**Power and Size analysis of Cointegration**

**tests in Conditional Heteroskedascity:**

**A Monte Carlo Simulation**

Osabuohien-Irabor Osarumwense

Ambrose Alli University, Ekpoma, Nigeria

Faculty of Physical Sciences, Dept. of Mathematics

osabuohien247@gmail.com, osabuohienosa@aauekpoma.edu.ng

**ABSTRACT**

This paper is an extension idea of Kosapattarapim et al (2013) as well as Lee et al (1996). It investigates several co-integration tests when the co-integration innovations are heteroskedastic. The bivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH(1,1)) model of Bollerslev (1986) process with Gaussian innovations used in the data generating process (DGP). The power and size properties of the cointegration tests (Co-integration Regression Durbin-Watson test (1983), Eagle-Granger test (1987), Dicky-Fuller (1979), Johansen likelihood ratio tests (1988), Phillips-Ouliaris(1990)) are compared across range of samples based on the frequency of rejection of the hypothesis of a no co-integration. This study answers the question of which co-integration test is better, particularly between the Eagle-Granger two-step test and the Johansen’s tests for co-integration. With two sets of parameter models (persistence and spiky), our simulation results reveals that there is size distortion in the different co-integration test considered. The Eagle-Granger two-step test shows good robustness with respect to heteroskedasticity for the different sample sizes applied. However, the Johansen’s test for co-integration still proves to be powerful in capturing co-integration relationship when the co-integration innovations are Gaussian, particularly for large sample size.

**Key words:** Co-integration test, Size, Monte-Carlos Simulation, Power, Heteroskedasticity

**JEL Classifications:** C12, C15 C32

**1. INTRODUCTION**

The idea of co-integration relationship between two or more time series was originally proposed by Granger (1983). It implies that $y\_{t}$ and $x\_{t}$ variables shares similar stochastic trend. If their difference ($e\_{t}$) are similar, they will never diverge too far from each other. The test for co-integration is effectively a test of the stationarity of the residuals. If the residuals are stationary, then $y\_{t}$ and $x\_{t}$ are said to be co-integrated. But if residual are non-stationary, then $y\_{t}$ and $x\_{t}$ are not cointegrated and any apparent regression relationship between them, is said to be spurious (Hill,et al (2011)). The concept and application of co-integration in financial time series and econometric analysis has received much attention over the years, particularly, from practitioners who test the stationarity of econometric and financial time series variables. Over the years, a number of tests have been developed and used to examine the behavior of these co-integration errors and the relationship among time series.

Homoskedasticity is one of the basic conditions underlying the applicability of many financial and econometric analyses, and in essence, the co-integration test is to avoid spurious results affected by heteroskedasticity. Therefore, the need to evaluate the adequacies of the different co-integration tests with regards to its power and size properties cannot be overemphasized. The novelty of this study is not mainly on power and size properties of the co-integration tests as numerous literatures abound, but on whether the Eagle-Granger two-step approach and the Phillips-Ouliaris tests for co-integration analysis can replace, compete or be used as close substitute to the powerful Johansen’s test for co-integration. Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model of Bollerslev (1986) are today the most popular model among the Non-linear models. It captures the basic stylized facts such as persistence in financial data. GARCH (1,1) has also been proven to be sufficient in capturing these stylized fact.

 However, the aims and objective of this paper is to evaluate the power and size of different co-integration test when the co-integration innovations follow the Gaussian distribution. This paper is an extension of ideas of previous work by Kosapattarapim et al (2011), and Lee et al (1996) on the performance and properties of power and size of co-integration test.

This rest of this paper is organized as follows; Section 2 deals with the literature review of previously related studies on co-integration tests and analysis. In section 3, we present the methodology of the study using simulation-based data generating process (DGP), while section 4 and 5 contains the analysis of results of our simulation experiment with regards to power and size of co-integration test.

**2. LITERATURE REVIEW**

The performance of some co-integration tests have been examined with different models and distribution innovations. Hence, numerous literatures abound. Boswijk etal (1999) consider the performance of several contemporary tests based on Gaussian quasi-likelihoods using either the kernel estimation or semi-nonparametric density approximation. They observed that, in small samples, the overall performance of the semi-nonparametric approach appears best in terms of size and power. In large samples and for heavily skewed or mul-timodal distributions,the kernel based adaptive method dominates.

