

# Moduli Identities and Cycles Cohomologies by Integral Transforms in Derived Geometry\*

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## 1. Introduction.

Factorizations of the moduli space  $\mathcal{M}({}^L G, C)$ , exhibit the flatness of the Cousin complexes that appears in this factorable process (*and that are re-interpreted by the Penrose transform as varieties whose zeros are roots of the corresponding polynomials on a bundle of lines*) due to the holonomicity and conformability of  $\mathbb{M}$ , in the field theory scale nearest to the Higgs fields. Then by local cohomology [1] we can inquire using the corollary given in [2], the cohomology of  $\mathbb{P}^3$  through moduli spaces modulo  $\mathcal{M}(\mathbb{P}^3, 0)$ . Then for geometrical Langlands program, ramifications correspond to extensions of induced moduli stacks where meromorphic connections are induced to holomorphic connections. Thus in the context of the Penrose transforms is available to obtain the different cohomological solution classes of the field equations including the singularities of the space-time in an algebraic frame with a geometrical image on twisted lines bundles. Likewise, when set  $\tilde{\mathcal{L}}_\lambda = \mathcal{L}_\lambda \otimes p^*(K^{1/2})$ , we are establishing a corresponding sheaf  $\mathcal{D}_{k, I_y}^\lambda$  of  $\tilde{\mathcal{L}}_\lambda$ -twisted differential operators on the moduli space  $\text{Bun}_{G, I_y}$  well-defined, which is our deformed sheaf necessary to establish the geometrical correspondences between objects of moduli stacks and differential operators that require meromorphic connections to determine the holomorphic connections of the corresponding derived category and their geometry. Other method to establish a justification on the nature of the our twisted derived category and their elements as ramifications of a field (to the field equations) is the followed through the Yoneda algebra [3], [4] where is searched extends the action of the endomorphism  $\text{End}(\mathbb{V}_{critical})$ , through the Lie algebra action  $\hat{\mathfrak{g}}$ , that is the degree zero part that we want, that is to say, the first member of the Penrose transform  $H^0({}^L G, \Gamma(U, \mathcal{O})) \cong \text{Ker}(U, p^*\nabla + \tau(\nabla))$ , of their isomorphism, which must be  $H^0(\hat{\mathfrak{g}}[[z]], \mathbb{V}_{critical})$ . We identify in the final part of the demonstration of the *theorem 4. 1*, [5], that with functions on  $\text{Op}_{LG}(\mathcal{D}^\times)$ , central elements as  $I_F K$ , act via their restriction to the sub-variety  $\text{Op}_{LG}$ , of *opers* on  $\Sigma$ . Then the Yoneda extension algebra must be understood as a projective Harish-Chandra module to the pair  $(\hat{\mathfrak{g}}, G[[z]])$  (to  $z$ , a singular point of manifold  $Z$ ). Then  $H^0(\hat{\mathfrak{g}}[[z]], \mathbb{V}_{critical}) = \mathbb{C}[\text{Op}_{LG}(\mathcal{D}^\times)]$ .

## 2. Moduli Identities and their Stacks as Divisors.

All begins with the relation

$$\mathcal{M}_{Higgs}(G, C) = T_V^\vee \text{Bun}_C(\Sigma), \quad (1)$$

obtained inside the procedure followed to the obtaining of the induced equivalences inside the moduli space  $\mathcal{M}_H(G, C)$ . Then is necessary to define cer-

tain ramification corresponding to the connection  $\nabla_s$  such that having a vector bundle  $p_c^*V$ , on  $C \times T_V^\vee Bun_c(\Sigma)$ , that comes equipped with a Higgs field  $\phi \in H^0(C \times T_V^\vee Bun_C(\Sigma))$ , characterized uniquely by the property that for every  $\theta \in T_V^\vee Bun_C(\Sigma)$ , we have  $\phi|_{C \times \{\theta\}} = \theta$  which is due to the spectral cover equipped with a natural lines bundle  $\tilde{\mathcal{L}}_\lambda = \mathcal{L}_\lambda \otimes p^*(V)$ , as has been mentioned in the introduction, where  $V$ , is a complex vector space. As we want projective Harish-Chandra module to the deformed part of our induced connection (*ramification*), where must be induced said lines bundle on the part of  $\mathcal{D}_{G/H}$  -modules which is a sheaf of certain lines bundle that is divisor of a lines bundle on  $Bun_G$ , then the component of lines bundle, given by  $p_c^*V$ , is factor of a canonical lines bundle on  $Bun_G$ , corresponding to the critical level that is required.

