**Dynamic Correlations and Volatility effects at the Nigerian Naira Exchange rates markets**

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**Abstract**

This paper investigates the presence of time-varying co-volatilities and dynamic correlations in Nigerian naira exchange rates markets. Since asset returns and correlation dynamics are important ingredients in asset pricing, portfolio management and risk hedging, emphasis is therefore placed on the respective constant and dynamic equity market correlations produced by variants of multivariate volatility models such as the Diag-BEKK, Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC) and its Asymmetric types (ADCC), Dynamic Equicorrelation (DCC-DECO), Corrected DCC and Corrected ADCC (cADCC). The daily Nigerian Naira exchange rates with United States Dollars (Naira-USD), British Pound (Naira-GBP), European Euro (Naira-Euro) and Japanese Yen (Naira-JPY), French CFA (Naira-CFA) and West Africa Unit of Account (Naira-WAUA) were applied on the models. Each time series data span between 2 January, 2009 and 6 June, 2015. The volatility dynamics of naira exchange rates markets is found to exhibit time-varying correlations. A very interesting results in terms of parameter estimates was obtained, and the variants of the DCC models proved to be superior to Diag-BEKK model in terms of estimates of log-likelihood and minimum information criteria. The work therefore recommends the DCC modelling frameworks in empirical multivariate volatility modelling of exchange rates returns in Nigeria. Our findings therefore provide quantified responses to international exchange rates marketers and the stakeholders as well as economic decision makers in the country.

**Keywords:** Diag-BEKK; DCC; Exchange rate; GARCH; MGARCH **JEL Classification:** C22

1. **Introduction**

Over the last two decades a greater interest has been born to understand the impact of volatilities and dynamics correlations on the exchange rates of emerging economies. This is because understanding the volatility of the exchange rate means having an indirect explanation of the distribution of the returns especially in a volatile economy like Nigeria where many financial institutions and investors in recent times have become more interested in the time varying movements of the naira exchange rates. Modelling the conditional distribution of a large group of assets is an important challenge of modern financial time series analysis. Empirical evidence indicates that both the conditional volatilities and correlations of assets change over time and these have practical implications for risk management and asset pricing. It is widely accepted that financial volatilities move together over time across assets and markets. This phenomenon is called volatility spillover which is characterized by a process where markets tend to move closer together especially when markets become agitated.

The analysis of financial market integration, co-movement and dynamic correlation between assets plays a vital role in many financial decisions for market participants such as international trading companies and financial institutions. For instance, the increased correlation between foreign assets may diminish the diversity of an international investment portfolio (Puja and Lagesh, 2012). However, the pronounced features of financial asset prices are well-recognized and documented by economists and these have result in a problem of obtaining an accurate estimation of financial co-movement and correlations. This limitation has been handled by a multivariate modelling framework, and this has opened doors to better decision tools in various areas such as asset pricing, portfolio selection, option pricing, edging and risk management (Bauwens, Laurent and Rombouts, 2006). Large time-varying covariance matrices are then needed in portfolio management and optimization, as well as large vector for autoregressions.

Instability in the Nigerian naira exchange rates major component is exhibited by the time varying conditional variance (volatility) of the exchange rates figures. The traditional time series model, Autoregressive Moving Average (ARMA), of Box and Jenkins (1970) has long been applied in predicting the mean value of the time series and the Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982), with the Generalized ARCH (GARCH) model version given in Bollerslev (1986)[[1]](#footnote-1) have been used to study volatilities extensively in literature.

Since correlations between asset returns and markets are important in many financial applications, our objective is to investigate time varying volatility movements, its implication and the dynamic correlations in the naira exchange rates with United States Dollars, British Pound, European Euro and Japanese Yen, French CFA and West Africa Unit of Account. Multivariate volatility models have been extended to describe the time–varying feature of the correlations in recent years. The most obvious application of MGARCH (Multivariate GARCH) models is the study of the relations between the volatilities and co-volatilities of several markets. Is the volatility of a market leading the volatility of other markets? Is the volatility of an asset transmitted to another asset directly (through its conditional variance) or indirectly (through its conditional co-variances)? Does a shock in a market increase the volatility on another market, and by how much? Is the impact the same for negative and positive shocks of the same amplitude? A related issue is whether the correlations between asset returns change over time (see Bollerslev, 1990; Longin and Solnik, 1995). Are they high during periods of higher volatility, period which are sometimes associated with financial crises? Are they increasing in the long run, perhaps because of the globalization of financial markets? Such issues can be studied directly by using a multivariate model.

