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# Optimal Portfolio with Sustainable Attitudes under Cumulative Prospect Theory

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## Abstract

In the last five years, extreme events such as the COVID-19 pandemic and the Ukrainian crisis have highlighted the importance of corporate social responsibility and sustainable principles. Consequently, the investment process is changing toward more ethical choices. In this context, we extend the classical optimization framework under the cumulative prospect theory (CPT) in two directions. We first consider an agent who maximizes a financial CPT-value function preselecting the assets to be included in the portfolio based on their environmental, social, and governance (ESG) scores. Then, we develop a bi-objective model that optimizes financial and sustainable CPT-value functions at the same time. Numerical results obtained on an investable universe from the constituents of the STOXX Europe 600 show that introducing ESG information improves the portfolio's financial performance.

**JEL classification numbers**: C63, G11, G17

**Keywords**: Cumulative prospect theory, ESG scores, portfolio optimization, genetic algorithm, European stock exchange

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# 1 Introduction

The portfolio selection process usually involves two stages. The first consists in analyzing the most promising assets from a pool. The second focuses on the most desirable combination of the selected securities, ending with a portfolio choice.

The first theoretical framework, developed to support investor's decisions under uncertainty, is Von Neumann and Morgenstern's representation theorem [1], which represents the cornerstone of the so-called expected utility theory. According to this result, the choices of a rational investor can be described by a utility function, which is concave since agents are usually assumed to be risk-averse. Furthermore, when choosing among alternative investments, agents select the ones that maximize their expected utility. In the same direction, Markowitz [2] introduced the so-called mean-variance (MV) analysis, which has become the foundation of the modern portfolio theory. Following this principle, investors should maximize the expected return while retaining a given level of volatility. The MV framework can be derived as a particular case of the expected utility maximization when a quadratic utility function approximates the agent's behavior in the risk-return trade-off.

Over the years, the expected utility theory has become the normative model of rational choice, and the MV approach has been widely accepted as a practical tool for portfolio optimization among researchers and practitioners [3]. However, several financial paradoxes have evidenced that, beyond risk aversion, some behavioral aspects deviate from the implications of expected utility theory (see, for instance, [4] and [5]). To solve these puzzles, Kahneman and Tversky [6] have developed the so-called prospect theory (PT), which incorporates human psychology into financial decisions and asserts that investors consider the deviations of their terminal wealth from a subjective reference point as gains and losses. Agents react differently with respect to negative and positive outcomes, following a non-smooth, asymmetrical S-shaped value function that is concave in the domain of gains (risk-aversion) and convex in the domain of losses (risk-propensity). Later, Tversky and Kahneman [7] have refined the original PT formulation and developed the so-called cumulative prospect theory (CPT). This extension adds a transformation for the probability of the realizations of gains and losses, which overweights small probabilities and underweights higher ones.

Recently, researchers have shown more interest in behavioral finance, applying the CPT theoretical framework to the asset allocation practice. De Giorgi and Hens [8] have built up a theoretical analysis of an asset allocation problem where they have maximized an S-shaped utility function that generalizes the one introduced by Tversky and Kahneman. The resulting optimization problem is non-smooth, and calculating its solutions can be challenging. Thus, to overcome this problem, De Giorgi et al. [9] have developed a robust technique to compute optimal CPT-based portfolios. Pirvu and Shutze [10] have considered how PT-investors allocate their wealth in a single period between a riskless bond and multiple risky assets. Hens and Mayer [11] have compared the financial performance of the CPT-based asset allocation strategy with respect to the classical mean-variance portfolio. Consigli et al. [12] have performed a sensitivity analysis of the impact of CPT parameters on the mean-risk efficient frontier.

It is worth mentioning that the above-quoted papers rely on restrictive assumptions on the number of assets and on the distribution of their returns to compute the solutions of the CPT-based portfolio optimization problems. To avoid these issues, several authors have used stochastic optimization algorithms, that do not require any assumptions on the objective function to optimize. In particular, genetic algorithms (GAs) have proved efficient for solving portfolio selection problems. Some significant examples are the papers of Chang et al. [13], Yang [14], Sefiane and Benbouziane [15], and Rankovic et al. [16], which have used GAs to compute the optimal solutions of MV models. Moreover, GAs have also been adopted for solving PT-based portfolio problems. Grishina et al. [17] have proposed several intelligent algorithms to obtain an accurate solution in optimization problems involving the original PT. Gong et al. [18] have extended this research by considering the re-weighted probabilities and have introduced a hybrid optimization method that combines a bootstrap technique and a genetic algorithm.

Environmental, social, and governance (ESG) issues and non-financial disclosure have recently gained considerable attention. In 2015, the European Union has introduced Directive 2014/95/EU to improve the quality of corporate non-financial disclosure. Therefore, political institutions, portfolio managers, and new generations of investors show an increasing sensitivity toward the impact of investment choices on society and environment. Debate on green and sustainable transitions in finance and investments is growing. Several studies have extended the traditional MV model with the inclusion of a third criterion related to sustainability (see, for instance, Hirschberger et al. [19] and Utz et al. [20]). Jessen [21] has incorporated investors' ESG tastes into the Markowitz model by adding the weighted sum of the constituents' ESG scores as a constraint of the portfolio optimization problem. Pedersen et al. [22] have computed the ESG-efficient frontier, displaying the highest possible Sharpe ratio for each ESG score. Then, they have observed that ethical investors should be willing to accept lower portfolio returns in exchange for a more sustainable portfolio choice. Mimicking the stock-picking strategies used by portfolio managers to select the most promising assets based on their financial performance, Liagkouras et al. [23] have developed a screening procedure to identify a subset of ESG-compliant stocks as constituents of a MV portfolio. Then, they have tested this approach through a real-world application on the European market. In parallel with the above study, Yu et al. [24] have selected the most promising assets according to their ESG score from the Chinese stock market to construct an ethical CPT-based portfolio.

