**Misspecification of Generalized Autoregressive Score Models: Monte Carlo Simulations and Real Life Applications**

**OlaOluwa S. Yaya** Department of Statistics, University of Ibadan, Nigeria Email: os.yaya@ui.edu.ng

**Oluwagbenga T. Babatunde** Department of Statistics, University of Ibadan, Nigeria Email: babatundegbenge03@gmail.com

**Olusanya E. Olubusoye** Department of Statistics, University of Ibadan, Nigeria Email: oe.olubusoye@ui.edu.ng

**Abstract**

The specification and misspecification of a new class of volatility model that is robust to jumps and outliers is investigated via Monte Carlo experiment and real life examples. The class includes the Generalized Autoregressive Score (GAS) model derived from the classical Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS) models form the variants of the GAS model. Using three different levels of volatility persistence and GARCH probability distributions, with estimates of Akaike Information Criterion (AIC) and kurtosis as criteria, we obtained useful information for studying the dynamics and tail behaviour of the newly proposed volatility model.

**Keywords:** Misspecification, Beta distribution, Beta-t-GARCH, Generalized Autoregressive Score.

1. **Introduction**

The Generalized Autoregressive Score (GAS) class is a variant of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of Bollerslev (1986), developed for capturing jumps/outliers effects in the returns series. Following Harvey (2013), the classical GARCH model is not robust to capturing these abnormalities; hence Harvey and Chakravarty (2008) and Harvey (2013) proposed the GAS class variants.

 The driving mechanism of the GAS models and its variant is the scaled score of the likelihood function, and this makes the model class unique among other earlier proposed volatility models. It combines the ability to capture asymmetry with occasional jumps detection. The GAS model encompasses other well-known models such as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH), the Autoregressive Conditional Duration (ACD), the Autoregressive Conditional Intensity (ACI), and the single source of error models. In addition, the GAS specification provides a wide range of new observation driven models. Examples include observation driven analogues of unobserved components time series models, multivariate point process models with time-varying parameters and pooling restrictions, new models for time-varying copula functions, and models for time-varying higher order moments. Based on these appealing properties of this new model, we were therefore motivated in investigating further the model class.

The literature, Mandelbrot (1963) and Fama (1970) define volatility and volatility clustering in stocks, and following these definitions, several parametric volatility models have been developed. The first is the Autoregressive Conditional Heteroscedasticity (ARCH) model earlier proposed in Engle (1982), and the generalized version as GARCH model, has gained many applications in empirical financial time series literature. (see Bollerslev, 1986; Xekalaki and Degiannakis, 2010). These literature have extended to studying the asymmetric behaviour and jumps in stocks and other asset prices. Different asymmetric robust volatility models have also been applied. The jump behaviour of stocks has recently been studied, and nonparametric approaches to detecting jumps have been applied (see Andersen, Bollerslev and Diebold, 2007). Jump robust volatility model is introduced in Harvey and Chakrarvarty (2008) and Harvey (2013). There, the authors proposed the Generalized Autoregressive Score (GAS) models and two variants, the Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS) models for predicting the conditional volatility with occasional jumps. As a result of newness of this model, there are fewer applications so far, though small sample properties have been investigated in Harvey (2013) and Creal, Koopman and Lucas (2008, 2011, 2013), there is need to study the property of this model class using simulation approach, with emphasis on the fitness ability and returns distributions. The fitness ability is achieved by the estimates of information criterion and the tail effects achieved by the estimates of the kurtosis.

The aim of this paper is to investigate misspecification of GAS models and its variants using Monte Carlo simulation approach. The work is extended to real life crude oil and natural gas prices. Literature has shown that these financial time series data display series of jumps over the historical years (Elder, Miao and Ramchander, 2013; Charles and Darne (2014). Hence, they serve as good applicable examples in this paper.

 The rest of the paper is structured as follows: Section 2 reviews literature on the volatility modelling and model misspecification. Section 3 presents the volatility models as well as misspecification testing approach. Section 4 presents the Monte Carlo experiment and results, while Section 5 renders the concluding remarks.

**2.0 Review of Literature**

Halunga and Orme (2004) developed a framework for the construction and analysis of misspecification tests for GARCH models and they proposed new asymptotically valid and locally optimal tests of asymmetry and nonlinearity. They argued that the asymmetry test of Engle and Ng (1993) and nonlinearity test of Lundbergh and Terasvirta (2002) are neither asymptotically valid (since they ignore asymptotically non-negligible estimation effects) nor locally optimal (since they ignore the recursive nature of the conditional variance structure). Their framework encompasses conditional mean specification estimated by the Ordinary Least Squares (OLS), Nonlinear Least Squares (NLS) or Quasi Maximum Likelihood (QML) method, and that the GARCH misspecification tests can be asymptotically sensitive to unconsidered misspecification of the conditional mean. The Monte Carlo results indicate that the new tests are very powerful when compared with the previous tests proposed by Engle and Ng (1993) and Lundbergh and Terasvirta (2002).

Posedel (2005) studied in depth the properties of the GARCH (1,1) model and the assumptions on the parameter space under which the process is stationary. In particular, the ergodicity and strong stationarity for the conditional variance (squared volatility) of the process was proved. He showed under which conditions higher order moments of the GARCH (1,1) process exist and concluded that GARCH processes are heavy tailed. Bellini and Bottolo (2007) investigated the impact of misspecification on the innovations in fitting GARCH (1,1) models through a Monte Carlo approach and showed that an incorrect specification of the innovations together with the reduction of the parameter space to the weak stationarity region, could give rise to a spurious Integrated GARCH (IGARCH) effect. They also analysed the impact of misspecification on forecasted volatilities, showing that innovations with light tails can lead to a remarkable over-estimation of volatilities. Caporin and Lisi (2010) analysed the real size and power of the likelihood ratio and the Lagrange Multiplier (LM) misspecification tests when periodic long memory GARCH models are involved. The performance of these tests was studied by means of Monte Carlo simulations with respect to the class of generalized long memory GARCH models, and by means of a Monte Carlo analysis the real size and power of these tests were derived, evidencing their reliability apart from some special and limited cases. The test performances were however influenced by the sample length with about a thousand observations needed to obtain reliable conclusions.

 On GAS modelling and its variants specification, Blasques, Koopman and Lucas (2008) characterized the dynamic properties of Generalized Autoregressive Score (GAS) models by identifying the regions of the parameter space that implied stationarity and ergodicity of the corresponding nonlinear time series process. They showed how these regions are affected by the choice of parameterization and scaling, which are key factors for the class of GAS models compared to other observation driven models.