The first MGARCH model is given in Engle et al (1986) with vech (.) operator. Though the model quite has many parameters to estimate, and the authors also noted that the model could not yield a positive definite covariance matrix unless nonlinear inequality restrictions are imposed. The diagonal specification allows for a relatively straightforward interpretation, as each series has a GARCH-like specification. With this Diag-Vech (.) imposition on the model by Bollerslev, Engle and Wooldridge (1988) to reduce the elements in the off-diagonal of some matrices, that only reduced the number of parameters to be estimated, as no interaction is allowed between the different conditional variances and co-variances. Bollerslev, Engle and Wooldridge (1988) then applied the model in analysing returns on bills, bonds and stocks, while Baillie and Myers (1991), Bera, Garcia and Roh (1991) and Myers (1991) employed it in estimating hedge ratios in commodity markets[[2]](#footnote-2). The MGARCH models were initially developed in the late 1980’s and the first half of the 1990s, and after a period of tranquillity in the second half of the 1990s, this area seemed to be experiencing again a quick expansion phase. Another generalized MGARCH model is given in Baba, Engle, Kraft and Kroner (1990). It is named after the authors as BEKK model of order (p,q) (that is BEKK(p,q)), and the modified version of the model by Engle and Kroner (1995) is implemented in software packages. Moschini and Myers (2002) presented the model version for estimating time-varying hedge ratios in commodity markets. A parsimonious BEKK (p,q) model is then presented to reduce the number of parameters in the model. This was achieved by reducing off-diagonal elements of the matrices to zeros.[[3]](#footnote-3)

Bollerslev (1990) introduced a special MGARCH model in which the conditional covariances are proportional to the product of the corresponding conditional standard deviations. This is the Constant Conditional Correlation (CCC) MGARCH model. The modelling strategy allows first the univariate GARCH models to be estimated for each asset returns and then the correlation matrix is estimated using the standard closed form of Maximum Likelihood Estimate (MLE) correlation estimator using transformed residuals. The restriction of constant correlation highly reduces the number of parameters in the earlier MGARCH models, ensures positive definiteness of the parameters and further simplifies the estimation. This simply requires each univariate conditional variance to be non-zero and the correlation matrix to be of full rank. Bollerslev (1990) therefore finds the motion of constant correlation plausible, while Tsui and Yu (1999) found that constant correlation can be rejected for certain assets returns, therefore Engle (2002) and Tse and Tsui (2002) proposed a generalization of the conditional correlation model, making the correlations to be dynamic, and tests are proposed to investigate this (see Engle and Sheppard (2001) and Tse (2000) among others).[[4]](#footnote-4) Following Engle and Shephard (2001), analysing and understanding how the univariate GARCH works is fundamental for the study of the Dynamic Conditional Correlation (DCC) MGARCH model. The DCC model is a nonlinear combination of univariate GARCH modelling framework and its matrix is based on how the univariate GARCH process works. Other symmetric variants of the Dynamic Conditional Correlation (DCC) model are the DCC-Dynamic Equicorrelation (DCC-DECO) and Corrected DCC (cDCC) models proposed in Engle and Kelly (2008) and Aielli (2009), respectively.[[5]](#footnote-5) In order to introduce asymmetry in the form of leverage effect as in the univariate asymmetric GARCH modelling, Cappielo et al. (2006) and Aielli (2013) proposed Asymmetric DCC (ADCC) and Asymmetric cDCC (cADCC) models, respectively.

This paper therefore investigates the presence of time-varying co-volatilities and dynamic correlations among Nigerian naira exchange rates markets using variants of multivariate GARCH models. The applicability and implications of the results from this work will serve as eye opener to policy makers, financial analysts and individuals in applying better decision tools in various areas such as asset (foreign exchange) pricing, portfolio selection, option pricing, hedging and risk management. Many indications have shown that naira-US dollar exchange rates behaviour influences other naira exchange rates, and even in their volatilities.

This paper is further structured as follows. In the second section, a critical review of existing literature is presented, showing the gaps in dynamics correlations and volatility effects in the naira exchange rates markets. The third section explicitly discusses the specifications of MGARCH models applied in the paper and the estimation procedure. The fourth section covers the discussion of empirical results, while section five renders the concluding remarks.

1. **The Review of Literature**

In April 2014, the most populous black nation became Africa’s largest economy after rebasing its GDP figure to more than $500 billion. However, In recent times, the Nigerian economy has been in a dwindling state where it has been characterized with; negative development in the oil sector due to fall in international oil price, consistent devaluation of naira, instability in the financial markets and the faith of the Naira hangs in the balance as it continues its downward slides. In recent times, the naira has witnessed depreciation in the parallel market and the role of the CBN is usually to intervene in foreign exchange market through its monetary policy actions and operations in the money market in other to influence the exchange rate movement in the desired direction such that it ensures the competitiveness of the domestic economy. The introduction of managed floating rate regime tends to increase the uncertainty in exchange rates, thus, increasing the volatility of exchange rate by the regime shifts. Hence the exchange rate is the most important asset price in the economy. (Olowe, 2009).