While these literature contributions build on deterministic quantities of sustainability (e.g., the ESG scores), Dorfleitner and Utz [25] have introduced the concept of sustainable returns obtained from firms' ESG scores. An ethical investor would prefer positive sustainable returns rather than negative ones. Then, they have implemented this novelty in the MV portfolio selection framework. Later, Dorfleitner and Nguyen [26] have integrated the concept of sustainability returns into the utility theory setting. More specifically, they have proposed an asset allocation model maximizing a convex combination of two utility functions linked to financial and sustainable returns, respectively.

There is a gap in the current literature concerning the extension of CPT-based portfolio models to account for investors' ESG attitudes. This paper aims to fill this issue. On the one hand, we propose two optimization models that extend the CPT-based portfolio selection framework. In the first model, we adopt several preselection techniques that identify a subset of ESG-compliant stocks based on their ESG scores. Then, we find the optimal portfolio weights according to the CPT-based asset allocation strategy. This approach generalizes the studies of Liagkouras et al. [23] and Yu et al. [24]. In the second model, we account for investors who could be sensitive to the ESG scores and the evolution of these values over time. To formalize this insight, we propose a novel portfolio design that involves CPT-value functions for the financial and sustainable returns as objectives to maximize simultaneously.

On the other hand, we present a real-world case study that involves a set of securities from the constituents of the STOXX Europe 600 index, covering the period from 2015 to 2021. The choice to use a European market index and to start the analysis in 2015 is coherent with the introduction of disclosure rules in Europe and the increase of their importance, especially for listed companies, that have to follow specific and strong rules on non-financial disclosure. We perform an ex-post analysis with a rolling window approach that covers the period from 2019 to 2021, involving the recent phases of market downturns due to the COVID-19 outbreak.

Following the literature, we exploit evolutionary computation to find the optimal portfolio weights for our models. In particular, for the bi-objective case, we use a variant of the GA suited for multi-objective portfolio problems, the so-called improved NSGA-II (shortly, iNSGA-II), proposed by Kaucic et al. [27].

We organize this paper as follows. Section 2 introduces the theoretical framework and presents the proposed CPT-based asset allocation models. Section 3 describes the GAs used to solve the optimization problems, whereas Section 4 presents the experimental results of the out-of-sample analysis implemented on the European stock market. Section 5 concludes with some remarks and future research directions.

# 2 Theoretical framework

## 2.1 Financial and sustainable returns

In this study, we consider a frictionless financial market where investors act as price takers and short selling is not allowed. The investable universe is represented by $n$ risky assets, and a portfolio is denoted by the vector $\boldsymbol{x} = (x_1, \ldots, x_n)^\top$ of its asset weights. We observe the market over a time window of $T + 1$ months $\{0, 1, \ldots, T\}$. We denote by $p_{i,t}$

the observed price at time $t$ of asset $i$, where $i = 1, \dots, n$ and $t = 0, 1, \dots, T$. Then, the realized rate of return at time $t$ for the $i$-th stock is defined as follows:

$$r_{i,t} = \frac{p_{i,t}}{p_{i,t-1}} - 1 \tag{1}$$

where $i = 1, \dots, n$ and $t = 1, \dots, T$. Next, we define the portfolio rate of return at time $t$ as $r_{p,t} = \sum_{i=1}^{n} x_i r_{i,t}$.

Following [25] and [28], we introduce a performance ratio that uses ESG information in order to obtain the sustainable counterpart of the financial rate of return. To this end, we consider the so-called relative sustainability value, which is calculated as the ratio between the company ESG score and the average ESG score of all companies in the current investable universe, that is:

$$s_{i,t} = \frac{ESG_{i,t}}{\overline{ESG}_{n,t}} \tag{2}$$

with $i = 1, \dots, n$, $t = 0, \dots, T$ and $\overline{ESG}_{n,t} = \frac{1}{n}\sum_{i=1}^{n} ESG_{i,t}$. This ratio represents how a company is performing in comparison to the average in terms of ESG score.

It is worth noting that there is no uniformity among companies in publishing non-financial reports, so firms' ESG scores are not updated at the same date in a given year. To guarantee a fair interpretation, we define the sustainability rate of return of firm $i$ at time $t$ by comparing the sustainability performance score in $t$ with the one of the previous year, as follows:

$$sr_{i,t} = \frac{s_{i,t}}{s_{i,t-12}} - 1\,. \tag{3}$$

A company with a positive sustainability rate of return at time $t$ reflects its capability to improve its relative value (2) in a year. This is an indicator of a successful implementation of an ESG-compliant business. Conversely, a negative value of (3) could indicate the mismatching of a company with the market standards of ESG practices.

Similar to the financial case, we also introduce the portfolio sustainability rate of return at time $t$ as:

$$sr_{p,t} = \sum_{i=1}^{n} x_i sr_{i,t}\,. \tag{4}$$

## 2.2 Cumulative prospect theory

According to the PT approach developed by Kahneman and Tversky in [6], individual preferences are modeled through a value function of the aggregate gains and losses from a reference point $z_{ref} \in \mathbb{R}$, which is usually fixed at the beginning of the investment period. Thus, the investor decides the values of these outcomes according to the following piecewise value function:

$$v(z) = \begin{cases} (z - z_{ref})^{\alpha}, & z \geq z_{ref} \\ -\beta(z_{ref} - z)^{\alpha}, & z < z_{ref} \end{cases} \tag{5}$$

with $0 < \alpha < 1$, $\beta > 0$ and $z \in \mathbb{R}$.

The concavity of (5) in the domain of gains and its convexity in the domain of losses imply that the investor is risk averse for gains and risk seeking for losses.

Two weighting functions are then introduced to modify the outcome probabilities in order to underweight higher outcomes and overweight smaller ones. Formally, in the domain of gains, we have:

$$w_{+}(p) = \frac{p^{\delta_{+}}}{(p^{\delta_{+}} + (1-p)^{\delta_{+}})^{\frac{1}{\delta_{+}}}} \tag{6}$$

while, in the domain of losses:

$$w_{-}(p) = \frac{p^{\delta_{-}}}{(p^{\delta_{-}} + (1-p)^{\delta_{-}})^{\frac{1}{\delta_{-}}}} \tag{7}$$

where $\delta_{+}, \delta_{-} > 0$ and $p \in [0,1]$ is the probability of an outcome.

