**Modified TOPSIS Method for Ranking the Financial Performance of Deposit Banks in Turkey**

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Decision-making, defined as the selection of the best among the various alternatives, is called Multi-Criteria Decision Making (MCDM) when there are multiple criteria. The MCDM methods, which presented solution proposals for the correct and useful decisions that can be made in many areas have begun to develop from the beginning of 1960's. The main purpose of using the methods is to control the decision making mechanism in cases where there are a lot of alternative and criterion numbers and to make the decision result as easy and quick as possible.

There are many multi criteria decision making methods in the literature. One of them is the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) introduced by Hwang and Yoon (1981). The method is briefly based on the principle that the selected alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution.

In this study, as an alternative to the Euclidean distance measure used in the calculation of the positive and negative ideal solutions at the traditional TOPSIS method a different approach has been proposed by using *Lp* Minkowski family and *𝐿1* Family distance measures. With the modified TOPSIS method, the financial performance of the deposit bank operating in the Turkish Banking Sector was examined. From the results obtained, it has been tried to emphasize the importance of the distance measures used in the TOPSIS method in order of alternatives.

***Keywords:*** *MCDM; TOPSIS, Lp Minkowski family distance, 𝐿1 Family distance.*

**1. INTRODUCTION**

We have to make a choice among alternatives for actions we think we are doing or are considering to do in many of our lives. The selection process starts with the formation of at least two alternatives, the process works and one of the alternatives is selected. This selection process is one of the simplest definitions of decision making.

Decision making is called Multi Criteria Decision Making (MCDM), which has a lot of criteria in particular. It started to develop from the beginning of 1960's. The objective of the MCDM is to keep the decision-making mechanism under control and to make the decision as easy and quick as possible, in cases where the alternative and criterion numbers are too great.

It is to be able to control the decision-making mechanism and to make the decision as easy and quick as possible, in cases where the objective of the MCDM is that there are a lot of alternative and criterion numbers.

There are a variety of methods to make correct and useful decisions in cases where there are many criteria in the literature. Some of the methods have been developed based on the concept of vector distance or similarity. The most popular of these is undoubtedly TOPSIS method. The TOPSIS method is a MCDM method that works with finding the closest points to the ideal positive point and the farthest points to the negative ideal point.

In this study, the TOPSIS method is considered and focused on the distance measure used in the method. For this purpose, as an alternative to Euclidean distance used in traditional TOPSIS method, *Lq* family that include Euclidean distance and *L1* family distances are considered. The effects of the modified TOPSIS method were applied to the data compiled by the Turkish Banking Unity (TBU) to measure the performances of the deposit banks operating in Turkey.

**2. MULTI CRITERIA DECISION MAKING**

It is possible for businesses to adapt to rapidly changing environmental conditions and make effective decisions in parallel with this change by using scientific methods that can assess a large number of qualitative and quantitative factors in the decision-making process (Taha, 1997).

Many problems encountered in real life fit the definition of multi-criteria decision making. People find their individual preferences while they are present in evaluative judgments in multi-criteria decision making problems. It may not be difficult to decide when there are few criteria or few alternatives. However, as the subject becomes more complex, the information processing capacity of people is restricted, decision making becomes more difficult and help may be needed. In such cases, instead of trying to integrate too much knowledge and trying to decide, applying simple rules and procedures and evaluating the problem gradually will make it easier to decide. Such approaches will also facilitate decision makers to make rational decisions, and the decision will be appropriate within the constraints (Taha, 1997).

It is possible to classify the MCDM problems as Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM). The MADM problems have a predefined number of alternatives and the success levels of each of these alternatives are determined. Decisions in the MADM problems are made by comparing the qualities that exist for each alternative. On the other hand, in the MODM problems, the number of alternatives can not be determined in advance and the aim of the model is to determine the "best" alternative. The number of alternatives that will give the optimal solution in quantitative decision making techniques can not be decided in advance.

In the decision analysis phase, the following four-stage general process is followed:

• Determination of criteria and alternatives,

• Assignment of numerical measures of relative importance to criteria,

• Assigning numerical measures to alternatives according to each criterion,

• Numerical values ​​for sorting alternatives.

The MCDM methods have been developed to effectively carry out the fourth stage of this process. There are different methods used in the literature for the solution of MCDM problems and none of these methods gives a complete advantage over others. The most important advantage of these methods is that they allow us to evaluate quantitative and qualitative criteria together.

MCDM methods; Weighted total method, Weighted product method, Analytic network process method, Analytical Hierarchy Process method, ELECTRE, TOPSIS, VIKOR, PROMETHEE, ORIENTE, MAPPAC, WSA etc. sorting is possible (Hwang, C. L., and Yoon, K., 1981).

As the TOPSIS method has been considered in the study, the details will be mentioned in the following section.

**2.1. TOPSIS Method**

**Step1.** Determining the decision matrix:

 (1)

A general multiple criteria analysis problem is represented as a decision matrix which consists of a set of alternatives *Ai* (*i=1,2,…,n*) to be evaluated against a set of criteria *Cj* (*j=1,2, …,m*).

**Step2.** Determining the weighting vector as below:

 (2)

In which the relative importance of criterion *Cj* with respect to the overall objective of the problem is represented as *wj*.

**Step 3.** Normalizing the decision matrix through Euclidean normalization:

 (3)

As a result, a normalized decision matrix can be determined as:

 (4)

**Step 4.** Calculating the performance matrix:

The weighted performance matrix which reflects the performance of each alternative with respect to each criterion is determined by multiplying the normalized decision matrix (11) by the weight vector (9).

 (5)

**Step 5.** Determining the PIS and the NIS:

The positive (negative) ideal solution consists of the best (worst) criteria values attainable from all the alternatives. Deng (2007) enumerated the advantages of using these two concepts as: their simplicity and comprehensibility, their computational efficiency, and their ability to measure the relative performance of the alternatives in a simple mathematical form.

 (6)



and,

 (7)



Both formulas  show benefit (maximization) and  show loss (minimization) value.

**Step 6.** Calculating the degree of distance of the alternatives between each alternative and the PIS and the NIS:

By using Euclid Distance.

  (8)

  (9)

**Step 7.** Calculating the overall performance index for each alternative across all criteria:

This index can be calculated based on the concept of the degree of similarity of alternative *Ai* relative to the ideal solutions.

 (10)

. The *Pi* value indicates the absolute closeness of the ideal solution. If  then Ai is the ideal solution, if  then Ai is the negative ideal solution.

**Step 8.** Ranking the alternatives in the descending order of the performance index value.

**3. DISTANCE MEASURE**

From the scientific and mathematical point of view, distance is defined as a quantitative degree of how far apart two objects are. Synonyms for distance include dissimilarity. Those distance measures satisfying the metric properties are simply called metric while other non-metric distance measures are occasionally called divergence. Synonyms for similarity include proximity and similarity measures are often called similarity coefficients (http://journocode.com/2016/03/10/similarity-and-distance-part-1/).

The choice of distance/similarity measures depends on the measurement type (nominal, ordinal, interval, or proportional scale) or representation (continuous or discreate) of the data.

Properties of Distance Function;

1. The distance of two objects *x* and *y* can not be less than zero.

 (11)

2. Two perfectly similar objects have distance zero.

 (12)

3. The distance between *x* and *y* is the same as between *y* and *x*, it doesn’t matter which way you go.

 (13)

4. If you take a detour via *y* on your way from *x* to *z*, your path can not be shorter than if you had taken the direct route. This is called the triangle inequality.

 (14)

In the following tables, the formulas of distances/similaries, including: *Lp* Minkowski family, *L1* family, Intersection family, Inner Product family, Fidelity family or Squared-chord family, Squared *L2* family or χ2 family, Shannon’s entropy family and family derived from combinations of different methods are shown (Sung-Hyuk Cha, 2007).

