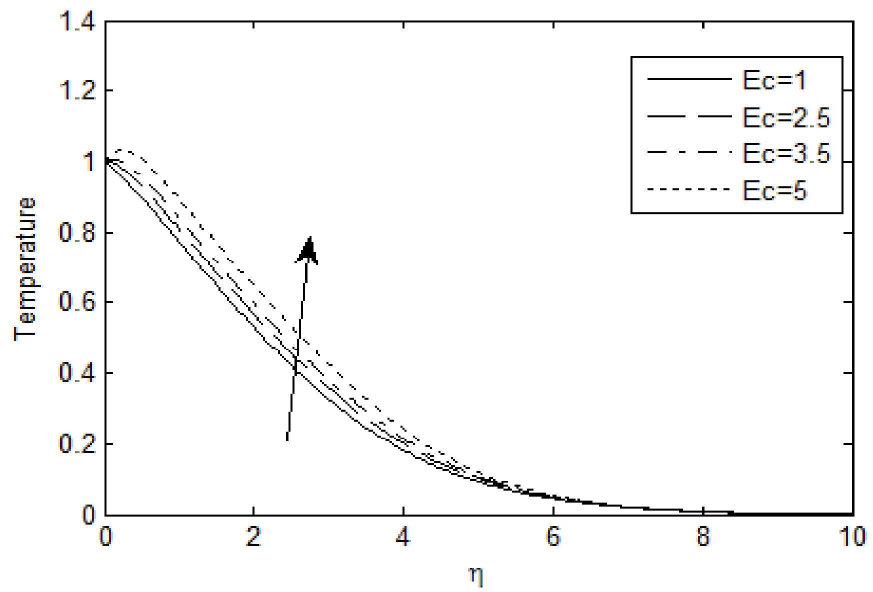


**Fig 21:** Variation at different values of $Ec$ for temperature profile

Temperature profiles for local Grashof number, Prandtl number, magnetic field parameter and Eckert number are presented in figures 18 – 21. It is reported in figure 18 and 19, that temperature $(\theta)$ decreases with increase in $Pr$ and $Gr$. Figure 20 and 21, shows that increase in the magnetic parameter and Ecker number lead to increase in temperature $(\theta)$ profile and it shows that $Ec$ has significant effect on the boundary layer growth and plays an important factor for heat transfer. Hence, from figures 14 – 21, temperature profile increases with increase in $M$ and $Ec$, while it decreases with increase in $Pr$ and $Gr$. Similarly, velocity decreases with increase in $Pr$ while $f'(\eta)$ increases with $Gr$, $M$ and $Ec$ increasing also.

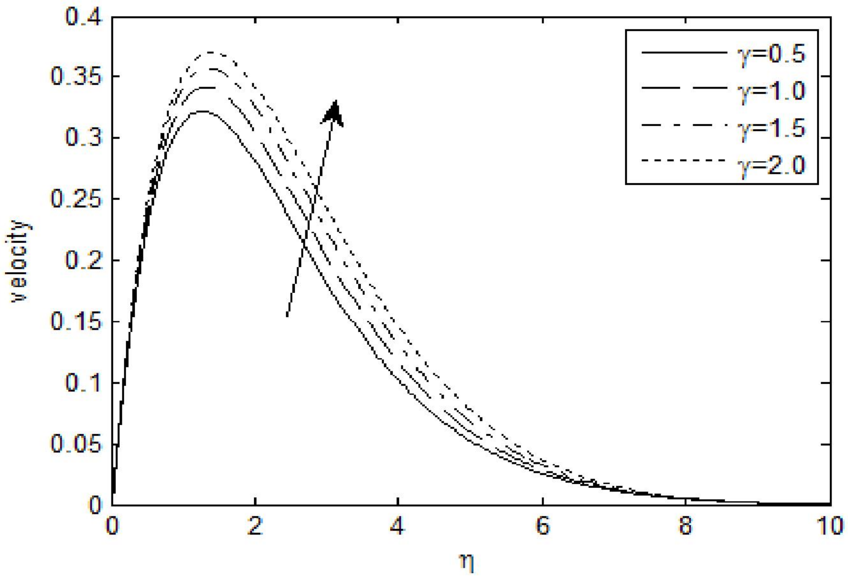


**Figure 22:** Velocity profile for thermal conductivity variation

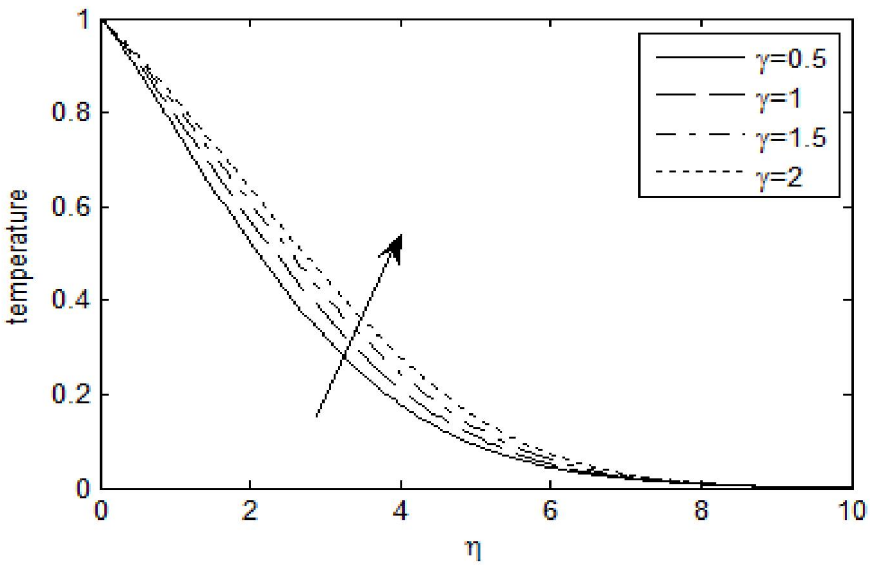


**Figure 23:** Temperature profile for thermal conductivity variation

Velocity and temperature profiles for variable thermal conductivity against $\eta$ are shown in figures 22 and 23. It is observed that as $\gamma$ increases in value, both velocity and temperature increases also away from the plate towards the free stream values.

## 4 Conclusion

The problem on effects of some thermo-physical properties on free convective heat and mass transfer of a reacting fluid flow vertical plate is considered. The governing partial differential equations of the problem, using similarity transformations, were reduced to a couple nonlinear differential equations and solved numerically using Runge-Kutta fourth order method with shooting technique. The main objective is to obtain a modified model for free convective flow over a vertical plate and numerical solution for the problem as well as establish and discuss the thermo-physical properties of the problem. To this end, the resulting coupled nonlinear ordinary differential equations (13) and (14) subject to the boundary conditions (15) was investigated under two cases and the following conclusions were drawn:

*i).* Skin-friction increases with increase in local Grashof number, variable thermal conductivity and modified Frank-Kamenetskii parameter but, decreases with increasing values in Prandtl number, magnetic parameter and activation energy in Case I. Similarly, increase in Prandtl number, activation energy and Grashof number, leads to a decrease in the value of Nusselt number but increases with increase in magnetic, variable thermal conductivity and modified Frank-Kamenetskii parameters respectively in Case I.

*ii).* In case II, increasing values in Eckert number, thermal conductivity variation, local Grashof number and magnetic field parameters, also increases the rate of heat transfer but decreases with increasing values in Prandtl number. Also, from case II, the skin friction coefficient increases with increasing values in $Ec$, $\gamma$ and $Gr$ but decreases with increase in values of $Pr$ and $M$

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