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GEOMETRICAL MOTIVES CATEGORIES TO DETERMINE CO-CYCLES AS SOLUTIONS IN FIELD THEORY

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Resumen

The principal goal is generalize the field theory on spaces that admit decomposing in components that can be manageable in the complex Riemannian manifolds whose complex varieties can be part of those components called motives, creating a decomposition in the derived category of its spectrum where solutions of the field equations are defined in a hypercohomology. The derived tensor product $\otimes^{tr}_{\mathbb{L},ét}$ induces a tensor-triangulated structure to a derived category more general than $\mathrm{D}^{-}R(\mathcal{A})$, as for example, $\mathrm{DM}^{eff,-}_{ét}(k, \mathbb{Z}/m)$, which is our objective. In this case, we want geometrical motives, where this last category $\mathrm{DM}^{eff,-}_{ét}(k, \mathbb{Z}/m)$, can be identified for the derived category $\mathrm{DM}^{-}_{gm}(k, R)$. Then the solutions to the category DQFT (the category of complexes to quantum field equations $d\mathrm{da} = 0$) are integrals of certain cohomology group. For other way, the category DQFT can be defined as of the motives in a hypercohomology of the type $H^{p,q}_{L}(X, \mathbb{Q}) = \mathbb{H}^{p,q}(X, \mathbb{Q})$, from the category Sm_k, on $\mathbb{Q}$- modules with transfers.

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1. Introduction

The intention of this study is establish a methodology through of commutative rings and their construction of a total tensor product $\otimes^{\mathbb{L}}$, [2] on the category PST(k), considering extensions of the tensor products $\otimes_{R(\mathcal{A})}$, to obtain resolution in the projective sense of infinite sequences of modules of Étale sheaves, being these, fundamental sheaves to the construction of derived categories of geometrical motives. These sheaves are pre-sheaves of Abelian groups on the category of smooth separated schemes restricted to scheme X.

Likewise, the immediate application of the derived tensor products will be the determining of the tensor trianguled category $\mathrm{DM}^{-}_{\acute{e}tt}(k, \mathbb{Z}/m)$, of Étale motives to be equivalent to the derived category of discrete $\mathbb{Z}/m$- modules over the Galois group $G = \mathrm{Gal}(k_{sep}, /k)$, which says on the equivalence of functors of tensor triangulated categories[3] . This is very useful to characterize a derived category in quantum field theory on k- modules that can be finer in equivalence classes of the category Sm_k, through the corresponding derived tensor product $\otimes^{tr}_{\acute{e}t}$, of pre-sheaves.

[2] $\mathbb{L}$, is a Lefschetz motive $\mathbb{Z}$ (1) [1].

[3] **Theorem.** If $1/m \in k, (\mathscr{L}, \otimes_{\mathscr{L}})$, is a tensor trianguled category and the functors

$$\mathrm{D}^{-}(G, \mathbb{Z}) \xrightarrow{\pi^*} \mathscr{L} \longrightarrow \mathrm{D}(W_{\mathbb{A}}^{-1}) = \mathrm{DM}^{eff,-}_{\acute{E}t}(k, \mathbb{Z}/m),$$

are equivalences of tensor triangulated categories. Here π^*, is a trangulated functor from $\mathrm{D}^{-}(G, \mathbb{Z}/m)$, until the category $\mathrm{D}^{-}(Sh_{\acute{e}t}(Cor_k, \mathbb{Z}/m))$.

Then the mean result of derived tensor products will be in tensor triangulated category $\mathrm{DM}_{Nis}^{eff,-}(k,R)$, of effective motives and their subcategory of effective geometric motives $\mathrm{DM}_{gm}^{eff,-}(k,R)$. Likewise, the motive $M(X)$, of a scheme X, is an object of $\mathrm{DM}_{Nis}^{eff,-}(k,R)$, and belongs to $\mathrm{DM}_{gm}^{eff,-}(k,R)$, if X, is smooth. However, this requires the use of cohomological properties of sheaves associated with homotopy invariant pre-sheaves with transfers for Zariski topology, Nisnevich and cdh topologies.