**Theorem(F. Bulnes)2.1** *Considering (1) and  $\phi|_{C \times \{\theta\}} = \theta$ , defined before we have*

$$\mathcal{M}({}^L G, C) = \mathcal{M}_{Higgs}({}^L G, C)K^{1/2}, \quad (2)$$

where  $K^{1/2}$ , is the square root of the bundle of lines on  $Bun_G$ , corresponding to the critical level.

Proof.[5].<sup>▪</sup>

#	Moduli Identity	Derived Geometry
1	$\mathcal{M}_{Flat}({}^L G / H, C) \cong \mathcal{M}_H(G, C; \omega_K)$	Dualities in Mirror Theory
2	$\mathcal{M}_\chi \cong (\mathbb{C}^\times)^k / G^a$	$\chi$ , is the dimension of the brane space
3	$\text{br}(\mathcal{M}_{g,0}(\mathbb{P}^1, \mu)) = \mathbb{P}^r$	Stable Curves in $\mathbb{P}^1$
4	$\mathcal{M}_{Higgs}(G, C) = T_V^\vee Bun_c(\Sigma)$	lines bundles $\tilde{\mathcal{L}}_\lambda$ of Higgs fields = Higgs bundles
5	$\mathcal{M}_{0,0}(\mathbb{P}^1, 1) \cong G_{2,4}(\mathbb{C})$	Space-Time(Minkowski Space)
6	$\mathcal{M}(G, C) = \mathcal{M}_{Higgs}(G, C)K^{1/2}$	Strings, $D$ -branes
7	$\mathcal{M}_H(G, C) = \mathcal{M}_{Flat}K$ ,	$S^1$ , Cones, Celestial Spheres

<sup>a</sup> This is a Khählerian moduli space.

From the theorem 2. 1, is clear that the ramification to the part of connection  $\nabla_s$ , must be inside the context of the moduli space  $\mathcal{M}_{Higgs}({}^L G, C)$  The induced lines bundle must be one from  $T_V^\vee Bun_C(\Sigma)$ , with the condition of that it must be a divisor of holomorphic vector bundle.

One immediate consequence of this theorem 2. 1, and the application of a meromorphic extension given for Pantev [6], but in the circumstance of a divisor factor of the moduli space  $\mathcal{M}_H(LG, C)$ , is the following result:

**Theorem (F. Bulnes) 2. 2.** If  $\nabla_s$ , has moduli stack  $\tilde{\mathcal{L}}_\lambda = \mathcal{L}^{\otimes 2}$ , where  $\mathcal{L}^{\otimes 2}$ , is the sub-bundle of lines

$$\mathcal{L}^{\otimes 2} \cong \tilde{\mathcal{L}}_{[\bar{C}_{h_V}(\theta)]} \otimes \zeta^{\otimes -(n-1)}, \quad (3)$$

where  $\bar{C}_V \rightarrow C \times T_V \text{Bun}$ , is simply the cover of  $(p_c^* V, \phi)$ , and hence comes equipped with a natural line bundle  $\tilde{\mathcal{L}}_\lambda$ , such that  $\pi_{V*} \tilde{\mathcal{L}}_\lambda = p_c^* V$ , then their generalized Penrose transform (which is a Penrose-Ward transform) comes given by

$$H^0(LG, \Gamma(U, \mathcal{O})) \cong \text{Ker}(U, p^* \nabla + \tau(\nabla)), \quad (4)$$