Cavaliere et al, (2007), analysed the vector autoregression with non stationary volatility of general form. Their analysis revealed that the conventional rank statistics of Johansen are potentially unreliable. They also showed that large sample distribution depends on the integrated covariation of the underlying multivariate volatility process which impacts on both the size and power of the associated co-integration tests.

 Lee and Tse (1996), fitted GARCH (1,1) model with normal and student’s-t distribution to examined the performance of Johansen’s likelihood ratio, DF, and CRDW co-integration tests for co-integration. The Johansen tests over-reject the null hypothesis of no co-integration, and proved to be a higher test than the other two tests. Cheung and Lai (1993), examined the effect of non Gaussian error distribution on the performance of Johansen’s co-integration test. Statistical analysis shows that the Johansen tests are still very roboust to both skewness and excess kurtosis of co-integration errors.

Other researchers include; Kim and Schmidt (1993), showed that the DF tests tend to overreject the null hypothesis of no co-integration in the presence of GARCH (1,1) errors, for investigation involving the size test. However, the problem is not very serious except when the variance process is nearly degenerate (in the sense that the ratio of the GARCH intercept to the initial variance is near zero) and the volatility parameter is large. Gerolimetto and Procidano (2003), investigated and compared the Wild Bootstrap test (proposed by Procidamo et al (1999, 2000) with the DF test, CRDW test and the Johansen’s test when co-integration errors follows GARCH(1,1) model with normal random error distribution. They also showed that the Wild Bootstrap test is roboust to heteroskedasticity.

This paper is an extension idea of Kosapattarapim et al (2013) as well as Lee and Tse (1996), with increase in sample sizes, addition of other popularly use co-integration tests like the Eagle-Granger two-step test, the Phillips-Ouliaris etc, use of different simulation design, use of two sets of co-integration innovations from different GAR CH(1,1) models with Gaussian innovation distribution.

**3. METHODOLOGY**

This paper applied simulation-based approach to assess the power and size properties of co-integration tests (CRDW, Eagle-Granger two-step test, DF, Johansen, Phillips-Ouliaris) when innovation is Gaussian GARCH (1,1) distributed. The comparison of co-integration test is done with the frequency of rejection of the null hypothesis of a no co-integration.

***Design of the Monte-Carlo simulation-based experiment***

The Monte-Carlo Simulation is carried out by generating artificial data series of ‘daily’ time series $y\_{t}$ and $x\_{t}$.The data generating process (DGP) will be repeated for and the first value of  observations are discarded. This help in removing the initial value effect. An independent simulation of ** replications are carried out for a variety of sample sizes respectively from the designed system and apply to the six cointegration tests (CRDW, Eagle-Granger two-step test, DF, Johansen (trace test and the maximum eigenvalue test , Phillips-Ouliaris). The comparison of the different co-integration tests are considered based on the frequency of rejection of the null hypothesis of no co-integration.

***The Size test***

The size of a test is the probability that the null hypothesis is rejected when it is true. It is evaluated by the frequency of the null hypothesis stating that truly non co-integrated series is not co-integrated is rejected in  trials. To investigate the size properties of co-integration test, a simulated sample sizes from bivariate non co-integrated system $y\_{t}=\left(y\_{1t}, y\_{2t}\right),$ with GARCH (1,1) Gaussian innovations are generated $ε\~NID(0,H\_{t})$. Where; $H\_{t}$ is the variance from the GARCH (1,1) model.

Let be an $NXN$ vector of cointegration series.

The bivariate system is given as;

 

 

The errors and follows GARCH (1,1) model

 

***The Power test***

The power of a test is the probability that the null hypothesis is rejected when the null hypothesis is false. The frequencies of rejection of no co-integration from a co-integrated system are counted. To examine the power of co-integration test, a co-integrated system is simulated. In this paper, the Phillips’ (1991) triangular representation equations as simulated and applied by Zivot (2000) and Fassati (2013), are used in simulating a bivariate co-integration system. It’s of the form;

 

 

Where and follows GARCH (1,1) Gaussian innovation model

Therefore;

 

 

 

Where are co-integrated residual with AR(1) of 

The hypotheses to be tested are;