Olowe (2009), investigated volatility persistence and asymmetric properties of the naira/dollar exchange rate in Nigeria using the asymmetry GARCH models and his result shows that volatility is persistence in the naira/dollar exchange rate. Foreign exchange market fluctuations have always attracted considerable attention in literatures, simply because exchange rate and its volatility are key indicators that influence economic activities, in Nigeria particularly. Bala and Asemota (2013) examined exchange rates volatility with GARCH models using monthly exchange rates series for Naira/US dollar, Naira/British Pounds and Naira/Euro returns. The study compare estimates of variants of GARCH models with break in respect of the US dollar rates with exogenously determined break points. Their results reveal presence of volatility in the three currencies and equally indicate that most of the asymmetric models rejected the existence of a leverage effect except for models with volatility break as in the case of Olowe (2009).

Financial market integration, co-movement, co-volatilities and the degree of correlation between assets plays a vital role in many financial decisions for market participants such as multinational companies and financial institutions. Yaya et;al (2016) investigated volatility persistence and returns spillovers between oil and gold markets before and after the global crisis using daily historical data from 1986 to 201. The Constant Conditional Correlation (CCC) modelling framework was applied to investigate the spillover effects and the volatility in the gold market was found to be less than that at the oil market before and after the crisis periods. MGARCH models were initially developed in the late 1980’s and in the 1990s to study co-movements and correlations of assets. Since then, the area has experienced a quick expansion phase. Another generalized MGARCH model is given in Baba, Engle, Kraft and Kroner (1990). It is named after the authors as BEKK model of order (p,q) (that is BEKK(p,q)), and the modified version of the model by Engle and Kroner (1995) is implemented in software packages.

Moschini and Myers (2002) presented the model version for estimating time-varying hedge ratios in commodity markets. A parsimonious BEKK (p,q) model is then presented to reduce the number of parameters in the model. This was achieved by reducing off-diagonal elements of the matrices to zeros.[[6]](#footnote-6) Bollerslev (1990) introduced a special MGARCH model in which the conditional covariances are proportional to the product of the corresponding conditional standard deviations. This is the Constant Conditional Correlation (CCC) MGARCH model. The modelling strategy allows first the univariate GARCH models to be estimated for each asset returns and then the correlation matrix is estimated using the standard closed form of Maximum Likelihood Estimate (MLE) correlation estimator using transformed residuals. The restriction of constant correlation highly reduces the number of parameters in the earlier MGARCH models, ensures positive definiteness of the parameters and further simplifies the estimation. This simply requires each univariate conditional variance to be non-zero and the correlation matrix to be of full rank.

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More recently, Tasi, et;al (2014) investigated the Nigerian naira against some major currency in the world to determine the volatility spillover of the nine selected countries against the U.S. Dollar simultaneously via Multivariate GARCH models. Their result indicated that the restricted BEKK, DVECH and CCC results exhibit rather similar behaviour for each countries and the BEKK model is considered to be the best because it has least number of parameters. Other symmetric variants of the Dynamic Conditional Correlation (DCC) model are the DCC-Dynamic Equicorrelation (DCC-DECO) and Corrected DCC (cDCC) models proposed in Engle and Kelly (2008) and Aielli (2009), respectively.[[8]](#footnote-8) In order to introduce asymmetry in the form of leverage effect as in the univariate asymmetric GARCH modelling, Cappielo et al. (2006) and Aielli (2013) proposed Asymmetric DCC (ADCC) and Asymmetric cDCC (cADCC) models, respectively.

1. **The MGARCH Model Specification and Estimation**

Consider a vector stochastic process {} of dimension N x 1 condition on the sigma field, and denoting a finite vector of parameters by ,

we have,

, (1)

where is the conditional mean vector and,

(2)

where is an N x N positive definite matrix, obtained by Cholesky factorization of . We assume that the N x N random vector has the first two moments as,

E () = 0 (3)

Var () = , (4)

where is the identity matrix of order N. the conditional variance matrix of defined as,

Var ( / ) = () = () = ()() () = (5)

The BEKK (p,q) model of Engle and Kroner (1995) is then defined as,

, (6)

where C, the A’s and the G’s matrices are of dimension N x N but C is upper triangular. Estimation of the BEKK(p,q) model bears large computations due to several matrix transpositions. The number of parameters of a complete BEKK(p,q) model is  whereas the Diag-BEKK (p,q) with only diagonal entries for both matrices and gives fewer parameters of . In the application of BEKK forms of model, we often assume in most cases. The Diag-BEKK (p,q) model is covariance stationary if

, (7)

These conditions are imposed during the estimation. When it exists, the unconditional variance matrix of the BEKK model is then given by,

Vec (8)

where vec denotes the operator that stacks the columns of a matrix as a vector.

In order to model the conditional correlations in the volatilities, Bollerslev (1990) proposes a class of MGARCH models in which the conditional correlations are constant and thus the conditional covariances are proportional to the product of the corresponding conditional standard deviations. This is the CCC model defined as,

, (9)

where,

(10)

and is any univariate GARCH model[[9]](#footnote-9), and

R = ( (11)

is a symmetric positive definite matrix with , i. *R* is the matrix containing the constant conditional correlations . The original CCC model for each conditional variance has a GARCH(1,1) specification,

 (12)

Thus, the matrix is positive definite if and only if all the N conditional variances are positive and R is positive definite[[10]](#footnote-10).