Finally, all this treatment goes in-walked to develop a motivic cohomology to establish a resolution in the field theory to solutions in the field equations $d\mathrm{da} = 0$, incorporating singularities in the complex Riemannian manifolds where singularities can be studied with deformation theory through operads, motives and deformation quantization [1].

2. Derived Triangulated Categories with Structure by Pre-Sheaves $\otimes^{\mathbb{L}}$ and $\otimes_{\mathbb{L},\acute{e}t}^{tr}$

We start giving some definitions for after establish some lemmas on the triangulated structure of derived categories with pre-sheaves in derived tensor products to build the derived category of the total tensor product that will determine the hypercohomology under the structure of a derived category of geometrical motives on k-modules. Here, k must be the numerical field more adequate to quantum field theory, that is to say, on $\mathbb{Q}$- modules.

Def. 2.1. A pre-sheaf with transfers is a contravariant additive functor:

$$F = \mathrm{Cor}_k \to \mathrm{Ab}, \tag{1}$$

Then we write

$$\mathrm{Pre}Sh(\mathrm{Cor}_k) \to \mathrm{PST}(k) = \mathrm{PST}, \tag{2}$$

to describe the functor category on the field k, whose objects are pre-sheaves with transfer and whose morphisms are natural transformations.

Analogously we can define to the tensor product $\otimes^{tr}$, and their extension to $\otimes^{tr}_{ét}$.

Then we have the definition.

Def. 2.2. If F, and G, are pre-sheaves of R- modules with transfers, we write:

$$(F \otimes^{tr} G)_{ét} \to F \otimes^{tr}_{ét} G, \tag{3}$$

to establish that the Étale sheaf associated to $F \otimes^{tr} G$, is $(F \otimes^{tr} G)_{ét}$.

If C, and D, are bounded above complexes of pre-sheaves with transfers, we shall write $C \otimes^{tr}_{ét} D$, for $(C \otimes^{tr} D)_{ét}$, and

$$(C \otimes^{tr}_{L} D) \cong P \otimes^{tr}_{ét} Q, \tag{4}$$

where P, and Q, are complexes of representable sheaves with transfers, $P \cong C$, and $Q \cong D$. Then there is a natural mapping

$$(C \otimes^{tr}_{L,ét} D) \to C \otimes^{tr}_{ét} D, \tag{5}$$

induced by

$$(C \otimes^{tr}_{L} D) \to C \otimes^{tr} D, \tag{6}$$

Lemma 2. 1. If F, and F', are Étale sheaves of R- modules with transfers, and F, is locally constant, the mapping:

$$h_X(U) \otimes_R h_Y(U) = \mathrm{Hom}_{\mathcal{A}}(U,X) \otimes \mathrm{Hom}_{\mathcal{A}}(U,Y) \xrightarrow{\otimes} \mathrm{Hom}_{\mathcal{A}}(U \otimes U, X \otimes Y)$$
$$\xrightarrow{\Delta'} \mathrm{Hom}_{\mathcal{A}}(U, X \otimes Y) = h_{X \otimes Y}(U), \tag{7}$$

induces an isomorphism

$$F \otimes_{ét} F' \xrightarrow{\cong} F \otimes_{ét}^{tr} F', \tag{8}$$

Remember that a pre-sheaf is defined as:

Def. 2. 3. A pre-sheaf F, of Abelian groups on Sm$/k$, is an Étale sheaf if it restricts to an Étale sheaf on each X, in Sm$/k$, that is if:

1. The sequence

$$0 \to F(X) \xrightarrow{diag} F(U) \xrightarrow{(+,-)} F(U \times_X U), \tag{9}$$

   is exact for every surjective Étale morphism of smooth schemes,

$$U \to X, \tag{10}$$

2. $F(X \underline{\cup} Y) = F(X) \oplus F(Y), \forall X, Y$, schemes.

We demonstrate the lemma 2. 1.

Proof. We want that the tensor product $\otimes_{\mathbb{L},ét}^{tr}$, induces to a tensor triangulated structure on the derived category of Étale sheaves of R- modules with transfers [4] defined in other expositions [4]. We consider the proposition [5] then we have:

$$(C \otimes_{L,ét}^{tr} D) \to D \otimes_{L,ét}^{tr} C, \tag{11}$$

Then is sufficient to demonstrate that $\otimes_{\mathbb{L},ét}^{tr}$, preserve quasi-isomorphisms. The details can be consulted in [5].