 ($No cointegration)$

 ($Cointegration)$

**4. RESULTS**

The results shown in tables $1,2,3$ and $4$ clearly report the two different GARCH (1,1) parameter models used in the analysis of the power and size properties of the several co-integration test considered in this paper. The probability distribution of the innovations of GARCH (1,1) models (and ) are Gaussian distributed.

|  |
| --- |
|   **Table 1: Proportion of Rejects for Size of the Test**  |
| **TEST** | **T=100** | **T=500** | **T=750** | **T=1500** | **T=2000** |
| **CRDW** | 0.094 | 0.097 | 0.003 | 0.081 | 0.091 |
|

|  |
| --- |
|  **Engle-Granger** |

 | 0.186 | 0.002 | 0.016 | 0.091 | 0.272 |
|  | 0.272 | 0.091 | 0.284 | 0.364 | 0.182 |
|  | 0.351 | 0.023 | 0.014 | 0.025 | 0.454 |
|  | 0.091 | 0.004 | 0.029 | 0.007 | 0.364 |
| **Phillips-Ouliaris** | 0.175 | 0.179 | 0.011 | 0.161 | 0.187 |
|  |  |  |  |  |  |

  

 The rejection frequencies are at 5% critical value for sets parameter, with

 Garch(1,1) Gaussian innovations distributed. The co-integration tests are

 applied in  replication from the designed experiment for size test analysis

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|  **Table 2: Proportion of Rejects for Power of the Test**  |
|  **TEST** | **T=100** | **T=500** | **T=750** | **T=1500** | **T=2000** |
|  **CRDW** | 0.636 | 0.182 | 0.636 | 0.545 | 0.364 |
|

|  |
| --- |
|  **Engle-Granger** |

 | 1.000 | 1.000 | 1.000 | 1.000 | 0.636 |
|  | 0.454 | 0.091 | 1.180 | 0.545 | 0.273 |
|  | 0.636 | 1.000 | 0.983 | 1.000 | 1.000 |
|  | 0.272 | 0.919 | 0.954 | 1.000 | 1.000 |
|  **Phillips-Ouliaris** | 0.185 | 0.096 | 0.018 | 0.166 | 0.273 |
|  |  |  |  |  |  |

  

 The rejection frequencies are at 5% critical value for sets parameter, with

 Garch(1,1) Gaussian innovations distributed. The co-integration tests are

 applied in  replication from the designed experiment for power test anlysis

When the GARCH parameter models is persistence, there is size distortion in all the co-integration tests across the different sample sizes – see Table 1. The Dickey Fuller test (1979) has the largest values of size distortion when co-integrated innovations are Gaussian distributed. However, in the power test, the Eagle-Granger two-step test and the Johansen’s test are much higher and consistence when compare to the other tests. In small sample sizes, the Eagle-Granger two-step co-integration test shows to be more powerful than the Johansen test. Results are reported in Table 2.

Just like in the persistence parameter models, there is irregularity and distortion in size as the sample size increases, when the model is spiky. Table 3 also reveals, that the CRDW and the Johansen’s Max co-integration test have smaller size distortion when compare to the other co-integration tests. The simulation analysis proves that the Eagle-Granger two-step test is as powerful as the Johansen’s trace test, but slightly more powerful than the Johansen’s Max test. Besides, the Phillips-Ouliaris test shows to be poor co-integration test when co-integration innovations are distributed Gaussian − see Table 4

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|  **Table 3: Proportion of Rejects for Size of the Test**  |
|  **TEST** | **T=100** | **T=500** | **T=750** | **T=1500** | **T=2000** |
|  **CRDW** | 0.162 | 0.027 | 0.060 | 0.004 | 0.013 |
|

|  |
| --- |
|  **Engle-Granger** |

 | 0.195 | 0.250 | 0.137 | 0.182 | 0.100 |
|  | 0.182 | 0.167 | 0.014 | 0.091 | 0.102 |
|  | 0.092 | 0.177 | 0.011 | 0.182 | 0.013 |
|  | 0.087 | 0.018 | 0.063 | 0.010 | 0.044 |
|  **Phillips-Ouliaris** | 0.182 | 0.250 | 0.007 | 0.318 | 0.100 |
|  |  |  |  |  |  |

  