The CCC model is limited as a result of unrealistic constant conditional correlations, therefore Engle (2002) and Tse and Tsui (2002) generalized the CCC model by making the conditional correlation matrix to be time dependent. The model is then called a DCC model, which is specified by re-defining,

(13)

where is defined in (10), is any univariate GARCH model, and

(14)

where and are non-negative parameters satisfying R is a symmetric N x N positive definite parameter matrix with , and is the N x N correlation matrix of for = t - *M*, *t – M* + 1, … ,*t*-1, with its (*i*, element given as,

 , (15)

where = / . The matrix is then expressed as,

(16)

where is a N x N diagonal matrix with diagonal element given by and is a N x M matrix, with .

Another specification of the DCC model is given by Engle (2002) which specifies,

(17)

where the N x N symmetric positive definite matrix is given as,

(18)

with as defined in Tse and Tsui (2002), is the unconditional variance matrix of , and are non-negative scalar parameters satisfying .

The DCC(p,q) models of Tse and Tsui (2002) and Engle (2002) are limited in its ability as a result of the conditional correlations of the same dynamics generated by the scalar values of  that are necessary to ensure that  is positive definite for every *t*. Then, DCC-DECO model was proposed in Engle and Kelly (2008). In the model,

(19)

(20)

where is the equicorrelation, is the element of in (18). is the *N*- dimensional identity matrix and is an  matrix of ones.  exists if and only if  and  and  is positive definite if and only if . For correlation estimation, one typically estimates as the empirical correlation matrix of , then the parameter in the DCC model are estimated by guassian Quasi Maximum Likelihood (QML). Generally, in the DCC(p,q) model, if the covariance matrix is not positive definite, then it is impossible to invert the covariance matrix which is essential in portfolio optimization. To guarantee a positive definite for all *t*, simple condition on the parameters are imposed, that is, the condition of the univariate GARCH model have to be satisfied.

Aielli (2009) has shown that the estimation of as the empirical correlation matrix of is inconsistent because,

E () = E {E()} = E () () (21)

Then, the author proposed cDCC(p,q) model in order to circumvent the problem. The cDCC model is similar to DCC model of Engle (2002) except the different specification for the N x N symmetric positive definite matrix, given by,

(22)

where is the N x N unconditional variance matrix of , and are non-negative scalar parameters satisfying . To obtain , we need however a first step estimator of the diagonal elements of . Aielli (2009) therefore proposes obtaining these values as,

, for i=1 ,…, N (23)

a a result of the fact that the diagonal elements of  do not depend on . Given and , one can compute and thus one can estimate  as the empirical covariance of .

A limitation of the DCC model (17-18) is that products of jointly positive and negative standardized returns have the same impact on the evolution of the future correlation matrix. Cappiello et al. (2006) relax this assumption by introducing different types of asymmetric specifications, being motivated by leverage effect in univariate volatility modelling. Moreover, it is of interest to assess if strings of joint common negative shocks are able to increase correlation significantly more than positive ones, implying, for instance that market drops can significantly reduce the ability to diversify investments.

The asymmetric variant for DCC model is then named the Asymmetric DCC (ADCC) model, while the extension of the asymmetry to cDCC model was achieved by Aielli (2013) in the model, cADCC. The difference between the symmetric and asymmetric variants of the models is in the specification of the matrix, which is redefined as,



where  and  are the intercept matrices, and  is used to capture the asymmetric impact of jointly negative news. Necessary and sufficient conditions for  to be positive definite are  and  together with the positive definiteness of the intercept , which is ensured by the condition  where  is the maximum eigenvalue of  (see Cappiello et al. 2006).

Both the Maximum Likelihood Estimation (MLE) and the Two-Step approaches are employed in this paper. The MLE method was used to estimate the Diag-BEKK model, while the Two-Step estimation approach was considered for the cDCC model.

The commonest distributional assumption for MGARCH model is the multivariate normal[[11]](#footnote-12). The resulting multivariate log-likelihood function is then given as,

(24)

As shown by Bollerslev and Wooldridge (1992), a consistent estimator of is then obtained by maximizing (24) with respect to **,** where is the r-dimensional parameter vector for the conditional mean, , conditional variance matrix, and conditional distribution p( ), where = (, ). This is the guassian QML and is consistent provided the conditional mean and the conditional variance are specified correctly.

Two-step estimation approach is considered for the CCC and DCC models. Engle and Shephard (2001) have shown that log-likelihood of the DCC model can be written as the sum of a mean and volatility part and a correlation part. The quasi-log-likelihood function is then written as the sum of the log-likelihood functions of N univariate models, given as,

QL (25)

where and are the set of unknown parameters and correlation estimates. Given and under appropriate regularity conditions, a consistent, but inefficient, estimator of can be obtained by maximizing,

QL (26)

where . Note, that the sum of the likelihood functions in (25) and (26), plus half of the total sum of squared standardized residuals is equal to the log-likelihood function in (24).