As a follow-up by Creal, Koopman and Lucas (2013), the observation driven time series models used the scaled score of the likelihood function as the mechanism for updating the parameters over time. This approach provides a unified and consistent framework for introducing time varying parameters in a wide class of non-linear models. They developed a framework for time varying parameters which is based on the score function of the predictive model density at time t and concluded that by scaling the score function appropriately, standard observation driven models such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH), Autoregressive Conditional Duration (ACD) and Autoregressive Conditional Intensity (ACI) models can be recovered.

Calvori, Cipollini and Gallo (2013) proposed a novel GAS model for predicting volume of shares (relative to the daily total), inspired by empirical regularities of the observed series (intra-daily periodicity pattern, residual serial dependence). An application of the proposed GAS model to New York Stock Exchange (NYSE) tickecters confirmed the suitability of the proposed model in capturing the features of intra-daily dynamics of volume shares.

Huang, Wang and Zhang (2014) proposed a new observation-driven time-varying parameter framework to model the financial return and realized variance jointly. The latent dynamic factor was updated by the scaled local density score as a function of past daily return and realized variance. The proposed GAS variant adapted quickly to drastic volatility changes by incorporating realized measures of volatility based on high frequency data and they demonstrated the promising performance of the proposed model by applying it to a number of equity returns, even during the 2008 financial crisis.

Blasques, Koopman and Lucas (2014a) studied the consistency and asymptotic normality of the Maximum Likelihood Estimators (MLE) for a class of time series models driven by score function of the predictive likelihood. They formulated primitive conditions, and asymptotic normality under correct specification and under misspecification of the GAS models.

 Blasques, Koopman and Lucas (2014b) investigated the theoretic optimality properties of the score function of the predictive likelihood as a device to update parameters in GAS models. Their results provided a new theoretical justification for the class of GAS models, which covers the GARCH model as a special case. Their main contribution was to show that only parameter updates based on the score always reduce the local Kullback-Leibler divergence between the true conditional density and the model implied conditional density and they found out that it holds irrespective of the severity of the model misspecification. They concluded that updates based on the score function minimized the local Kullback-Leibler divergence between the true conditional data density and the model implied conditional density.

Bernadi and Cantania (2015) developed a new class of flexible Copula models where the evolution of the dependence parameters follows a Markov-Switching Generalized Autoregressive Score (SGASC) dynamics. Maximum Likelihood Estimation is consistently performed using the Inference Function for Margins (IFM) approach and a version of the Expectation-Maximisation (EM) algorithm specifically tailored to this class of models. They used their developed SGASC model to estimate the Conditional Value-at-Risk (CoVaR), which is defined as the VaR of a given asset conditional on another asset (or portfolio) being in financial distress, and the Conditional Expected Shortfall (CoES). Their empirical investigation shows that the proposed SGASC models are able to explain and predict the systemic risk contribution of several European countries. Also, they found out that the SGASC models outperformed competitors using several CoVaR back testing procedures.

1. **The GAS Models and their Variants**

The GAS model specification was derived from the classical GARCH model of Bollerslev (1986) which is given as,

  (1)

  (2)

where  is the returns time series decomposed as in (1), ,  and  are the parameters defined with the conditions , ,  and  to ensure covariance stationarity of the model in (2). Harvey and Chakravarty (2008) and Harvey (2013) proposed the jump volatility model by re-writing GARCH (1,1) as,

 

and,

 ,

which is finally written as,

  (3)

where  and  is proportional to the score of the conditional distribution of  with respect to . This is Beta-GARCH model because and  has a Beta distribution, and the innovations  are given as,

  for Normal distribution, ; (4)

  for Student-t distribution, ; (5)

 for Generalized Error Distribution (GED),  (6)

and  for Skewed Student-t distribution,  where

 ,  ,

  and  (7)

Now, combining (3) with (4) gives the Beta-Normal-GARCH model; combining (3) with (5) gives the Beta-t-GARCH model; combining (3) with (6) gives the Beta-GED-GARCH model, and combining (3) with (7) gives the Beta-Skew-t-GARCH model.

 Harvey and Chakravarty (2008) also considered the Exponential GARCH (EGARCH) and Asymmetric Exponential GARCH (AEGARCH) types of the Beta-GARCH models, each with the four distributional assumptions applied. The Beta-EGARCH model is given as,

  (8)

specified without the leverage effect.[[1]](#footnote-1) Introducing the leverage effect, we have the Beta-AEGARCH model,

  (9)

where  when Normal, Student-t and GED distributions are considered, and  for the Skewed Student-t distribution. [[2]](#footnote-2)

**3.1 Model Misspecification tests**

Each model under the distributional assumption is evaluated using the Akaike (1973) Information Criterion (AIC),

 ; (10)

where  is the maximized log-likelihood function, simplified using numerical derivatives,  is the ML estimator of the parameter vector  based on a sample of size *N*, and gives the dimension of . The excess kurtosis is then computed based on the formula,

  (11)

where  is the estimate of unconditional variance, and the fourth moment about the mean,  and  is the excess kurtosis from the assumed GARCH distribution pro cess .

1. **Monte Carlo Experiment and Results**

Though the structural and distributional properties of classical GARCH model have been investigated theoretically and by simulations but the properties of GAS model and its variants are yet to be established. The Monte Carlo (MC) simulations experiment carried out in this work investigated both the fitness performance of the models as well as the measure of tail effect of the model residuals. Four Data Generating Processes (DGPs) considered are:

(1). GARCH(1,1) : $σ\_{t}^{2}= ω+ α\_{1}ε\_{t-1}^{2}+ β\_{1 }σ\_{t-1}^{2}$ (12)

 (2). GAS(1,1) : $σ\_{t}^{2}= ω+ α\_{1}μ\_{t-1}σ\_{t-1}^{2}+ (α\_{1}+β\_{1})σ\_{t-1}^{2}$ (13)

 (3). EGAS(1,1): $logσ\_{t}^{2}= ω+ α\_{1}μ\_{t-1}σ\_{t-1}^{2}+ (α\_{1}+β\_{1})logσ\_{t-1}^{2}$ (14)

