Relative Advantage of Leasing: A Blended Methods Approach

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Abstract
Real estate decisions, such as leasing, buying, or selling properties, play a crucial role for tenants, landlords, and real estate developers. The relative advantages of leasing compared to buying or selling are influenced by various factors, including financial considerations, market conditions, and individual preferences. This paper proposes a methodology to analyze and quantify the relative advantage of leasing from the perspectives of tenants, landlords, and real estate developers. By employing a mixed methods approach and considering key valuation approaches, valuable insights into the dynamics and trade-offs associated with leasing in the real estate sector are provided. The methodology aims to assist stakeholders in making informed decisions by comprehensively assessing the benefits and drawbacks of leasing compared to other options.

1 Introduction

Real estate decisions, such as whether to lease, buy, or sell a property, are critical considerations for tenants, landlords, and real estate developers. These decisions are often influenced by various factors, including financial considerations, market conditions, and individual preferences. Understanding the

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relative advantages of leasing compared to buying or selling is essential in making informed decisions in the real estate market. In this paper, we propose a methodology that aims to analyze and quantify the relative advantage of leasing from the perspectives of tenants, landlords, and real estate developers. By considering key valuation approaches and employing a mixed methods approach, we provide valuable insights into the dynamics and trade-offs associated with leasing versus other options in the real estate sector.

1.1 Considerations about renting or owning status

The decision between renting or buying a property depends on various factors and individual circumstances. There is no one-size-fits-all answer to determine which option is better, as each person will have different needs, financial goals, and preferences.

This being said, some points to consider when making this decision include:

- Financial stability: Evaluate your current and future financial stability. Buying a house requires a significant upfront investment, including a down payment and additional costs such as taxes, legal fees, and maintenance. Make sure you have adequate financial resources to cover these costs and maintain regular payments.

- Real estate market: Consider the conditions of the real estate market. Research house price trends in the area where you intend to live. If prices are high and supply is limited, it may be more advantageous to rent while waiting for a more favorable buying opportunity.

- Flexibility: Reflect on your flexibility and future plans. If you anticipate significant changes in your personal or professional life, such as job or location changes, renting may be a more suitable option as it offers greater mobility and fewer long-term commitments.

- Residential stability: If you seek to establish roots and have long-term residential stability, buying a house may be the right option. Owning a property allows for customization, building equity, and the possibility of investing in a real estate asset.

- Costs and responsibilities: Consider the costs and responsibilities associated with buying and renting. In addition to the monthly rent amount, tenants are typically not responsible for maintenance and repair costs, whereas homeowners are responsible for these costs in their own property. Additionally, homeowners are subject to taxes and additional charges, such as municipal taxes and condominium expenses.

- Financial goals: Evaluate your personal financial goals. Buying a house can be a long-term investment strategy as it can build equity over time. However, it is important to analyze the potential profitability, expected returns, and market conditions before making a decision.

The valuation of urban buildings is commonly conducted using three distinct approaches:

- The market analysis approach relies on methodologies supported by market values and utilizes comparative techniques. This approach is commonly known as the Comparative Method, Market Method, or Comparative Market Method. In this approach, the value of the property is assessed as the Presumed Transaction Value (PVT).
- The income-based evaluation assigns value based on a given or determined income, known as the Value Based on Income (VBR). This approach generally involves two main sets of methods: Capitalization Methods and Discounted Cash Flow Methods, which consider the determination and conceptualization of yields, capitalization rates, and capital opportunity costs.
- The cost-based approach to property valuation primarily involves adding the value of the land to the cost of replacing the building that performs the same functions as the evaluated property. This approach is referred to as the Cost Method, and the resulting value is known as the Value Based on Cost (VBC). This value can be estimated using the static cost method, which defines or estimates a profit margin value, or through the dynamic cost method, which considers the replacement cost of the building.

It is important to note that these three approaches provide different perspectives on property valuation, and their application depends on various factors such as the purpose of the valuation, availability of data, and specific market conditions. The integration of these approaches can offer a more comprehensive and accurate assessment of the value of urban buildings.numa abordagem pseudo-dinâmica, fazendo intervir a taxa de retorno do investimento (ROI) delineando prazos e tempos de tarefas.

2 Cost-Based Approach

This approach involves determining the costs associated with the entire real estate investment process, including the land, project development, construction and marketing costs, as well as indirect costs. In the static method, these costs are added to a profit margin to derive the valuation, VBC (Value Based on Cost)[1].

Indirect costs correspond to expenses related to administrative fees and permits, mostly associated with the construction process itself.

The general formulation for estimating the value based on cost, VBC, is written in implicit form:

[1] For the sake of simplicity, the pseudo-dynamic method is not considered in this text.

$$\begin{aligned} VBC - C_{\text{inv}} \; - \; L &= 0 \\ C_{\text{inv}} \; &= T_{\text{inf}} + D_{\text{proj}} + C + K + M \end{aligned} \tag{1}$$

where:

C_{inv} – Represents the cost of investment required for the formation of the asset.

L – Represents the return of the developer, often expressed as a profit margin[2].

T_{inf} – Corresponds to the rental cost of the property, including not only the cost or value of the land itself and the acquisition expenses but also the fees related to studies and infrastructure projects resulting from the type of occupancy. It includes the cost of installed or to-be-installed infrastructure works.

D_{Proj} – Costs related to the design project and technical assistance from designers.

C – Costs associated with the necessary operations for the construction or rehabilitation of a property. Its value is estimated based on the project phase, using appropriate methodologies.

K – Costs associated with the expenses related to the process of forming the property to be built or replaced.

M – Costs associated with the operations necessary for the marketing and sale of the asset.

By substituting C_{inv} in VBC from Equation 1, we obtain:

$$VBC - \Big[T_{\text{inf}} + D_{\text{proj}} + C + K\Big] - [M + L] = 0 \tag{2}$$

2.1 Land Cost

The cost of an infrastructured land, T_{inf}, is obtained by adding the cost of the undeveloped land to the cost of infrastructure works. By adding the value of the land to the fees of consultants, designers, and construction managers (cf. Equation 2), the land cost, T, is obtained:

$$T = T_{\text{inf}} + D_{\text{proj}} = L_{\text{and}} + (\eta_{\text{inf}} + \delta_{\text{proj}}) \cdot C \tag{3}$$

By dividing the land cost by VBC, the relative value of the land, ν_T, is defined:

$$\nu_T = \frac{T}{VBC} \tag{4}$$

[2]In the case of a static approach.

2.2 Construction Cost

Accurate estimation of construction costs is crucial for development investment analysis. It encompasses the costs directly associated with the construction process and related expenses. In the case of buildings, the following subsections discuss both aspects. Considering the wide range of building construction systems, even within the same usage category, cost estimates are rough approximations that can only be obtained under specific and well-defined conditions.