Proof.[5].▪

Then we can to analyze through cohomology of cycles these moduli identities from the Hitchin mappings extended to deformations of the stacks  $T^\vee \text{Bun}_G$ , and  $T^\vee \text{Bun}_{LG}$ , in analogue manner. Likewise these cohomological versions, can give a factorization result of the solution classes to field equations to a corresponding dimension of the cohomology spaces considering as proper ramification the used in the stack moduli  $T^\vee \text{Bun}_{LG}$ , using the images of Cousin complexes [7], [8] (the corresponding to the Cousin cohomology) due to the Penrose transforms framework.

### 3. Results through Cohomology of Cycles and Moduli problems.

**Theorem 3. 1 (F. Bulnes) [9].** If we consider the category  $M_{K_F}(\hat{\mathfrak{g}}, Y)$ , then a scheme of their spectrum  $V_{critical}^{Def}$ , where  $Y$ , is a Calabi-Yau manifold comes given as:

$$\text{Hom}_{\hat{\mathfrak{g}}}(X, V_{critical}^{Def}) \cong \text{Hom}_{\text{Loc}_{LG}}(V_{critical}, M_{K_F}(\hat{\mathfrak{g}}, Y)), \quad (5)$$

Proof.[9].▪

Then we can to establish the following results considering the moduli problems between objects of an algebra.

Studies realized using commutative rings extended by the Yoneda algebra say that:

**Theorem. 3. 2.** The Yoneda algebra  $\text{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$ , is abstractly  $A_\infty$ -isomorphic to  $\text{Ext}_{\text{Loc}_G}^\bullet(\mathcal{O}_{Op_{LG}}, \mathcal{O}_{Op_{LG}})$ .

This result bring in particular that formal deformations of the sheaf  $\mathcal{D}^s$ , can be consigned in  $\mathcal{D}_{Bun_G}^s$ -mod, which in the stack moduli language can be re-written using the theorem 3. 1, as

$$\text{Spec}_{Bun_G} TBun_G = T^\vee Bun_G,$$

Through of the consideration of Frenkel on the necessity of compute the cohomologies of higher dimension and prove that

$$H^\bullet(T^\vee Bun_G, \mathcal{O}) \cong \Omega^\bullet[Op_{LG}(D)], \quad (6)$$

We can establish a long sequence where the correspondence between moduli stacks and cohomological classes as products of the generalized Verma modules (see table [7]) where precisely the cohomological space  $H^\bullet(T^\vee Bun_G, \mathcal{O})$ , has their corresponding version with coefficients in the Verma module at critical level <sup>1</sup> as  $H^\bullet(\mathfrak{g}[[t]], \mathfrak{g}; \mathbb{V}_{crit})$ . Of fact, this appears inside moduli identity of the theorem 2. 1. where

**Theorem (F. Bulnes) 3. 3.** The following resolution of cohomological spaces is a geometrical resolution to the lines bundles given in (3) and that gives moduli stacks in (4):

$$\begin{aligned} & \mathbb{C}[Op_{LG}(D)] \\ & \cong \\ H^0(T^\vee Bun_G, \mathcal{O}) & \rightarrow H^1(T^\vee Bun_G, \Omega^1) \rightarrow H^2(T^\vee Bun_G(\Sigma)\Omega^1) \rightarrow \dots \\ & \dots \rightarrow H^\bullet(?, \Omega^\bullet) \rightarrow \dots, \end{aligned}$$

One question that arises immediately is, who is '?', and which is the corresponding dimension of the cohomological space  $H^\bullet(?, \Omega^\bullet)$ , and their cotangent bundle  $\Omega^\bullet$ ?

*Proof.* We consider the following lemma published in [9].