1. **Data, Results and Discussion**

The data used in this work are the daily Nigerian Naira exchange rate with United States Dollars (Naira-USD), British Pound (Naira-GBP), European Euro (Naira-Euro) and Japanese Yen (Naira-JPY), French CFA (Naira-CFA) and West Africa Unit of Account (Naira-WAUA). The data were sourced from Central Bank of Nigeria website ([www.cenbank.org](http://www.cenbank.org)) and spanning between 2 January, 2009 and 6 June, 2015.

Each of the time series was first transformed using logarithm of difference process to obtain the log-returns series which is often used as proxy volatility series. As it is the usual practice in volatility modelling, volatility is easily noticed using the log-returns series than using the original time series. Figure 1 therefore presents the plots of each of the log-returns series. We notice some similarities in the returns particularly around 2012 and 2015 where the returns are exceptionally of larger magnitudes. This is to show that the returns respond together as a result of the response of exchange rates at the markets.

 

 

 

***Figure 1: Log-returns of Naira Exchange rates***

Now, looking at the co-volatilities in the returns via Diag-BEKK(1,1) model[[12]](#footnote-13). The results are presented in Table 1 in three matrices. The first is the matrices containing estimates of constants, with only the upper triangular entries. The remaining two are the Ai and Gj  matrices for the ARCH and GARCH parameters. Altogether, 33 parameters were estimated based on the formula for number of parameters expected for a Diag-BEKK model[[13]](#footnote-14). Most of the ARCH/GARCH parameters are significant at 5% level, this implying a good representation of the co-volatility by the Diag-BEKK(1,1,) model.

**Table 1: Results of Diagonal BEKK model**

Coefficient Std.Error t-value t-prob

C\_11 0.069021 0.034067 2.026 0.0429

C\_12 0.094618 0.041422 2.284 0.0225

C\_13 0.025600 0.021882 1.170 0.2422

C\_14 0.064609 0.046023 1.404 0.1606

C\_15 0.010089 0.079683 0.1266 0.8993

C\_16 0.051586 0.045271 1.139 0.2547

C\_22 0.025512 0.012733 2.004 0.0453

C\_23 0.025590 0.026373 0.9703 0.3321

C\_24 0.046606 0.043291 1.077 0.2818

C\_25 -0.079746 0.12114 -0.6583 0.5105

C\_26 -0.225817 0.13611 -1.659 0.0973

C\_33 0.015198 0.019441 0.7817 0.4345

C\_34 -0.060380 0.032509 -1.857 0.0635

C\_35 0.189482 0.34409 0.5507 0.5819

C\_36 -0.199126 0.19988 -0.9962 0.3193

C\_44 0.041395 0.062138 0.6662 0.5054

C\_45 0.382937 0.18680 2.050 0.0405

C\_46 0.090850 0.14790 0.6143 0.5391

C\_55 0.000000 0.0032369 0.00 1.0000

C\_56 0.000159 0.0011001 0.1445 0.8851

C\_66 0.000193 0.00052005 0.3714 0.7104

A\_1.11 0.246978 0.058208 4.243 0.0000

A\_1.22 0.295026 0.46217 0.6384 0.5233

A\_1.33 0.222565 0.030866 7.211 0.0000

A\_1.44 0.258626 0.12574 2.057 0.0399

A\_1.55 0.339564 0.10398 3.266 0.0011

A\_1.66 0.318511 0.14863 2.143 0.0323

G\_1.11 0.959060 0.021850 43.89 0.0000

G\_1.22 0.880047 0.10602 8.300 0.0000

G\_1.33 0.974913 0.010173 95.83 0.0000

G\_1.44 0.927873 0.052662 17.62 0.0000

G\_1.55 0.181731 0.16493 1.102 0.2707

G\_1.66 0.014997 0.069083 0.2171 0.8282

Log Likelihood: 2263.429

Akaike 2.947919 Shibata 2.947046

Schwarz 3.061262 Hannan-Quinn 2.990063

We also noticed that the sums of the ARCH and GARCH parameters are greater than 1 in the volatility involving Naira-USD, Naira-GBP, Naira-EURO and Naira-JPY returns, this implying nonstationary high persistence of volatility and IGARCH effect. The persistence for the remaining series (Naira-CFA and Naira-WAUA) are quite low. We then proceed to studying the correlation between the conditional standard deviations of returns using the CCC and variants of DCC models.

Due to large number of parameters estimated, and the results which are similar to that of Diag-BEKK(1,1) model above, we have therefore reported only the results for the second step of estimation as presented in Table 2.