Table 1. *Lp* Minkowski family

|  |  |  |
| --- | --- | --- |
| 1. Euclidean *L2*  |  | (1) |
| 2. City block *L1* |  | (2) |
| 3. Minkowski *Lp* |  | (3) |
| 4. Chebyshev *L∞* |  | (4) |

Table 2. *L1* family

|  |  |  |
| --- | --- | --- |
| 5. Sørensen |  | (5) |
| 6. Gower |  | (6)(7) |
| 7. Soergel |  | (8) |
| 8. Kulczynski *d* |  | (9) |
| 9. Canberra |  | (10) |
| 10. Lorentzian |  | (11) |
| \* *L1* family ⊃ {Intersectoin (13), Wave Hedges (15), Czekanowski (16), Ruzicka (21), Tanimoto (23), etc}. |

Table 3. Intersection family

|  |  |  |
| --- | --- | --- |
| 11. Intersection |  | (12) |
|  |  | (13) |
| 12. Wave Hedges |  | (14)(15) |
| 13. Czekanowski |  | (16) |
|  |  | (17) |
| 14. Motyka |  | (18) |
|  |  | (19) |
| 15. Kulczynski s |  | (20) |
| 16. Ruzicka |  | (21) |
| 17. Tani-moto |  | (22)(23) |

Table 4. Inner Product family

|  |  |  |
| --- | --- | --- |
| 18. Inner Product |  | (24) |
| 19. Harmonic mean |  | (25) |
| 20. Cosine |  | (26) |
| 21.KumarHassebrook (PCE) |  | (27) |
| 22. Jaccard |  | (28) |
|  |  | (29) |
| 23. Dice |  | (30) |
|  |  | (31) |

Table 5. Fidelity family or Squared-chord family

|  |  |  |
| --- | --- | --- |
| 24. Fidelity |  | (32) |
| 25. Bhattacharyya |  | (33) |
| 26. Hellinger |  | (34)(35) |
| 27. Matusita |  | (36)(37) |
| 28. Squared-chord |  | (38) |
| ssqc = 1-dsqc |  | (39) |

Table 6. Squared *L2* family or χ2 family

|  |  |  |
| --- | --- | --- |
| 29. Squared Euclidean |  | (40) |
| 30. Pearson χ2 |  | (41) |
| 31. Neyman χ2 |  | (42) |
| 32. Squared χ2 |  | (43) |
| 33. Probabilistic Symmetric χ2 |  | (44) |
| 34. Divergence |  | (45) |
| 35. Clark |  | (46) |
| 36. Additive Symmetric χ2 |  | (47) |
| \* Squared L2 family ⊃ {Jaccard (29), Dice (31)} |

Table 7. Shannon’s entropy family

|  |  |  |
| --- | --- | --- |
| 37. Kullback– Leibler |  | (48) |
| 38. Jeffreys |  | (49) |
| 39. K divergence |  | (50) |
| 40. Topsøe |  | (51) |
| 41. Jensen-Shannon |  | (52) |
| 42. Jensen difference |  | (53) |

Table 8. Combinations

|  |  |  |
| --- | --- | --- |
| 43. Taneja |  | (54) |
| 44. Kumar Johnson |  | (55) |
| 45. Avg(L1 ,L∞) |  | (56) |

While most measures can be efficiently computed using simple vector operators, some measures prone to the division by zero and the log of zero cases deserve careful attention. It is briefly summarized in Table 9 that the cases mentioned in the methods can be confronted (Sung-Hyuk Cha, 2007).

It should be noted that 0/0 are treated as 0. Similarly, 0 log0 is treated as 0 as well. For the division by zero and log of zero group cases, the zero is replaced by a very small value.

Table 9. Grouping of distance/similarity measures by caveats to implementation

|  |  |
| --- | --- |
| Vector Ops | Eqns (1~9), (11~13), (16~19), (21~23), (26~40), and (56~57) |
| 0 / 0 | Canberra (10), Wave Hedges (14), Harmonic mean (25), Squared χ2 (43), Probabilistic Symmetric χ2 (44), Divergence (45), Clark (46), and Additive Symmetric χ2 (47) |
| division by zero | Kulczynski (9) (20), Pearson χ2 (41), Neyman χ2 (42), KL (48), Jeffreys (49), Taneja (54), and Kumar-Johnson (55) |
| 0 log0 | KL (48), K divergence (50), Topsøe (51), Jensen-Shannon (52), Jensen difference (53), and Taneja (54) |
| Log of 0 | Jeffreys (49) |

In this study, TOPSIS method has been modified by considering distances in the *Lp* Minkowski family and *L1* family from the distance/similarity families mentioned above briefly.

**3.1. Proposed Method**

In the proposed method, Steps 6 and 7 of the traditional TOPSIS method are modified as follows.

**Proposed Step 6.** Calculating the degree of similarity of the alternatives between each alternative and the PIS and the NIS:

***Lp Minkowski family***

 (57)

  (58)

For *p = r* ≥1, it ist he ***lp*-metric**, including the **Euclidean, Manhattan** (or magnitude, or city block) and **Chebyshev** (or maximum value, dominance) **metrics** for *p* = 2, 1 and ∞, respectively. The case (*p, r*) = (2, 1) corresponds tot he **squared Euclidean distance**.

***L1 family***

Table 10. Seperation Measures (Anandan V. and Uthra G., 2017).

|  |  |  |
| --- | --- | --- |
| Sorensen (or) Bray-Curtis |  | (59) |
|  |  | (60) |
| Gower |  | (61) |
|  |  | (62) |
| Soergel |  | (63) |
|  |  | (64) |
| Kulczynski |  | (65) |
|  |  | (66) |
| Canberra |  | (67) |
|  |  | (68) |
| Lorentzian |  | (69) |
|  |  | (70) |

The average *d+* and *d-* values for the *Lq* and *L1* families were calculated by taking the average of the *d+* and *d-* values obtained from the distances of *Lq* and *L1* families.

 (71)

 (72)

Where *k* is represent the number of methods.

**Proposed Step 7**. Calculating the overall performance index for each alternative across all criteria:

The *Pi* value is obtained from the *D+* and *D-* value.

 (73)

**4. EVALUATION OF THE METHODS**

In this study, as an alternative to the Euclidean distance measure used in the calculation of the positive and negative ideal solutions at the traditional TOPSIS method a different approach has been proposed by using *Lp* Minkowski family and *𝐿1* Family distance measures. With the modified TOPSIS methods, the financial performance of the deposit bank operating in the TBU was examined. From the results obtained, it has been tried to emphasize the importance of the distance measures used in the TOPSIS method in order of alternatives. For this purpose, data on deposit banks for the period of 2017 published by the TBU were used. A total of 28 banks were considered as alternatives in the study. The list of these banks is as in Table 11.

Table 11: List of the deposit banks in Turkey.

|  |  |
| --- | --- |
| The name of the Bank | Code |
| Adabank A.Ş. | A1 |
| Akbank T.A.Ş. | A2 |
| Alternatifbank A.Ş. | A3 |
| Anadolubank A.Ş. | A4 |
| Arap Türk Bankası A.Ş. | A5 |
| Bank of Tokyo-Mitsubishi UFJ Turkey A.Ş. | A6 |
| Birleşik Fon Bankası A.Ş. | A7 |
| Burgan Bank A.Ş. | A8 |
| Citibank A.Ş. | A9 |
| Denizbank A.Ş. | A10 |
| Deutsche Bank A.Ş. | A11 |
| Fibabanka A.Ş. | A12 |
| Finans Bank A.Ş. | A13 |
| HSBC Bank A.Ş. | A14 |
| ICBC Turkey Bank A.Ş. | A15 |
| ING Bank A.Ş. | A16 |
| Odea Bank A.Ş. | A17 |
| Rabobank A.Ş. | A18 |
| Şekerbank T.A.Ş. | A19 |
| Turkish Bank A.Ş. | A20 |
| Turkland Bank A.Ş. | A21 |
| Türk Ekonomi Bankası A.Ş. | A22 |
| Türkiye Cumhuriyeti Ziraat Bankası A.Ş. | A23 |
| Türkiye Garanti Bankası A.Ş. | A24 |
| Türkiye Halk Bankası A.Ş. | A25 |
| Türkiye İş Bankası A.Ş. | A26 |
| Türkiye Vakıflar Bankası T.A.O. | A27 |
| Yapı ve Kredi Bankası A.Ş. | A28 |

There are various ratios that measure the financial performance of the banks. As a result of literature review made from these ratios; a set of criteria was established by considering the 15 financial ratios observed most frequently under six main headings as capital adequacy, exchange rate risk, asset quality, liquidity status, profitability status and income-expenditure structure. These criteria are listed in Table 12 (Taşabat SE, Cinemre N, Şen S. 2015).