**Lemma (F. Bulnes) 3. 1.** Twisted derived categories corresponding to the algebra of functions  $\mathbb{C}[Op_{LG}(\mathcal{D}^\times)]$ , are the images obtained by the composition  $\mathcal{P}(\tau)$ , on  $\tilde{\mathcal{L}}_\lambda, \forall \lambda \in \mathfrak{h}^*$ , and such that their Penrose transform is:

$$\mathcal{P} : H^0(LG, \Gamma(Bun_G, \mathcal{D}^\times)) \cong \text{Ker}(U, \tilde{\mathcal{D}}_{\lambda, y}),$$

The lemma plays an important role to exhibit the influence of twistor transform to the obtaining the twisted nature of the derived categorie  $\mathcal{D}^\times$ , starting from the line bundle  $\mathcal{L}_\lambda$ .

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<sup>1</sup> $\mathbb{V}_{crit} = U_{crit} \hat{\mathfrak{g}} \otimes_{\mathfrak{g}[[z]]} \mathbb{C}$ .

Proof. It is other form to write the twistor transform treatment followed in [7]. The image that stays is naturally a Penrose transform image. ■

Now we demonstrate the theorem 3. 3. To it, we consider the Yoneda algebra  $\text{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$ , and as “quantum” version of  $\text{Sym } T$ , the moduli stack  $Bun_G = G[[z]] \backslash X$ ,<sup>2</sup> then by the theorem 3. 2., a Harish-Chandra module of the type  $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{crit})$ , implements an  $A_\infty$ -isomorphism of the module  $H^\bullet(Bun_G, \mathcal{D}^s)$ , considering a skew-commutative sub-algebra of  $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{crit})$ . But  $H^\bullet(Bun_G, \mathcal{D}^s)$ , is the corresponding cohomology  $\mathbb{H}_{G[[z]]}^q(X, (\wedge^\bullet \mathfrak{g}[\Sigma^0] \otimes \mathbb{V}_{crit}; \partial))$ , where  $\Sigma^0 = \Sigma \setminus \{\sigma\}$ ,  $\forall \sigma \in \Sigma$ , and  $\partial$ , is the Chevalley differential for the fiber-wise Lie algebra action of  $\mathfrak{g}[\Sigma^0]$ , on  $\mathbb{V}_{crit}$ , twisted at the point  $\phi \bullet G[\Sigma^0] \epsilon X$ , by the adjoint action of the loop group element  $\phi$ . We need to use Hodge theory over classes  $\phi \bullet G[\Sigma^0] \epsilon X$ . We want to extend the Deligne connection with Penrose transform on each ramification  $\bar{\partial} + d$ , to schemes as [5] of spectrum  $\mathbb{V}_{critical}^{Def}$ , of the category  $M_{K_F}(\hat{\mathfrak{g}}, Y)$ , which are applications in deformation theory [10],[11],[12]. But, by the lemma A. 1, we have the functors in the space  $\text{Fun}(\mathcal{QCoh}(Y), \mathfrak{F}) \epsilon \text{FunOp}_{L_G}$ ,<sup>3</sup> that by integral transforms as in [4], their kernels are in a sheaf  $\mathcal{O}_{Op^L_G}$ , [13] having as cohomological space  $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \text{End} \mathbb{V}_{crit})$ , which has a “quantum version”  $H^\bullet(T^\vee Bun_G, \mathcal{O})$ , where

$$H^\bullet(T^\vee Bun_G, \mathcal{O}) = H^\bullet(?, \Omega^\bullet), \quad (7)$$

But in  $\mathcal{O}_{Op^L_G}$  we have  $\text{Spec}_{Bun_G} TBun_G \epsilon T^\vee(\text{Op}_{L_G}(D))$ , and the quantum version of this is obtained in the cohomology space, re-taking the non-commutative Hodge theory to a Higgs context [6] we have;  $H^\bullet(T^\vee Bun_G \gamma, \mathcal{O}) \cong \mathbb{C}[H] \otimes H^\vee$ , where  $H^\vee = T^\vee(\text{Op}_{L_G}(D))$ , that is to say, the corresponding extension of the derived category  $\mathbb{C}[H]$ , to the operator  $\partial$ ,<sup>4</sup> is  $H^\vee$ . For other side, by (4) or (5)