2.2.1 Estimating Construction Costs

The accuracy of cost estimation depends on the level of project detail. Therefore, cost estimation methodologies are often tailored to different project stages, which can be categorized as follows:

- *Preliminary Program* - Information gathered by the owner that defines project objectives, functional needs, required spaces, and desired timeframes. This information, combined with knowledge of the site's topography, establishes a "baseline program."
- *Baseline Program* - A detailed version of the preliminary program, prepared by the designer, including alternative solutions suitable for local conditions. Once approved by the owner, it serves as the basis for subsequent phases, including the "Baseline Project," "Specialty Projects," and "Execution Project."
- *Preliminary Study* - The initial phase where the project begins to take shape, exploring alternative layouts, plans, elevations, sections, structural types, and materials. Cost estimates are typically made at this stage.
- *Anteproject or Licensing Project* - Defining spaces, structure, and general principles for material selection. Cost estimates are usually made.
- *Architecture Project* - Production of written and drawn documents that convey the developed idea. The project, in the form of a Licensing Project, is submitted to authorities and organizations for approval and licensing.
- *Specialty Projects* - Projects developed by specialists related to structural safety, electrical power supply, gas installation, water and sewage systems, exterior arrangements, telecommunications, thermal behavior, electromechanical installations, fire safety, acoustic performance, and mechanical HVAC systems (*heating*, *ventilation*, *and air conditioning*).
- *Execution Project* - A coordinated set of written information and drawings that can be easily interpreted by entities involved in project execution. It includes measurements of work items, segmented by trades, along with their respective budgets. Together with specifications, it constitutes the documentation for construction companies interested in the project's award.

Cost estimation involves predicting the cost required for a specific construction project. As the project progresses, the precision of cost estimation improves, ranging from a simple estimate to a budget value.

Various institutions provide categories or levels for construction cost estimates. For example, the Association for the Advancement of Cost Engineering (AACE) and the American Society of Professional Estimators (ASPE) present five levels of cost estimation [1]. The Canadian Construction Association (ACA) [2] recommends four categories based on the level of project detail or phase, with cost control during and after execution falling into another level.

The degree of project definition is a key factor used to classify types of cost estimates, with other characteristics such as the purpose of the estimate, the adopted methodology, and the required resources (cost and time) considered secondary.

A Class 5 estimate according to AACE or a Category D according to CCA refers to cases where the level of project information and definition is low, while Class 2 or Category B corresponds to the level of control and deviation detection in relation to the budgeted values agreed upon between the owner and the contractor.

Detailed information on construction cost estimation methodologies can be found in the Real Estate Evaluation Manual by M. Miranda and R. Camposinhos [3].

2.3 Indirect Costs

A portion of the investment cost is directly related to the construction cost. However, what is commonly referred to as "Indirect Costs" includes expenses associated with the licensing and management of construction works. These costs are associated with the procedural and administrative aspects of the construction process, excluding land acquisition costs and those related to asset marketing and sales. The value of these costs can range from 5

The relationship between the value of indirect costs and the construction cost is generally expressed as a percentage of the construction cost:

$$\kappa_K = \frac{K}{C} \tag{5}$$

These costs are further detailed in the following sections.

2.3.1 Management and Supervision

Management and supervision are distinct activities. In summary, construction management involves the planning, direction, coordination, and centralized control of activities required for the implementation of a project. It encompasses the proficient administration of the entire construction site.

On the other hand, supervision is an activity that encompasses services aimed at ensuring the oversight of activities related to the execution of a con-

struction project. It involves ensuring compliance with the engineering specialty projects and specifications until the provisional acceptance of the work, guaranteeing the quality and safety of the construction, including the final acceptance of the project.

Regardless of the type of work - construction, reconstruction, expansion, alteration, adaptation, conservation, restoration, repair, rehabilitation, improvement, or demolition - the execution is entrusted to a contractor. The responsibilities of the management and supervision teams include, among others:

- Supervision and control of construction execution;
- Quality control;
- Environmental monitoring;
- Construction safety control;
- Planning control.

The legal framework for urban development and building regulations[3] requires that all licensed works or those subject to "prior notification" must have a construction logbook kept at the site, recording all relevant facts related to the execution of the work. The format and content of the logbook must comply with the requirements defined in a regulation, which also regulates the characteristics of the electronic logbook.

Both the supervisors and the construction management personnel produce documentation that justifies and verifies the execution of the aforementioned procedures. This documentation is recorded in the logbook, following the established format. The logbook should contain the following information:

- Identification of the license holder or the entity submitting the prior notification for the execution of the work, or the permit holder for demolition or excavation and peripheral containment works;
- Identification of the technical supervisor responsible for overseeing the construction, including their registration number with a public professional association, where applicable;
- Identification of the project coordinator and the authors of the projects, including their registration numbers with a professional association or officially recognized public entity, where applicable;

[3] *The European Union launched the International Urban Cooperation Programme (IUC) programme to promote international urban cooperation in 2016, and decided to launch a second phase in 2021 to extend the programme to regional authorities and to additional countries. The International Urban and Regional Cooperation Programme entails cooperation on common urban and territorial challenges, with particular attention to the green and digital transitions and the inclusive post-pandemic recovery, namely by pairing up EU with non-EU partner cities or regions in order to share knowledge and best practices. Cities and regions will cooperate in three thematic networks: the ecological transition and green deal; urban and regional renewal and social cohesion; and innovative sustainable and carbon-neutral ecosystems and strategic sectors.*

- Identification of the construction company responsible for carrying out the work, including the company's construction license number or registration title enabling them to carry out the work;
- Identification of the works director, whether integrated into the technical staff of the construction company responsible for the execution or, in the case of a registered company, the entrepreneur or legal representative who has signed a liability statement for the proper execution of the work, including their registration numbers with a professional association or officially recognized public entity, where applicable.

The costs associated with management and supervision activities can range from 2% to 3% of the Construction Cost, C [4, 3].

2.3.2 Licensing and Fees

The costs associated with licenses and fees, L_{tax}, are related to project approvals, municipal licensing of the construction works, as well as property registration and land registry costs[4].

The licensing fees depend on specific Municipal Regulations and vary from one municipality to another. They include costs for demolition and/or construction permits, inspection fees and/or technical inspections, registration of horizontal property, fees for connecting to water supply networks, sewage systems, installation of water, gas, and electricity meters, among others.

In general, the most significant expenses are related to construction and occupancy licenses and fees for licensing urban infrastructure works. Each municipality periodically sets a table with the values of the applicable fees. The available information allows estimating a value as a percentage of the construction cost, C, ranging from 3

2.4 Marketing and Sales

The costs associated with marketing and sales are either assumed by the developer themselves or commissioned to an intermediary. Regardless of the option chosen, different marketing fees should not be applied. Even though the second option may have higher costs, the commission margins for intermediation should be included in the appraisal value. The developer must bear all the procedures associated with direct marketing, such as advertising expenses, scheduling visits and negotiations, assistance in document preparation, post-sales support, etc. In Portugal, agency fees can range from 3% to 6%, which are similar to those practiced in Spain and France but much higher than those in Anglo-Saxon countries.

