**RELIABILITY IN THE ESTIMATES AND COMPLIANCE TO INVERTIBILITY CONDITION OF STATIONARY AND NONSTATIONARY TIME SERIES MODELS**

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ABSTRACT

In this paper, we fit models to stationary and non-stationary series for comparison of the estimates of the data, considering invertibility condition for the models. The condition requires that every parameter of a time series model should lie between -1 and 1 exclusive. The distribution of autocorrelation and partial autocorrelation functions as shown Appendixes 1A, 1B, 2A and 2B, suggested AR(1) model for the non-stationary series and ARIMA(2,1,2) for the stationary series. The two models have given good estimates for the series, with residuals which are independently and identically distributed. This paper has established the fact that not until a series is stationary, it becomes invertible. This is affirmation of assertion by Box and Jenkins (1976) that invertibility is independent of stationarity. The models of non-stationary series that are not invertible are those whose data series are absolutely explosive in nature.

KEY WORDS: Autoregressive Model, Moving Average Model, Invertibility Condition, Stationarity and Non-Stationarity.

1. INTRODUCTION: In time series analysis, there are two processes which explain the nature and distribution of time series data. There are autoregressive and moving average processes. The processes are identified on the basis of the distribution of autocorrelation and partial autocorrelation functions. Box and Jenkins (1976) described a process to be autoregressive, if it exhibits exponential decay or sine wave pattern in autocorrelation function and a cut off at a certain lag in partial autocorrelation function. While, moving moving average process is described by the exhibition of exponential decay or sine wave pattern in partial autocorrelation function and cut off at certain lag in the autocorrelation function. It is a popular practice in time series that stability of data has to be ensured before a suitable model is suggested to the time series data. This is so because parameter(s) of the fitted time series model is expected to have values that will give room for invertibility. The assumption of stationarity means the mean and variance of the series are constant over time and that the structure of the series depends only upon the relative position in time of the observations, Kendell and Ord (1993). Box and Cox(1964) introduced the class of variance stability transformation. The condition of stationarity is clearly fundamental to the statistical analysis of time series, but it is not an assumption that can be made automatically. For the assumption of stationarity, condition of weak, second order or covariance stationary should be satisfied at least to a reasonable degree. This fact does not negate fitting time series models to non-stationary series so as to ascertain if stability is required in every non-stationary series. Usoro and Omekara (2008) fitted Bilinear Autoregressive Vector models to non-stationary revenue data. The fitted models gave good estimates with uncorrelated error term. Multivariate time series models were fitted to non-stationary series, with a response and two predictor vectors. Estimates obtained from the models were good and autocorrelation functions were uncorrelated, Usoro and Omekara (2007). The motivation behind this work is to fit time series models to both stationary and non-stationary series for comparison of estimates and checking if the parameters of both models give room for invertibility.

2. STATIONARY AND NON-STATIONARY MODELS:

 Kendall and Ord (1973) stated the general autoregressive time series model as, φ(B)Yt = Єt - - - model 2.1

By expansion, the model becomes,

(1 -φ1B – φ2B2 – φ3B3 - … - φpBp)Yt = Єt

 => Yt - φ1BYt – φ2B2Yt – φ3B3Yt - … - φpBpYt = Єt

 => Yt - φ1Yt-1 – φ2Yt-2 – φ3Yt-3 - … - φpYt-p = Єt

=> Yt = φ1Yt-1 + φ2Yt-2 + φ3Yt-3 + … + φpYt-p + Єt - - - - model2.2

 where Yt is the time series process, φ1 φ2, …,φp are the parameters of the model and B, B2, …, Bp are the backward shift operators.