$$H^1(Bun_G, \text{Sym} T) \cong \text{Ker}(U, \tilde{\mathfrak{g}}; \partial + d) = \Omega^\bullet(\text{Op}_{L_G}(D)), \quad (8)$$

and the Higgs stack bundle is  $\mathbb{C}[H] \otimes H^\vee = \Omega^1[H]$ . But  $H^\bullet(T^\vee Bun_G, \mathcal{O})$ , is generated by a copy of  $H^\vee$ , over  $H^0 = \mathbb{C}[\text{Op}_{L_G}]$ , being associate with the graded algebra  $\text{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$ , but is had the exact long sequence when the sheaves  $\mathcal{O}$ , are analytic sheaves:

$$\begin{aligned} & a \in \mathbb{C}[\text{Op}_{L_G}] \\ & \cong \\ & \text{Ext}^0(\mathbb{V}_{crit}, \mathbb{V}_{crit}) \xrightarrow{a} \text{Ext}_{HC}^1(\mathbb{V}_{crit}, \mathbb{V}_{crit}) \xrightarrow{da} \\ & \text{Ext}^2(\mathbb{V}_{crit}, \mathbb{V}_{crit}) \xrightarrow{d(da)} \dots \xrightarrow{\bar{\partial}+d} \text{Ext}_{LocL_G}^\bullet(\mathcal{O}_{Op_{L_G}}, \mathcal{O}_{Op_{L_G}}) \xrightarrow{\partial} \dots, \\ & \parallel \\ & \text{Ext}_{HC(\hat{\mathfrak{g}}, G[[z]])}^\bullet(\mathbb{V}_{crit}, \mathbb{V}_{crit}) \cong \Omega^\bullet[\text{Op}_{L_G}(D)] \end{aligned}$$

survival only cohomology generators  $H^1$ . Then the dimension of the hyper-

<sup>2</sup> $X := G((z))/G[[\Sigma^0]]$ , is the thick flag variety obtained through “quantum” version of  $\text{Sym } T$ .

<sup>3</sup>Here  $\mathfrak{F}$ , is a sheaf of ramifications.

<sup>4</sup> $\mathfrak{g}[\Sigma^0] \xrightarrow{\partial=Ad_\phi} \mathfrak{g}((z))/\mathfrak{g}[[z]]$

cohomological space  $\mathbb{H}^q$ , is at least  $q = 1$ , and due to that

$$H^\bullet(H^\vee, \Omega^\bullet) = H^\bullet(?, \Omega^\bullet), \quad (9)$$

we have that  $?=H^\vee = T^\vee[\text{Op}_{L_G}(D)]$ , which is included in the quasi-coherent category  $M_{K_F}(\hat{\mathfrak{g}}, Y)$ . This proves the theorem 3.3.  $\blacksquare$

In Stein varieties language, the before quantum version takes the form  $T^\vee X \subset Y, \forall X, Y$  stein varieties.

We consider the application of the theorem 3.2, in the moduli spaces context of the deep space-time  $\mathbb{M}$ :

**Example 3. 1.** In BRST-cohomology, the field equations

$$\begin{aligned} b_0 &= \phi a, & \forall a \\ b_{1\bar{l}} d\bar{z}^{\bar{l}} &= \bar{\partial} a, \end{aligned}$$

have solutions such that  $b_0 \bmod \text{Im } \phi \in H^0(D, \mathcal{O}(D^\vee) |_D \otimes \mathcal{O}_D) = \text{Ext}^1(\mathcal{O}_D, \mathcal{O})$ , with  $D$ , a divisor on the complex line  $\mathbb{C}$ , <sup>5</sup> that is to say,

$$0 \longrightarrow \mathcal{O}(-D) \xrightarrow{\phi} \mathcal{O} \longrightarrow \mathcal{O}_D \longrightarrow 0,$$