The scaling of these probabilities is based on the following procedure. Given a random outcome $Z$, let $\boldsymbol{z} = (z_1, \dots, z_n)^{\top} \in \mathbb{R}^{\boldsymbol{n}}$ be the vector of its realizations and denote by $N_{+}$ and $N_{-}$ the number of positive and negative values in $\boldsymbol{z}$. We sort the elements of $\boldsymbol{z}$ in ascending order:

$$z_{(1)} \leq \cdots \leq z_{(N_{-})} < 0 \leq z_{(N_{-}+1)} \leq \cdots \leq z_{(n)}.$$

Then, we define the positive and negative re-weighted probabilities as:

$$\pi_{+,j} = \begin{cases} w_{+}\left(\frac{(N_{+} - j + 1)}{n}\right) - w_{+}\left(\frac{(N_{+} - j)}{n}\right) & j = 1, \dots, N_{+} - 1 \\ w_{+}\left(\frac{1}{n}\right) & j = N_{+} \end{cases}$$

and

$$\pi_{-,j} = \begin{cases} w_{-}\left(\frac{(N_{-} - j + 1)}{n}\right) - w_{-}\left(\frac{(N_{-} - j)}{n}\right) & j = 1, \dots, N_{-} - 1 \\ w_{-}\left(\frac{1}{n}\right) & j = N_{-}. \end{cases}$$

Hence, according to the CPT, the utility value of $Z$ is the weighted average of the value function (5) calculated in the positive and negative outcomes using the re-weighted probabilities $\pi_{+,j}$ and $\pi_{-,j}$:

$$V(\boldsymbol{z}) = \sum_{j=1}^{N_-} \pi_{-,j}\, v(z_{(j)}) + \sum_{j=1}^{N_+} \pi_{+,j}\, v(z_{(N_-+j)}).$$

In the experimental section, we will adopt the same parameter setting as in [6] and [7], that is $\alpha = 0.88$, $\beta = 2.25$ and $z_{ref} = 0$ for the value function (5), $\delta_+ = 0.61$, and $\delta_- = 0.68$ for the weighting function (6) and (7), respectively. Figure 1 displays the graphs of these functions.

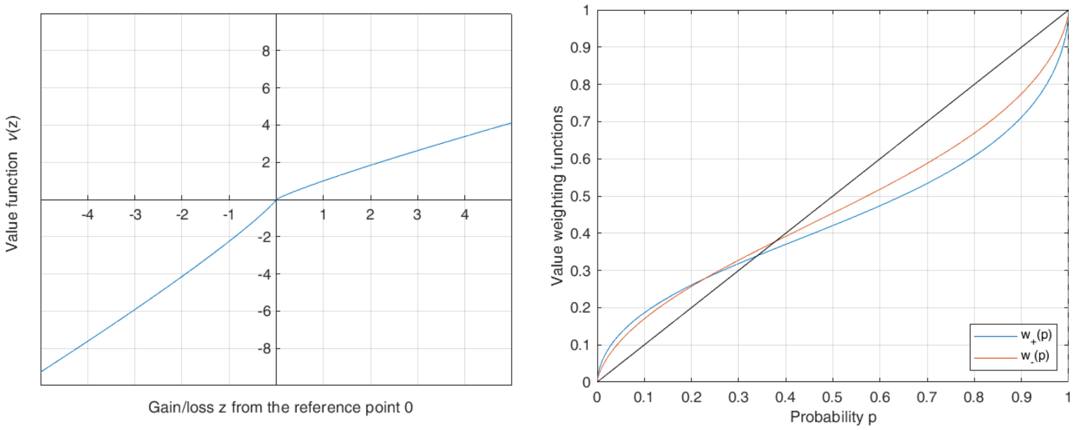


*Figure 1: Cumulative prospect theory piecewise value function and weighting functions with parameters as in [6] and [7]*

## 2.3 Portfolio optimization problems under CPT and ethical principles

In this part, we first present the standard CPT-based portfolio optimization model. Then, we describe the two extensions we propose to include ESG criteria.

We consider the $T$ observed portfolio rates of return $\{r_{p,t}\}_{t=1,\dots,T}$ as realizations of the random variable $R_p$, which expresses the stochastic rate of return of portfolio $\boldsymbol{x}$ with a monthly investment horizon. Following the steps described in the previous section, we calculate the vector of the re-weighted probabilities of the realized rates of return $\boldsymbol{\pi}^{(\mathbf{r})} = \{\pi_1^{(r)}, \dots, \pi_T^{(r)}\}$. Then, the standard CPT-based portfolio optimization problem can be written as:

$$\max_{x \in \mathbb{R}^n} \sum_{t=1}^{T} \pi_t^{(r)} v\left(\sum_{i=1}^{n} x_i r_{i,t}\right)$$
$$s.t.\ x_i \geq 0, \qquad i = 1, \dots, n$$
$$\sum_{i=1}^{n} x_i = 1. \tag{8}$$

Problem (8) does not explicitly account for the attitude of investors towards firms' sustainable targets. To overcome this issue, we present two novel asset allocation strategies that directly exploit ESG scores. In the first model, we consider a CPT-based investor who preselects a subset of promising assets using ESG information to solve problem (8). The other model extends the previous one by including, besides the preselection based on ESG scores, a second CPT-value function for the sustainability rates of return, as an additional objective to maximize, in order to take advantage of the evolution of ESG scores over time.

### 2.3.1 CPT-based model with preselection using ESG scores

To include ethical goals in the asset allocation process, we adopt a stock-picking technique to preselect the subset of assets that go beyond a given ESG threshold $\overline{\theta}$. We denote by $U_T(\overline{\theta})$ the subset which includes $k$ out of the $n$ available assets with an ESG score higher than the threshold $\overline{\theta}$ at the end of the observed period $T$. Then, the portfolio selection model can be reformulated as follows:

$$\max_{x \in \tilde{X}} V_1(\boldsymbol{x}) = \sum_{t=1}^{T} \pi_t^{(r)} v\left(\sum_{i \in U_T(\overline{\theta})} x_i r_{i,t}\right)$$
$$s.t.\ x_i \geq 0, \qquad i \in U_T(\overline{\theta})$$
$$\sum_{i \in U_T(\overline{\theta})} x_i = 1 \tag{9}$$

where $\tilde{X} = \{\boldsymbol{x} \in \mathbb{R}^k : i \in U_T(\overline{\theta})\}$.
Note that if we do not adopt a preselection, problem (9) is equivalent to (8).