[4] **Definition.** A presheaf with transfers is a contravariant additive functor from the category Cor$_k$, to the category of Abelian groups Ab.

[5] **Proposition.** The derived category $D^{-}R(\mathcal{A})$, equipped with $\otimes^{\mathbb{L}}$- structure is a tensor-triangulated category.

In the demostration of this proposition was used the particular fact of that Hom$(F, \bullet)$ is left exact and $F \otimes \bullet$, is right exact.

Then the tensor product $\otimes_{ét}^{tr}$, as pre-sheaf to Étale sheaves can to have a homology space of zero dimension that is vanished in certain component right exact functor $\Phi(F) = R_{tr}(Y) \otimes_{ét}^{tr} F$, from the category $\mathrm{PST}(k, R)$, of pre-sheaves of R- modules with transfers to the category of the Étale sheaves of R-modules and transfers. Then every derived functor $L_n\Phi$, vanishes on $H_0(\tilde{C})$, to certain complex of Étale.

Then all right exact functor $R_{tr}(Y) \otimes_{ét}^{tr} F$, is acyclic. This is the machinery to prove the functor exactness and resolution in modules through of induce from $\otimes_{\mathbb{L},ét}^{tr}$, a tensor-triangulated structure to a derived category more general that $\mathrm{D}^{-}R(\mathcal{A})$.

In addition, we have:

Lemma 2. 2. Fix Y, and set $\Phi = R_{tr}(Y) \otimes_{ét}^{tr}$. If F, is a pre-sheaf of R-modules with transfers such that $F_{ét} = 0$, then $L_n\Phi(F) = 0, \forall n$.

Proof. [2].

To establish a triangulated tensor category on the Étale pre-sheaves defined in $\mathbb{Q}$- modules, we require related sheaves to the Étale pre-sheafs such that their homotopy invariance as also their cohomology can be equivalent. Because to the Nisnevich sheaves we have the following property: If F, is a homotopy invariant pre-sheaf with transfer and k, is a perfect field then the associated Nisnevich sheaf, which we can denote as F_{Nis}, is homotopy invariant and also complies to their cohomologies.

Also considering the lemma 2. 1, we have that the tensor product $\otimes_{\mathbb{L},ét}^{tr}$, endows to Étale derived category to be a tensor triangulated category, which will be relevant to obtaining through the Étale sheaves of a tensor triangulated category of motives. Likewise, we have:

Corollary. 2. 1. The tensor product $\otimes_{\mathbb{L},ét}^{tr}$, endows $\mathrm{DM}_{ét}^{eff,-}(k, \mathrm{R})$, with the structure of a tensor triangulated category.

Proof. [2-4].

Then in the motives context we have the following lemma.

Lemma 2. 2. Let F, be a Zariski sheaf of $\mathbb{Q}$- modules with transfers. Then F, is also an Étale sheaf with transfers [4].

The association of sheaves with Zariski topology and establish their equivalence to Étale sheaves on $\mathbb{Q}$- modules help us to determine a structure of derived categories with geometrical motives even in singularities. Now in homotopy invariance obtained through Nisnevich sheaves we have the following consequence from lemma 2. 1, and from the tensor triangulated categories of motives.

Corollary. 2. 2. If F, is a pre-sheaf of $\mathbb{Q}$- modules with transfers, then we have $F_{Nis} = F_{ét}$.

Then also their homologies:

Proposition 2. 1. If F, is an Étale sheaf of $\mathbb{Q}$- modules, the homologies comply that:

$$H^n_{ét}(\_, F) = H^n_{Nis}(\_, F), \tag{12}$$

Then consequently we can enunciate the following functor.

Let $\mathscr{L}_{ét}$, denote the full subcategory of derived category $\mathrm{D}^-_{ét} = \mathrm{D}^{,-}(\mathrm{Sh}_{ét}$ $(\mathrm{Cor}_k, \mathrm{R})$, consisting of complexes with homotopy invariant cohomology sheaves.