 (4). AEGAS(1,1): $logσ\_{t}^{2}= ω+ α\_{1}μ\_{t-1}σ\_{t-1}^{2}+γ\_{1}τ\_{t-1 }+ (α\_{1}+β\_{1})logσ\_{t-1}^{2}$ (15)

where $τ\_{t-1 }=sign (-z\_{t })(μ\_{t }+1)$. For each of the DGP in (14)-(17), a sample of 1000 time series was generated after making control for the initialization error, and each generated following Normal, Student-t and Skewed Student-t distributions. The sum $α+β $is referred to as the persistence of the conditional variance process. For financial return series, estimates of $α and β$ are often in the ranges [0.02,0.25] and [0.75,0.98], respectively with $α$ often in the lower part of the interval and $β$ in the upper part of the interval, such that the persistence is close but rarely exceeding 1 (Bauwens, Hafner and Laurent, 2012). We can then make classification into low, medium and high persistence. Halunge and Orme (2004) varied the parameters in the models and classified them as low, medium and high volatility persistence realizations as given below:

Low Persistence: ******

Medium Persistence: 

High Persistence: 

where the values of the intercept  and asymmetric parameter  remained constant throughout, and these do not affect volatility persistence. The value of  for the case of GAS(1,1), EGAS(1,1) and AEGAS(1,1) models.

The estimates for Akaike Information Criterion (AIC) and Excess kurtosis were estimated for each experimental run and the full results are presented in Tables 4.1 – 4.6 below:

**Table 4.1 When the DGP is GAS-N**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 0.9176 | -0.1157 | 2.0708 | 0.1722 | 4.6456 | -0.0104 |
| *T* | 0.9225 | -0.1210 | 2.0716 | 0.1722 | 4.6476 | -0.0104 |
| *Skew t* | 0.9211 | -0.1264 | 2.0693 | 0.1734 | 4.6468 | -0.0106 |
| *GAS(1,1)* | *Normal* | 0.9180 | -0.1115 | 2.0708 | 0.1715 | 4.6455 | -0.0108 |
| *T* | 0.9208 | -0.1139 | 2.0725 | 0.1749 | 4.6475 | -0.0107 |
| *Skew t* | 0.9194 | -0.1220 | 2.0697 | 0.1767 | 4.6462 | -0.0127 |
| *EGAS(1,1)* | *Normal* | 0.9185 | -0.1077 | 2.0630 | 0.0672 | 4.6433 | -0.0369 |
| *T* | 0.9252 | -0.1124 | 2.0649 | 0.0688 | 4.6454 | -0.0365 |
| *Skew t* | 0.9200 | -0.1210 | 2.0623 | 0.0750 | 4.6438 | -0.0384 |
| *AEGAS(1,1)* | *Normal* | 0.9198 | -0.1226 | 2.0646 | 0.0573 | 4.6446 | -0.0544 |
| *T* | 0.9276 | -0.1281 | 2.0665 | 0.0589 | 4.6466 | -0.0540 |
| *Skew t* | 0.9222 | -0.1343 | 2.0639 | 0.0643 | 4.6447 | -0.0605 |

Table 4.1 presents the results for the case when the DGP was GAS-N model. In terms of model fitness, at low persistence, the least AIC for the GAS (1,1) under the three assumed distribution is 0.9180 which corresponds to the GAS (1,1)-N model. This implies that specifying GAS (1,1)-N model as GAS (1,1)-N model fits or performs better than specifying it as either GAS (1,1)-T model or GAS (1,1)-Skew-T. Likewise, at medium and high persistence, specifying GAS (1,1)-N model as GAS (1,1)-N model fits or performs better than specifying it as either GAS (1,1)-T model or GAS (1,1)-Skew-T model since the least AICs 2.0708 (at medium persistence) and 4.6455 (at high persistence) correspond the GAS (1,1)-N model. At low persistence, the AIC of GARCH (1,1)-N model (0.9176) is lower than that of GAS (1,1)-N model while the AICs of the other models are higher than that of GAS (1,1)-N model. This implies that, misspecifying GAS (1,1)-N model as GARCH (1,1)-N model fits or performs better whereas misspecifying GAS (1,1)-N model as any of the rest model, performs lesser than the GAS (1,1)-N model. At medium persistence, misspecifying GAS (1,1)-N model as GARCH (1,1)-N model performs equally as the GAS (1,1) since they both have the same AIC (2.0708) while misspecifying GAS (1,1)-N model as GARCH (1,1)-T model performs lesser since its AIC (2.0716) is higher than that of GAS (1,1)-N model. Whereas misspecifying GAS (1,1)-N model an any of the rest model performs better, the best being EGAS (1,1)-Skew-T model with the least AIC (2.0623). At high persistence, misspecifying GAS (1,1)-N model as GARCH (1,1)-{Normal, T and Skew-T} models performs lesser since their AICs are higher than that of GAS (1,1)-N model. Whereas, misspecifying GAS (1,1)-N as EGAS (1,1)-{ Normal, T and Skew-T} models or AEGAS (1,1)-{ Normal, T and Skew-T} models perform better since their AICs are lower than that of GAS (1,1)-N model and the best being EGAS (1,1)-N with the least AIC (4.6433).

 Looking at the estimates of excess kurtosis, we observed low negative estimates of excess kurtosis at both low and high persistence which implies closeness to normality of the estimated residual for the four models. As the persistence is increase to medium level, the estimates of kurtosis became positive values, and at very high persistence, the reverted back to negative values. The results here imply that kurtosis of GARCH and GAS models is actually a function of persistence measures.

**Table 4.2 When the DGP is EGAS-N (Beta-N-EGARCH)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 3.0700 | -0.1166 | 3.4392 | 0.2114 | 7.5658 | 0.0250 |
| *T* | 3.0779 | -0.1070 | 3.4396 | 0.2117 | 7.5678 | 0.0249 |
| *Skew t* | 3.0760 | -0.1126 | 3.4371 | 0.2115 | 7.5668 | 0.0247 |
| *GAS(1,1)* | *Normal* | 3.0712 | -0.1068 | 3.4391 | 0.2107 | 7.5658 | 0.0246 |
| *T* | 3.0743 | -0.1074 | 3.4409 | 0.2176 | 7.5678 | 0.0251 |
| *Skew t* | 3.0735 | -0.1086 | 3.4379 | 0.2169 | 7.5663 | 0.0240 |
| *EGAS(1,1)* | *Normal* | 3.0860 | -0.0698 | 3.4251 | 0.0283 | 7.5628 | 0.0020 |
| *T* | 3.0722 | -0.1100 | 3.4271 | 0.0290 | 7.5648 | 0.0026 |
| *Skew t* | 3.0723 | -0.1108 | 3.4242 | 0.0367 | 7.5632 | 0.0038 |
| *AEGAS(1,1)* | *Normal* | 3.0720 | -0.1157 | 3.4259 | 0.0145 | 7.5642 | -0.0135 |
| *T* | 3.0819 | -0.1179 | 3.4279 | 0.0154 | 7.5662 | -0.0130 |
| *Skew t* | 3.0783 | -0.1188 | 3.4237 | 0.0156 | 7.5645 | -0.0148 |