Reciprocally  $\text{Ext}^1(\mathcal{O}_D, \mathcal{O}) = H^0(D, \mathcal{O}(D^\vee) |_D \otimes \mathcal{O}_D)$ , with the field equations

$$\begin{aligned} \bar{\partial}(b_{1\bar{l}} d\bar{z}^{\bar{l}}) &= 0, \\ \bar{\partial} b_0 &= -\phi(b_1 d\bar{z}) \end{aligned}$$

which have solutions as the extended field  $\mathcal{Q}_{BRST} = \bar{\partial} + \Sigma \phi^{\alpha\beta} = \bar{\partial} + \tilde{\varphi} = \text{Oper}(\mathcal{Q}_{BRST})$ . Here precisely,  $\mathcal{Q}_{BRST}$ , is the solution to the field equation with differential operators in  $\mathcal{O}(D^\vee) |_D \otimes \mathcal{O}_D$ .

### Specialized Notation

$\mathcal{P}$  -Penrose tranform.

$\mathcal{D}^\times$  -Twisted sheaf of differential operators to our Oper, given by  $\text{Loc}_{L_G}(\mathcal{D}^\times)$ .

$K^{1/2}$  - Root square of the canonical line bundle on  $\text{Bun}_G$ , corresponding to the critical level. This is a divisor vector bundle.

$\text{Bun}_G(X)$  -Category of principal  $G$  - bundles over  $C \times X$ . Also is the moduli stack of principal  $G$  -bundles over  $C$ .

$\text{Loc}_{L_G}(\mathcal{D}^\times)$  -Set of equivalence classes of  $L_G$  -bundles with a connection on  $\mathcal{D}^\times$ . This space shapes a bijection with the set of gauge equivalence classes of the

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<sup>5</sup> $\mathbb{C}/D = \mathcal{O}_D \cup \mathcal{O}(-D)$

ramified operators, as defined in [14],[15].

$\mathcal{D}_{BRST}$  - the derived category on  $\mathcal{D}$  -modules of  $\mathcal{Q}_{BRST}$  -operators applied to the geometrical Langlands correspondence to obtain the “quantum” geometrical Langlands correspondence.

$\mathcal{H}_G-$   $\cong (B \backslash G / B)$ , of bi-equivariant  $\mathcal{D}$  -modules on a complex reductive group  $G$ .

$\mathcal{D}^\times(Bun_G(\Sigma))$  -It’s the category of the twisted Hecke categories  $\mathcal{H}_{G,\lambda}$ .

$Ch_{G,[\lambda]}$  -Character sheaves used as Drinfeld centers in derived algebraic geometry. Their use connects different cohomologies in the Hecke categories context.

$\mathcal{M}_{Higgs}({}^L G, C)$  -Moduli space of the dualizing of the Higgs fields, that is to say, quasi-coherent  $D$ -modules. Usually said quasi-coherent  $D$ -modules are coherent  $D$ -modules as  $D$ -branes.

$\mathcal{M}_{Higgs}(G, C)$  -Moduli space of the Higgs fields. Their fields are the  $\theta\epsilon T_V^\vee Bun_C(\Sigma)$

## Appendix A.

**Lemma(F.Bulnes, I. Verkelov) A. 1.** Let  $\mathcal{C}$ , a derived category whose functor belongs to the space  $\text{Fun}(\mathcal{D}^\times, \mathcal{C})$ <sup>6</sup>. Then the cycles and co-cycles in the scheme (7.3.7) of the theorem 7.3.1., [16] are calculated by the Penrose transform on each ramification  $\partial + d$ , of  $\mathcal{O}_{Op^L G}$ , having:

$$\text{End}_{\mathfrak{g}}(V_{crit}) \cong \text{FunOp}_{L_G}, \quad (A,1) \quad kpv16 : kpv16$$

*Proof.* [13].

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<sup>6</sup>Of tact we have in the Oper, language that  $\text{FunOp}_{L_{G\lambda}} \subset \text{FunOp}_G(\mathcal{D}^\times)$  [7]

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