The general autoregressive moving average model is given by,

 φ(B)Yt =Ѳ(B)Єt - - - - - model 2.3

By expansion, the model becomes

(1 -φ1B – φ2B2 – φ3B3 - … - φpBp)Yt = (1- Ѳ1B – Ѳ2B2- …- ѲqBq)Єt

=> Yt - φ1BYt – φ2B2Yt – φ3B3Yt - … - φpBpYt = Єt - Ѳ1B – Ѳ2B2- …- ѲqBq

=> Yt - φ1Yt-1 – φ2Yt-2 – φ3Yt-3 - … - φpYt-p = Єt - Ѳ1Єt-1 – Ѳ2Єt-2 - …- ѲqЄt-q

=> Yt = φ1Yt-1 + φ2Yt-2 + φ3Yt-3 + … + φpYt-p + Єt - Ѳ1Єt-1 – Ѳ2Єt-2 - …- ѲqЄt-q model 2.4

Model ‘1.4’ is ARMA model for non-difference series, Johnston and Dinardo (1997)

If a series is differenced, model 1.1 and becomes,

 φ(B)(1-B)Yt = Єt

=> (1 -φ1B – φ2B2 – φ3B3 - … - φpBp)(1-B)Yt = Єt - - model 2.5

While model 1.3 becomes,

 φ(B)Yt =Ѳ(B)Єt

=> (1 -φ1B – φ2B2 – φ3B3 - … - φpBp)(1-B)Yt = (1- Ѳ1B – Ѳ2B2- …- ѲqBq)Єt model 2.6

Where (1-B) is the difference operator

3. ESTIMATION OF PARAMETERS OF NON-STATIONARY AND STATIONARY SERIES

 Before the parameters estimated, there must be a choice of a model through the distribution of correlogram. From appendix 1a, 1b, 2a and 2b, the distribution of autocorrelation and partial autocorrelation functions have suggested AR (1) model for the non-stationary series and ARIMA (2, 1, 2) for the stationary series.

3.1 THE AR (1) MODEL:

The AR (1) model is given by,

 Yt = φ1Yt-1 + Єt - - - - - model 3.1

where φ1 is the parameter of the model, Єt is the error term assumed to independently and identically distributed with zero mean and constant variance.

The fitted model is Yt = 0.9989Yt-1. The graph of original with estimated values is shown in figure 1. The estimates from the model are in appendix 3.

3.2 THE ARIMA (2,1,2) MODEL:

The ARIMA (2, 1, 2) model for the stationary series is given by,

 (1 - φ1B - φ2B2)(1-B)Yt = (1- Ѳ1B - Ѳ2B2)Єt

=> (1 – B - φ1B – φ1B2 – φ2B2 + φ2B3)Yt = Єt - Ѳ1Єt-1 – Ѳ2Єt-2

=> Yt – Yt-1- φ1Yt-1 + φ1Yt-2 – φ2Yt-2 + φ2Yt-3 = Єt - Ѳ1Єt-1 – Ѳ2Єt-2

=> Yt – Yt-1= φ1(Yt-1 - Yt-2) + φ2(Yt-2 - Yt-3) + Єt - Ѳ1Єt-1 – Ѳ2Єt-2 - - - model 3.2

If Yt – Yt-1 = yt, Yt-1 – Yt-2 = yt-1, Yt-2 – Yt-3 = yt-2, model 3.2, becomes

yt = φ1yt-1 + φ2yt-2 + Єt - Ѳ1Єt-1 – Ѳ2Єt-2­ - - - - - - model 3.3

where yt is the difference series.

Therefore Ŷt( estimate Yt) can be obtained in either of the following ways:

(1) fitting ARMA (2,0,2) to yt, so that ŷt + Yt-1 = Ŷ. (2) fittingARIMA (2,1,2) to Yt.

The fitted ARMA (2,0,2) to yt yields,

ŷt = 0.5244yt-1 + 0.1991yt-2 + Єt – 0.4294Єt-1 – 0.0516Єt-2.

The graph of original with estimated values is shown in figure 2. The estimates from the model are in appendix 3.

CONCLUSION

There is no gainsaying the fact that stationarity of time series data is very expedient in building autoregressive moving average model. This is due to the condition of invertibility and of course duality between the autoregressive and moving average processes. The invertibility condition provides that the parameter of a model, say AR(1) should neither be less than -1 nor greater than 1. It is the fear of the unknown explosive or evolutionary behavior of non-stationary series that motivates stationarity of a series before model building. However, in this paper, we have been able to show that non-stationary series can be invertible (that is the roots of φ(B) = 0 lie outside the unit circle, as the parameters lie within the unit circle). It is an indisputable fact that any non-stationary series that is absolutely explosive in nature must have a parameter lying outside the unit circle. That is a clear case of violation of invertibility condition. The exhibition of such explosive behavior calls for differencing for stability of the process. Therefore, it is not every non-stationary series that violet invertibility condition.