### 2.3.2 CPT-based model with financial and ethical objectives

Since the variations of the firms' ESG scores over time could be a signal of the company's capability to implement an ESG-compliant business, an ESG-aware investor may also be sensitive to this information. Assuming that investors present the same decision-making behavior in approaching financial and sustainable returns, we define the following CPT-type value function for the sustainability rates of return:

$$V_2(\boldsymbol{x}) = \sum_{t=1}^{T} \pi_t^{(s)} v\left( \sum_{i \,\in\, U_T(\overline{\theta})} x_i \, sr_{i,t} \right) \tag{10}$$

where $sr_{i,t}$ is the sustainability rate of return of asset $i \in U_T(\overline{\theta})$ at time $t$ and $\boldsymbol{\pi}^{(s)} = \{\pi_1^{(s)}, \dots, \pi_T^{(s)}\}$ is the vector of the re-weighted probabilities associated to the sustainability rates of return.

In this extended formulation of the portfolio optimization problem, the agent simultaneously maximizes the value functions $V_1$ and $V_2$ with a preliminary ESG-based screening of the constituents. The resulting bi-objective optimization problem is:

$$\begin{aligned} &\max_{\boldsymbol{x} \in \widetilde{\boldsymbol{X}}} \; \left(V_1(\boldsymbol{x}), V_2(\boldsymbol{x})\right) \\ &s.t. \; x_i \geq 0, \qquad i \in U_T(\overline{\theta}) \\ &\sum_{i \,\in\, U_T(\overline{\theta})} x_i \; = 1. \end{aligned} \tag{11}$$

In such a setting, portfolio choices are made with respect to the following preference relation.

**Definition 1**. *Given two portfolios* $\boldsymbol{x}^{(1)}$, $\boldsymbol{x}^{(2)}$, *we say that* $\boldsymbol{x}^{(1)}$ *is preferred to (or non-dominated by)* $\boldsymbol{x}^{(2)}$ *if and only if* $V_1(\boldsymbol{x}^{(1)}) \geq V_1(\boldsymbol{x}^{(2)})$ *and* $V_2(\boldsymbol{x}^{(1)}) \geq V_2(\boldsymbol{x}^{(2)})$, *with at least one strict inequality*.

In this manner, an agent will prefer portfolios with higher financial and sustainable values. The set of non-dominated solutions to problem (11) forms the so-called Pareto front or efficient frontier, denoted by $\boldsymbol{P}$. At this point, we can choose on $\boldsymbol{P}$ the portfolio which best-fit investor's attitude toward financial and sustainable objectives as follows. First, the values of the two objective functions for each candidate portfolio $\boldsymbol{x}$ in $\boldsymbol{P}$ are normalized using the formula:

$$V_j^{(n)}(\boldsymbol{x}) = \frac{V_j(\boldsymbol{x}) - V_j^{min}}{V_j^{max} - V_j^{min}} \qquad \text{for } j = 1, 2$$

where $V_j^{min} = \min_{\boldsymbol{x} \in \boldsymbol{P}} V_j(\boldsymbol{x})$ and $V_j^{max} = \max_{\boldsymbol{x} \in \boldsymbol{P}} V_j(\boldsymbol{x})$. Next, we define the vector of preferences $\boldsymbol{w}_{pref} = (w_{ESG}, w_{fin})^{\mathsf{T}} \in [0,1]^2$ such that $w_{ESG} + w_{fin} = 1$. Finally, we calculate the weighted sum of the objective functions and maximize the result on the set of the efficient portfolios:

$$\max_{\boldsymbol{x} \in \boldsymbol{P}} \left\{ w_{fin} V_1^{(n)}(\boldsymbol{x}) + w_{ESG} V_2^{(n)}(\boldsymbol{x}) \right\}.$$

The solution to this single-objective problem identifies an efficient portfolio tailored to the investor's financial/ESG profile.

# 3 Description of the optimizers

In this paper, we consider two evolutionary algorithms to solve the above optimization problems, which are nonlinear and non-smooth. More specifically, to tackle the single-objective problems (8), (9), we employ a genetic algorithm (GA), while for the bi-objective one (11) we implement the iNSGA-II developed in [27]. In Figure 2, we report the flowcharts of the two solvers. In both cases, an initial set of candidate solutions, forming the so-called population, is randomly initialized and is evolved through the following three steps.

1. Two sub-populations of parents are created by uniform selection. The first sub-population is involved in the recombination phase, while the second is subject to the mutation strategy. It is worth noting that the two sub-groups are not disjointed. In this way, the same individual may enter into both the crossover step and the mutation stage.
2. The crossover is applied to the first sub-population of parents to give rise to a set of offspring. This operator produces two children $\bar{\boldsymbol{x}}_1, \bar{\boldsymbol{x}}_2 \in \mathbb{R}^{\boldsymbol{n}}$ for each pair of parents $\boldsymbol{x}_1, \boldsymbol{x}_2 \in \mathbb{R}^{\boldsymbol{n}}$:

$$\bar{x}_{1i} = c_i x_{1i} - (1 - c_i) x_{2i}$$

$$\bar{x}_{2i} = c_i x_{2i} - (1 - c_i) x_{1i}$$

   where $c_i$ is the scaling factor randomly chosen in $[-1,2]$ with $i = 1, \dots, n$. The second sub-group of parents is modified by a Gaussian mutation.
3. The sub-populations of children are picked together to forge the offspring population.

Due to the requirement of full investment, we also introduce a constraint-handling procedure based on the repair mechanism developed in [29]. First, in order to satisfy the non-negativity constraint, each candidate solution $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$ is clamped in $[0,1]^n$ as:

$$\tilde{x}_i = \begin{cases} 0 & if\ x_i < 0 \\ 1 & if\ x_i > 1 \\ x_i & otherwise \end{cases}$$

where $i = 1, \dots, n$. Then, the projected vector $\tilde{\boldsymbol{x}}$ is adjusted by normalisation to satisfy the budget constraint in the following way:

$$\tilde{\tilde{\boldsymbol{x}}} = \frac{\tilde{x}_i}{\sum_{j=1}^{n} \tilde{x}_j} \quad for\ i = 1, \dots, n.$$

The above-quoted procedure makes feasible the individuals in the search space.