**Table 2: Results of Constant Conditional Correlation (CCC) model**

Coefficient Std.Error t-value t-prob

ρ\_21 0.614896 0.032567 18.88 0.0000

ρ\_31 0.126321 0.048242 2.618 0.0089

ρ\_41 0.384234 0.037244 10.32 0.0000

ρ\_51 0.033738 0.023537 1.433 0.1519

ρ\_61 0.045211 0.034825 1.298 0.1944

ρ\_32 0.142873 0.056142 2.545 0.0110

ρ\_42 0.552959 0.043867 12.61 0.0000

ρ\_52 0.009258 0.021299 0.435 0.6638

ρ\_62 0.032477 0.040327 0.805 0.4207

ρ\_43 0.084716 0.032666 2.593 0.0096

ρ\_53 0.018552 0.024239 0.765 0.4441

ρ\_63 -0.036577 0.032791 -1.115 0.2648

ρ\_54 0.009436 0.022508 0.419 0.6751

ρ\_64 0.057634 0.040152 1.435 0.1514

ρ\_65 0.062045 0.038760 1.601 0.1096

Log Likelihood: 1456.699

Akaike 1.912323 Shibata 1.911450

Schwarz 2.025666 Hannan-Quinn 1.954467

The estimates of constant correlations are positive except for the correlation between the 6th and 3rd series, that is, the conditional standard deviations series of Naira-EURO and Naira WAUA. Between conditional standard deviation series 2 (Naira-GBP) and conditional standard deviation series 1 (Naira-USD), we observed highest constant correlation of 0.61 and this is highly significant, even at 1% level. We also observed significant correlations between Naira-USD conditional standard deviations and those of Naira-EURO (series 3) and Naira-YEN (series 4). The remaining correlations that are not significant at 5% level are actually positive.

From the results of dynamic correlations from the DCC model in Table 3, we observed more negative correlations that are not significant at 5% level. For those correlations that were significant when CCC model was applied, we observed improved values. The and parameters were estimated at 0.009398 and 0.966894, which are significant at 1% level, implying that the estimated DCC(1,1) model of Engle (2002)[[14]](#footnote-15) is preferred to the CCC(1,1) model, and it is implied from the estimated information criteria that are smaller for the DCC(1,1) model.

**Table 3: Results of Dynamic Conditional Correlation (DCC) (Engle) model**

Coefficient Std.Error t-value t-prob

ρ\_21 0.657896 0.034295 19.18 0.0000

ρ\_31 0.231306 0.10191 2.270 0.0234

ρ\_41 0.448967 0.042977 10.45 0.0000

ρ\_51 0.007741 0.048687 0.1590 0.8737

ρ\_61 -0.003717 0.076760 -0.0484 0.9614

ρ\_32 0.257724 0.13151 1.960 0.0502

ρ\_42 0.612151 0.044773 13.67 0.0000

ρ\_52 -0.017819 0.058984 -0.302 0.7626

ρ\_62 -0.018355 0.10744 -0.171 0.8644

ρ\_43 0.168775 0.090793 1.859 0.0632

ρ\_53 0.005108 0.033244 0.154 0.8779

ρ\_63 -0.077139 0.065586 -1.176 0.2397

ρ\_54 -0.017556 0.044978 -0.390 0.6963

ρ\_64 0.030927 0.080428 0.385 0.7006

ρ\_65 0.064147 0.065045 0.986 0.3242

0.009398 0.0032159 2.923 0.0035

0.966894 0.0099457 97.22 0.0000

Log Likelihood: 1394.475

Akaike 1.835013 Shibata 1.834033

Schwarz 1.955225 Hannan-Quinn 1.879712

With the estimates of cDCC model in Table 4, we observed improved results in terms of parameter estimates and level of significance over the classical DCC model. This is also very glaring from the log-likelihood and information criteria. The and parameters are also significant at 5% level implying the applicability of dynamic correlation modelling.

**Table 4: Results of Corrected Dynamic Conditional Correlation (cDCC) model**

Coefficient Std.Error t-value t-prob

ρ\_21 0.664124 0.039228 16.93 0.0000

ρ\_31 0.245474 0.11007 2.230 0.0259

ρ\_41 0.455772 0.045603 9.994 0.0000

ρ\_51 0.006608 0.051101 0.129 0.8971

ρ\_61 -0.002691 0.086996 -0.031 0.9753

ρ\_32 0.271417 0.14054 1.931 0.0536

ρ\_42 0.625312 0.047432 13.180 0.0000

ρ\_52 -0.016321 0.061018 -0.2675 0.7891

ρ\_62 -0.014726 0.11875 -0.1240 0.9013

ρ\_43 0.178771 0.10007 1.786 0.0742

ρ\_53 0.004844 0.034514 0.1404 0.8884

ρ\_63 -0.081214 0.067050 -1.211 0.2260

ρ\_54 -0.017966 0.047084 -0.3816 0.7028

ρ\_64 0.035689 0.088422 0.4036 0.6865

ρ\_65 0.063025 0.066406 0.9491 0.3427

0.011705 0.0052231 2.241 0.0252

0.965276 0.010673 90.44 0.0000

Log Likelihood: 1392.712

Akaike 1.832750 Shibata 1.831770

Schwarz 1.952962 Hannan-Quinn 1.877448

Now, using the DCC-DECO model which represents all the dynamic correlations in the DCC modelling with only a single estimate, we have the Dynamic Equicorrelation estimated as 0.485528 which is significant at 5% level.