[4]In simple terms, the property registration number is to a property what the taxpayer identification number is to an individual, and the matricial article number is to the identity card. The cost values for registrations vary depending on the type of registration and are listed in a specific table by the IRN (Institute of Registries and Notaries)

Table 1: Net Margin in Real Estate 2005-2015 [5]

Asset Type	**Average Value**
Retail	28.7%
Apartment	23.7%
Office	16.5%
Industrial	10.8%

Administrative costs related to certificates, licenses, property and land registration represent approximately 1% of the declared transaction values in Portugal.

The percentage values mentioned in this section are based on the final value, VBC. The appraiser may choose to index these costs to the construction cost, C, in which case they should be reflected proportionally to the VBC/C ratio.

The value of λ_M represents the weight of the costs associated with marketing and sales, M:

$$\lambda_M = \frac{M}{VBC} \tag{6}$$

2.5 Profit Margin

The gross profit margin, L, of an investment is usually expressed in relative terms in relation to the investment value through a rate, μ_L:

$$\mu_L = \frac{L}{VBC} \tag{7}$$

Although it is considered difficult to know the specific profit margins of builders and/or developers, there are historical data regarding profit margins in the United States and Canada. Average values of pre-tax operating profit margins in the real estate sector in the United States during the period 2005-2015 are presented in Table 1 based on studies by E. Yardeni[5].

2.6 Explicit Formulation

In many cases, appraisals are based on the most quantifiable and influential component, the construction cost, C. Thus, the following expression or its equivalent is often encountered:

$$VBC = \frac{C \cdot (1 + \kappa_K)}{1 - (\nu_T + \lambda_M + \mu_L)} \tag{8}$$

This expression requires knowledge of the relative values of the investment cost and the absolute value of the construction cost of the new building, C. If,

in addition to the construction cost, C, the value of the land cost, T, is known, Equation 8 can be simplified to:

$$VBC = \frac{T + C \cdot (1 + \kappa_K)}{1 - (\lambda_M + \mu_L)} \tag{9}$$

Expressing the construction cost, C, as a percentage of the cost based value, VBC:

$$\tau_C = \frac{C}{VBC} \tag{10}$$

Equation 9 can be further simplified when only the absolute value of the land cost, T, is known:

$$VBC = \frac{T}{1 - (\gamma_M + \mu_L) - \tau_C \cdot (1 + \kappa_K)} \tag{11}$$

These formulations provide explicit expressions for calculating the cost based value, VBC, based on the construction cost, C, and land cost, T.

3 Income Approach

In this approach, the appraiser (or investor) updates, or as it is sometimes said, discounts the future net incomes generated (or prudently projected to be generated) by the property to its present value. The discount acknowledges that a monetary unit in the future is worth less than a monetary unit in hand [3].

The economic value may lead, depending on the conditions imposed in its calculation, either to an investment value or a market value. When determining the market value, the appraiser relies solely on evidence derived from that market. However, if the intention is to calculate the investment value, other references may be used, often indicated by the interested party. In any case, income approaches provide much better tools for determining the investment value rather than the market value of a given property [3].

As is known, the value of a capital at a given point in time, equivalent to a certain periodic income over a given period, depends on the discount rate and the net value of the incomes.

In the case of a property that can be leased, the corresponding incomes can be transformed into an equivalent capital, as defined in the above-mentioned paragraph, which is referred to as the income-based value (VBR).

Therefore, at a given moment, specifically the date of the valuation, it is necessary to know the net operating income for a given time horizon, update it at a rate that reflects the market situation and the risk of income return. The value based on income corresponds to the sum of the present value of the net incomes plus the potential residual value of the appraised property, usually updated at the same rate, for the considered time horizon.

In this regard, it should be noted that *"... the benefits being appraised are future benefits, which is why any method included in the Income Approaches cannot*

be solely based on past revenues or expenses. Both are important, but they cannot be the only sources, due to the natural tendency to consider the history and present state of a business or investment, the appraiser must exercise special care and be aware that the values are not a reflection of the past or even the present. They are an anticipation of the future. Moreover, since the net incomes to be considered are future incomes, both revenues and expenses will always have to be estimated, although compared to any existing ones. Precisely for this reason, there will be more certainty in assessing an investment value and less certainty when assessing market value because, in the former situation, the appraiser relies on information that the investor has collected or had collected, while in the assessment of market value, the appraiser assumes the role of a generic investor and usually does not have as much information available." [6].

The net operating income is equal to the effective gross income minus the operating expenses and sometimes capital expenses, as well as replacement, maintenance, and improvement investments[5].

3.1 Simplified Approach

In the case of urban buildings, it is common to equate lease rents to a series of constant and perpetual monthly incomes. Thus, in its simplified form, the current expression for determining the value based on income, VBR, is given by:

$$VBR = \frac{12 \times R_{mi}}{t_{ai}} = \frac{R_{ai}}{t_{ai}}; \tag{12}$$

where:

R_{mi} – Gross monthly rental value;

R_{ai} – Gross annual income;

t_{ai} – Conversion factor; discount rate, or simply yield[6].

This formulation implicitly assumes:

- The residual value of the building is zero, or the discounting period is sufficiently high.
- The rents are monthly.

[5] In this regard, taking into account the provisions of the European Valuation Standards 2016 [6], when the income approach is used to calculate the Market Value, both the Municipal Property Transfer Tax (IMT), Stamp Duty (IS), income taxes (IRS or IRC), and other transaction expenses are deducted from the calculated income value.

[6] The yield rate is a simple comparison between the gross annual rent of a real estate asset and its purchase price or market value. The rent-to-price ratio is an indicator of profitability for the housing market, often used by analogy to the stock market. Just as the price of a stock should equal the discounted value of future dividends, the price of a property should reflect the future benefits (dividends) of owning it; both from the perspective of an investor's profitability or profit and from the perspective of a homeowner's rental savings while also enjoying the property.

- The conversion factor t_{ai} represents a certain annual rate that includes the risk premium plus the risk-free interest rate value.

This approach results from comparing, based on market rental indicators, the value of the gross annual income R_{ai} and the valuation value. This comparison is made using the defined relationship in equation (12), which can express a certain "discount rate".

3.2 Discounted Sum of Incomes

The value of a property, VBR, is associated with a given income. By discounting the net operating income, i.e., the revenues minus the costs, VBR is simply the Net Present Value (NPV) of the cash flow generated by the property, at a given rate 't'. It is necessary to estimate the revenues and costs, discount them at a certain rate, bearing in mind that:

- The gross income of a property, usually monthly, directly comes from the rental value and the conditions of the lease contract under which a particular unit or building is leased. Thus, if the discount rate of the income is annually for practical convenience, the monthly rental value must be transformed into an annual income value.
- The gross rental value is determined by the supply and demand dynamics, reflecting, in the absence of distortions, the rental market conditions throughout the life of the property.
- The rental value estimation should reflect not only the market conditions but also the condition of the building, its location, functional capacity, state of conservation, etc.