Table 12: Financial ratios used as criteria.

|  |  |  |  |
| --- | --- | --- | --- |
| **Rate****Group** | **Criteria****Code** | **Ratios** | **Performance****Effect** |
| CapitalAdequacy | C1 | Equity / (Credit + Market + Operational Ratio) | Positive |
| C2 | Equity / Total Assets | Positive |
| C3 | (Equity - Fixed Assets) / Total Assets | Positive |
| C4 | Equity / (Deposits + Non-Deposit Sources) | Positive |
| Exchange Rate | C5 | Net Balance Sheet Position / Equity (\*) | Negative |
| C6 | (Net Balance Sheet Position + Net Off-Balance Sheet Position) / Equity (\*) | Negative |
| ActiveQuality | C7 | Total Loans / Equity | Negative |
| C8 | Credits in the Sector (Gross) / Total Loans | Negative |
| C9 | Consumer Loans / Total Loans | Positive |
| Liquidity | C10 | Liquid Assets / Short Term Liabilities | Positive |
| C11 | Liquid Assets / (Deposits + Non-Deposit Sources) | Positive |
| Profitability | C12 | Net Period Profit (Loss) / Total Assets | Positive |
| C13 | Net Period Profit / Equity | Positive |
| Income/ExpenseStructure | C14 | Net Interest Income After Specific Provisions / Total Operating Income | Positive |
| C15 | Non-interest Income / Other Operating Expenses | Positive |

 (\*) Calculated by taking absolute values.

The performance of the criterion analyzed at the analysis stage has been made positive by applying the conversion to the ones that are negative. There are two different methods used in the literature for the transformation process. The first is the  transformation and the second is the transformation. In this study, by applying the second approach, the criterions with negative effect on performance and the criterions with negative value were transformed into TOPSIS method.

**Weighting Criteria**

Determining the weight of performance criteria in multi-criteria decision making is another important step. This is due to the fact that the decision makers of performance evaluation give different weights (importance) to each criterion. Different weights give different performance orders.

Different techniques are used in the literature to weight the criteria. In this study, "Equal Weighted Method" is considered. Equal Weighted Method is a method based on equal weighting of the criteria considered in evaluating alternatives.

 (74)

At this formula *n* is the number of criteria. The formula is used when the weight for the criteria is determined since the weights must be "1".

The application of the modified TOPSIS methods are explaned step by step below.

**Step1.** Determining the decision matrix

In this matrix 28 banks which have been investigated in the TBU report, are considered to be alternatives and the 15 ratios formed the criteria. The decision matrix is shown at Table 13.

Table 13: The decision matrix with original data

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 |
| A1 | 211,3 | 84,6 | 79,9 | 711,3 | 0,0 | 0,0 | 0,0 | - | - | 775,6 | 791,1 | 3,1 | 3,6 | 94,7 | 7,6 |
| A2 | 14,6 | 11,4 | 10,2 | 13,6 | -32,7 | -0,8 | 60,4 | 2,4 | 27,9 | 61,3 | 39,4 | 1,3 | 11,2 | 53,8 | 71,8 |
| A3 | 15,5 | 7,6 | 4,2 | 9,2 | -57,8 | 5,8 | 71,1 | 5,1 | 2,8 | 64,9 | 30,8 | 0,5 | 6,0 | 49,5 | 46,8 |
| A4 | 14,5 | 12,4 | 8,6 | 14,9 | -26,5 | -5,6 | 62,1 | 3,7 | 4,5 | 62,4 | 40,1 | 1,3 | 10,6 | 61,4 | 44,9 |
| A5 | 18,6 | 15,0 | 12,6 | 17,9 | -5,1 | 2,1 | 33,8 | 1,0 | 0,1 | 116,5 | 67,8 | 1,7 | 11,6 | 62,0 | 81,7 |
| A6 | 14,1 | 9,7 | 9,0 | 11,0 | -218,7 | -0,4 | 53,5 | 0,0 | 0,0 | 92,3 | 51,7 | 0,3 | 2,8 | 143,7 | -69,5 |
| A7 | 61,0 | 19,6 | 17,9 | 70,3 | 3,3 | 3,3 | 44,6 | 3,9 | 0,2 | 89,9 | 90,0 | 0,7 | 3,5 | 84,7 | 30,6 |
| A8 | 16,0 | 9,5 | 5,5 | 11,5 | -53,8 | 3,1 | 76,7 | 2,6 | 1,7 | 42,0 | 22,9 | 0,5 | 5,2 | 67,0 | 25,4 |
| A9 | 17,6 | 14,9 | 14,7 | 18,3 | -74,0 | -2,8 | 44,1 | 2,2 | 0,0 | 64,4 | 62,2 | 2,0 | 13,7 | 85,7 | 25,1 |
| A10 | 16,1 | 9,8 | 3,6 | 12,6 | -102,4 | 6,3 | 61,0 | 5,2 | 31,9 | 51,3 | 36,2 | 0,9 | 9,2 | 55,9 | 32,8 |
| A11 | 20,7 | 17,0 | 16,4 | 21,0 | 5,1 | 7,7 | 57,9 | 0,0 | 0,0 | 122,6 | 50,1 | 2,4 | 14,2 | 65,1 | 66,8 |
| A12 | 13,6 | 9,3 | 6,8 | 10,9 | -15,4 | -10,9 | 77,0 | 1,7 | 5,6 | 44,8 | 23,9 | 0,7 | 7,9 | 72,7 | 23,4 |
| A13 | 15,4 | 10,5 | 7,0 | 14,3 | -141,8 | -2,6 | 66,8 | 6,6 | 37,8 | 40,1 | 28,4 | 0,8 | 7,8 | 63,5 | 31,5 |
| A14 | 15,7 | 8,4 | 6,4 | 10,6 | -105,1 | 2,9 | 64,7 | 6,1 | 36,1 | 56,9 | 40,0 | -1,0 | -12,5 | 29,7 | 50,7 |
| A15 | 12,8 | 8,9 | 7,4 | 9,9 | -112,0 | -2,1 | 61,9 | 4,4 | 11,8 | 99,7 | 40,6 | -0,3 | -2,9 | 78,1 | 5,8 |
| A16 | 15,8 | 9,1 | 7,3 | 11,1 | -159,0 | 1,4 | 71,5 | 3,0 | 32,1 | 46,6 | 28,4 | 0,2 | 2,6 | 70,4 | 15,3 |
| A17 | 12,2 | 4,2 | 2,4 | 4,7 | -232,0 | 8,8 | 68,0 | 2,2 | 10,1 | 57,6 | 32,6 | 0,2 | 3,7 | 87,8 | -20,9 |
| A18 | 227,9 | 82,8 | 82,0 | 536,6 | -0,6 | -0,6 | 13,9 | 0,0 | 0,0 | 43.449,7 | 550,5 | 3,2 | 3,8 | 95,1 | 10,1 |
| A19 | 13,7 | 10,3 | 3,6 | 12,5 | -98,5 | -0,5 | 68,5 | 6,0 | 9,8 | 35,0 | 23,4 | 0,4 | 4,1 | 51,8 | 36,7 |
| A20 | 19,9 | 15,6 | 10,3 | 19,1 | 8,2 | 7,8 | 67,0 | 1,5 | 0,9 | 67,2 | 31,1 | 0,3 | 2,0 | 81,3 | 14,2 |
| A21 | 15,6 | 12,8 | 8,9 | 15,1 | 4,0 | 0,0 | 68,1 | 6,1 | 0,2 | 59,5 | 35,1 | 0,2 | 2,0 | 61,0 | 30,5 |
| A22 | 13,9 | 9,7 | 7,8 | 11,9 | -90,2 | 8,0 | 73,9 | 2,3 | 28,2 | 37,7 | 27,4 | 1,2 | 12,7 | 60,0 | 36,4 |
| A23 | 15,1 | 10,4 | 7,5 | 12,1 | -2,6 | 2,3 | 61,7 | 1,7 | 26,9 | 51,2 | 36,8 | 1,7 | 16,4 | 73,0 | 51,6 |
| A24 | 15,0 | 12,2 | 8,7 | 15,2 | -25,1 | 5,5 | 62,6 | 2,8 | 33,0 | 46,0 | 29,2 | 1,3 | 11,0 | 64,0 | 47,1 |
| A25 | 13,8 | 10,3 | 7,1 | 12,0 | -7,3 | 1,5 | 67,5 | 3,1 | 21,1 | 37,6 | 23,1 | 1,2 | 11,9 | 63,0 | 56,9 |
| A26 | 15,6 | 11,6 | 6,2 | 14,4 | -51,4 | -7,3 | 64,5 | 2,0 | 26,1 | 53,1 | 34,1 | 1,1 | 9,6 | 62,2 | 50,3 |
| A27 | 14,5 | 9,2 | 6,3 | 11,0 | 2,2 | 10,7 | 67,7 | 3,9 | 29,6 | 44,4 | 29,2 | 1,1 | 11,5 | 58,5 | 59,0 |
| A28 | 13,8 | 10,5 | 6,0 | 13,4 | -35,2 | 3,3 | 67,5 | 4,1 | 29,4 | 46,5 | 30,8 | 0,8 | 8,1 | 55,4 | 52,2 |