APPENDIX 1A: ACF OF ORIGINAL DATA



APPENDIX 1B: PACF OF ORIGINAL DATA



APPENDIX 2A: ACF OF DIFFERENCE DATA



APPENDIX 2B: PACF OF DIFFERENCE DATA



120

100

80

60

40

20

1450

1350

1250

1150

1050

950

850

750

Index

Xt

 **Figure1: Graph of Original with Estimates of Non-Stationary Data**

Plot of Original in circle

Plot of Estimates in plus

120

100

80

60

40

20

1500

1400

1300

1200

1100

1000

900

800

700

Index

Xt

 **Figure2: Graph of Original with Estimates of Stationary Data**

Plot of Original in circle

Plot of Estimates in plus

APPENDIX 3: Original and Estimates From Stationary and Non-Stationary Models

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| S/N |  Yt |  SŶt |  NSŶt | S/N |  Yt |  SŶt |  NSŶt |
| 1 | 1393 | 1389.99 |  - | 61 | 1269 | 1234.66 | 1242.53 |
| 2 | 1382 | 1391.49 | 1390.99 | 62 | 1281 | 1238.66 | 1245.37 |
| 3 | 1369 | 1380.51 | 1379.16 | 63 | 1246 | 1259.64 | 1266.23 |
| 4 | 1362 | 1367.52 | 1364.82 | 64 | 1263 | 1278.62 | 1287.43 |
| 5 | 1355 | 1360.53 | 1357.48 | 65 | 1246 | 1289.60 | 1298.31 |
| 6 | 1346 | 1353.54 | 1351.14 | 66 | 1306 | 1296.60 | 1303.81 |
| 7 | 1287 | 1344.54 | 1342.22 | 67 | 1301 | 1304.59 | 1310.66 |
| 8 | 1261 | 1285.61 | 1278.24 | 68 | 1342 | 1299.59 | 1304.01 |
| 9 | 1236 | 1259.64 | 1245.87 | 69 | 1351 | 1340.55 | 1346.69 |
| 10 | 1240 | 1234.66 | 1222.84 | 70 | 1368 | 1349.54 | 1360.07 |
| 11 | 1240 | 1238.66 | 1230.26 | 71 | 1397 | 1366.52 | 1375.08 |
| 12 | 1173 | 1238.66 | 1235.73 | 72 | 1402 | 1395.49 | 1405.77 |
| 13 | 1173 | 1171.73 | 1159.16 | 73 | 1404 | 1400.48 | 1410.88 |
| 14 | 1027 | 1025.89 | 1006.73 | 74 | 1435 | 1402.48 | 1409.20 |
| 15 | 1079 | 1077.83 | 1052.99 | 75 | 1418 | 1410.47 | 1415.74 |
| 16 | 1094 | 1092.82 | 1090.88 | 76 | 1392 | 1416.47 | 1421.63 |
| 17 | 1081 | 1079.83 | 1079.29 | 77 | 1394 | 1390.50 | 1392.16 |
| 18 | 1083 | 1081.83 | 1080.38 | 78 | 1392 | 1392.49 | 1390.61 |
| 19 | 1081 | 1079.83 | 1079.89 | 79 | 1356 | 1390.50 | 1390.66 |
| 20 | 1078 | 1076.83 | 1079.89 | 80 | 1357 | 1354.53 | 1351.53 |
| 21 | 1106 | 1076.83 | 1076.81 | 81 | 1378 | 1355.53 | 1349.80 |
| 22 | 1091 | 1104.80 | 1107.65 | 82 | 1318 | 1376.51 | 1376.82 |
| 23 | 1109 | 1089.82 | 1094.35 | 83 | 1233 | 1316.58 | 1314.52 |
| 24 | 1092 | 1107.80 | 1110.02 | 84 | 1195 | 1231.67 | 1214.52 |
| 25 | 1044 | 1090.82 | 1093.65 | 85 | 1035 | 1193.71 | 1170.74 |