These conditions vary over time, and therefore, the validity of the valuation is limited. It is practically impossible to accurately predict the fluctuations and changes that affect the estimation of future values.

Let:

R – the value of each constant and post-dated receipt over a given period of time;

t – discount rate;

n – the number of corresponding periods, equal to the discounting period;

T – Land value or residual value.

Thus, the general expression for calculating the sum of the values of a series of receipts, V is given by:

$$V = R \times \frac{1-(1+t)^{-n}}{t} + T \times (1+t)^{-n} \tag{13}$$

This expression represents the sum of values of a series of regular receipts, each corresponding to the same period. Notice that Equation (12) results from taking the limit as $n \to \infty$ in Equation (13):

$$V = R \times \lim_{n\to\infty}\left[\frac{1-(1+t)^{-n}}{t} + T \times (1+t)^{-n}\right] = R \times \frac{1}{t} + 0; \tag{14}$$

Notice that the deduction applies regardless of the value of $\boldsymbol{T}$ and $t > 0$. In fact, for the current values of interest rates t and the lifespan of buildings (several decades), the difference between the two expressions is often negligible[7]

3.2.1 Periods and Equivalent Rate

For financial congruence, if the rents are monthly, they should be subjected to monthly rates. When using annual rates for monthly rents, considering the rent as annual requires that this fact be taken into account in the calculation of VBR[8].

Keeping in mind the equivalence between the nominal annual rate, t_a, and the effective annual rate, t_{ae}:

$$t_{ae} = \left(1 + \frac{t_a}{12}\right)^{12} - 1; \tag{15}$$

the congruent expression for VBR to be used instead of the expression (12) is:

$$VBR = \frac{12 \times R_m \cdot \left(\left(1 + \frac{t_a}{12}\right)^{12} - 1\right)}{t_a^2} = \frac{R_a \times t_{ae}}{t_a^2}; \tag{16}$$

This 'correction' has a greater impact as the rate values increase. In fact, as can be seen in Figure 1, where the value based on income calculated by expression (16), VBR_m, is compared to the value of VBR_a given by expression (12).

The Value Based on Income (VBR) should be obtained from the net operating income (net cash flow) by deducting the costs associated with the rental value.

In current cases, the discount rate does not consider the effect of inflation since the rents are assumed to be constant. However, when considering the effect of inflation in the VBR calculation, the discount rate should be adjusted. It is important to note that the value obtained will be different when rents and operating costs have different growth rates.

In situations where the market is not stable, and the effect of inflation is significant, the forecast of net operating income is made at current prices. In

[7] Note that for a zero discount rate, it would result in an absurdity:

$$V = R \times \lim_{t\to 0}\left[\frac{1-(1+t)^{-n}}{t} + T \times (1+t)^{-n}\right] = R \times n + T$$

[8] It should be noted that, in general, this rule is not followed.

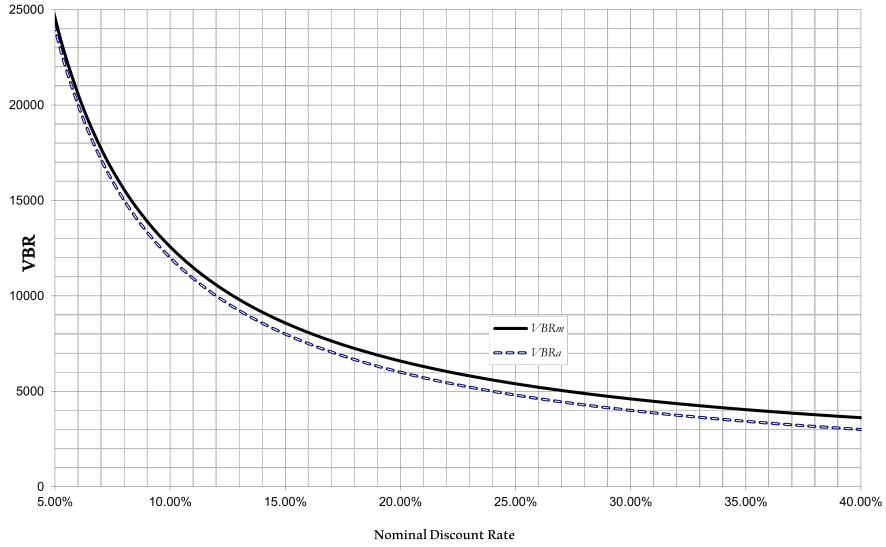

Figure 1: Value based on income considering monthly capitalization of rents (VBR_m) and without (VBR_a)

such circumstances, the discount rate should be adjusted based on the inflation rate. The net operating income is then capitalized at a real discount rate:

$$t_{ra} = \frac{1 + t_a}{1 + i} - 1; \tag{17}$$

When measuring real estate returns in real terms, adjusted for inflation, a combination of high economic growth and low inflation is generally more favorable than having high (or low) growth and high inflation. This is due to the direct relationship between the economy, i.e., rental demand/growth, and expectations for rental property growth. Additionally, higher inflation typically results in higher interest rates, increasing the cost of capital and slowing economic growth. Overall, profitability in the real estate sector is more sensitive to economic growth than to inflation.

3.3 Net Income

Net income, R_l, is equal to the actual gross income[9] minus operating expenses and sometimes capital expenses, replacement investments, maintenance, and rehabilitation costs.

[9]The potential gross income refers to the sum of all incomes generated by the property assuming it is fully utilized. The actual gross income is obtained by deducting uncollectible revenues and losses due to occupancy rates below 100%, and it can be considered as the investor's expectation of gross income [3].

Table 2: Coefficients for Calculation of Operating Expenses [7]

Property Type	Coefficient
Residential and Office	20%
Same with concierge services	30%
Other purposes	10%

3.3.1 Operating Expenses

Operating expenses are the costs that the tenant has to bear for the property to effectively fulfill the functions that justify the investment objective.

Among the operating expenses, there are those that are permanent, regardless of whether the operation is fully utilized or not. These are the fixed expenses. For example, fixed expenses may include costs for cleaning, security personnel, property management, insurance, property tax (MPT), municipal fees, etc.

On the other hand, there are variable expenses that depend on the level of utilization, such as costs for minor repairs, energy, maintenance and replacement works, etc. Although variable expenses, by definition, do not occur every year, they are sometimes considered evenly distributed over time for ease of analysis.

Quantifying projected operating expenses is usually very difficult, especially in market valuation. Under such circumstances, values resulting from the application of empirical coefficients can be used. For example, NEBREDA ET AL [7] suggest, in an expedient way, that operating expenses can be approximately obtained by applying coefficients to the actual gross income as shown in Table 3.3.1.