**Step2.** Determining the weighting vector

Since 15 criteria were used in this study, the criterial weights were determined as 1/15 = 0.06667.

Table 14: Weight of the criteria.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|   | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 |
| wi | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 | 0,067 |

**Step 3.** Normalizing the decision matrix through Euclidean normalization:

Before normalizing data the criterions with negative effect on performance and the criterions with negative value were transformed. And then data are normalized. For this purpose the formula (18) is applied. The result is shown in Table 15.

Table 15: Normalized data

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|   | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 |
| A1 | 0,648 | 0,640 | 0,648 | 0,794 | 0,238 | 0,272 | 0,585 | 0,000 | 0,000 | 0,018 | 0,802 | 0,348 | 0,151 | 0,247 | 0,137 |
| A2 | 0,045 | 0,086 | 0,083 | 0,015 | 0,205 | 0,254 | 0,126 | 0,204 | 0,261 | 0,001 | 0,040 | 0,196 | 0,222 | 0,140 | 0,251 |
| A3 | 0,048 | 0,057 | 0,034 | 0,010 | 0,179 | 0,128 | 0,045 | 0,074 | 0,026 | 0,001 | 0,031 | 0,126 | 0,173 | 0,129 | 0,207 |
| A4 | 0,045 | 0,094 | 0,070 | 0,017 | 0,211 | 0,133 | 0,113 | 0,140 | 0,042 | 0,001 | 0,041 | 0,199 | 0,216 | 0,160 | 0,203 |
| A5 | 0,057 | 0,113 | 0,102 | 0,020 | 0,233 | 0,219 | 0,328 | 0,272 | 0,001 | 0,003 | 0,069 | 0,235 | 0,226 | 0,162 | 0,269 |
| A6 | 0,043 | 0,073 | 0,073 | 0,012 | 0,014 | 0,263 | 0,178 | 0,319 | 0,000 | 0,002 | 0,052 | 0,111 | 0,143 | 0,374 | 0,000 |
| A7 | 0,187 | 0,148 | 0,145 | 0,078 | 0,235 | 0,190 | 0,246 | 0,132 | 0,002 | 0,002 | 0,091 | 0,147 | 0,150 | 0,221 | 0,178 |
| A8 | 0,049 | 0,072 | 0,044 | 0,013 | 0,183 | 0,194 | 0,002 | 0,195 | 0,016 | 0,001 | 0,023 | 0,130 | 0,165 | 0,174 | 0,169 |
| A9 | 0,054 | 0,113 | 0,119 | 0,020 | 0,162 | 0,204 | 0,250 | 0,213 | 0,000 | 0,001 | 0,063 | 0,261 | 0,245 | 0,223 | 0,168 |
| A10 | 0,049 | 0,074 | 0,029 | 0,014 | 0,133 | 0,114 | 0,122 | 0,068 | 0,298 | 0,001 | 0,037 | 0,165 | 0,203 | 0,146 | 0,182 |
| A11 | 0,064 | 0,128 | 0,133 | 0,023 | 0,233 | 0,079 | 0,145 | 0,319 | 0,000 | 0,003 | 0,051 | 0,291 | 0,250 | 0,170 | 0,242 |
| A12 | 0,042 | 0,070 | 0,055 | 0,012 | 0,222 | 0,000 | 0,000 | 0,237 | 0,052 | 0,001 | 0,024 | 0,150 | 0,191 | 0,189 | 0,165 |
| A13 | 0,047 | 0,080 | 0,057 | 0,016 | 0,093 | 0,208 | 0,078 | 0,000 | 0,353 | 0,001 | 0,029 | 0,158 | 0,190 | 0,165 | 0,180 |
| A14 | 0,048 | 0,064 | 0,052 | 0,012 | 0,130 | 0,200 | 0,093 | 0,027 | 0,338 | 0,001 | 0,041 | 0,000 | 0,000 | 0,077 | 0,214 |
| A15 | 0,039 | 0,067 | 0,060 | 0,011 | 0,123 | 0,221 | 0,114 | 0,108 | 0,110 | 0,002 | 0,041 | 0,067 | 0,090 | 0,203 | 0,134 |
| A16 | 0,048 | 0,069 | 0,059 | 0,012 | 0,075 | 0,239 | 0,042 | 0,175 | 0,300 | 0,001 | 0,029 | 0,108 | 0,141 | 0,183 | 0,151 |
| A17 | 0,037 | 0,032 | 0,020 | 0,005 | 0,000 | 0,051 | 0,068 | 0,211 | 0,094 | 0,001 | 0,033 | 0,102 | 0,152 | 0,229 | 0,086 |
| A18 | 0,699 | 0,626 | 0,665 | 0,599 | 0,237 | 0,258 | 0,479 | 0,319 | 0,000 | 1,000 | 0,558 | 0,356 | 0,153 | 0,248 | 0,141 |
| A19 | 0,042 | 0,078 | 0,029 | 0,014 | 0,137 | 0,261 | 0,064 | 0,029 | 0,091 | 0,001 | 0,024 | 0,124 | 0,155 | 0,135 | 0,189 |
| A20 | 0,061 | 0,118 | 0,084 | 0,021 | 0,230 | 0,077 | 0,076 | 0,246 | 0,008 | 0,002 | 0,032 | 0,115 | 0,136 | 0,212 | 0,149 |
| A21 | 0,048 | 0,097 | 0,072 | 0,017 | 0,234 | 0,272 | 0,067 | 0,026 | 0,002 | 0,001 | 0,036 | 0,109 | 0,135 | 0,159 | 0,178 |
| A22 | 0,043 | 0,073 | 0,063 | 0,013 | 0,146 | 0,072 | 0,023 | 0,207 | 0,264 | 0,001 | 0,028 | 0,192 | 0,236 | 0,156 | 0,188 |
| A23 | 0,046 | 0,079 | 0,061 | 0,014 | 0,235 | 0,216 | 0,116 | 0,238 | 0,252 | 0,001 | 0,037 | 0,232 | 0,270 | 0,190 | 0,215 |
| A24 | 0,046 | 0,092 | 0,070 | 0,017 | 0,212 | 0,136 | 0,109 | 0,186 | 0,309 | 0,001 | 0,030 | 0,201 | 0,220 | 0,167 | 0,207 |
| A25 | 0,042 | 0,078 | 0,058 | 0,013 | 0,231 | 0,234 | 0,072 | 0,168 | 0,198 | 0,001 | 0,023 | 0,192 | 0,228 | 0,164 | 0,225 |
| A26 | 0,048 | 0,088 | 0,050 | 0,016 | 0,185 | 0,090 | 0,095 | 0,221 | 0,244 | 0,001 | 0,035 | 0,183 | 0,207 | 0,162 | 0,213 |
| A27 | 0,045 | 0,069 | 0,051 | 0,012 | 0,236 | 0,003 | 0,071 | 0,131 | 0,277 | 0,001 | 0,030 | 0,177 | 0,225 | 0,152 | 0,228 |
| A28 | 0,042 | 0,079 | 0,049 | 0,015 | 0,202 | 0,190 | 0,072 | 0,121 | 0,275 | 0,001 | 0,031 | 0,159 | 0,192 | 0,144 | 0,216 |

**Step 4.** Calculating the performance matrix:

The weighted performance matrix which reflects the performance of each alternative with respect to each criterion is determined by multiplying the normalized decision matrix (3) by the weight vector (2).