The main differences between the two algorithms are the following. In the GA, the best individual at each generation represents a sub-optimal solution to the problem at hand. Instead, the iNSGA-II algorithm generates good approximations of the Pareto front. To this end, iNSGA-II exploits a two-step ranking scheme. The first ranking is based on the non-domination relation given in Definition 1. If the individuals have the same position in that stage, then one applies the second-level ranking, which exploits a diversity-preserving mechanism, the so-called crowding distance. The elitist individuals are chosen from the current population and the offspring set based on the ranking scheme. The candidates, saved in the following population, correspond to the higher level of non-domination fronts and the higher values of crowding distance.

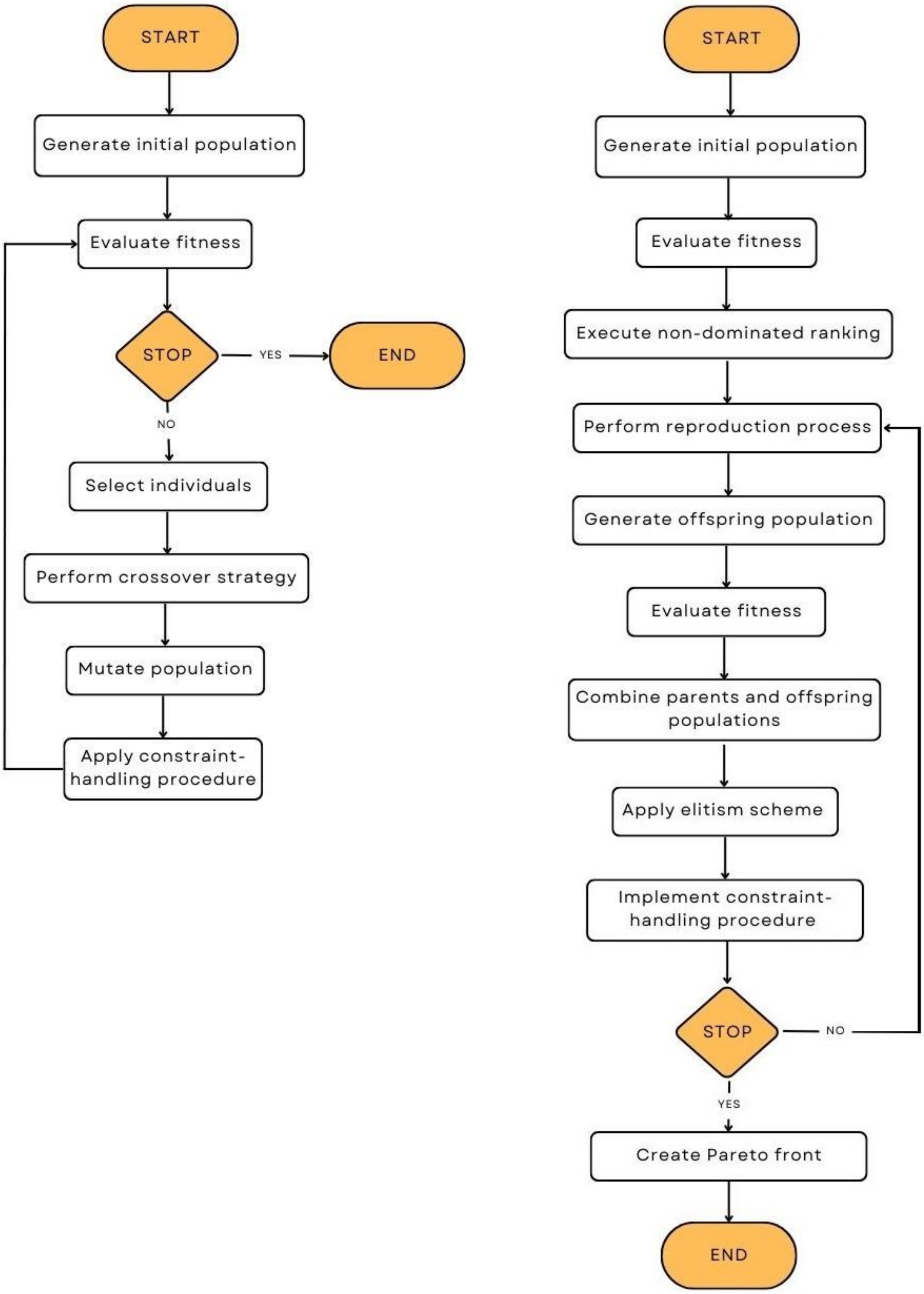


*Figure 2: Flowcharts of the two GAs used in the paper. On the left, the GA for the single-objective problems and on the right the iNSGA-II for the bi-objective problem*

# 4 Experimental part

Our analysis focuses on the European market due to the advanced disclosure rules in this zone. More specifically, we consider a subset of the constituents from the STOXX Europe 600 index as the investable universe. We obtain the monthly rates of return and the monthly ESG ratings from 01/01/2015 to 31/12/2021 for 538 companies from Refinitiv Datastream. The ESG scores are scaled between 1 and 100 and represent an assessment of how the company operates on the side of sustainability. The choice of the time window is motivated by Directive 2014/95/EU, which has imposed the disclosure of non-financial information for this type of company since 2015.

In the first instance, our goal is to highlight the role in portfolio choices of a preselection technique based on the sustainable score of firms. In the second step, we analyze the benefits that derive from the inclusion in the investment process of a criterion involving the dynamics of the ESG scores.

To this end, we consider an investment plan with a one-month horizon and we employ a rolling window scheme based on historical data. The out-of-sample window covers the period from 31/11/2019 to 31/12/2021, for a total of 25 months. The procedure also exploits an in-sample window of 47 months to set the model parameters.

