#  NORMAL FORM FOR MODULE OF EQUIVARIANTS

# ABSTACT

The subject of dynamical systems is concerned with:

1. Given an matrix describe the behavior , in a neighborhood of the origin the solutions of all systems of differential equations having a rest point at the origin with the linear part
2. Describe the be the behavior (near origin) of all systems close to a system of the type described in I above.

The normal form is intended to be the ‘’simplest ‘’form in which any system of intended type can be transformed by changing the coordinates in a prescribed manner. But if a a normal for is thought as the ‘’simplest’’ form into which a system can be placed there might be disagreement as to what is considered simplest. A systematic policy for deciding what accounts for simplest is called normal form style. The important normal form styles are Semisimple, Innerproduct and SL(2) Or Triad styles.

The unfolding of the normal form is intended to be the simplest form in which all systems close to the original system can be transformed

I will present the procedure for obtaining the normal form using the inner product style and explain the structure of normal form using the language of equivariants over a ring of invariants and give an algorithm suitable for use in symbolic computation systems procedure

# 1.STANLEY DECOMPOSITION

Give the Grobner basis for the ker is **G **2 2 2}

We write down the Stanley decomposition of the ring of invariants. We know that the Groebner basis on the ideal of relations 

Let be the ideal generated by the leading terms of the Groebner basis for , which is a monomial ideal. It is a fact that the Stanley decomposition of is the same as the Stanley decomposition of and so it suffices to work  [7]

Where



The Stanley decomposition of the normal form with linear part 

Where 

The only generator, which forms a Groebner basis for the ideal of relations among the basic invariants, is

2

This relation can be used to eliminate the combination 2 (the leading term of the relation from any polynomial.

Consider the ring = of polynomial in the known basic invariants. The representation of an element in is not unique but this equation itself can be used to restore uniqueness by excluding  or any higher power of that is where  appear in the polynomial in it can be replaced by the right hand of the equation and the expression obtained this way will be unique. Thus the Stanley decomposition of is  (i)

Another way is to say that this polynomials incan be written uniquely as (ii)

Where and are polynomials in five variables. Still another way to say this is that each element is uniquely expressible as a linear combination of standard monomials which are monomials in basic invariants that contain at most one factor of (equivalently, not divisible by 2). Everything stated in this paragraph remains true if the polynomial is replaced by formal power series provided equation (i) is written as **R**= and linear combination is understood as infinite linear combinations.

The Stanley decomposition of (ii) can be abbreviated as. The function and will be referred as coefficients and and being called Stanley basis elements.

# 1.2 Table of functions

To generate the table of functions of Stanley decomposition, we replace each term in (ii) by a rational function  in and (symbols standing for degree in and weight) constructed as follows;

1. For each basic invariant appearing in a coefficient, the dominator is to contain the factor  where and are degree and weight of the invariants
2. For each term the numerator will be  where and are the degree and weight of the Stanley basis element for that term.

When the table of function is fully expanded as a power series in and, it contains the term  if and only if there exists standard monomials that have total degree and weights. Thus, the table of function is encoded version of that part of the top weight list, carried to all orders that can be generated by , To prove =ker , it would suffice to prove that the number of elements in the chains having these top weights in degree is equal to dim .

The following lemma gives a method to check that enough basic invariants have been found.

**Lemma 1.2.1** *Let { X, Y, Z} be a triad of  matrices, let { X, Y, Z} be the induced triad, and suppose that Il* ,…,*It* *is a finite set of polynomials in ker X, let be a subring of; suppose that the relations among the* *have been found, and that the Stanley decomposition and its associated table function have been determined.*

Then if and only if

 

By lemma 1.1.1.we will note that



Note

We have chosen a term ordering on the symbols such that 2 is the leading term of +- we associate the monomial ideal generated by its leading term (. A standard monomial is any monomial in these symbols that does not belong to this monomial ideal.

# 2. NORMAL FORM MODULE OF EQUIVARIATS

Here we obtain the Stanley decomposition of the module ker r, given the Stanley decomposition of ker x. To do so we shall use the following theorem theorem,

**Theorem 2.1** *The image of*  is ker  and the mapping: ker *X* →  is an isomorphism of modules over the ring ker *.*

Proof. Observe that if *f* ker and g  *P* (n, ), then

(*f*g) = .

It follows that if g ker  (for any ) then, :that is ker is a module over ker.