Theorem 2. 1. The natural functor

$$\mathscr{L}_{\acute{e}t}, \rightarrow \mathrm{DM}_{\acute{e}t}^{eff,-}(k, \mathrm{R}), \tag{13}$$

is an equivalence of triangulated categories if $\mathbb{Q} \subseteq \mathrm{R}$.

This is the part of the theorem described in the note foot 2, that we use in the demonstration of the main theorem in this paper.

3. Considerations to the Field equations.

Remember that in the derived geometry we work with structures that must support R- modules with characterizations that should be most general to the case of singularities, where is necessary to use an irregular connection, if was the case, for example in field theory in mathematical physics when are studied the quantum field equations on a complex Riemann manifold with singularities.

Characterizing connections through derived tensor products [2] we search precisely generalize the connections through pre-sheaves with certain special properties, as can be the Étale sheaves [4].

Remember we want generalize the field theory on spaces that admits decomposing in components that can be manageable in the complex manifolds whose complex varieties can be part of those components called motives, creating a decomposition in the derived category of its spectrum considering the functor Spec, and where solutions of the field equations are defined in a hypercohomology [6] . Likewise, this goes focused to obtain a good integrals theory (solutions) in the hypercohomology context considering the knowledge of the

[6]**Definition.** A hyperhomology or hypercohomology of a complex of objects of an Abelian category is an extension of the usual homology of an object to complexes. The mechanism to give a hypercohomology is suppose that $\mathcal{A}$, is an Abelian category with enough injectives and Φ, a left exact functor to another Abelian category $\mathcal{B}$. If C, is a complex of objects of $\mathcal{A}$, bounded on the left, the hypercohomology $H^i(C)$, of C, (for an integer i) is calculated as follows: is taken a quasi-isomorphism $\psi : C \rightarrow I$, where I, is a complex of injective elements of $\mathcal{A}$. The hypercohomology $H^i(C)$, of C, is then the cohomology $H^i(\Phi(I))$, of the complex $\Phi(I)$.

spectral theory of the cycle sequences in theory that searches the solution of the field equation even including singularities of the complex Riemann manifold.

We can demonstrate that $\otimes_{L,ét}^{tr}$, induces a tensor-triangulated structure to a derived category more general than $\mathrm{D}^{-}R(\mathcal{A})$, as for example, $\mathrm{DM}_{ét}^{eff,-}(k, \mathbb{Z}/m)$, which is our objective. In this case, we want geometrical motives, where this last category $\mathrm{DM}_{ét}^{eff,-}(k, \mathbb{Z}/m)$ can be identified for the derived category DM_{gm}^{-} (k, R).

We consider and fix Y, and the right exact functor $\Phi(F) = R_{tr}(Y) \otimes_{ét}^{tr} F$, from the category $\mathrm{PST}(k, R)$, of pre-sheaves of R- modules with transfers to the category of the Étale sheaves of R- modules and transfers. Likewise, their left functors $L_p\Phi(F)$, are the homology sheaves of the total left derived functor $\Phi(F) = R_{tr}(Y) \otimes_{L,ét}^{tr} F$.

Considering a chain complex C, the hypercohomology spectral sequence is:

$$E_{p,q}^{2} = L_p\Phi(H_qC), \tag{14}$$

then

$$\mathbb{L}_{p+q}\Phi(C) = 0, \tag{15}$$

Then the corresponding infinite sequence is exact (and are had integrals to the complexes of field equations [1]).

We consider A, and $B \in \mathcal{A}$, where $\mathcal{A}$, is a category as has been defined before.

We have the following propositions demonstrated in [5].

Proposition 3. 1. There is equivalence between categories $\mathrm{Ab}(\mathrm{CRing}_{A//B})$ $\cong \mathrm{Mod}_B$.

Then a hypercohomology as given to $d\mathrm{da} = 0$, can be obtained through double functor work $A \rightarrow B \rightarrow B$, through an inclusion of a category Mod_B, in $\mathrm{CRing}_{A//B}$. Then is had the result.

Theorem 3. 1. The left adjoint to the inclusion functor $\mathrm{Mod}_B \hookrightarrow \mathrm{CRing}_{A//B}$, is defined by $X \mapsto \Omega_{X/A} \otimes_X B$. In particular, the image of $A \to B \to B$, under this functor is $B \mapsto \Omega_{X/A}$.