The results for the case of EGAS-N DGP are presented in Table 4.2. In terms of model fitness, at low persistence, the least AIC for the EGAS (1,1) models under the three assumed distributions is 3.0722 which corresponds to the EGAS (1,1)-T model. This implies that specifying EGAS (1,1)-N model as EGAS (1,1)-T model fits better than specifying it as EGAS (1,1)-N or EGAS (1,1)-Skew-T. At medium persistence, the least AIC for the EGAS (1,1) models under the three assumed distributions is 3.4242 which corresponds to the EGAS (1,1)-Skew-T model. This implies that specifying EGAS (1,1)-N model as EGAS (1,1)-Skew-T model fits better than specifying it as EGAS (1,1)-N model or EGAS (1,1)-T model. At high persistence, the least AIC for the EGAS (1,1) models under the three assumed distributions is 7.5626 which corresponds to the EGAS (1,1)-N model. This implies that specifying EGAS (1,1)-N model as EGAS (1,1)-N model fits better than specifying it as EGAS (1,1)-T model or EGAS (1,1)-Skew-T model.

 For misspecification, we observed that at low persistence, the AICs of GARCH (1,1)-N model (3.0700), GAS (1,1)-N model (3.0712) and AEGAS (1,1)-N model (3.0720) are lower than that of EGAS (1,1)-N model while the AICs of the rest models are higher than that of EGAS (1,1)-N model. This implies that misspecifying EGAS (1,1)-N model as either GARCH (1,1)-N model or GAS (1,1)-model or AEGAS (1,1)-N model fits better than EGAS (1,1)-N model and GARCH (1,1)-N model fits best while misspecifying EGAS (1,1)-N as any of the rest model fits lesser. At medium persistence, misspecifying EGAS (1,1)-N model as AEGAS (1,1)-Skew-T model fits better than EGAS (1,1)-N model since the AIC of AEGAS (1,1)-Skew-T (3.4237) is lower than that of EGAS (1,1)-N model (3.4242) while misspecifying EGAS (1,1)-N model as any of the rest models fits lesser since their AICs are higher than that of EGAS (1,1)-N model. At high persistence, misspecifying EGAS (1,1)-N model as any of the rest models fits lesser since all the AICs realized at high persistence are all higher than that of EGAS (1,1)-N model (7.5628).

 In Table 4.2, we also observed low negative estimates of excess kurtosis at low persistence and this implies closeness to normality of the estimated residuals for the four models. At medium persistence, low positive estimates of excess kurtosis were observed for the four models and this implies closeness to normality of the estimated residuals. At high persistence, the estimates of the excess kurtosis are positively low except for AEGAS (1,1) models under the three assumed distributions which are negatively low and this implies closeness to normality of the estimated residuals for the four models.

**Table 4.3 When the DGP is AEGAS-N (Beta-N-AEGARCH)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 3.0744 | -0.0873 | 3.4454 | 0.2378 | 7.5973 | 0.0580 |
| *T* | 3.0788 | -0.0861 | 3.4454 | 0.2388 | 7.5991 | 0.0579 |
| *Skew t* | 3.0795 | -0.0919 | 3.4430 | 0.2387 | 7.5981 | 0.0574 |
| *GAS(1,1)* | *Normal* | 3.0755 | -0.0802 | 3.4453 | 0.2370 | 7.5972 | 0.0576 |
| *T* | 3.0775 | -0.0821 | 3.4469 | 0.2474 | 7.5993 | 0.0585 |
| *Skew t* | 3.0770 | -0.0874 | 3.4439 | 0.2455 | 7.5978 | 0.0571 |
| *EGAS(1,1)* | *Normal* | 3.0885 | -0.0626 | 3.4301 | 0.04058 | 7.5939 | 0.0309 |
| *T* | 3.0875 | 0.0619 | 3.4321 | 0.0416 | 7.5959 | 0.0315 |
| *Skew t* | 3.0764 | -0.0897 | 3.4293 | 0.0495 | 7.5945 | 0.0333 |
| *AEGAS(1,1)* | *Normal* | 3.0869 | -0.0959 | 3.4291 | 0.0113 | 7.5931 | -0.0102 |
| *T* | 3.0865 | -0.1038 | 3.4311 | 0.0126 | 7.5952 | -0.0094 |
| *Skew t* | 3.0853 | -0.0466 | 3.4264 | 0.0104 | 7.5933 | -0.0120 |

When the DGP was AEGAS-N model, in terms of model fitness, at low persistence, the least AIC for the AEGAS (1,1) models under the three assumed distributions is 3.0853 which corresponds to the AEGAS (1,1)-Skew-T model. This implies that specifying AEGAS (1,1)-N model as AEGAS (1,1)-Skew-T model fits better than specifying it as either AEGAS (1,1)-N model or AEGAS (1,1)-T model. Also, at medium persistence, specifying AEGAS (1,1)-N model as AEGAS (1,1)-Skew-T model fits better than specifying it as either AEGAS (1,1)-N model or AEGAS (1,1)-T model since the AIC of AEGAS (1,1)-Skew-T model (3.4264) is the least among the AEGAS (1,1) models. At high persistence, specifying AEGAS (1,1)-N as AEGAS (1,1)-N model fits better than specifying it as either AEGAS (1,1)-T model or AEGAS (1,1)-Skew-T model since the AIC of AEGAS (1,1)-N model is the least among the AEGAS (1,1) models. For misspecification, at low persistence, misspecifying AEGAS (1,1)-N model as any of the rest models except for EGAS (1,1)-N model and EGAS (1,1)-T model fits better than AEGAS (1,1)-N model since their AICs are lower than that of AEGAS (1,1)-N model(3.0869) and the best being GARCH (1,1)-n model with the least AIC (3.0744). At medium persistence, misspecifying AEGAS (1,1)-N model as AEGAS (1,1)-Skew-T model fits better since the AIC of AEGAS (1,1)-Skew-T model (3.4264) is lower than the AIC of AEGAS (1,1)-N model (3.4291) whereas misspecifying AEGAS (1,1)-N model as any of the rest model fits lesser since the AIC of AEGAS (1,1)-N model is the least among the rest models. At high persistence, misspecifying AEGAS (1,1)-N model as any of the rest models fits lesser since the AIC of AEGAS (1,1)-N model (7.5931) is lower than the AICs of the rest models.