**Table 5: Results of Dynamic Equicorrelation (DCC-DECO) Model**

Coefficient Std.Error t-value t-prob

ρ 0.485528 0.23003 2.111 0.0350

0.268775 0.21829 1.231 0.2184

0.697976 0.20873 3.344 0.0008

Log Likelihood: 2013.144

Akaike 2.611224 Shibata 2.610868

Schwarz 2.683352 Hannan-Quinn 2.638043

Tables 6 and 7 present the results for the asymmetric variants of DCC and cDCC models, that is the ADCC and cADCC models.

**Table 6: Results of Asymmetric Dynamic Conditional Correlation (ADCC) model**

Coefficient Std.Error t-value t-prob

ρ\_21 0.715261 0.042666 16.76 0.0000

ρ\_31 0.212029 0.10927 1.940 0.0525

ρ\_41 0.459366 0.044722 10.27 0.0000

ρ\_51 -0.034569 0.047552 -0.7270 0.4674

ρ\_61 -0.013033 0.066807 -0.1951 0.8453

ρ\_32 0.293118 0.13464 2.177 0.0296

ρ\_42 0.672519 0.046618 14.43 0.0000

ρ\_52 0.000399 0.057120 0.006994 0.9944

ρ\_62 0.020464 0.098494 0.2078 0.8354

ρ\_43 0.131678 0.10926 1.205 0.2283

ρ\_53 -0.051827 0.043540 -1.190 0.2341

ρ\_63 -0.135134 0.070806 -1.908 0.0565

ρ\_54 -0.043020 0.039800 -1.081 0.2799

ρ\_64 0.023270 0.078265 0.2973 0.7663

ρ\_65 0.032611 0.061772 0.5279 0.5976

0.008299 0.0028875 2.874 0.0041

0.961562 0.0074047 129.9 0.0000

γ 0.027169 0.016001 1.698 0.0897

Log Likelihood: 1380.433

Akaike 1.818271 Shibata 1.817235

Schwarz 1.941918 Hannan-Quinn 1.864246

The results obtained in the two tables are quite similar to those obtained earlier only with the newly computed asymmetric parameter, γ that is positive. This is only significant at 10% level for the ADCC and but is significant at even 1% level for the cADCC model. Both and parameters are also highly significant.

**Table 7: Results of Corrected Dynamic Conditional Correlation (cADCC) model**

Coefficient Std.Error t-value t-prob

ρ\_21 0.625993 0.037926 16.51 0.0000

ρ\_31 0.263048 0.089566 2.937 0.0034

ρ\_41 0.424504 0.036273 11.70 0.0000

ρ\_51 0.026750 0.042295 0.6325 0.5272

ρ\_61 0.031196 0.067900 0.4594 0.6460

ρ\_32 0.290618 0.12154 2.391 0.0169

ρ\_42 0.584354 0.037869 15.43 0.0000

ρ\_52 0.000751 0.053667 0.01399 0.9888

ρ\_62 0.010678 0.093215 0.1146 0.9088

ρ\_43 0.181743 0.094125 1.931 0.0537

ρ\_53 0.031847 0.038210 0.8335 0.4047

ρ\_63 -0.070501 0.051017 -1.382 0.1672

ρ\_54 0.008788 0.039956 0.2200 0.8259

ρ\_64 0.058028 0.069153 0.8391 0.4015

ρ\_65 0.084452 0.050569 1.670 0.0951

0.008585 0.0037628 2.281 0.0227

0.961047 0.0081098 118.5 0.0000

γ 0.028129 0.0097113 2.897 0.0038

Log Likelihood: 1369.429

Akaike 1.804146 Shibata 1.803109

Schwarz 1.927792 Hannan-Quinn 1.850121

The summary of the results from Table 1-7 is given in Table 8. Based on maximum likelihood estimates obtained by the MaxSQP algorithm and the corresponding minimum Akaike and Schwarz information criteria computed, cADCC(1,1) model emerged as the optimal model among the competing MGARCH models, particularly the DCC variants.

**Table 8: Models Evaluation Results.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Models | Log-Likelihood | Akaike | Schwarz |  |
| Diag-BEKK(1,1) | -2263.429 | 2.947919 | 3.061262 |  |
| CCC(1,1) | -1456.699 | 1.912323 | 2.025666 |  |
| DCC(1,1) | -1394.475 | 1.835013 | 1.955225 |  |
| cDCC(1,1) | -1392.712 | 1.832750 | 1.832750 |  |
| DCC-DECO(1,1) | -2013.144 | 2.611224 | 2.611224 |  |
| ADCC(1,1) | -1380.433 | 1.818271 | 1.818271 |  |
| cADCC(1,1) | ***-1369.429*** | ***1.804146*** | ***1.804146*** |  |

The constant correlation model variants performed better than Diag-BEKK model as applied in this paper.