The derived tensor product is a regular tensor product.

Theorem 3. 2. The character to an adjoint lifts to a homotopicaly meaningful adjunction complies:

$$\mathrm{Ch(B)}_{\geqslant B} \longleftrightarrow s\mathrm{CRing}_{A//B}, \tag{16}$$

Meaning that is an adjunction of categories, which induces an adjunction to level of homotopy categories.

We define the cotangent complex required in derived geometry and QFT.

Def. 3. 1. The cotangent complex $\mathbb{L}_{A/B}$, is the image of functor $A \to B \to B$, under the left functor of the Kähler differentials module $M \otimes^L_{R(\mathcal{A})}$. Likewise, if $P_\bullet \to B$, be a free resolution then

$$\mathbb{L}_{A/B} = \Omega_{P_\bullet/A} \otimes_{P_\bullet} B, \tag{17}$$

The cotangent complex as defined in (17) lives in the derived category Mod_B. We observe that choosing the particular resolution of B, then $\Omega_{P_\bullet/A}$, is a co-fibrant object in the derived category $\mathrm{Mod}_{P_\bullet}$, which no exist distinction between the derived tensor product and the usually tensor product. Then to any representation automorphic of $G(\mathbb{A})$, the $G(F)/G(\mathbb{A})$, can be decomposed as the tensor product $\otimes^n_{i=1}\pi_I$. This last fall in the ramification theory to Langlands ramifications.

Example 3. 1. This, in the context of solution of field equations as $d\mathrm{da} = 0$, has solution in the hypercohomology of a spectral sequence of $\mathrm{D}^- R(\mathcal{A})$, (established on the infinite sequence $... \to F^n \to 0 \to ...$[1]) when its functors whose image $\Omega_{B/A}$, have as its cotangent complex the image under of the functor $\mathbb{L}_{A/B}$, which is the functor image $A \to B \to B$, under the left derived functor of Kähler differentials.

To demonstrate this, is necessary to define an equivalence between derived categories in the level of derived categories $\mathrm{D}({}^L\mathrm{Bun}, \mathcal{D})$, and $\mathrm{D}({}^L Loc, \mathcal{O})$, where geometrical motives can be risked with the corresponding moduli stack to holomorphic bundles. The integrals are those whose functors image will be in $\mathrm{Spec}_H\mathrm{SymT}(\mathrm{OP}_{{}^LG}(D))$, is the variety of opers on the formal disk D, or neighborhood of all point in a surface Σ, in a complex Riemannian manifold [1].

4. Main theorem.

As was showed the geometrical motives required in our research are result of embed the derived category $\mathrm{DM}^{-}_{gm}(k, R)$, (geometrical motives category) in the $\mathrm{DM}^{eff,-}_{\acute{e}t}(k, \mathbb{Z}/m)$, considering the category of smooth schemes on the field k.

We consider the following functors. For each $F \in \mathrm{D}^{-}(\mathrm{Sh}^{Nis}(\mathrm{Cor}(k)))$, there is $L_{\mathbb{A}^1}F \in \mathrm{D}^{eff}_{-}(k)$, the resulting functor is:

$$L_{\mathbb{A}^1} : \mathrm{D}^{-}(\mathrm{Sh}^{Nis}(\mathrm{Cor}(k))) \to \mathrm{D}^{\mathrm{eff}}_{-}(k), \tag{18}$$

which is exact and left-adjoint to the inclusion

$$\mathrm{D}^{eff}_{-}(k) \to \mathrm{D}^{-}(\mathrm{Sh}^{Nis}(\mathrm{Cor}(k))), \tag{19}$$

Also the functor (18) decends to an equivalence of triangulated categories. This is very useful to make $\mathrm{D}^{eff}_{-}(k)$, into a tensor category in the way as follows. We consider the Nisnevich sheaf $\mathbb{Z}_{tr}(k)$, with transfer $tr : Y \to c(Y, X)$. We define

$$\mathbb{Z}_{tr}(k) \otimes \mathbb{Z}_{tr}(k) := \mathbb{Z}_{tr}(X \times_k Y), \tag{20}$$