 From the results, we observed low negative estimates of excess kurtosis for all the models except for EGAS (1,1)-T model which is positive but also low. This implies closeness to normality of the estimated residuals for the four models. At both medium and high persistence, low positive estimates of excess kurtosis were observed for all the models except for AEGAS (1,1) models under the three assumed distributions which were negative but also low. This implies closeness to normality of the estimated residuals for all the models.

**Table 4.4 When the DGP is GAS- Skew-*T* (Beta-Skew-T-GARCH)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 0.8564 | 2.2057 | 1.9514 | 1.9395 | 4.2391 | 3.0753 |
| *T* | 0.8074 | 2.3125 | 1.9049 | 1.9583 | 4.1653 | 3.5141 |
| *Skew t* | 0.8059 | 2.3975 | 1.9077 | 1.9746 | 4.1653 | 3.4908 |
| *GAS(1,1)* | *Normal* | 0.8599 | 2.4268 | 1.9551 | 2.0437 | 4.2391 | 3.0750 |
| *T* | 0.8063 | 2.7751 | 1.9064 | 2.2196 | 4.1579 | 3.2160 |
| *Skew t* | 0.8046 | 2.7747 | 1.9152 | 3.5880 | 4.1573 | 3.2151 |
| *EGAS(1,1)* | *Normal* | 0.8615 | 2.4927 | 1.9550 | 2.0518 | 4.2471 | 2.9393 |
| *T* | 0.8068 | 2.7799 | 1.9149 | 3.7194 | 4.1564 | 3.2423 |
| *Skew t* | 0.8060 | 2.7795 | 1.9141 | 3.7928 | 4.1557 | 3.2349 |
| *AEGAS(1,1)* | *Normal* | 0.8633 | 2.5131 | 1.9577 | 2.0563 | 4.2491 | 2.9296 |
| *T* | 0.8106 | 2.7206 | 1.9151 | 3.5320 | 4.1578 | 3.1636 |
| *Skew t* | 0.8080 | 2.7699 | 1.9142 | 3.5624 | 4.1572 | 3.1568 |

When the DGP was GAS-Skew-T model, in terms of model fitness, at low persistence, specifying GAS (1,1)-Skew-T model as GAS (1,1)-Skew-T model fits better since the GAS (1,1)-Skew-T model has the least AIC (0.8046) among the GAS (1,1) models. At medium persistence, specifying GAS (1,1)-Skew-T model as GAS (1,1)-T model fits better since the GAS (1,1)-T model has the least AIC (1.9064) among the GAS (1,1) models. At high persistence, specifying GAS (1,1)-Skew-T model as GAS (1,1)-Skew-T model fits better since the GAS (1,1)-Skew-T model has the least AIC (4.1573) among the GAS (1,1) models. For misspecification, at low persistence, misspecifying GAS (1,1)-Skew-T model as any of the misspecified models fits lesser since the AIC of the GAS (1,1)-Skew-T model (0.8046) is lower than the AICs of all the misspecified models. At medium persistence, misspecifying GAS (1,1)-Skew-T model as GARCH (1,1)-T model or GARCH (1,1)-Skew-T model or EGAS (1,1)-T model or EGAS (1,1)-Skew-T model or AEGAS (1,1)-T model or AEGAS (1,1)-Skew-T model fits better since the AICs of the models above are all lower than the AIC of GAS (1,1)-Skew-T model (1.9152) with GARCH (1,1)-T model having the AIC (1.9049) as the best whereas misspecifying GAS (1,1)-Skew-T model as GARCH (1,1)-N model or EGAS (1,1)-N model or AEGAS (1,1)-N model fits lesser since the AIC of GAS (1,1)-Skew-T model is lower than the AICs of the three models. At high persistence, misspecifying GAS (1,1)-Skew-T model as EGAS (1,1)-T or EGAS (1,1)-Skew-T model or AEGAS (1,1)-Skew-T model fits better since the AICs of the three models are lower than the AIC of GAS (1,1)-Skew-T model (4.1573) with EGAS (1,1)-Skew-T model having the least AIC (4.1557) as the best whereas misspecifying GAS (1,1)-Skew-T model as any of the rest misspecified models fits lesser since the AIC of GAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models.

 We observed positive estimates of excess kurtosis throughout table 4.10 which are all greater than zero. This implies that the estimated residuals for all the models deviate from normal distribution and they have fatter tails than normal distribution.

**Table 4.5 When the DGP is EGAS- Skew-*T* (Beta-Skew-T-EGARCH)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 3.0116 | 2.3782 | 3.3123 | 1.8936 | 7.1875 | 3.0959 |
| *T* | 2.9597 | 2.3719 | 3.2647 | 1.8707 | 7.1134 | 3.5439 |
| *Skew t* | 2.9578 | 2.3739 | 3.2636 | 1.8705 | 7.1136 | 3.5244 |
| *GAS(1,1)* | *Normal* | 3.0119 | 2.4088 | 3.3140 | 1.9335 | 7.1875 | 3.0957 |
| *T* | 2.9602 | 2.7622 | 3.2739 | 3.6663 | 7.1074 | 3.1897 |
| *Skew t* | 2.9572 | 2.7622 | 3.2731 | 3.7394 | 7.1067 | 3.1844 |
| *EGAS(1,1)* | *Normal* | 3.0429 | 2.8475 | 3.3132 | 1.9195 | 7.2006 | 3.0071 |
| *T* | 2.9566 | 2.7593 | 3.2694 | 3.9312 | 7.1067 | 3.2583 |
| *Skew t* | 2.9546 | 2.7577 | 3.2681 | 3.9415 | 7.1060 | 3.2477 |
| *AEGAS(1,1)* | *Normal* | 3.0447 | 2.8466 | 3.3164 | 1.9496 | 7.2025 | 2.9693 |
| *T* | 2.958 | 2.7413 | 3.2709 | 3.8455 | 7.1081 | 3.1824 |
| *Skew t* | 2.9574 | 2.7421 | 3.2698 | 3.8799 | 7.1074 | 3.1653 |