The reader can find a detailed breakdown of operating expense values in the Manual of Real Estate Valuation [3]. However, the following paragraphs highlight the most important ones:

Tax Costs are determined by the state according to social policies. The following are currently considered:

- Property Income Tax (IR)
- Municipal Property Tax (MPT)

Income Tax The personal income tax (IRS) rates applied to rental income in the European Union can vary from country to country, as tax rates and regulations are set by individual member states. There is no uniform rate that applies across the entire European Union.

Each country has its own tax laws and rates, so the specific rate applied to rental income will depend on the country where the rental income is earned. Generally, rental income is subject to progressive income tax rates, meaning

that the tax rate increases as the income level rises. Additionally, some countries may have specific tax deductions or allowances for rental income.

However, it is important to note that there are situations where reduced rates or exemptions may apply, depending on the specific circumstances of the taxpayer, the type of lease agreement, the location of the property, among other factors[10].

Municipal Property Tax Property taxes imposed by municipal authorities are common in many European countries. However, it's important to note that the specific name, regulations, and rates of the property tax can vary between countries and even within different municipalities within a country. Each country has its own tax system and local governance structure, which may influence how property taxes are implemented.

Property taxes are generally levied on real estate properties, including residential, commercial, and industrial properties. The rates and calculation methods can differ, with some countries basing the tax on the property's market value, while others use assessed values or other criteria.

In Portugal, the Municipal Property Tax (MPT) is a fee paid to the municipal entities of the area where the property is located. It is calculated based on a percentage of the value stated in the property register, known as the taxable asset value. It's worth noting that this value may not always coincide with the current value of the property, as updates typically occur during purchase and sale transactions. The MPT applies to rural, urban, and mixed properties, with different rates depending on the property's location and use.

For urban properties, including houses, apartments, and land for construction, the MPT rates in Portugal can range between 0.3% and 0.45% of the property's taxable asset value. For rural properties, such as agricultural land, the rates can range between 0.8% and 7.5% of the property's taxable asset value[11].

It is advisable to consult the specific tax laws and regulations of the country and municipality where the property is located to understand the details and obligations regarding municipal property taxes in a particular European location.

Sewage Conservation Fee The Sewage Conservation Fee (SCF) is a charge imposed by certain municipalities or utility companies to cover the costs associated with wastewater treatment and conservation efforts. It is typically applied to properties connected to the municipal sewer system.

The primary purpose of the fee is to provide funding for the operation, maintenance, and enhancement of sewage treatment plants and related infrastructure. It assists in offsetting the expenses incurred in treating and process-

[10] It should be noted that the taxable amount is obtained by deducting maintenance and conservation expenses from the gross income. It is recommended to consult the tax laws of the specific country within the European Union where the rental income is generated to determine the applicable tax rate.

[11] It is important to note that the MPT rates can be defined by the municipalities within ranges established by law, so the exact value of the rates may vary from one municipality to another.

ing wastewater to meet environmental standards before its discharge into water bodies or its reuse for irrigation or other purposes.

The specific name, regulations, and rates of the Sewage Conservation Fee can vary among municipalities and utility providers. Usually, it is determined based on factors such as the property's water usage or size. The fee may be listed as a separate line item on water bills or property tax bills.

To obtain a comprehensive understanding of the particulars and obligations associated with the Sewage Conservation Fee in a particular jurisdiction, it is advisable to consult the regulations and guidelines established by the local municipal or utility authorities.

In some municipalities, it is worth considering the Sewage Conservation Fee (SCF), which is typically around 1/8 of the Municipal Property Tax (MPT) rate[12].

In Portugal, for instance, the responsibility for paying the Sewage Conservation Fee (SCF) typically rests with the property owner, i.e., the landlord. However, it is important to note that financial responsibilities can be negotiated and stipulated in the lease agreement between the landlord and the tenant. In certain cases, the tenant may assume the responsibility of paying the SCF in addition to expenses related to water consumption and sanitation. Therefore, the exact determination of who bears the SCF payment may vary depending on the specific agreement outlined in the lease contract.

Operating Costs Operating costs, as the name suggests, result from maintenance costs and those associated with the set of operations necessary to keep the common areas that are the landlord's responsibility in operation. Costs that are the tenant's responsibility, such as cleaning and security, are not considered.

Maintenance and Conservation Costs The RICS (Royal Institution of Chartered Surveyors) provides indicators for building maintenance costs through the Building Maintenance Cost Information Service (BMCIS). The values are presented as a percentage of the new construction cost, C, based on the age and type of building, ranging from 0.1% to 1%.

These costs depend fundamentally on the type and age of the building. Generally, maintenance costs are higher in the early years of a building's life when it is necessary to correct anomalies that are only detected after a few months of operation. The costs decrease in the following years, for about 20 years, and then increase progressively more or less continuously until reaching the end of its useful life (50-60 years), as shown in Table 3.

[12] In Portugal, the SCF is implemented to finance sewage conservation and treatment services. Its rate varies from one municipality to another, as each city council has the autonomy to define its own rate. Generally, it is calculated based on water consumption as a percentage applied to the water bill. Typically, the fee ranges from 30% to 70% of the water bill, with variations depending on the municipality.

Table 3: Maintenance Costs – BMCIS

Building Type	**Age** (years)	T_m
Single-family houses	≤ 3	0.50%
	3 – 20	0.12%
	≥ 20	1.00%
Multi-family buildings with elevator	≤ 3	0.34%
	3 – 20	0.09%
	≥ 20	0.60%
Buildings with elevator: housing; commercial; public buildings	≤ 2	0.60%
	2 – 20	0.11%
	≥ 20	0.52%

Fire Insurance The premium for mandatory fire insurance results from competition between insurance companies and is based on the age and type of construction, total construction area, proximity to higher-risk areas, etc. The commercial premium is subject to a 3.9% fee that goes to the National Civil Protection Authority (ANPC) and Stamp Duty (IS) equal to 9% of that premium. In total, as an indicative measure, it can be said that the premium will vary between 0.1% and 0.5% of the insured capital and does not include the value of the land [3].

Net Income Calculation Once the operational and fiscal costs are known, the net income, R_l, can be expressed as the difference between the gross or gross income and the fiscal and operational costs:

$$R_l = R_i - (C_F + C_M); \tag{18}$$

The fiscal costs C_F are quantified as follows:

$$C_F = T_{IR} \times R_i + T_{IM} \times VBR; \tag{19}$$

where:

T_{IR} - Annual rate applied to the gross annual income;

T_{IM} - Annual rate that includes municipal taxes and is applied to the property value.

The annual operational costs related to maintenance, conservation, and insurance, C_M, are expressed as a function of the construction cost, C. Let T_M represent the annual relative value of these costs in relation to C. Then:

$$C_M = T_M \times C; \tag{20}$$

By substituting equations (19) and (20) into equation (18), we obtain the net income value, R_l:

$$R_l = R_i - (T_{IR} \times R_i + T_{IM} \times VBR + T_M \times C); \tag{21}$$

4 Blended Approach

The content presented can be considered a blended approach, aiming to compare, from a value perspective, the viewpoints of investors and/or landlords with those of users or tenants. It assesses the relative advantages between two options: buying (selling) or leasing.