Then can be demonstrated that the operation realised in (18) can be extended to give $\mathrm{D}^{-}(\mathrm{Sh}^{Nis}(\mathrm{Cor}(k)))$, with the structure of a triangulated tensor category. Then the functor $L_{\mathbb{A}^1}$, induces a tensor operation on $\mathrm{D}^{-}_{\mathbb{A}^1}\mathrm{Sh}^{Nis}(\mathrm{Cor}(k)))$, making that the itself $\mathrm{D}^{-}_{\mathbb{A}^1}\mathrm{Sh}^{Nis}(\mathrm{Cor}(k)))$, is a triangulated tensor category. Likewise, explicitly in $\mathrm{DM}^{\mathrm{eff}}_{-}(k)$, this gives us the functor

$$m : \mathrm{Sm}_k \to \mathrm{DM}^{eff}_{-}(k), \tag{21}$$

defined by

$$m(X) := C^{Sus}(Z_{tr}(X)), \tag{22}$$

where we have the formula

$$m(X \times_k Y) = m(X) \otimes m(Y), \tag{23}$$

If we consider the embedding theorem, then we can establish the following triangulated scheme

$$\begin{array}{ccc} \mathrm{Sm}_k & \longrightarrow & \mathrm{DM}_{gm}^{eff}(k) \\ & m \searrow & \updownarrow \mathrm{Id} \\ & & \mathrm{DM}_{gm}^{eff}(k) \end{array} \tag{24}$$

which has implications in the geometrical motives applied to bundle of geometrical stacks in mathematical physics.

Theorem 4. 1 (F. Bulnes). Suppose that $\mathbb{M}$, is complex Riemannian manifold with singularities. Let be X, and Y, smooth projective varieties in $\mathbb{M}$[7]. We know that solutions of the field equations $d\mathrm{da} = 0$, are given in a category $\mathrm{Spec}(\mathrm{Sm}_k)$, (see example 4). Solutions of the quantum field equations for $d\mathrm{da} = 0$, are defined in a hyper-cohomology on $\mathbb{Q}$- coefficients from the category Sm_k, defined on a numerical field k, considering the derived tensor product $\otimes_{\acute{e}t}^{tr}$, of pre-sheaves. Then the following triangulated tensor category scheme is true and commutative:

$$\begin{array}{ccc} & \mathrm{DQFT} & \\ i \swarrow & & \searrow F \\ \mathrm{MD}_{gm}(\mathbb{Q}) & \longleftrightarrow & \mathrm{MD}(\mathcal{O}_Y) \end{array} \tag{25}$$

[7]Singular projective varieties useful in quantization process of the complex Riemannian manifold. The quantization condition to compact quantizable Kähler manifolds can be embedded into projective space.

The category $\mathrm{DM}_{gm}^{eff}(k, R)$, has a tensor structure and the tensor product of its motives is defined in (23) as $m(X) \otimes m(Y) = m(X \times Y)$.

Triangulated category of geometrical motives $\mathrm{DM}_{gm}(k, R)$, or written simply as $\mathrm{DM}_{gm}(k)$, is defined formally inverting the functor of the Tate objects[8] (are objects of a motivic category called Tannakian category) $\mathbb{Z}(1)$, to be image of the complex $[\mathbb{P}^1] \to [\mathrm{Spec}(k)]$, where the motive in degree $p = 2, 3$, will be $m(p) = m \otimes \mathbb{Z}(1)^{\otimes_p}$, or to any motive $m \in \mathrm{DM}_{gm}^{eff}(k), \forall p \in \mathbb{N}$.

Likewise, the important fact is that the canonical functor $\mathrm{DM}_{gm}^{eff}(k), \to \mathrm{DM}_{gm}(k)$, is full embedding [6]. Therefore, we work in the category $\mathrm{DM}_{gm}(k)$.

Likewise, for X, and Y, smooth projective varieties and for any integer i, exist an isomorphism:

$$\mathrm{Hom}_{\mathrm{DM}_{gm}^{eff}(k)}(m(X), m(Y)(i)[2i]) \cong A^{m+i}(X \times Y), \quad m = \mathrm{dim}Y, \tag{26}$$

We demonstrate the theorem 4. 1.