 The rejection frequencies are at 5% critical value for sets parameter, with

 Garch(1,1) Gaussian innovations distributed. The co-integration tests are

 applied in  replication from the designed experiment for size test analysis

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|  **Table 4: Proportion of Rejects for Power of the Test**  |
| **TEST** | **T=100** | **T=500** | **T=750** | **T=1500** | **T=2000** |
| **CRDW** | 0.727 | 0.636 | 0.300 | 0.727 | 0.918 |
|

|  |
| --- |
|  **Engle-Granger** |

 | 0.888 | 0.988 | 1.000 | 1.000 | 1.000 |
|  | 0.182 | 0.182 | 0.300 | 0.019 | 0.020 |
|  | 0.909 | 0.999 | 1.000 | 1.000 | 1.000 |
|  | 0.818 | 0.901 | 1.000 | 1.000 | 1.000 |
|  **Phillips-Ouliaris** | 0.061 | 0.273 | 0.178 | 0.242 | 0.113 |
|  |  |  |  |  |  |

 

 The rejection frequencies are at 5% critical value for sets parameter, with

 Garch(1,1) Gaussian innovations distributed. The co-integration tests are

 applied in  replication from the designed experiment for power test analysis

Figs 1- 6, shows some selected series, variance series, and GARCH series for both the cointegrated and non-cointegrated series for the different sample sizes. Trials of 3,000 replications in five different sample sizes are voluminous, hence the author showing seletced copies. Request of full analysis with graphical details, kindly email the author.

**5. CONCLUSION**

A Monte Carlo simulation experiment was performed to assess the power and size properties of six different co-integration tests in five different sample sizes, with Gaussian innovation. Two different sets of GARCH parameter models were experimented. We can summarize that there is size distortion in the several tests analyzed. And the power of the Eagle-Granger two-step test is quiet robust to heteroskedicity and as powerful as the Johansen’s trace test. The Phillip-Ouliaris co-integration test proves not to be good co-integration test.

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**APPENDIX I**

 Non cointegrated series for sample size 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

-5.0

-2.5

0.0

2.5

5.0

7.5

10.0

12.5

15.0

 Cointegrated series for sample size 100of

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

0.0

2.5

5.0

7.5

10.0

12.5

 Non cointegrated series for sample size 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-15

-10

-5

0

5

10

15

20

25

30

 Cointegrated series for sample size 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-15

-10

-5

0

5

10

15

20

25

 Non cointegrated series for sample size 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-20

-10

0

10

20

30

40

50

 Cointegrated series for sample size 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-20

-15

-10

-5

0

5

10

15

 Non cointegrated series for sample size 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-50

-40

-30

-20

-10

0

10

20

30

 Cointegrated series for sample size 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-50

-40

-30

-20

-10

0

10

20

 Non cointegrated series for sample size 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-30

-20

-10

0

10

20

30

 Cointegrated series for sample size 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-15

-10

-5

0

5

10

15

20

25

30

Fig 1: Graphs showing some selected co-integrated and non co-integrated series with different sample sizes for the parameter sets 

 Simulated variance for second series with sample size of 100

 Simulated variance for first series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

0.6

0.7

0.8

0.9

1.0

1.1

1.2

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

0.6

0.7

0.8

0.9

1.0

1.1

1.2

1.3

1.4

1.5

 Simulated variance for first series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

0.6

0.8

1.0

1.2

1.4

1.6

1.8

2.0

 Simulated variance for second series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

0.6

0.8

1.0

1.2

1.4

1.6

1.8

2.0

2.2

2.4

 Simulated variance for first series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

0.6

0.8

1.0

1.2

1.4

1.6

1.8

2.0

2.2

2.4

Simulated variance for second series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

0.6

0.8

1.0

1.2

1.4

1.6

1.8

 Simulated variance for first series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

0.50

0.75

1.00

1.25

1.50

1.75

2.00

2.25

2.50

2.75

 Simulated variance for second series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

0.6

0.8

1.0

1.2

1.4

1.6

1.8

2.0

Simulated variance for first series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

0.50

0.75

1.00

1.25

1.50

1.75

2.00

2.25

2.50

2.75

 Simulated variance for second series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