When the DGP was EGAS-Skew-T model, in terms of model fitness, at low persistence, specifying EGAS (1,1)-Skew-T model as EGAS (1,1)-Skew-T model fits better since the EGAS (1,1)-Skew-T model has the least AIC (2.9546) among the EGAS (1,1) models. At medium persistence, specifying EGAS (1,1)-Skew-T model as EGAS (1,1)-Skew-T model fits better since the EGAS (1,1)-Skew-T model has the least AIC (3.2681) among the EGAS (1,1) models. Also, at high persistence, specifying EGAS (1,1)-Skew-T model as EGAS (1,1)-Skew-T model fits better since the EGAS (1,1)-Skew-T model has the least AIC (7.1060) among the EGAS (1,1) models. For misspecification, at low persistence, misspecifying EGAS (1,1)-Skew-T model as any of the misspecified models fits lesser since the AIC of EGAS (1,1)-Skew-T model (2.9546) is lower than the AICs of all the misspecified models. At medium persistence, misspecifying EGAS (1,1)-Skew-T model as GARCH (1,1)-T or GARCH (1,1)-Skew-T model fits better since the AICs of the two models are lower than the AIC of EGAS (1,1)-Skew-T model (3.2681) with GARCH (1,1)-Skew-T model having the least AIC (3.2636) as the best whereas misspecifying EGAS (1,1)-Skew-T model as any of the rest misspecified models fits lesser since the AIC of EGAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models. At high persistence, misspecifying EGAS (1,1)-Skew-T model as any of the misspecified models fits lesser since the AIC of EGAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models.

 We observed positive estimates of excess kurtosis throughout table 4.11 which are all greater than zero. This implies that the estimated residuals for all the models deviate from normal distribution and they have fatter tails than normal distribution.

**Table 4.6 When the DGP is AEGAS- Skew-*T* (Beta-Skewed-T-AEGARCH)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Estimated*** ***Model*** | ***Assumed*** ***Distribution*** | ***Low Persistence*** | ***Med. Persistence*** | ***High Persistence*** |
|  |  | ***AIC*** | ***Ex. Kurt.*** | ***AIC*** | ***Ex .Kurt.*** | ***AIC*** | ***Ex. Kurt.*** |
| *GARCH(1,1)* | *Normal* | 3.0104 | 2.1638 | 3.3226 | 1.9133 | 7.2633 | 3.0510 |
| *T* | 2.9617 | 2.2747 | 3.2733 | 1.8337 | 7.1902 | 3.5472 |
| *Skew t* | 2.9605 | 2.3509 | 3.2721 | 1.8386 | 7.1903 | 3.5291 |
| *GAS(1,1)* | *Normal* | 3.0140 | 2.3797 | 3.3225 | 1.9110 | 7.2633 | 3.0509 |
| *T* | 2.9612 | 2.7147 | 3.2819 | 3.6854 | 7.1843 | 3.1988 |
| *Skew t* | 2.9600 | 2.7220 | 3.2811 | 3.7601 | 7.1837 | 3.1927 |
| *EGAS(1,1)* | *Normal* | 3.0454 | 2.8095 | 3.3212 | 1.8843 | 7.2766 | 2.9267 |
| *T* | 2.9595 | 2.7200 | 3.2774 | 3.9293 | 7.1836 | 3.2731 |
| *Skew t* | 2.9574 | 2.7194 | 3.2761 | 3.9396 | 7.1829 | 3.2615 |
| *AEGAS(1,1)* | *Normal* | 3.0469 | 2.8090 | 3.3247 | 1.8892 | 7.2783 | 2.9942 |
| *T* | 2.9627 | 2.7837 | 3.2793 | 3.8896 | 7.1856 | 3.2639 |
| *Skew t* | 2.9606 | 2.7649 | 3.2781 | 3.9225 | 7.1849 | 3.2501 |

When the DGP was AEGAS-Skew-T model, in terms of model fitness, at low persistence, specifying AEGAS (1,1)-Skew-T model as AEGAS (1,1)-Skew-T model fits better since the AEGAS (1,1)-Skew-T model has the least AIC (2.9606) among the AEGAS (1,1) models. At medium persistence, specifying AEGAS (1,1)-Skew-T model as AEGAS (1,1)-Skew-T model fits better since the AEGAS (1,1)-Skew-T model has the least AIC (3.2781) among AEGAS (1,1) models. Also, at high persistence, specifying AEGAS (1,1)-Skew-T model as AEGAS (1,1)-Skew-T model fits better since the AEGAS (1,1)-Skew-T model has the least AIC (7.1849) among the AEGAS (1,1) models. For misspecification, at low persistence, misspecifying AEGAS (1,1)-Skew-T model as GARCH (1,1)-Skew-T model or GAS (1,1)-Skew-T model or EGAS (1,1)-T model or EGAS (1,1)-Skew-T model fits better since their AICs are lower than that of AEGAS (1,1)-Skew-T model (2.9606) with EGAS (1,1)-Skew-T model having the least AIC (2.9574) as the best whereas misspecifying AEGAS (1,1)-Skew-T model as any of the rest misspecified models fits lesser since the AIC of AEGAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models. At medium persistence, misspecifying AEGAS (1,1)-Skew-T model as GARCH (1,1)-T model or GARCH (1,1)-Skew-T model or EGAS (1,1)-T model or EGAS (1,1)-Skew-T model fits better since their AICs are lower than the AIC of AEGAS (1,1)-Skew-T model (3.2781) with GARCH (1,1)-Skew-T model having the least AIC (3.2721) as the best whereas misspecifying AEGAS (1,1)-Skew-T model as any of the rest misspecified models fits lesser since the AIC of AEGAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models. At high persistence, misspecifying AEGAS (1,1)-Skew-T model as GAS (1,1)-T model or GAS (1,1)-Skew-T model or EGAS (1,1)-T model or EGAS (1,1)-Skew-T model fits better since their AICs are lower than the AIC of AEGAS (1,1)-Skew-T model (7.1849) with EGAS (1,1)-Skew-T model having the least AIC (7.1829) as the best whereas misspecifying AEGAS (1,1)-Skew-T as any of the rest misspecified models fits lesser since the AIC of AEGAS (1,1)-Skew-T model is lower than the AICs of the rest misspecified models.

 We observed positive estimates of excess kurtosis throughout table 4.12 which are all greater than zero. This implies that the estimated residuals for all the models deviate from normal distribution and they have fatter tails than normal distribution.