1. **Concluding remark**

This paper investigated the presence of time-varying co-volatilities and dynamic correlations in Nigerian naira exchange rates markets using variants of MGARCH model. Starting from the parsimonious Diag-BEKK type of MGARCH model to constant (CCC) and dynamic conditional correlation (DCC) modelling frameworks, we obtained interesting results that are of importance in economic policy decision. The conditional correlation modelling framework allow one to model the conditional covariances through conditional variances (or standard deviations) and correlations. Apart from imposing fewer parameters, the CCC model variants also performed better than Diag-BEKK model as applied in this paper. The work therefore recommends the DCC modelling frameworks in empirical multivariate volatility modelling of exchange rates returns in Nigeria. Our findings therefore provide quantified responses to international exchange rates marketers and the stakeholders as well as economic decision makers in the country.

This paper emphasized model specification but quite a lot of applications of the CCC variants on asset prices particularly exchange rates can still be carried out in the areas of portfolio management and risk hedging, and these are of much practical applications to mere multivariate volatility forecasting which may not quite have meaningful policy implications. Our bit is to proposed and validate models, and it is left for policy makers, financial agencies and interested individuals to apply the models in solving policy economic issues.

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1. A survey on ARCH-type models is found in Bollerslev, Engle and Nelson (1994), Bera and Higgins (1993), Pagan (1996), Shepherd (1996), Xekalaki and Degiannakis (2010), among others. [↑](#footnote-ref-1)
2. Ding and Engle (2001) gave sufficient conditions for the diagonal MGARCH (1,1) model to be positive definite, and therefore proposed four models which are nested in the multivariate diagonal MGARCH (1,1). [↑](#footnote-ref-2)
3. It was noted that when the number of series to be estimated in the Diagonal BEKK (Diag-BEKK) is larger than three or four, quite a high number of unknown parameters would be estimated, in this regard, factor and orthogonal models are therefore proposed to circumvent this difficulty. The Factor-ARCH model is proposed in Engle et al. (1990), and the model is applied in predicting the conditional volatilities of treasury bills and stock returns in Ng, Engle and Rothchild (1992) and Bollerslev and Engle (1993). [↑](#footnote-ref-3)
4. The Bivariate and multivariate GARCH tests for testing significance of constant correlations in CCC model are documented in Tse (2000). [↑](#footnote-ref-4)
5. For comprehensive review on updates on MGARCH models and their applications, see Bollerslev (1990), Bollerslev (1994), Engle and Shephard (2001), Brooks (2002), Tse and Tsui (2002), Tsay (2002), Harris and Sollis (2003), Vrontos, Dellaportas and Politis (2003), Lutkepohl and Kratzig (2004), Lutkepohl (2005), McAleer (2005), Anderson et al. (2006), Lanne and Saikkonnen (2007), Silvennoinen and Terasvirta (2008), among others. [↑](#footnote-ref-5)
6. It was noted that when the number of series to be estimated in the Diagonal BEKK (Diag-BEKK) is larger than three or four, quite a high number of unknown parameters would be estimated, in this regard, factor and orthogonal models are therefore proposed to circumvent this difficulty. The Factor-ARCH model is proposed in Engle et al. (1990), and the model is applied in predicting the conditional volatilities of treasury bills and stock returns in Ng, Engle and Rothchild (1992) and Bollerslev and Engle (1993). [↑](#footnote-ref-6)
7. The Bivariate and multivariate GARCH tests for testing significance of constant correlations in CCC model are documented in Bera (1996) and Tse (2000). [↑](#footnote-ref-7)
8. For comprehensive review on updates on MGARCH models and their applications, see Bollerslev (1990), Bollerslev (1994), Engle and Shephard (2001), Brooks (2002), Tse and Tsui (2002), Tsay (2002), Harris and Sollis (2003), Vrontos, Dellaportas and Politis (2003), Lutkepohl and Kratzig (2004), Lutkepohl (2005), McAleer (2005), Anderson et al. (2006), Lanne and Saikkonnen (2007), Silvennoinen and Terasvirta (2008), among others. [↑](#footnote-ref-8)
9. This refers to any GARCH model with normally distributed errors which meet the requirements for suitable stationary and non-negative conditions. [↑](#footnote-ref-9)
10. The unconditional variances of CCC model are easily obtained, as in the univariate case but the unconditional covariances are difficult to calculate because of the nonlinearity in (9). [↑](#footnote-ref-10)
11. Though this distributional assumption may be rejected in some applications dealing with daily or weekly data. In that case, multivariate student-t distribution is considered. [↑](#footnote-ref-12)
12. For simplicity, we have considered MGARCH models of order (1,1) throughout in this paper, and OxGARCH software of Laurent and Peters (2006) and Laurent (2013) . [↑](#footnote-ref-13)
13.  and *N* = 6 for the number of series. Number of parameters to be estimated is given by . [↑](#footnote-ref-14)
14. MaxSQP algorithm implemented in GARCH software for estimating MGARCH models failed to converge while estimating the DCC(1,1) model of Tse and Tsui (2002). [↑](#footnote-ref-15)