## 4.1 Ex-post performance measures

In the sequel, we introduce the ex-post performance measures that will be used to assess the profitability of the proposed strategies. Let us denote by $\boldsymbol{x}_t$ the optimal portfolio at the ex-post month $t$, with $t = 1, \dots, 25$. Due to the time dependence of the considered investment plan, we calculate the value of turnover as follows:

$$\sum_{i=1}^{n} | \boldsymbol{x}_{t,i} - \boldsymbol{x}_{t-1,i}|$$

where $\boldsymbol{x}_{t-1} = \left(x_{t-1,1}, \dots, x_{t-1,n}\right)$ represents the portfolio to be rebalanced.
Now let $r_{p,t}^{out}$ be the ex-post portfolio rate of return realized at time $t$, with $t = 1, \dots, 25$. We consider the so-called ex-post Sharpe ratio, defined as:

$$SR^{out} = \frac{\mu^{out}}{\sigma^{out}}$$

where $\mu^{out}$ and $\sigma^{out}$ are the mean and the standard deviation of the ex-post portfolio rates of return, respectively.
Further, we compute the net wealth at time $t$ as:

$$W_t = W_{t-1}\left(1 + r_{p,t}^{out}\right) - \lambda(\boldsymbol{x_t}, \boldsymbol{x}_{t-1})$$

where $\lambda(\boldsymbol{x_t}, \boldsymbol{x}_{t-1})$ is the cost function, whose structure is presented in Table 1.

*Table 1: Structure of transaction costs*

| Trading segment (€) | Fixed fee (€) | Proportional cost (%) |
|---|---|---|
| 0 – 7,999 | 40 | 0 |
| 8,000 – 49,999 | 0 | 0.50 |
| 50,000 – 99,999 | 0 | 0.40 |
| 100,000 – 199,999 | 0 | 0.25 |
| ≥ 200,000 | 400 | 0 |

Moreover, we use the so-called compound annual growth rate, which in our case is calculated as:

$$CAGR = \left(\frac{W_{T_{end}}}{W_0}\right)^{\frac{12}{T_{end}}} - 1$$

where $W_0$ represents the initial wealth and $W_{T_{end}}$ is the final wealth.
Finally, we measure the downside risk by the so-called peak-to-valley drawdown:

$$DD_t = -\min\left\{0, \frac{W_t - W_{peak}}{W_{peak}}\right\}$$

where $W_{peak}$ is the maximum amount of wealth reached by the strategy until time $t$. In particular, we consider the maximum of the drawdowns over time.

## 4.2 Analysis of the proposed models

The preselection strategies involved at the time of investment consider six levels of sustainable-worthiness as reported in Table 2. In particular, the NO ESG strategy, without requiring any ESG information, represents the entire investable universe. Conversely, the second strategy requires only the availability of the ESG score at the investment time for the assets to be included in the portfolio. Strategies B and B+ consist of a pool of companies with good ESG performance with respect to their sector and an above-average degree of transparency in publicly reporting ESG material. Finally, the last two stock-picking levels translate excellent ESG performances of a firm compared to its sector and a maximum degree of transparency in disclosing sustainable materials.

*Table 2: Preselection strategies description*

| Stock-picking | Description |
|---|---|
| NO ESG | All firms in the dataset |
| ANY ESG | Firms with an ESG score |
| B | Firms with ESG score ≥ 58.3 |
| B+ | Firms with ESG score ≥ 66.7 |
| A- | Firms with ESG score ≥ 75 |
| A | Firms with ESG score ≥ 83.3 |

### 4.2.1 Financial-only CPT-based models

In models (8) and (9), agents first preselect the investment pool based on the strategies introduced above. Then, they maximize the financial CPT-value function $V_1$. In Figure 3 we can observe the evolution of this objective for the different stock-picking techniques. When we consider NO ESG and ANY ESG, we notice that the corresponding optimal portfolios generate similar CPT-values. The other screening procedures present significantly lower values. Overall, according to the literature, it is worth noting that when the ESG threshold increases, the financial utility progressively decreases.

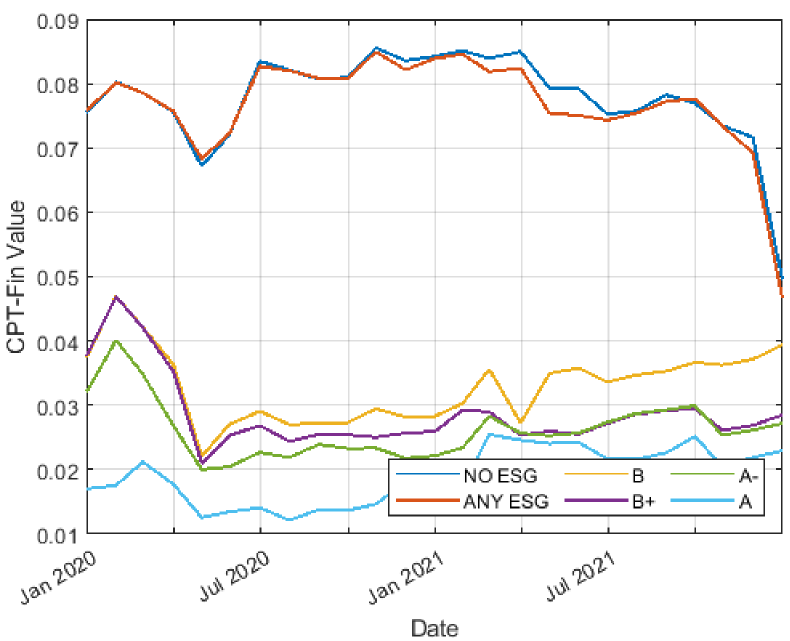


*Figure 3: Evolution of the financial CPT-value function over time*

Next, we assess the profitability of the different portfolio allocation strategies with an initial wealth $W_0 = 1{,}000{,}000$ €.
Looking at Table 3, one can infer that including an ESG preference for the investable pool is a positive discriminant for a CPT-type agent investing in the European market. More specifically, the best strategy is associated with the ANY ESG screening, which presents a Sharpe ratio three times more than the benchmark with controlled maximum drawdown and turnover. The portfolios derived from NO ESG and A screenings are the second-best alternatives. The former shows a higher SR value while the latter controls better risk and turnover.