**5.0 Results of the Real Life Data**

We apply both daily crude oil and Gas prices to test the effect of misspecification of volatility models. The crude oil prices are the European Brent prices (US dollars/barrel) while the gas prices are the Henry Hub Natural gas spot prices (US Dollars per Million Btu), both obtained from the website of US Energy Information Administrations (<http://www.eia.gov/>). The oil prices span between 20 May 1987 and 29 September 2014 while the natural gas series span between 07 January 1997 and 09 March 2015.

 The plot of the crude oil prices is given in Figure 5.1. We observe stability in the prices of crude oil from 1987 to 1999 with a major spike in 1990. We observe a gradual increase in the prices of crude oil from 2000 to 2008 with the prices of crude oil getting to its peak in 2008. We also observe a fall in 2008 and a gradual increase in the prices of crude oil from 2008 to 2011 and the prices were stable from 2011 to 2015.

 20 May 1987 29 September 2014

**Figure 5.1: Time Plot of Crude Oil Prices (US Dollar/Barrel )**

We present the result of model estimation in Table 5.1 below. Based on the Akaike Information Criterion, the model with the minimum AIC is chosen as the fitted model. From Table 5.1, it can be observed that the AEGAS (1,1)-Skew-T model has the minimum AIC which is -6.7482. Therefore, the fitted model to the crude oil prices is the AEGAS (1,1)-Skew-T model. We observed positive estimates of excess kurtosis throughout Table 5.2 which are all greater than zero. This implies that the estimated residuals for all the models deviate from normal distribution and they have fatter tails than the normal distribution. In terms of model fitness, specifying AEGAS (1,1)-Skew-T model as AEGAS (1,1)-Skew-T model fits better than specifying it as either AEGAS (1,1)-Normal or AEGAS (1,1)-T model since the AEGAS (1,1)-Skew-T model has the least AIC (-5.5516) among the AEGAS models. Also, misspecifying AEGAS (1,1)-Skew-T model as EGAS (1,1)-Skew-T model fits better than any of the rest models since the EGAS (1,1)-Skew-T model has the least AIC (-6.7476).

**Table 5.1: Specification and Misspecification tests for Models for Crude Oil prices**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Estimated Model | Distribution Assumed | LogLik | AIC | Ex.Kurtosis |
| GARCH | Normal | 23441.323 | -6.6928 | 1.9432 |
|  | *T* | 23621.291 | -6.7439 | 2.0256 |
|  | Skewed-*t* | 23626.203 | -6.7451 | 2.0341 |
| GAS | Normal | 23441.461 | -6.6929 | 1.9429 |
|  | *T* | 23625.438 | -6.7451 | 2.4417 |
|  | Skewed-*t* | 23630.391 | -6.7463 | 2.4535 |
| EGAS | Normal | 23436.872 | -6.6916 | 1.9525 |
|  | *T* | 23630.107 | -6.7465 | 2.3753 |
|  | Skewed-*t* | 23635.154 | -6.7476 | 2.3888 |
| AEGAS | Normal | 23438.531 | -6.6918 | 1.9381 |
|  | *T* | 23633.446 | -6.7471 | 2.4110 |
|  | Skewed-*t* | 23638.115 | -6.7482 | 2.4264 |

The plot of the natural gas prices is given in Figure 5.2. We observe major spike in the prices of natural gas in 2001, 2003, 2005, 2008, 2010 and 2014. We observe fall in prices of natural gas after each spike and stability of prices of natural gas before the next spike.

 07 January 1997 09 March 2015

**Figure 5.2: Time Plot of Gas Prices (US Dollar/btu )**

Using the Akaike Information Criterion, the model with the minimum AIC is chosen as the fitted model. From Table 5.1, it can be observed that the EGAS (1,1)-T model has the minimum AIC which is -5.5516. Therefore, the fitted model to the natural gas prices is the EGAS (1,1)-T model.

 We observed positive estimates of excess kurtosis throughout Table 5.1 which are all greater than zero. This implies that the estimated residuals for all the models deviate from normal distribution and they have fatter tails than the normal distribution. In terms of model fitness, specifying EGAS (1,1)-T model as EGAS (1,1)-T model fits better than specifying it as either EGAS (1,1)-Normal or EGAS (1,1)-Skew-T model since the EGAS (1,1)-T model has the least AIC (-5.5516) among the EGAS models. Also, misspecifying EGAS (1,1)-T model as EGAS (1,1)-Skew-T model fits better than any of the rest models since the EGAS (1,1)-Skew-T model has the least AIC (-5.5513).

***Table 5.2: Specification and Misspecification tests for Natural Gas model***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Estimated Model | Distribution Assumed | LogLik | AIC | Ex.Kurtosis |
| GARCH | Normal | 12436.730 | -5.4666 | 8.7823 |
|  | *T* | 12624.677 | -5.5488 | 10.341 |
|  | Skewed-*t* | 12624.833 | -5.5484 | 10.337 |
| GAS | Normal | 12436.725 | -5.4666 | 8.7822 |
|  | *T* | 12617.975 | -5.5458 | 8.6593 |
|  | Skewed-*t* | 12618.250 | -5.5455 | 8.6125 |
| EGAS | Normal | 12318.836 | -5.4147 | 10.506 |
|  | *T* | 12631.035 | -5.5516 | 8.6237 |
|  | Skewed-*t* | 12631.334 | -5.5513 | 8.5867 |
| AEGAS | Normal | 12338.316 | -5.4229 | 11.931 |
|  | *T* | 12631.269 | -5.5512 | 8.3575 |
|  | Skewed-*t* | 12631.549 | -5.5509 | 8.3358 |

**6.0 Conclusion**

This paper has investigated the misspecification of GAS models and its variants using Monte Carlo simulation approach. The work was extended to real life situation by using the daily prices of crude oil and natural gas prices, since these time series are known in the literature to possess occasional jumps. The estimation involved investigating the misspecification of GAS models and their variants assuming both normally and Skewed Student-t probability distributions for the GARCH variates. Model selection performance was then investigated using information criteria and tail coefficient (kurtosis). We therefore obtained the results for model fitness and residual tail behaviour.