Regarding lease agreements, the term 'landlord' refers to the property owner, also known as the 'lessor.' The landlord is the party that leases a property to another person, known as the tenant. The landlord holds the responsibility for the property, including its maintenance and compliance with legal and contractual obligations related to the lease. On the other hand, the tenant occupies the property and typically pays periodic installments over a specified period.

The 'landlord' refers to the property owner who, apart from being a real estate investor, may also be the developer[13].

In these instances, the company or real estate investor assumes the role of the landlord, being responsible for property management, lease agreement signings, and compliance with legal and contractual obligations, just like any other landlord.

The following sections present a methodology that enables quantifying what is referred to as the 'Relative Leasing Advantage' (VRA), from both the investor or landlord's perspective and the tenant's perspective.

4.1 VRA - Investor Perspective

In Equation 21, replace VBR with VBC, signifying that the property value based on cost equals the value based on income, VBR:

$$R_l = R_i - (T_{IR} \times R_i + T_{IM} \times VBC + T_M \times C); \tag{22}$$

Equation 22 captures the essence of the aforementioned blended method. The value based on income, VBR, is linked to the value based on cost, VBC. Assuming that the annual income is net, $R_a = R_i$, substituting its value into

[13] In some cases, real estate investment companies or developers act as landlords by leasing properties they own or have developed. Similarly, a real estate investor may acquire commercial properties such as offices or stores for lease purposes.

Table 4: Values of the example illustrated in Figure 2

VBC (€)	R_i (€)	t_a	T_{IR}	T_{IM}	T_M	τ_C
1,250,000	134,000	8.00%	20%	0.56%	0.50%	60%

Equation 16, we have[14]:

$$VBR = \frac{R_a \times t_{ae}}{t_a^2} \tag{16}$$

$$VBR = \frac{[R_i - (T_{IR} \times R_i + VBC \times T_{IM} + C \times T_M)] \times t_{ae}}{t_a^2} \tag{23}$$

This approach explicitly demonstrates the relative relationship between the value based on lease income, VBR, and the transaction value based on cost, VBC[15].

4.1.1 Relative Leasing Advantage

Defining the relative leasing advantage, VRA, as the ratio between the value based on income, VBR, and the value based on cost, VBC, we have:

$$VRA = \frac{(R_i - T_{IR} \times R_i - VBC \times T_{IM} + C \times T_M) \times t_{ae}}{t_a^2 \times VBC} \tag{24}$$

From the landlord's perspective, if $VRA > 1$, leasing is preferable to selling. Conversely, when $VRA < 1$, the landlord has an advantage in selling. Figure 2 represents the influence of the main parameters on this indicator. The comparison refers to a practical case based on the parameters defined in Table 4.

Minimum Rent Value for Leasing vs. Selling By setting $VRA = 1$ in Equation (24), we can obtain the minimum value of the gross rent, R_i, that the developer should request instead of the sale value, VBC:

$$R_i = \frac{\left(t_{ae} \times (T_{IM} + \tau_c \times T_M) + t_a^2\right) \times VBC}{t_{ae} \times (1 - T_{IR})} \tag{25}$$

Minimum Transaction Value for Selling vs. Leasing Similarly, by setting $VRA = 1$ in Equation (24), we can determine the minimum transaction value, VBC_{min}, based on a given rental value:

$$VBC_{min} = \frac{t_{ae} \times R_i\,(1 - T_{IR})}{t_{ae} \times (T_{IM} + \tau_c \times T_M) + t_a^2} \tag{26}$$

[14]Alternatively, using the relative construction cost τ_C according to Equation 10, we obtain: $VBR = \frac{[R_i - (T_{IR} \times R_i + VBC \times (T_{IM} + \tau c \times TM))] \times t_{ae}}{t_a^2}$.

[15]Note that the developer's profit margin is implicit in the case of a sale, while in the case of leasing, its profitability is implicit in the discount rate.

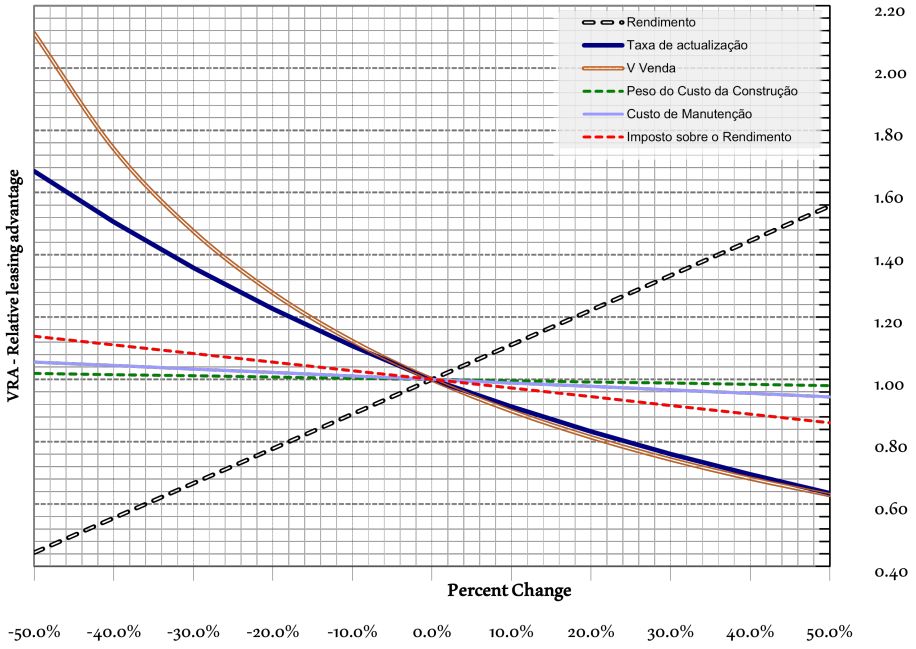

Figure 2: Variation of the relative leasing advantage for a practical case (cf. Table 4)