*Table 3: Performance measures of portfolio allocation models (8) and (9) for the different stock-picking strategies*

| | **NO ESG** | **ANY ESG** | **B** | **B+** | **A-** | **A** | **Benchmark** |
|---|---|---|---|---|---|---|---|
| **SR** | 0.3130 | 0.4501 | 0.2914 | 0.1639 | 0.1317 | 0.2100 | 0.1638 |
| **CAGR (%)** | 6.5549 | 28.2080 | -7.1879 | -12.8220 | -8.1606 | 4.1181 | 9.0256 |
| **Std** | 0.0978 | 0.0981 | 0.0651 | 0.0595 | 0.0553 | 0.0522 | 0.0539 |
| **Max DD** | 0.2922 | 0.2958 | 0.2910 | 0.2812 | 0.2480 | 0.2014 | 0.2303 |
| **Turnover** | 0.2891 | 0.2735 | 0.7386 | 0.7121 | 0.5976 | 0.5379 | − |

Figure 4 highlights the behavior of the six investment strategies in terms of the produced wealth. The mere inclusion of the ESG criterion leads to an increase in investment especially after 2021, if we do not consider transaction costs. However, focusing on the net wealth, only portfolios with NO and ANY ESG screenings still outperform the index. This is a consequence of the low turnover level of these portfolios.

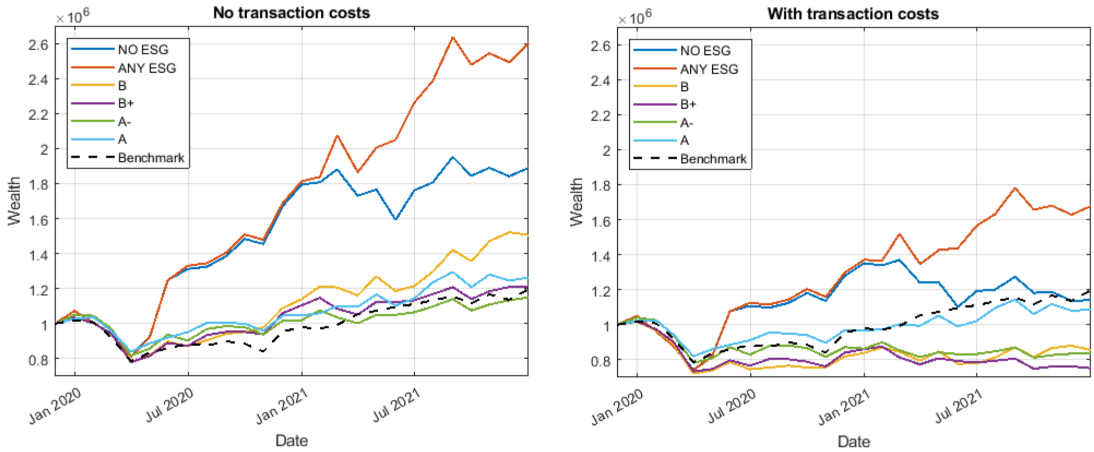


*Figure 4: Ex-post evolution of gross wealth (on the left) and net wealth (on the right) of the proposed stock-picking strategies*

### 4.2.2 CPT-based models with financial and ethical objectives

Concerning the bi-objective model (11), Figure 5 shows for each preselection strategy the corresponding efficient frontier at the end of each year in the out-of-sample window. It is worth noting that the non-dominated fronts can be ranked in decreasing order with respect to the ESG threshold of the screening. In particular, we can observe the clustering of the efficient frontiers into three groups: ANY ESG; B and B+; A- and A.

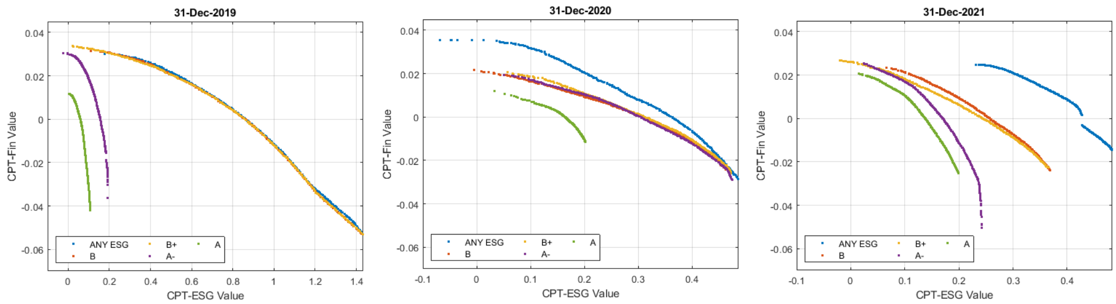


*Figure 5: Efficient frontiers of model (11) at the end of each year*

Similar to the single-objective instances, we assess the ex-post profitability of the portfolio allocation strategies for three representative investor attitudes, whose vectors of preferences are listed in Table 4. The first agent gives more importance to the financial criterion while Agent 3 to the sustainable one. Finally, Agent 2 reflects a moderate profile and perfectly balances financial and ESG preferences.

*Table 4: Description of the three representative agents*

| | **Weight of the financial preference** | **Weight of the ESG preference** |
|---|---|---|
| **Agent 1** | 0.75 | 0.25 |
| **Agent 2** | 0.50 | 0.50 |
| **Agent 3** | 0.25 | 0.75 |

The ex-post results for these ESG-aware agents are reported in Table 5. As in the previous section, we consider an initial wealth $W_0 = 1{,}000{,}000$ €, with the same transaction cost structure presented in Table 1. Each column refers to a specific ESG-based preselection. We observe that as the relevance of the CPT-type value function for the sustainability rates of return increases, the preselection threshold decreases in order to allow a broader range of ESG variations over time. Conversely, if there is a greater financial preference then the optimal ethical screening is attained at a higher ESG threshold.