Table 16: Performance matrix

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|   | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 |
| A1 | 0,043 | 0,043 | 0,043 | 0,053 | 0,016 | 0,018 | 0,039 | 0,000 | 0,000 | 0,001 | 0,053 | 0,023 | 0,010 | 0,016 | 0,009 |
| A2 | 0,003 | 0,006 | 0,006 | 0,001 | 0,014 | 0,017 | 0,008 | 0,014 | 0,017 | 0,000 | 0,003 | 0,013 | 0,015 | 0,009 | 0,017 |
| A3 | 0,003 | 0,004 | 0,002 | 0,001 | 0,012 | 0,009 | 0,003 | 0,005 | 0,002 | 0,000 | 0,002 | 0,008 | 0,012 | 0,009 | 0,014 |
| A4 | 0,003 | 0,006 | 0,005 | 0,001 | 0,014 | 0,009 | 0,008 | 0,009 | 0,003 | 0,000 | 0,003 | 0,013 | 0,014 | 0,011 | 0,014 |
| A5 | 0,004 | 0,008 | 0,007 | 0,001 | 0,016 | 0,015 | 0,022 | 0,018 | 0,000 | 0,000 | 0,005 | 0,016 | 0,015 | 0,011 | 0,018 |
| A6 | 0,003 | 0,005 | 0,005 | 0,001 | 0,001 | 0,018 | 0,012 | 0,021 | 0,000 | 0,000 | 0,003 | 0,007 | 0,010 | 0,025 | 0,000 |
| A7 | 0,012 | 0,010 | 0,010 | 0,005 | 0,016 | 0,013 | 0,016 | 0,009 | 0,000 | 0,000 | 0,006 | 0,010 | 0,010 | 0,015 | 0,012 |
| A8 | 0,003 | 0,005 | 0,003 | 0,001 | 0,012 | 0,013 | 0,000 | 0,013 | 0,001 | 0,000 | 0,002 | 0,009 | 0,011 | 0,012 | 0,011 |
| A9 | 0,004 | 0,008 | 0,008 | 0,001 | 0,011 | 0,014 | 0,017 | 0,014 | 0,000 | 0,000 | 0,004 | 0,017 | 0,016 | 0,015 | 0,011 |
| A10 | 0,003 | 0,005 | 0,002 | 0,001 | 0,009 | 0,008 | 0,008 | 0,005 | 0,020 | 0,000 | 0,002 | 0,011 | 0,014 | 0,010 | 0,012 |
| A11 | 0,004 | 0,009 | 0,009 | 0,002 | 0,016 | 0,005 | 0,010 | 0,021 | 0,000 | 0,000 | 0,003 | 0,019 | 0,017 | 0,011 | 0,016 |
| A12 | 0,003 | 0,005 | 0,004 | 0,001 | 0,015 | 0,000 | 0,000 | 0,016 | 0,003 | 0,000 | 0,002 | 0,010 | 0,013 | 0,013 | 0,011 |
| A13 | 0,003 | 0,005 | 0,004 | 0,001 | 0,006 | 0,014 | 0,005 | 0,000 | 0,024 | 0,000 | 0,002 | 0,011 | 0,013 | 0,011 | 0,012 |
| A14 | 0,003 | 0,004 | 0,003 | 0,001 | 0,009 | 0,013 | 0,006 | 0,002 | 0,023 | 0,000 | 0,003 | 0,000 | 0,000 | 0,005 | 0,014 |
| A15 | 0,003 | 0,004 | 0,004 | 0,001 | 0,008 | 0,015 | 0,008 | 0,007 | 0,007 | 0,000 | 0,003 | 0,004 | 0,006 | 0,014 | 0,009 |
| A16 | 0,003 | 0,005 | 0,004 | 0,001 | 0,005 | 0,016 | 0,003 | 0,012 | 0,020 | 0,000 | 0,002 | 0,007 | 0,009 | 0,012 | 0,010 |
| A17 | 0,002 | 0,002 | 0,001 | 0,000 | 0,000 | 0,003 | 0,005 | 0,014 | 0,006 | 0,000 | 0,002 | 0,007 | 0,010 | 0,015 | 0,006 |
| A18 | 0,047 | 0,042 | 0,044 | 0,040 | 0,016 | 0,017 | 0,032 | 0,021 | 0,000 | 0,067 | 0,037 | 0,024 | 0,010 | 0,017 | 0,009 |
| A19 | 0,003 | 0,005 | 0,002 | 0,001 | 0,009 | 0,017 | 0,004 | 0,002 | 0,006 | 0,000 | 0,002 | 0,008 | 0,010 | 0,009 | 0,013 |
| A20 | 0,004 | 0,008 | 0,006 | 0,001 | 0,015 | 0,005 | 0,005 | 0,016 | 0,001 | 0,000 | 0,002 | 0,008 | 0,009 | 0,014 | 0,010 |
| A21 | 0,003 | 0,006 | 0,005 | 0,001 | 0,016 | 0,018 | 0,004 | 0,002 | 0,000 | 0,000 | 0,002 | 0,007 | 0,009 | 0,011 | 0,012 |
| A22 | 0,003 | 0,005 | 0,004 | 0,001 | 0,010 | 0,005 | 0,002 | 0,014 | 0,018 | 0,000 | 0,002 | 0,013 | 0,016 | 0,010 | 0,013 |
| A23 | 0,003 | 0,005 | 0,004 | 0,001 | 0,016 | 0,014 | 0,008 | 0,016 | 0,017 | 0,000 | 0,002 | 0,015 | 0,018 | 0,013 | 0,014 |
| A24 | 0,003 | 0,006 | 0,005 | 0,001 | 0,014 | 0,009 | 0,007 | 0,012 | 0,021 | 0,000 | 0,002 | 0,013 | 0,015 | 0,011 | 0,014 |
| A25 | 0,003 | 0,005 | 0,004 | 0,001 | 0,015 | 0,016 | 0,005 | 0,011 | 0,013 | 0,000 | 0,002 | 0,013 | 0,015 | 0,011 | 0,015 |
| A26 | 0,003 | 0,006 | 0,003 | 0,001 | 0,012 | 0,006 | 0,006 | 0,015 | 0,016 | 0,000 | 0,002 | 0,012 | 0,014 | 0,011 | 0,014 |
| A27 | 0,003 | 0,005 | 0,003 | 0,001 | 0,016 | 0,000 | 0,005 | 0,009 | 0,018 | 0,000 | 0,002 | 0,012 | 0,015 | 0,010 | 0,015 |
| A28 | 0,003 | 0,005 | 0,003 | 0,001 | 0,013 | 0,013 | 0,005 | 0,008 | 0,018 | 0,000 | 0,002 | 0,011 | 0,013 | 0,010 | 0,014 |

**Step 5.** Determining the PIS and the NIS

The PIS and the NIS are attainable from all the alternatives (28 banks) across all 15 criteria according to formula (8)-(9). The PIS and the NIS are shown in Table 17.