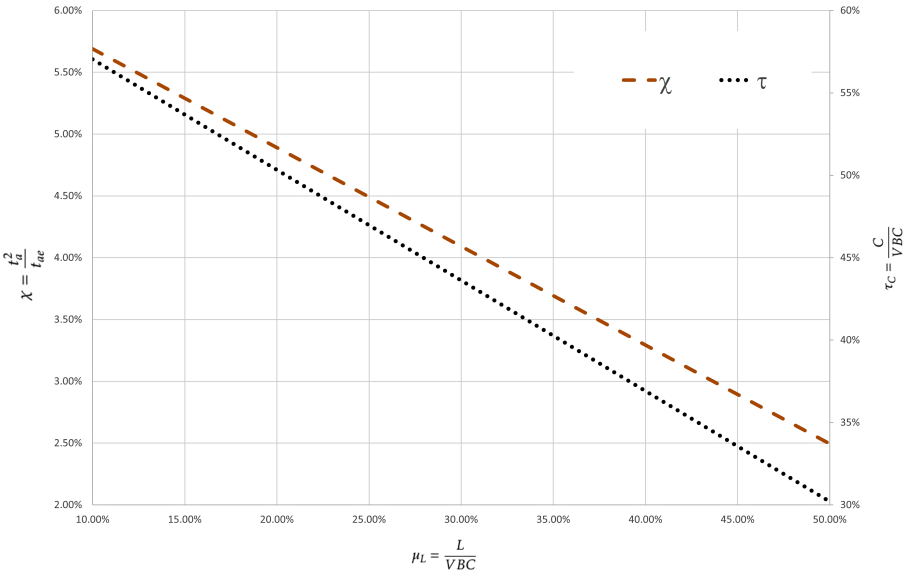

Figure 3: Minimum profit margin ν_L for $VRA = 1$

Minimum Profit Margin in Selling The minimum selling profit margin, as opposed to leasing, is obtained by equating the value based on cost, VBC, to the value based on income, VBR, as given in Equation (16), and explicitly expressing the value of the profit margin, λ_M, defined in Equation (9), which can be rewritten as:

$$VBC = \frac{T + C \cdot (1 + \kappa_K)}{1 - (\lambda_M + \mu_L)} \tag{9}$$

By equating the value based on cost, VBC, to the value based on income, VBR, in Equation (16), we have:

$$\frac{T + C \cdot (1 + k_k)}{1 - (\lambda M + \mu L)} - \frac{[R_i - (TIR \times R_i + VBC \times (TIM + \tau c \times T_M))] \times tae}{t_a^2} = 0 \tag{27}$$

By expressing ν_L, we obtain:

$$\nu L = 1 - \gamma M - \frac{[T + C \cdot (1 + k_k)] \cdot t_a^2}{tae \cdot [R_i - (R_i \cdot TIR + VBR \cdot (\tau C \cdot T_M + TIM))]} \tag{28}$$

Equation (28) allows us to determine the minimum percentage of profit, ν_L, for a landlord/investor to have an advantage in selling compared to the value based on income, i.e., when $VRA < 1$.

Figure 3[16] illustrates, as an example, the values of the minimum profit margin, ν_L, that a seller must have in order to compare to a gross annual rent, R_i, of € 84,000, with different discount rates ($2.5\% \lesssim t_a \lesssim 5.8\%$).

In the case of selling, the marketing cost is $\lambda_M = 5\%$, the construction cost is € 600,000[17], the cost of land with a project is € 216,000, and the indirect construction costs are estimated at € 144,000. The operational costs and rental rates are: $T_{IM} = 0.6\%$, $T_M = 0.5\%$, $T_{IR} = 15\%$.

To facilitate the explanation, we use χ to reference the discount rate[18] when considering the annual rent instead of the monthly rent:

$$\chi = \frac{t_a^2}{t_{ae}} = \frac{t_a^2}{\left(1 + \frac{t_a}{12}\right)^{12} - 1} \tag{29}$$

Maximum Discount Rates By setting Equation (28) to zero, we can determine the value of the discount rate χ for which the sale is carried out at the income-based value VBR without any loss, i.e., when $VRA \geq 1$.

$$\chi \geq \frac{[(R_i \cdot T_{IR} + VBR \cdot (\tau C \cdot T_M + TIM))] \cdot (\nu_L - 1 + \gamma_M) - R_i}{T + C \cdot (1 + k_k)} \tag{30}$$

Figure 4 represents the values of χ that correspond to the discount rates obtained from Equation (30)[19], for three scenarios derived from the example illustrated in Figure 3. Two additional cases were included, where the construction cost is € 700,000 and € 800,000 with $k = 14\%$ and 15% respectively.

The marked points – where the margin $\nu_L = 0$ – correspond to the values of χ on the ordinate axis[16].

4.2 VRA – Buyer or Tenant Perspective

Determining the relative advantage of leasing supports the decision between leasing and acquisition. This situation should be analyzed considering the possibility of financing, which requires considering (anticipating) many factors, including the financing cost rate and inflation.

The procedure consists of determining the net present value of the cash flows associated with each hypothesis and, from the tenant's perspective, choosing the one with the lowest value.

To do so, it is necessary to establish the cash flows that have occurred up to the moment of analysis and determine the corresponding relative advantage for each moment.

[16] On the ordinate axes, the left side represents the value of χ, and the right side represents the relative cost of construction, τ_C.

[17] With $57\% \lesssim \tau_C \lesssim 30\%$.

[18] Note that for discount rates $t_a < 10\%$, the rate χ is approximately equal.

[19] Using Equation 29.

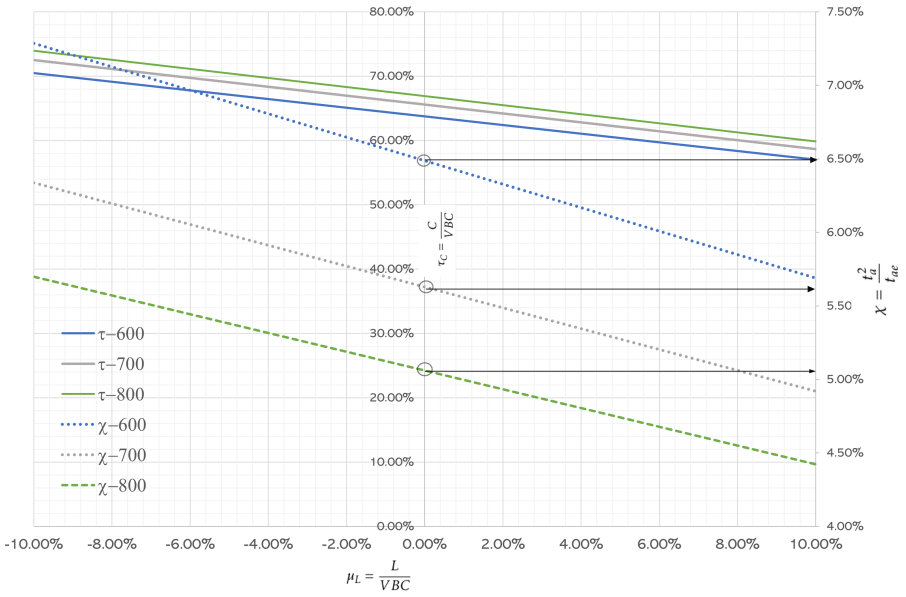

Figure 4: Limiting values of $\chi = t_a^2/t_{ae}$ for zero profit

Present value of rents For a given moment k, knowing the value of a given rent in the first period R_1 and the value of its real growth, let's determine the present value of rents $\mathrm{VA}_{(\mathrm{R})}$ at the end of k periods:

$$\mathrm{VA}_{(\mathrm{R})} = \sum_{j=1}^{k} R_1 \times (1 + t_{ra})^j \times (1 + t_a)^{-j}; \tag{31}$$

where t_{ra} is the real discount rate:

$$t_{ra} = \frac{(1 + t_a)}{(1 + i)} - 1; \tag{32}$$

with t_a as the current price discount rate and i as the inflation rate.