0.6

0.8

1.0

1.2

1.4

1.6

1.8

2.0

2.2

 Fig 2: Graphs showing some selected simulated variance series with different sample sizes for

 parameter sets  

 First simulated series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

-2

-1

0

1

2

3

4

Second simulated series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

-3

-2

-1

0

1

2

3

 First simulated series with sample size of 500

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-4

-3

-2

-1

0

1

2

3

4

 Second simulated series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-4

-3

-2

-1

0

1

2

3

 First simulated series with sample size of 750

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-4

-3

-2

-1

0

1

2

3

 Second simulated series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-4

-3

-2

-1

0

1

2

3

4

 First simulated series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-5

-4

-3

-2

-1

0

1

2

3

4

 Second simulated series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-4

-3

-2

-1

0

1

2

3

 First simulated series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-4

-3

-2

-1

0

1

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4

 Second simulated series with sample size of 2000

200

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600

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1200

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Fig 3: Graphs showing some selected simulated Garch series with different sample sizes for the parameter set 

 Non cointegrated series for sample size 100

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 Cointegrated series for sample size 100

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-4

-2

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4

 Non cointegrated series for sample size 500

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-10.0

-7.5

-5.0

-2.5

0.0

2.5

5.0

7.5

 Cointegrated series for sample size 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-15.0

-12.5

-10.0

-7.5

-5.0

-2.5

0.0

2.5

5.0

7.5

 Non cointegrated series for sample size 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-10

-5

0

5

10

15

20

25

30

 Cointegrated series for sample size 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-10

-5

0

5

10

15

20

25

30

35

 Non cointegrated series for sample size 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-30

-20

-10

0

10

20

30

40

 Cointegrated series for sample size 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-30

-20

-10

0

10

20

30

40

50

 Non cointegrated series for sample size 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-50

-40

-30

-20

-10

0

10

20

30

40

 Cointegrated series for sample size 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-60

-50

-40

-30

-20

-10

0

10

 Fig 4: Graphs showing some selected co-integrated and non co-integrated series for different

sample sizes for parameter sets 

 Simulated variance for first series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

0

1

2

3

4

5

6

 Simulated variance for second series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

0.0

0.5

1.0

1.5

2.0

2.5

 Simulated variance for first series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

0.0

0.5

1.0

1.5

2.0

2.5

3.0

3.5

4.0

4.5

 Simulated variance for second series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

0.0

2.5

5.0

7.5

10.0

 Simulated variance for first series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

0.0

2.5

5.0

7.5

10.0

 Simulated variance for second series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

0.0

2.5

5.0

7.5

10.0

12.5

15.0

17.5

20.0

 Simulated variance for first series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

0.0

2.5

5.0

7.5

10.0

12.5

 Simulated variance for second series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

0

10

20

30

40

50

60

 Simulated variance for first series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

0.0

2.5

5.0

7.5

10.0

12.5

 Simulated variance for second series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

0.0

2.5

5.0

7.5

10.0

12.5

15.0

17.5

20.0

 Fig 5: Graphs showing some selected simulated variance series for different sample sizes for

 parameter sets  

 First simulated series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

-3

-2

-1

0

1

2

3

 Second simulated series with sample size of 100

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

-2.0

-1.5

-1.0

-0.5

0.0

0.5

1.0

1.5

2.0

 First simulated series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-2.5

-2.0

-1.5

-1.0

-0.5

0.0

0.5

1.0

1.5

2.0

 Second simulated series with sample size of 500

25

50

75

100

125

150

175

200

225

250

275

300

325

350

375

400

425

450

475

500

-5.0

-2.5

0.0

2.5

5.0

 First simulated series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-3

-2

-1

0

1

2

3

4

5

 Second simulated series with sample size of 750

50

100

150

200

250

300

350

400

450

500

550

600

650

700

750

-7.5

-5.0

-2.5

0.0

2.5

5.0

7.5

10.0

12.5

 First simulated series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-5

-4

-3

-2

-1

0

1

2

3

 Second simulated series with sample size of 1500

100

200

300

400

500

600

700

800

900

1000

1100

1200

1300

1400

1500

-7.5

-5.0

-2.5

0.0

2.5

5.0

7.5

10.0

 First simulated series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-5

-4

-3

-2

-1

0

1

2

3

 Second simulated series with sample size of 2000

200

400

600

800

1000

1200

1400

1600

1800

2000

-6

-4

-2

0

2

4

6

Fig 6: Graphs showing some selected simulated Garch series with different sample sizes for the parameter sets  

**APPENDIX II**

**CODES FOR RATS PROGRAMMING**

**\***

\*

\*

seed

all 1500

compute n=2

\* u is the simulated GARCH process\*

dec vect[series] u(n)