Tablo 17: PIS and NIS value.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|   | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 |
| I+ | 0,699 | 0,640 | 0,665 | 0,794 | 0,238 | 0,272 | 0,585 | 0,319 | 0,353 | 1,000 | 0,802 | 0,356 | 0,270 | 0,374 | 0,269 |
| I- | 0,037 | 0,032 | 0,020 | 0,005 | 0,000 | 0,000 | 0,000 | 0,000 | 0,000 | 0,001 | 0,023 | 0,000 | 0,000 | 0,077 | 0,000 |

**Step 6.** Calculating the degree of distance of the alternatives between each alternative and the PIS and the NIS:

The abbreviations used in the following tables are as follows;

D\_1: Euclidean, D\_2: City blok, D\_3: Chebyshev, D\_4: Sorensen, D\_5: Gower, D\_6: Soergel, D\_7: Kulczyynski, D\_8: Canberra, D\_9: Lorentzian

Tablo 18: D+ and D- value *Lq* family.

|  |  |  |
| --- | --- | --- |
|  | Ideal Distances | Negative Distances |
| D\_I | D\_2 | D\_3 | D\_I | D\_2 | D\_3 |
| *D+* | *D+* | *D+* | *D-* | *D-* | *D-* |
| A1 | 0,07428 | 0,14053 | 0,06546 | 0,11605 | 0,35546 | 0,05256 |
| A2 | 0,12603 | 0,36709 | 0,06656 | 0,04184 | 0,12890 | 0,01740 |
| A3 | 0,13292 | 0,42445 | 0,06655 | 0,02571 | 0,07154 | 0,01377 |
| A4 | 0,12842 | 0,39680 | 0,06656 | 0,03248 | 0,09919 | 0,01439 |
| A5 | 0,12281 | 0,35512 | 0,06648 | 0,04647 | 0,14087 | 0,02190 |
| A6 | 0,12926 | 0,39852 | 0,06651 | 0,03824 | 0,09747 | 0,02124 |
| A7 | 0,11844 | 0,36555 | 0,06652 | 0,03812 | 0,13044 | 0,01641 |
| A8 | 0,13223 | 0,41364 | 0,06659 | 0,02935 | 0,08235 | 0,01302 |
| A9 | 0,12353 | 0,36919 | 0,06656 | 0,04066 | 0,12680 | 0,01738 |
| A10 | 0,12918 | 0,40004 | 0,06658 | 0,03316 | 0,09595 | 0,01988 |
| A11 | 0,12476 | 0,36695 | 0,06647 | 0,04328 | 0,12904 | 0,02124 |
| A12 | 0,13249 | 0,41502 | 0,06659 | 0,03051 | 0,08097 | 0,01577 |
| A13 | 0,12979 | 0,39877 | 0,06659 | 0,03576 | 0,09723 | 0,02356 |
| A14 | 0,13257 | 0,42258 | 0,06657 | 0,03187 | 0,07341 | 0,02250 |
| A15 | 0,13030 | 0,41626 | 0,06650 | 0,02587 | 0,07973 | 0,01475 |
| A16 | 0,12991 | 0,40021 | 0,06658 | 0,03359 | 0,09578 | 0,02003 |
| A17 | 0,13363 | 0,43423 | 0,06657 | 0,02353 | 0,06176 | 0,01408 |
| A18 | 0,03528 | 0,08649 | 0,02356 | 0,12467 | 0,40950 | 0,06660 |
| A19 | 0,13213 | 0,41753 | 0,06660 | 0,02834 | 0,07846 | 0,01738 |
| A20 | 0,12897 | 0,40468 | 0,06655 | 0,03050 | 0,09131 | 0,01642 |
| A21 | 0,13113 | 0,41216 | 0,06656 | 0,03053 | 0,08383 | 0,01812 |
| A22 | 0,12974 | 0,39540 | 0,06660 | 0,03509 | 0,10059 | 0,01757 |
| A23 | 0,12629 | 0,36214 | 0,06658 | 0,04351 | 0,13385 | 0,01801 |
| A24 | 0,12676 | 0,37545 | 0,06658 | 0,03954 | 0,12054 | 0,02060 |
| A25 | 0,12848 | 0,38047 | 0,06660 | 0,03838 | 0,11552 | 0,01562 |
| A26 | 0,12785 | 0,38652 | 0,06657 | 0,03610 | 0,10947 | 0,01629 |
| A27 | 0,12993 | 0,39521 | 0,06659 | 0,03629 | 0,10078 | 0,01843 |
| A28 | 0,12893 | 0,38972 | 0,06658 | 0,03585 | 0,10627 | 0,01832 |

Table 19: D+ and D- value for *L1* family.

|  |  |  |
| --- | --- | --- |
|   | Ideal Distances | Negative Distances |
|   | D\_4 | D\_5 | D\_6 | D\_7 | D\_8 | D\_9 | D\_4 | D\_5 | D\_6 | D\_7 | D\_8 | D\_9 |
|   | *D+* | *D+* | *D+* | *D+* | *D+* | *D+* | *D-* | *D-* | *D-* | *D-* | *D-* | *D-* |
| A1 | 0,160 | 0,009 | 0,276 | 0,381 | 3,840 | 0,138 | 0,932 | 0,024 | 0,965 | 27,289 | 12,104 | 0,349 |
| A2 | 0,564 | 0,024 | 0,721 | 2,587 | 7,287 | 0,359 | 0,832 | 0,009 | 0,908 | 9,896 | 10,480 | 0,128 |
| A3 | 0,715 | 0,028 | 0,834 | 5,019 | 9,666 | 0,416 | 0,733 | 0,005 | 0,846 | 5,492 | 9,695 | 0,071 |
| A4 | 0,639 | 0,026 | 0,780 | 3,536 | 8,487 | 0,389 | 0,792 | 0,007 | 0,884 | 7,615 | 10,569 | 0,099 |
| A5 | 0,536 | 0,024 | 0,698 | 2,307 | 7,234 | 0,348 | 0,844 | 0,009 | 0,915 | 10,815 | 11,419 | 0,140 |
| A6 | 0,643 | 0,027 | 0,783 | 3,607 | 9,598 | 0,390 | 0,789 | 0,007 | 0,882 | 7,483 | 8,935 | 0,097 |
| A7 | 0,560 | 0,024 | 0,718 | 2,548 | 7,615 | 0,359 | 0,834 | 0,009 | 0,909 | 10,014 | 12,465 | 0,130 |
| A8 | 0,684 | 0,028 | 0,813 | 4,337 | 9,195 | 0,405 | 0,760 | 0,005 | 0,863 | 6,322 | 9,805 | 0,082 |
| A9 | 0,569 | 0,025 | 0,725 | 2,640 | 7,672 | 0,362 | 0,830 | 0,008 | 0,907 | 9,735 | 10,294 | 0,126 |
| A10 | 0,647 | 0,027 | 0,786 | 3,671 | 8,675 | 0,392 | 0,786 | 0,006 | 0,880 | 7,366 | 9,910 | 0,095 |
| A11 | 0,564 | 0,024 | 0,721 | 2,583 | 7,712 | 0,359 | 0,832 | 0,009 | 0,908 | 9,906 | 10,542 | 0,128 |
| A12 | 0,688 | 0,028 | 0,815 | 4,415 | 9,519 | 0,407 | 0,757 | 0,005 | 0,861 | 6,216 | 0,000 | 0,081 |
| A13 | 0,644 | 0,027 | 0,783 | 3,617 | 8,868 | 0,391 | 0,789 | 0,006 | 0,882 | 7,464 | 9,074 | 0,097 |
| A14 | 0,710 | 0,028 | 0,830 | 4,889 | 10,226 | 0,414 | 0,738 | 0,005 | 0,849 | 5,636 | 7,809 | 0,073 |
| A15 | 0,692 | 0,028 | 0,818 | 4,487 | 9,337 | 0,408 | 0,754 | 0,005 | 0,860 | 6,121 | 10,454 | 0,079 |
| A16 | 0,648 | 0,027 | 0,786 | 3,678 | 8,710 | 0,392 | 0,786 | 0,006 | 0,880 | 7,353 | 10,060 | 0,095 |
| A17 | 0,744 | 0,029 | 0,853 | 5,806 | 10,499 | 0,426 | 0,703 | 0,004 | 0,826 | 4,741 | 0,000 | 0,061 |
| A18 | 0,093 | 0,006 | 0,170 | 0,205 | 2,249 | 0,086 | 0,940 | 0,027 | 0,969 | 31,437 | 13,170 | 0,402 |
| A19 | 0,695 | 0,028 | 0,820 | 4,564 | 9,407 | 0,409 | 0,751 | 0,005 | 0,858 | 6,024 | 9,406 | 0,078 |
| A20 | 0,660 | 0,027 | 0,795 | 3,878 | 9,011 | 0,397 | 0,778 | 0,006 | 0,875 | 7,010 | 9,973 | 0,091 |
| A21 | 0,680 | 0,027 | 0,810 | 4,255 | 9,395 | 0,404 | 0,763 | 0,006 | 0,866 | 6,436 | 10,543 | 0,083 |
| A22 | 0,635 | 0,026 | 0,777 | 3,480 | 8,464 | 0,387 | 0,794 | 0,007 | 0,885 | 7,722 | 9,887 | 0,100 |
| A23 | 0,552 | 0,024 | 0,711 | 2,466 | 7,112 | 0,354 | 0,837 | 0,009 | 0,911 | 10,276 | 10,327 | 0,133 |
| A24 | 0,584 | 0,025 | 0,738 | 2,811 | 7,617 | 0,368 | 0,822 | 0,008 | 0,902 | 9,254 | 10,307 | 0,120 |
| A25 | 0,597 | 0,025 | 0,747 | 2,960 | 7,741 | 0,372 | 0,816 | 0,008 | 0,899 | 8,869 | 9,816 | 0,115 |
| A26 | 0,612 | 0,026 | 0,759 | 3,155 | 8,044 | 0,379 | 0,808 | 0,007 | 0,894 | 8,404 | 10,294 | 0,109 |
| A27 | 0,635 | 0,026 | 0,776 | 3,473 | 8,677 | 0,387 | 0,795 | 0,007 | 0,886 | 7,737 | 8,875 | 0,100 |
| A28 | 0,620 | 0,026 | 0,766 | 3,267 | 8,131 | 0,382 | 0,803 | 0,007 | 0,891 | 8,159 | 9,988 | 0,106 |