*Table 5: Performance measures of the proposed portfolio optimization for the three representative agents*

| | **ANY ESG** | **B** | **B+** | **A-** | **A** |
|---|---|---|---|---|---|
| **Agent 1** | | | | | |
| **SR** | -0.0399 | 0.0567 | 0.2218 | 0.1652 | 0.3592 |
| **CAGR (%)** | -37.9350 | -23.4090 | -10.3210 | -4.5579 | 10.5370 |
| **Std** | 0.0552 | 0.0444 | 0.0470 | 0.0573 | 0.0408 |
| **Max DD** | 0.6595 | 0.4263 | 0.2616 | 0.1882 | 0.1041 |
| **Turnover** | 0.7305 | 0.9666 | 0.8976 | 0.7693 | 0.4197 |
| **Agent 2** | | | | | |
| **SR** | 0.1073 | 0.2117 | 0.3737 | 0.2232 | 0.2090 |
| **CAGR (%)** | -24.1850 | -11.7920 | 3.6240 | 3.3902 | 4.3580 |
| **Std** | 0.0458 | 0.0471 | 0.0521 | 0.0672 | 0.0498 |
| **Max DD** | 0.4383 | 0.2330 | 0.1672 | 0.1815 | 0.1753 |
| **Turnover** | 0.7053 | 0.6048 | 0.5100 | 0.5462 | 0.3957 |
| **Agent 3** | | | | | |
| **SR** | 0.3323 | 0.3308 | 0.3368 | 0.2978 | 0.0944 |
| **CAGR (%)** | 5.6427 | 9.1423 | 12.8160 | 10.3200 | -1.8447 |
| **Std** | 0.0703 | 0.0712 | 0.0711 | 0.0646 | 0.0560 |
| **Max DD** | 0.1645 | 0.1483 | 0.1471 | 0.1457 | 0.1810 |
| **Turnover** | 0.2102 | 0.2001 | 0.1632 | 0.3874 | 0.2648 |

We conclude this section by comparing the models presented above. In Figure 6, the best-performing model in terms of final wealth corresponds to the financial-only investor with the ANY ESG preselection criterion as it reaches 1,700,000 €. The bi-objective model for Agent 3 profile and preselection B+ is the second-best strategy having as final wealth around 1,300,000 €. We also stress the fact that for the same period, the buy-and-hold strategy finally shows 1,200,000 € as net wealth. Finally, the strategy that envisages no inclusion of ESG criteria underperforms the buy-and-hold strategy with a final wealth of 1,140,000 €.

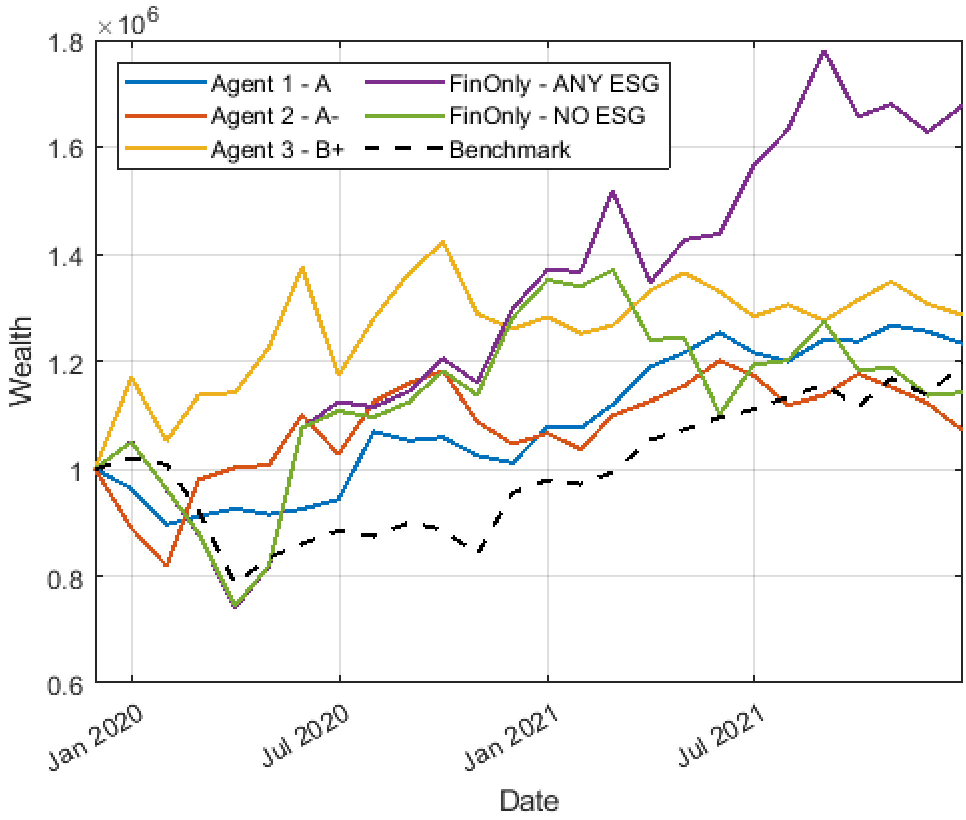


*Figure 6: Ex-post evolution of net wealth of the best-proposed portfolio strategies*

## 5 Conclusions

In this paper, we have presented two extensions of the classical cumulative prospect theory framework. In the first model, we have analyzed the impact of various ethical stock-picking strategies on the financial CPT-value function. In a second formulation, we have proposed a sustainable CPT-value function, which has been integrated into the objectives of the optimization problem. The aim has been to also include in the valuation framework of an ESG-aware investor the variations of the firms' ESG scores over time. We have introduced a scaling procedure based on the investor's preferences toward financial and ESG criteria to select the suitable portfolio on the associated efficient frontier.
Finally, we have assessed the profitability of the developed portfolio allocation models using an investment pool from the STOXX Europe 600 covering the period 2019 to 2021. The results show that using a stock-picking procedure based on the ESG information improves the financial performance of the investment. Moreover, we have considered the portfolio construction for three representative investor attitudes. The ex-post analysis reveals that the bi-objective models with higher ESG preferences have lower risk and better control of extreme losses with respect to the buy-and-hold strategy and the single-objective CPT-based models.

However, these findings could depend on the ESG rating evaluation methodology of the selected provider as well as the geographical area of the stocks employed in the investments. These factors offer an opportunity to develop further research by considering other ESG providers and different international markets.

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