The image of is ker r1 r2 ,…,ker rk and the mapping.

r1 r2 rk is an isomorphism of modules over the ring ker x.[5]

# 2.1 Stanley decomposition for the normal form module ker x

**Lemma 2 .1.1***Let be any sub ring of ker x generated by homogenous polynomials I,…,I, in which are weight vectors for the triad {and let Rik be the vector sub space of consisting of polynomials homogenous in x of degree I and weight k. Let a Groebner basis for the relations of II,…, Is be be selected. Then:*

1. *The standard monomials II,…, Is( with respect to the given Groebner basis) having degree i( in x) and weight k form a basis for Rik.*
2. *If the standard monomials of degree I form a set of chain tops for the chains in pi(n *

Acoording to this lemma the chain tops of n under the triad {can be taken to be the standard monomials in the basis invariants, say  with respect to the Stanley decomposition of ker . The chains under these chain tops are the Jordan chains of y and can be obtained by repeated application of y(regarded as a mapping downward), a vector space basis foe ker is obtained by computing the y iterates of depth r.

We now describe how to obtain the Stanley decomposition of ker r for any Stanley decomposition of ker .

We recall here the definition of prefix ring. Let be the leading monomials of the Groebner basis of the ideal of relations. Let be a stripped suffix or suffixes and let be the first basic invariant (from the left) appearing in g. if is a Standard monomial with respect to the ideal where then the suffix monomial associated with the given stripped suffix g are the standard monomials with respect to the (new) ideal (ordering the basic invariants by <if j<i). Now let be a prefix monomials associated with a given suffix S, then the collection of polynomials which are linear combination of such prefix monomials for a given suffix s is a ring called prefix ring for s which has a Stanley decomposition(defined by its standard prefix monomials).

We now consider N33 to illustrate the above steps.

Ordering the invariants by .

These are related by the single relation 2 + 2 2 with 2 as the leading monomial. To compute ker 3 it is necessary to apply y to depth 3.

A monomial of the first type has the form I where I is an arbitrary monomial not involving  that is I. The entries under the Chain top of this form will be I,I2   and I3 such an entry will be said to have prefix I and suffix ,2 or 3 . A suffix always begins with . The prefix ring for a given suffix is the set of admissible prefixes that can appear with that suffix; the prefix ring for each suffix 2 and 3 .

A monomial of the second type has the form Iwith I , the entries under it have prefix I suffix 2 and 3 . The monomial of the third type has the form Iwith I the entries under it have the prefix I and suffix 2 or 3 

Ker 3 =

2 

3 

2 3 

2 

3 

Having obtained the Stanley decomposition for ker 3 the normal form for ker x that is the module of equivariants is obtained by Theorem 2.0.1.i.e. ker r1 ker r2 rk

Hence for N33 we have ker 3ker 3

Ker x = (

2  }

3 

2 }

3 

2 3 

(

2 }

3  

2 3 

2 

3 

Where

,  , 2 , 3 

=, 2 , 3 }=, ,

2 =, 3 , , ,

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# 3.0 APPLICATION

# 3.1 Sositaisvilis theorem

Consider the parameter dependent vector field.

x,y,z,

x,y,z,

 x,y,z,

 C S U p

Where

(0, 0, 0) = 0

 

And and are r in some neighbours of the origin A, is C x C matrix having eigenvalues with zero real part, B is a S x S matrix having eigenvalues with positive real parts.

The centre manifold Theorem tells us that near the origin in C S U p, the flow generated by equations above is C0 conjugated to the flow generated by the following vector field.






 (x,y,z, C S U p

Where w() represents the Cr vector fields on the centre manifold. The results are due to Sositaisvili(1975)[2].

A wide area where this knowledge is also necessary is in Poincare maps where the idea is to reduce continuous time systems (flows) to discrete time system (maps)

Under these we have:

1. Poincare map near s periodic orbit.
2. Poincare maps of a time periodic ordinary differential equation.
3. The Poincare map near homoclinic orbit.
4. The Poincare map associated with two degree of freedom Hamiltonian systems.

# 3.2 Areas for Further Research

* Generalize to N33…..3 and mixed problems like N233.
* The role of transvectants in N33 in particular.
	+ Deriving the relation among the basic invariants from the trasvectant expression.
	+ Is there a way to obtain the Groebner basis work using the transvectants?
	+ Relation between the transvectant structure and the bracket aldgebra that Cushman and Sanders use.
* Obtaining the invariants using the cross – section method.

# 4.0 CONCLUSION

The method of normal forms provideds a way of finding a coordinate system in which the dynamical systems takes the “simplest” form. Three important characteristics are apparent.

1. The method is local in the sense that the coordinate transformations are generated in a neighbourhood of a known solution.
2. In general, the coordinate transformations will be non linear functions of the dependent variables. However, the important point is that these coordinate transformations are found by solving a sequence of linear problems.
3. The structure of the normal form is determined entirely by the nature of linear part of the vector field.

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