Proof. $\forall X \in \mathrm{Sm}_k$, the category Sm_k, extends to a pseudo-tensor equivalences of cohmological categories over motives on k, that is to say, $\mathrm{MM}(k)$, is the image of functors[9]

$$\mathrm{DM}^{\mathrm{eff}}(k) \to \mathrm{DM}_{\mathrm{gm}}(k), \tag{27}$$

[8]Let $\mathrm{MT}(\mathbb{Z})$, denote the category of mixed Tate motives unramified over $\mathbb{Z}$. It is a Tannakian category with Galois group $\mathrm{Gal}_{\mathrm{MT}}$.

[9]The duality between derived category of motives can be understood under homology and cohomology. Likewise we can define the derived category of geometrical motives $\mathrm{DM}_{gm}(k)$, as image from a homological category of motives under the functor which maps a derived category belonging to the category Sm_k^{Op}, called Levin′s derived category, to the derived category of geometrical motives. Both derived categories obtained homologically are dual. Likewise, we have the important result.

Theorem. Let k, be a field admitting resolution of singularities. Sending $\mathbb{Z}_{\cdot X}(n)$, in the Levine′s derived category $\mathrm{DM}_{Lev}(k)$, to $\mathrm{Hom}_{\mathrm{DM}_{gm}}(m(X), \mathbb{Z}(n))$, in $\mathrm{DM}_{gm}(k)$, for $X \in \mathrm{Sm}_k$, extends to pseudo-tensor equivalence of cohmological categories over motives on k, having $\mathrm{DM}_{Lev}(k) \to \mathrm{DM}_{gm}(k)$, that is to say, an equivalence of the underlying triangulated tensor categories, compatible with respective functors on Sm_k^{Op}.

which is an equivalence of the underlying triangulated tensor categories.

For other way, the category DQFT can be defined as of the motives in a hypercohomology from the category Sm_k, defined as:

$$\mathrm{Hom}_{\mathrm{DM}_{gm}^{eff}(k)}(m(X), \mathbb{Q}(q)[p]) \cong H^{\bullet}_{Nis}(X, \mathbb{Q}(q)) = \mathbb{H}^{p,q}(X_{Nis}, \mathbb{Q}(q)), \quad (28)$$

which comes from the hypercohomology

$$H_L^{p,q}(X, \mathbb{Q}) = \mathbb{H}^{p,q}(X, \mathbb{Q}), \quad (29)$$

We observe that if a Zariski sheaf of $\mathbb{Q}$- modules with transfers F, is such that $F = H^q C$, for all C, a complex defined on $\mathbb{Q}(q)$- modules, being a special case when $C = \mathbb{Q}(q)$, where the cohomology groups of the isomorphism $H^p_{\acute{e}t}(X, F_{\acute{e}t}) \cong H^p_{Nis}(X, F_{Nis})$, can be vanished for $p > \dim(Y)$.

Then survives a hypercohomology $\mathbb{H}^q(X, \mathbb{Q})$. We consider $\mathrm{Spec}(\mathrm{Sm}_k)$ we can to have the quantum version of this hyper-cohomology with an additional work on moduli stacks of the category Mod_B, in a study on equivalence between derived categories in the level of derived categories $D({}^L\mathrm{Bun}, \mathcal{D})$, and $D({}^L Loc, \mathcal{O})$, where geometrical motives can be risked with the corresponding moduli stack to holomorphic bundles[10] .

For other way, with other detailed work of quasi-coherent sheaves [1, 7] we can to obtain the category $\mathrm{MO}_{\mathcal{O}}(Y)$. The functors are constructed using the Mukai-Fourier transforms.

□

[10] We consider the functor F, defined as:

$$F : \mathrm{D}(X \times_Y X) \xrightarrow{F^{\gamma}\alpha F} \mathrm{Mod}_T(\mathrm{D}(X \times_Y X)) \cong^{\perp} \mathcal{K}(F^{\gamma})\tilde{\mathrm{D}}_{\mathrm{holomorphic}}(\mathcal{LY}),$$

where $\mathcal{K}(F^{\gamma})$ is the kernel space of the functor F^{γ}, is the funtor that induces the equivalence $\mathrm{Mod}_T(\mathrm{D}(X \times_Y X)) \cong^{\perp} \mathcal{K}(F^{\gamma})$, and the operator $T = F^{\gamma} \circ F$, acting on category $\mathrm{D}(X \times_Y X))$.