\* H is the series of variance matrices, UU is the series of lagged outer products of the residuals\*

dec series[symm] h uu

dec symm vc(n,n) va(n,n) vb(n,n)

compute vc=||.05|.01,.05||

compute va=||.90|.85,.90||

compute vb=||.05|.05,.05||

\* Compute the stationary variance\*

dec sym h0(n,n)

ewise h0(i,j)=vc(i,j)/(1-va(i,j)-vb(i,j))

dec frml[symm] hf hu

dec symm hx

dec vect ux

frml hf = vc+va.\*h{1}+vb.\*uu{1}

frml hu = hx=hf,ux=%ranmvnormal(%decomp(hx)),uu=%outerxx(ux),%pt(u,t,ux),hx

\* Initialize the UU and H series for the pre-sample values

gset uu = h0

gset h = h0

\* Generate starting in entry 2.

gset h 2 1500 = hu(t)

\* Graph the series and variance series

graph(footer="First simulated series")

# u(1)

graph(footer="Second simulated series")

# u(2)

set h11 = h(t)(1,1)

graph(footer="Simulated variance for first series")

# h11

set h22 = h(t)(2,2)

graph(footer="Simulated variance for first series")

# h22

set et1 = u(1)

set et2 = u(2)

\*

\* SIZE TEST

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

set(first=0.0), y1 = y1{1}+et1

set(first=0.0), y2 = y2{1}+et2

graph(footer=" NON COINTEGRATED SERIES "), 2

# y1

# y2

@DFUNIT y1

@DFUNIT y2

LINREG y1 / resids

# constant y2

@DFUNIT resids

\* (1)

\*\*\*COINTEGRATING REGRESSION DURBIN-WATSON TEST(CRDW)\*\*\*\*\*\*\*\*

linreg y1

# constant y2

\* (2)

\*\*\*ENGLE-GRANGER (E-G) TEST\*\*\*\*\*\*\*\*

@egtest(lags=5,method=aic)

# y1 y2

\* (3)

\*\*\*DICKEY-FULLER TEST\*\*\*\*\*\*\*\*\*\*\*

@DFUNIT(lags=5,nottest), y1

@DFUNIT(lags=5,nottest), y2

\* (4)

\*\*\* PHILLIPS-OULIARIS TEST\*\*\*\*\*\*\*\*\*

@potest(lags=5)

# Y1 Y2

\*

@varlagselect(crit=aic,lags=10)

# y1 y2

\*

\*\*\*(5)Trace (6) Maxeigen\*\*\*\*\*\*\*\*\*

\* \*\*\*\*\*\*JOHANSEN TEST(TRACE & MAXEIGEN)\*\*\*\*\*\*\*\*\*\*\*

@johmle(lags=1,det=rc,cv=cvector)

# y1 y2

\*

\*

\* POWER TEST

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

set(first=0.0), ut1 = 0.75\*ut1{1}+et1

set(first=0.0), y22 = y22{1}+et2

set(first=0.0), y11 = y22+ut1

graph(footer=" COINTEGRATED SERIES ") 2

# y11

# y22

@DFUNIT y11

@DFUNIT y22

LINREG y11 / resids

# constant y22

@DFUNIT resids

\*

\*

\* (1)

\*\*\*COINTEGRATING REGRESSION DURBIN-WATSON TEST(CRDW)\*\*\*\*\*\*\*\*

linreg y11

# constant y22

\* (2)

\*\*\*Engle-Granger (E-G) test with fixed lags\*\*\*\*\*\*\*\*

@egtest(lags=5,method=aic)

# y11 y22

\* (3)

\*\*\*DICKEY-FULLER TEST\*\*\*\*\*\*\*\*

@DFUNIT,(lags=5,nottest) y11

@DFUNIT,(lags=5,nottest) y22

\* (4)

\*\*\* PHILLIPS-OULIARIS TEST\*\*\*

@potest(lags=5)

# Y1 Y2

@varlagselect,(crit=aic,lags=10)

# y11 y22

\*

\*\*\*(5)Trace (6) Maxeigen\*\*\*\*\*\*\*\*\*

\*\*\*JOHANSEN TEST(TRACE & MAXEIGEN)\*\*\*

@johmle,(lags=1,det=rc,cv=cvector)

# y11 y22

\*

\*

\*

End