 In terms of model fitness, the results showed that when the residuals of the DGPs are assumed to be normally distributed, at low persistence misspecifying them as GARCH (1,1)-N model fits or performs better. At medium persistence, when the DGP is GAS (1,1), misspecifying it as EGAS (1,1)-SKT model fits better and when the DGPs are EGAS (1,1) and AEGAS (1,1), misspecifying them as AEGAS (1,1)-SKT model fits better. At high persistence, when the DGP is GAS (1,1) misspecifying it as EGAS (1,1)-N model fits better while when the DGPs are EGAS (1,1) and AEGAS (1,1) , misspecifying them as any of the estimated models fits or performs lesser. When the residuals of the DGPs are assumed to be skewed-student t distributed, the results showed that at low persistence, when the DGPs are GAS (1,1) and EGAS (1,1), misspecifying them as any of the estimated models fits or performs lesser while when the DGP is AEGAS (1,1), misspecifying it as GAS (1,1)-SKT model fits better. At medium persistence, when the DGPs are GAS (1,1), EGAS (1,1) and AEGAS (1,1), misspecifying them as GARCH (1,1) model with the residuals assumed to be student t, skewed-student t and skewed-student t respectively fits better. At high persistence, when the DGPs are GAS (1,1) and AEGAS (1,1), misspecifying them as EGAS (1,1)-SKT model fits better while when the DGP is EGAS (1,1), misspecifying it as any of the estimated models fits or performs lesser.

 The results of this paper also showed that when the residuals for the three DGP namely GAS, EGAS and AEGAS were assumed to be normally distributed, the estimated residuals from the DGPs behaved normally since all the excess kurtosis observed under the three DGPs, at low, medium and high persistence were either negatively low or positively low and close to zero. Also, when the residuals for the three DGPs, namely the GAS, EGAS and AEGAS were assumed to be skewed-student t distributed, the residuals of the models estimated from the DGPs deviated from normality since the excess kurtosis observed under the three DGPs levels (low, medium and high persistence) were positive and greater than zero.

**References**

Akaike, H. (1973). “Information theory and an extension of the maximum likelihood principle”. In B.N. Petrov and F. Csaki (eds), Proceedings of the Second International Symposium on Information Theory, pp. 267–281. Budapest: Akademiai Kiado´. Reproduced in S. Kotz and N.L. Johnson (eds), Breakthroughs in Statistics Vol I Foundations and Basic Theory, pp. 610–624. New York: Springer-Verlag 1992.

Andersen, T., Bollerslev, T. and Diebold, F. (2007). Roughing it up: including jump components in the measurement, modelling and forecasting of return volatility. The Review of Economics and Statistics, 89(4), 701–720.

Bauwens, L., Hafner, C. and Laurent, S. (2012). Volatility Models, In Handbook of Volatility Models and Their Applications (eds L. Bauwens, C. Hafner and S. Laurent), John Wiley and Sons, Inc., NJ, U.S.A.

Bellini, F. and Bottolo, L. (2007). Misspecification and domain issues in fitting Garch (1,1) models: a Monte Carlo investigation. Working paper. University of Milano-Bicocca, Italy and Imperial College London, UK.

Bernadi, M. and Cantania, L. (2015a). Switching-GAS copula models for systemic Risk Assessment. arxiv.org/abs/1504.03733vi.

Blasques, F., Koopman, S.J. and Lucas, A. (2008). Stationarity and Ergodicity of Univariate GAS Process. Electronic Journal of Statistics. Vol. 8, pp. 1088-1112.

Blasques, F., Koopman, S.J. and Lucas, A. (2014a). Maximum Likelihood Estimation for correctly specified Generalized Autoregressive Score Models: Feedback Effects, Contraction Conditions and Asymptotic Properties. Tinbergen Institute Discussion Paper No TI 2014-074/III.

Blasques, F., Koopman, S. J., and Lucas, A. (2014b). Information theoretic optimality of observation driven time series models. Discussion Paper Tinbergen Institute TI 14-046/III.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics 31: 307–27.

Calvori, F., Cipollini, F. and Gallo, G.M. (2013). Go with the flow: A GAS model for predicting intra-daily volume shares. Social Science Research Network. [http://ssrn.com/abstract=2363483](http://ssrn.com/abstract%3D2363483)

Caporin, M. and Lisi, F. (2010). Misspecification tests for periodic long memory GARCH models. Statistical Method and Application, Vol. 19, 47-62.

Charles, A. and Darne, O. (2014). Volatility persistence in Crude oil markets. Energy Policy, 65: 729-742.

Creal, D., Koopman, S.J. and Lucas, A. (2008). A General Framework for Observation Driven Time-Varying Parameter Models. Tinbergen Institute Discussion Paper. TI 2008-108/4.

Creal, D., Koopman, S. J., and Lucas, A. (2011). A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations. Journal of Business and Economic Statistics, 29(4):552-563.

Creal, D., Koopman, S.J. and Lucas, A. (2013). Generalized Autoregressive Score (GAS) Models With Applications. Journal of Applied Econometrics, Vol 28,777-795.

Elder, J., Miao, H. and Ramchander, S. (2013). Jumps in Oil Prices: The role of economic news. The Energy Journal, 34(3): 217-237.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom. Econometrica, 50: 987-1008.

Engle, R.F. and Ng, V.K. (1993). Measuring and Testing the Impact of News on Volatility. The Journal of Finance, Vol. 48, pp. 1749-1778.

Fama, E.F. (1970). Efficient Capital Markets: A Review and Theory and Emperical Work. The Journal of Finance, Vol. 25, pp. 383-417.

Halunga, A.G. and Orme, C.D. (2004). Misspecification tests for GARCH models. School of Social Sciences, University of Manchester, Working Paper, pp. 1-35.

Harvey, A. and Chakravarty, T. (2008). Beta-t-(E)GARCH. Working paper series. University of Cambridge.

Harvey, A. (2013). Dynamic Models for volatility and Heavy tails: with applications to financial and economic time series. Cambridge University Press, London.

Huang, Z., Wang, T. and Zhang, X. (2014). GAS model with realized measures of volatility. [http://ssrn.com/abstract=2461831](http://ssrn.com/abstract%3D2461831).

Lundbergh and Terasvirta (2002). Evaluating GARCH Models. Journal of Econometrics, 110, 417-435.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. Journal of Business, Vol. 36, 394-419.

Nelson, D. B. 1991. “Conditional Heteroscedasticity in Asset Returns: A New approach.” Econometrica. 59(2): 347–70.

Posedel, P. (2005). Properties and Estimation of GARCH (1,1) model. Metodoloski Zvezki, vol 2,243-257.

Xekalaki, E. and Degiannakis, S. (2010). ARCH Models for Financial Applications. John Wiley and Sons Ltd.

1. The Beta-EGARCH specification has no asymmetric parameter, unlike the classical EGARCH model of Nelson (1991). [↑](#footnote-ref-1)
2. Note,  in the three symmetric distributions, while  for the Skewed Student-t distribution. [↑](#footnote-ref-2)