**Step 7.** Calculating the overall performance index for each alternative across all criteria:

This index can be calculated based on the concept of the degree of distance of alternative *Ai* relative to the ideal solutions.

Table 20: *Pi*value for *Lq* and *L1* family

|  |  |  |
| --- | --- | --- |
|     | *Lq* Family Distance | *L1* Family Distance |
| D\_I | D\_2 | D\_3 | D\_4 | D\_5 | D\_6 | D\_7 | D\_8 | D\_9 |
| *Pi* | *Pi* | *Pi* | *Pi* | *Pi* | *Pi* | *Pi* | *Pi* | *Pi* |
| A1 | 0,61 | 0,72 | 0,45 | 0,85 | 0,72 | 0,78 | 0,99 | 0,76 | 0,72 |
| A2 | 0,25 | 0,26 | 0,21 | 0,60 | 0,26 | 0,56 | 0,79 | 0,59 | 0,26 |
| A3 | 0,16 | 0,14 | 0,17 | 0,51 | 0,14 | 0,50 | 0,52 | 0,50 | 0,15 |
| A4 | 0,20 | 0,20 | 0,18 | 0,55 | 0,20 | 0,53 | 0,68 | 0,55 | 0,20 |
| A5 | 0,27 | 0,28 | 0,25 | 0,61 | 0,28 | 0,57 | 0,82 | 0,61 | 0,29 |
| A6 | 0,23 | 0,20 | 0,24 | 0,55 | 0,20 | 0,53 | 0,67 | 0,48 | 0,20 |
| A7 | 0,24 | 0,26 | 0,20 | 0,60 | 0,26 | 0,56 | 0,80 | 0,62 | 0,27 |
| A8 | 0,18 | 0,17 | 0,16 | 0,53 | 0,17 | 0,52 | 0,59 | 0,52 | 0,17 |
| A9 | 0,25 | 0,26 | 0,21 | 0,59 | 0,26 | 0,56 | 0,79 | 0,57 | 0,26 |
| A10 | 0,20 | 0,19 | 0,23 | 0,55 | 0,19 | 0,53 | 0,67 | 0,53 | 0,20 |
| A11 | 0,26 | 0,26 | 0,24 | 0,60 | 0,26 | 0,56 | 0,79 | 0,58 | 0,26 |
| A12 | 0,19 | 0,16 | 0,19 | 0,52 | 0,16 | 0,51 | 0,58 | 0,00 | 0,17 |
| A13 | 0,22 | 0,20 | 0,26 | 0,55 | 0,20 | 0,53 | 0,67 | 0,51 | 0,20 |
| A14 | 0,19 | 0,15 | 0,25 | 0,51 | 0,15 | 0,51 | 0,54 | 0,43 | 0,15 |
| A15 | 0,17 | 0,16 | 0,18 | 0,52 | 0,16 | 0,51 | 0,58 | 0,53 | 0,16 |
| A16 | 0,21 | 0,19 | 0,23 | 0,55 | 0,19 | 0,53 | 0,67 | 0,54 | 0,20 |
| A17 | 0,15 | 0,12 | 0,17 | 0,49 | 0,12 | 0,49 | 0,45 | 0,00 | 0,13 |
| A18 | 0,78 | 0,83 | 0,74 | 0,91 | 0,83 | 0,85 | 0,99 | 0,85 | 0,82 |
| A19 | 0,18 | 0,16 | 0,21 | 0,52 | 0,16 | 0,51 | 0,57 | 0,50 | 0,16 |
| A20 | 0,19 | 0,18 | 0,20 | 0,54 | 0,18 | 0,52 | 0,64 | 0,53 | 0,19 |
| A21 | 0,19 | 0,17 | 0,21 | 0,53 | 0,17 | 0,52 | 0,60 | 0,53 | 0,17 |
| A22 | 0,21 | 0,20 | 0,21 | 0,56 | 0,20 | 0,53 | 0,69 | 0,54 | 0,21 |
| A23 | 0,26 | 0,27 | 0,21 | 0,60 | 0,27 | 0,56 | 0,81 | 0,59 | 0,27 |
| A24 | 0,24 | 0,24 | 0,24 | 0,58 | 0,24 | 0,55 | 0,77 | 0,58 | 0,25 |
| A25 | 0,23 | 0,23 | 0,19 | 0,58 | 0,23 | 0,55 | 0,75 | 0,56 | 0,24 |
| A26 | 0,22 | 0,22 | 0,20 | 0,57 | 0,22 | 0,54 | 0,73 | 0,56 | 0,22 |
| A27 | 0,22 | 0,20 | 0,22 | 0,56 | 0,20 | 0,53 | 0,69 | 0,51 | 0,21 |
| A28 | 0,22 | 0,21 | 0,22 | 0,56 | 0,21 | 0,54 | 0,71 | 0,55 | 0,22 |

Table 21: Ranking

|  |  |  |
| --- | --- | --- |
|  | *Lq* Family Distance | *L1* Family Distance |
| D\_I | D\_2 | D\_3 | D\_4 | D\_5 | D\_6 | D\_7 | D\_8 | D\_9 |
| Rank | Rank | Rank | Rank | Rank | Rank | Rank | Rank | Rank |
| A1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| A2 | 6 | 7 | 16 | 7 | 7 | 7 | 7 | 6 | 7 |
| A3 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 23 | 27 |
| A4 | 19 | 15 | 25 | 15 | 15 | 15 | 15 | 12 | 15 |
| A5 | 3 | 3 | 5 | 3 | 3 | 3 | 3 | 4 | 3 |
| A6 | 11 | 16 | 7 | 16 | 16 | 16 | 16 | 25 | 16 |
| A7 | 8 | 5 | 20 | 5 | 5 | 5 | 5 | 3 | 5 |
| A8 | 24 | 22 | 28 | 22 | 22 | 22 | 22 | 20 | 22 |
| A9 | 7 | 8 | 17 | 8 | 8 | 8 | 8 | 9 | 8 |
| A10 | 18 | 18 | 10 | 18 | 18 | 18 | 18 | 16 | 18 |
| A11 | 4 | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 6 |
| A12 | 23 | 23 | 22 | 23 | 23 | 23 | 23 | 27 | 23 |
| A13 | 15 | 17 | 3 | 17 | 17 | 17 | 17 | 21 | 17 |
| A14 | 20 | 26 | 4 | 26 | 26 | 26 | 26 | 26 | 26 |
| A15 | 26 | 24 | 24 | 24 | 24 | 24 | 24 | 18 | 24 |
| A16 | 17 | 19 | 9 | 19 | 19 | 19 | 19 | 15 | 19 |
| A17 | 28 | 28 | 26 | 28 | 28 | 28 | 28 | 28 | 28 |
| A18 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| A19 | 25 | 25 | 18 | 25 | 25 | 25 | 25 | 24 | 25 |
| A20 | 21 | 20 | 19 | 20 | 20 | 20 | 20 | 19 | 20 |
| A21 | 22 | 21 | 13 | 21 | 21 | 21 | 21 | 17 | 21 |
| A22 | 16 | 14 | 15 | 14 | 14 | 14 | 14 | 14 | 14 |
| A23 | 5 | 4 | 14 | 4 | 4 | 4 | 4 | 5 | 4 |
| A24 | 9 | 9 | 8 | 9 | 9 | 9 | 9 | 8 | 9 |
| A25 | 10 | 10 | 23 | 10 | 10 | 10 | 10 | 11 | 10 |
| A26 | 12 | 11 | 21 | 11 | 11 | 11 | 11 | 10 | 11 |
| A27 | 13 | 13 | 11 | 13 | 13 | 13 | 13 | 22 | 13 |
| A28 | 14 | 12 | 12 | 12 | 12 | 12 | 12 | 13 | 12 |