5. Conclusions.

The determination of a hypercohomology as cohomology group where are defined the solutions of the field equations obeys to the triangulated derived categories that permit an scheme (triangle) commutative whose integrals are solutions of the field equations. The determination of this hypercohomology arise of the fact of that derived motivic category $\mathrm{DM}_{gm}(k)$, which is of the motivic objects whose image is under $\mathrm{Spec}(k)$, that is to say, an equivalence of the underlying triangulated tensor categories, compatible with respective functors on Sm_k^{Op}. The geometrical motives will be risked with the moduli stack to holomorphic bundles. By the lemma 2. 3, corollary 2. 2, proposition 2. 1, and Theorem 2. 1, the special case where complexes $C = \mathbb{Q}(q)$, are obtained when cohomology groups of the isomorphism $H^p(X, F_{ét}) \cong H^p_{Nis}(X, F_{Nis})$, can be vanished for $p > \dim(Y)$. We observe also the Beilinson-Soul$\acute{e}$ vanishing conjectures [8-10] where we have the vanishing $H^p(F, \mathbb{Q}(q)) = 0$, if $p \leqslant 0$, and $q > 0$, which confirms the before established. Then survives a hypercohomology $\mathbb{H}^q(X, \mathbb{Q})$. Then their objects are in $\mathrm{Spec}(\mathrm{Sm}_k)$. Likewise for the complex Riemannian manifold the integrals of this hypercohomology are those whose functors image will be in $\mathrm{Spec}_H\mathrm{SymT}(\mathrm{OP}_{L_G}(D))$, which is the variety of opers on the formal disk D, or neighborhood of all point in a surface Σ [1].

Technical Notation

$\mathrm{DM}_{gm}^{eff}(k)$- Derived category of effective geometrical motives. This is a fundamental derived category to build the tensor-triangulated category of motives.

$\mathbb{H}^q(X, \mathbb{Q})$- Hypercohomology space of dimension q. This is our hypercohomology considered as base to obtain its their quantum version with additional work in the moduli stack Mod_B, to obtaining of solutions of the equations $d\mathrm{da} = 0$.

$\mathbb{Q}$- modules- These modules are $k-$ modules obtaining of the equivalence of the category Sm/k. These are our modules to define our hypercohomology to the field equations.

$\otimes^{tr}_{\mathbb{L},ét}$- Tensor product of complexes of Étale sheaves with transfers.

Sm/k- Category of smooth separated schemes.

$\mathbb{H}^n_{Nis}(X, K)$- Nisnevich hypercohomology of complex of sheaves K.

Cor_k- Category of finite correspondences.

$\mathrm{DM}^{eff,-}_{ét}(k, R)$- Category of effective Étale motives.

$\mathrm{Hom}(M, N)$- Internal Hom, in DM_{gm}.

$\mathrm{DM}^-(k, R)$- Category of motives.
$\otimes^{tr}_{L,Nis}$- Tensor product on the derived categories $\mathrm{D}^-_{\mathbb{A}^1}(\mathrm{Sh}^{Nis}(\mathrm{Cor}(k)))$.

$H^{p,q}(X, \mathbb{A})$- Étale motivic homology.

$\mathrm{Hom}(F, G)$- Hom, pre-sheaf.

$\mathbb{M}$- Complex Riemannian manifold with singularities. Model of the space-time that includes quantum field phenomena.

$\otimes^{tr}_{ét}$- Tensor product of Étale sheaves with transfers.

$\otimes^{\mathbb{L}}$- Total tensor product.

$\mathcal{L}_{ét} - \mathbb{A}^1$- local object in $\mathrm{D}^-(\mathrm{Sh}_{ét}(\mathrm{Cor}_k, R))$.

$\mathrm{D}^-(G, \mathbb{Z}/n)$- Derived category of discrete $\mathbb{Z}/n$- modules over G.

$d\mathrm{da} = 0$- Field equations of the field $\mathrm{a} \in \mathbb{C}(\mathrm{Op}_L(D))$, which is in a hypercohomology of the type $\mathbb{H}^q(X, \mathbb{Q})$.

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