| 26 | 1039 | 1042.87 | 1037.69 | 86 | 1023 | 1033.88 | 1002.82 |
| 27 | 1052 | 1037.88 | 1028.82 | 87 | 972 | 1021.89 | 983.19 |
| 28 | 1028 | 1050.86 | 1047.80 | 88 | 974 | 970.95 | 946.63 |
| 29 | 1039 | 1026.89 | 1025.31 | 89 | 944 | 972.95 | 953.72 |
| 30 | 1052 | 1037.88 | 1035.13 | 90 | 951 | 942.98 | 931.42 |
| 31 | 1047 | 1050.86 | 1053.06 | 91 | 940 | 949.97 | 940.79 |
| 32 | 1021 | 1045.87 | 1048.70 | 92 | 951 | 938.98 | 934.96 |
| 33 | 1053 | 1019.90 | 1018.57 | 93 | 957 | 949.97 | 947.73 |
| 34 | 1045 | 1051.86 | 1051.25 | 94 | 944 | 955.97 | 957.53 |
| 35 | 966 | 1043.87 | 1048.08 | 95 | 966 | 942.98 | 943.71 |
| 36 | 938 | 964.96 | 958.55 | 96 | 989 | 964.96 | 966.08 |
| 37 | 973 | 936.99 | 920.65 | 97 | 985 | 987.93 | 994.45 |
| 38 | 946 | 971.95 | 964.36 | 98 | 956 | 983.94 | 990.36 |
| 39 | 943 | 944.98 | 943.99 | 99 | 986 | 954.97 | 955.23 |
| 40 | 959 | 941.98 | 937.42 | 100 | 965 | 984.93 | 984.52 |
| 41 | 1008 | 957.96 | 957.58 | 101 | 937 | 963.96 | 966.75 |
| 42 | 1013 | 1006.91 | 1014.12 | 102 | 943 | 935.99 | 931.92 |
| 43 | 1028 | 1011.90 | 1023.26 | 103 | 931 | 941.98 | 937.35 |
| 44 | 993 | 1026.89 | 1034.88 | 104 | 926 | 929.99 | 928.06 |
| 45 | 1003 | 991.93 | 995.37 | 105 | 917 | 925.00 | 922.20 |
| 46 | 996 | 1001.92 | 1000.16 | 106 | 929 | 916.01 | 913.62 |
| 47 | 1014 | 994.92 | 995.71 | 107 | 889 | 928.00 | 927.17 |
| 48 | 1027 | 1012.90 | 1014.41 | 108 | 806 | 888.04 | 886.01 |
| 49 | 1047 | 1025.89 | 1031.05 | 109 | 769 | 805.13 | 790.83 |
| 50 | 1133 | 1045.87 | 1052.58 | 110 | 788 | 768.17 | 746.58 |
| 51 | 1156 | 1131.78 | 1146.73 | 111 | 794 | 787.15 | 773.94 |
| 52 | 1178 | 1154.75 | 1177.05 | 112 | 788 | 793.14 | 790.18 |
| 53 | 1152 | 1176.73 | 1193.23 | 113 | 803 | 787.15 | 785.95 |
| 54 | 1140 | 1150.75 | 1160.40 | 114 | 782 | 802.13 | 802.46 |
| 55 | 1143 | 1138.77 | 1139.42 | 115 | 811 | 781.15 | 781.88 |
| 56 | 1146 | 1141.76 | 1141.70 | 116 | 822 | 810.12 | 810.58 |
| 57 | 1204 | 1144.76 | 1146.14 | 117 | 796 | 821.11 | 827.14 |
| 58 | 1215 | 1202.70 | 1209.95 | 118 | 810 | 795.14 | 797.33 |
| 59 | 1245 | 1213.69 | 1227.16 | 119 | 786 | 809.12 | 808.33 |
| 60 | 1261 | 1222.68 | 1232.01 | 120 | 763 | 785.15 | 785.14 |

Key:

Yt = Original Series

SŶt = Estimates from Stationary Model

NSŶt = Esimates from Non-Stationary Model

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