The proposed *D+* and *D-* values and also *Pi* value are calculated according to formulas (71)-(72). The solutions are sumarized in Table 22 and shown in Figure 1.

Table 22: Proposed method solutions.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Lq*Family |  | *L1* Family |  |
| *D+* | *D-* | *Pi* | Rank | *D+* | *D-* | *Pi* | Rank |
| A1 | 0,093 | 0,175 | 0,652 | 1 | 0,265 | 9,534 | 0,973 | 1 |
| A2 | 0,187 | 0,063 | 0,252 | 18 | 1,222 | 3,644 | 0,749 | 18 |
| A3 | 0,208 | 0,037 | 0,151 | 5 | 2,090 | 2,136 | 0,506 | 5 |
| A4 | 0,197 | 0,049 | 0,198 | 23 | 1,568 | 2,866 | 0,646 | 23 |
| A5 | 0,181 | 0,070 | 0,278 | 11 | 1,118 | 3,957 | 0,780 | 7 |
| A6 | 0,198 | 0,052 | 0,209 | 2 | 1,593 | 2,820 | 0,639 | 11 |
| A7 | 0,184 | 0,062 | 0,251 | 7 | 1,208 | 3,684 | 0,753 | 2 |
| A8 | 0,204 | 0,042 | 0,169 | 9 | 1,852 | 2,422 | 0,567 | 9 |
| A9 | 0,186 | 0,062 | 0,248 | 24 | 1,242 | 3,589 | 0,743 | 24 |
| A10 | 0,199 | 0,050 | 0,200 | 25 | 1,616 | 2,781 | 0,632 | 25 |
| A11 | 0,186 | 0,065 | 0,257 | 26 | 1,221 | 3,648 | 0,749 | 26 |
| A12 | 0,205 | 0,042 | 0,172 | 28 | 1,879 | 2,386 | 0,559 | 28 |
| A13 | 0,198 | 0,052 | 0,208 | 6 | 1,597 | 2,814 | 0,638 | 27 |
| A14 | 0,207 | 0,043 | 0,170 | 13 | 2,044 | 2,186 | 0,517 | 22 |
| A15 | 0,204 | 0,040 | 0,164 | 27 | 1,904 | 2,353 | 0,553 | 4 |
| A16 | 0,199 | 0,050 | 0,200 | 22 | 1,619 | 2,776 | 0,632 | 6 |
| A17 | 0,211 | 0,033 | 0,135 | 16 | 2,362 | 1,876 | 0,443 | 13 |
| A18 | 0,048 | 0,200 | 0,805 | 10 | 0,153 | 10,936 | 0,986 | 10 |
| A19 | 0,205 | 0,041 | 0,168 | 4 | 1,931 | 2,320 | 0,546 | 16 |
| A20 | 0,200 | 0,046 | 0,187 | 20 | 1,690 | 2,659 | 0,611 | 20 |
| A21 | 0,203 | 0,044 | 0,178 | 21 | 1,823 | 2,461 | 0,575 | 21 |
| A22 | 0,197 | 0,051 | 0,206 | 12 | 1,548 | 2,903 | 0,652 | 8 |
| A23 | 0,185 | 0,065 | 0,260 | 14 | 1,177 | 3,773 | 0,762 | 12 |
| A24 | 0,190 | 0,060 | 0,241 | 8 | 1,305 | 3,425 | 0,724 | 15 |
| A25 | 0,192 | 0,057 | 0,228 | 19 | 1,360 | 3,294 | 0,708 | 19 |
| A26 | 0,194 | 0,054 | 0,218 | 15 | 1,431 | 3,135 | 0,687 | 14 |
| A27 | 0,197 | 0,052 | 0,208 | 3 | 1,545 | 2,908 | 0,653 | 3 |
| A28 | 0,195 | 0,053 | 0,215 | 17 | 1,471 | 3,052 | 0,675 | 17 |

Figure 1. *Lq* Family Distance, *L1* Family Distance and proposed method

In Figure 1 the following definitions apply. Series 1: Euclidean, Series 2: City blok, Series 3: Chebyshev, Series 4: Sorensen, Series 5: Gower, Series 6: Soergel, Series 7: Kulczyynski, Series 8: Canberra, Series 9: Lorentzian, Series 10: Proposed Method for *Lq* family, Series 11: Proposed Method for *L1* family.

The first ten banks which are ranked through modified methods are reported in Table 23.

Table 23: Top five banks with high performans value.

|  |
| --- |
| *Lq* Family Distance |
| D\_I | D\_2 | D\_3 | Proposed\_D |
| A18 | A18 | A18 | A1 |
| A1 | A1 | A1 | A18 |
| A5 | A5 | A13 | A5 |
| A11 | A23 | A14 | A23 |
| A23 | A7 | A5 | A11 |

|  |
| --- |
| *L1*Family Distance |
| D\_4 | D\_5 | D\_6 | D\_7 | D\_8 | D\_9 | Proposed\_D |
| A18 | A18 | A18 | A18 | A18 | A18 | A1 |
| A1 | A1 | A1 | A1 | A1 | A1 | A18 |
| A5 | A5 | A5 | A5 | A7 | A5 | A5 |
| A23 | A23 | A23 | A23 | A5 | A23 | A23 |
| A7 | A7 | A7 | A7 | A23 | A7 | A7 |

**5. Conclusions**

With this study, it has been tried to present a different approach to evaluate alternatives in cases where there are many criteria for decision makers, by modifying the TOPSIS method, one of the well known MCDM methods.

Performance evaluation, which has a very important role in the banking sector, was dealt with and a model was proposed to evaluate different alternatives within the framework of the various criteria considered to affect the financial performance of the banks.

The proposed model was evaluated on a real sample and the results obtained were interpreted. For this purpose, the financial performance of 28 deposit banks, which account for more than 90% of assets size in the Turkish banking sector and operate in Turkey as of 2017, has been evaluated.

In order for the banks to be ranked according to their financial performances in the study, 15 financial ratios (criteria) used in many applications in the literature, calculated from year-end financial tables related to capital adequacy, exchange rate risk, asset quality, profitability and income-expenditure status were used. The evaluation of the performance of the banks is not based on the individual criteria; all the criteria were taken together with the modified versions of the TOPSIS method, which were taken into account together. The criteria discussed in the methods are weighted by Equal weighting technique.

In the study, distances of Manhattan and Chebyshev in the *Lq* family and distances of Sorensen, Gower, Soergel, Kulczyynski, Canberra and Lorentzian in the *L1* family were used as an alternative to the distance of Euclid included in the *Lq* family used in the traditional TOPSIS method. Also in the study, the average *d+* and *d-* values for the *Lq* and *L1* families are proposed, taking the averages of the distance measures considered.

According to the proposed method, it is seen that the banks receiving the first four orders from the results obtained for the families *Lq* and *L1* are the same. The banks in question are respectively: Adabank A.Ş. (A1), Rabobank A.Ş. (A18), Arab Türk Bankası A.Ş. (A5), Türkiye Cumhuriyeti Ziraat Bankası A.Ş (A23).

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