Present value of financial flows with a loan In financing for the purchase[20], the credit is mortgage-based, with the property value being the main loan guarantee.

The costs associated with financing include, in addition to the cost of money, expenses related to borrower and property insurance premiums, mortgage registry fees, etc.

These costs can be reflected in an overall annual loan cost rate t_{aeg}. In current cases in Europe, the most commonly used loan repayment system is the French system, which consists of constant-value post-dated installments P.

[20]Usually for owner-occupied housing.

Thus, for a given borrowed capital, D, the value of the installment P[21] for a given effective annual interest rate t_{aeg} and a given amortization period n is given by:

$$P = \frac{D \times t_{aeg}}{1 - \left(1 + t_{aeg}\right)^{-n}}; \tag{33}$$

The constant installment value is equal to the sum of the amortization value A and the interest value J. Therefore, at the end of each period k, the outstanding loan balance is obtained:

$$\begin{aligned} D_k &= D_{k-1} - A_k \\ D_k &= D_{k-1} - (P - J_k) \\ D_k &= D_{k-1} - (P - t_{aeg} \times D_{k-1}) \end{aligned} \tag{34}$$

In other words:

$$D_k = D - A_1 \cdot \left(\frac{\left(1 + t_{aeg}\right)^k - 1}{t_{aeg}} \right); \tag{35}$$

where A_1 is the amortization value in the first period.

The present value of financial flows, including financing, $\mathrm{PV}_{(\mathrm{E})}$ at the end of a given period k, corresponds to the present value of installments, minus the difference between the estimated transaction value of the property PVT at the end of period k at current values, and the outstanding loan balance $\mathrm{D_k}$ at that date:

$$\mathrm{PV}_{(\mathrm{E})} = P \cdot \frac{1 - (1 + ta)^{-k}}{t_a} - \frac{\mathrm{PVT}k - \mathrm{D_k}}{(1 + t_a)^k}; \tag{36}$$

where t_a is the current price discount rate.

The estimated value for the transaction at the end of period k, PVT_k, is equal to the purchase value PVT_1, considering the appreciation and inflation that have occurred in the meantime, assuming them to be constant over k periods:

$$\mathrm{PVT}_k = \mathrm{PVT}_1 \times \left(\frac{1 + p}{1 + i} \right)^k; \tag{37}$$

where p is the appreciation rate and i is the inflation rate.

Equation 36 corresponds to an acquisition made without using own capital. The percentage of financing is 100%.

However, in general, the financing entity "requires" participation in the investment with own capital - the difference between the purchase value, PVT1, and the borrowed capital D[22].

[21] D for *Debt* and P for *Payment*

[22] The percentage of financing corresponds to the ratio C_E/PVT1.

Table 5: Evaluation Values Based on Costs - Example 1

Construction	50%	12 T3	600,000
Land	T	18%	216,000
Indirect Costs	K	12%	144,000
Profit Margin	M	20%	240,000
VBC (€)			1,200,000

Under these conditions, the value of own capital, $(\mathrm{PVT1} - D)$, is introduced at the beginning of the cash flow, so that Equation 36 takes the form[23]:

$$\mathrm{PV}_{(\mathrm{E})} = P \cdot \frac{1-(1+ta)^{-k}}{t_a} + (\mathrm{PVT}_1 - D) - \frac{\mathrm{PVT}_k - \mathrm{D_k}}{(1+ta)^k}; \tag{38}$$

The value of the Relative Advantage of Renting compared to buying with a loan, at the end of a given period, k, VRA_{Ek} corresponds to the ratio between the value of Equation (38) and Equation (31).

$$VRA_{Ek} = \frac{\mathrm{VA}_{(\mathrm{E})}}{\mathrm{VA}_{(\mathrm{R})}} = \frac{P \cdot \frac{1-(1+t_a)^{-k}}{t_a} + (\mathrm{PVT}_1 - D) - \frac{\mathrm{PVT}_k - D_k}{(1+t_a)^k}}{\sum j = 1^k R_1 \times (1+t_{ra})^j \times (1+t_a)^{-j}}; \tag{39}$$

If $VRA_{Ek} \geq 1$, there is an advantage in renting; otherwise, it is beneficial to purchase with a potential loan. Note that the value of $VRA_{Ek} \geq 1$ may change signs in certain periods k.

4.3 Application Examples

Here are some examples to illustrate the analysis of the relative advantage of renting from the perspective of the investor or the tenant.

4.3.1 Example 1 - New Building

This concerns a recently completed building entirely dedicated to housing, with 12 T3-type units of approximately 120 m².

The investor wants to decide on the potential advantage of renting it instead of selling the apartments.

The risk of renting is that the rental income depends on the market situation, introducing some uncertainty in achieving a return. The risk premium is based on comparable situations in some European countries, so the discount rate is assumed to be 3.0% plus a risk premium of 6.0%.

Considering the construction costs and a post-tax profit margin of 20%, the following transaction value is obtained based on costs, summarized in Table 5:

[23] Recalling that D and Dk are the loan amount and the outstanding loan balance at the end of period k, respectively.

Table 6: Custos e taxas no arrendamento – Exemplo 1

t_a	T_{IR}	T_{IM}	T_M	τ_C
9,00%	15%	0,60%	0,50%	50%

The question is to find a minimum annual rent for the building, considering that:

- Annual maintenance costs are estimated at 0.5% of construction costs.
- The weight of construction in the value, τ_C, is 50%.
- From the investor's perspective, the profit margin, $\nu_L = 0$ (cf. Equation 30).
- The remaining data is presented in Table 6.

Using equation (25), the minimum value for the rent is obtained as follows:

$$R_i \geq \frac{\left(t_{ae} \times (T_{IM} + \tau_c \times T_M) + t_a^2\right) \times VBC}{t_{ae} \times (1 - T_{IR})} =$$

$$R_i \geq \frac{\left(0.09381 \times (0.006 + 0.5 \times 0,005) + 0,09^2\right)}{0.09381 \times (1 - 0.15)} \times 1\ 200\ 000 \approx 133\ 902\ €$$

In the case of an equal distribution of rent among the 12 units, the monthly rent per unit amounts to € 930, as opposed to a selling price of € 100,000.

4.3.2 Example 2 - Used Building

In this case, it is a building located in a consolidated area where there is some real estate pressure for land occupation.

The building is outdated but in good condition. It has 10 "identical" units rented for an average monthly value of € 400 per unit.

In this case, the investor considers the uncertainties in rent collection to be irrelevant. Therefore, they assign a risk premium of 2% to the investment and consider a discount rate of 5% appropriate. The data for the problem is presented in Table 7.

Table 7: Data for Example 2

t_a	T_{IR}	T_{IM}	T_M	τ_C
5.00%	20%	0.20%	0.60%	40%

The objective is to determine the potential sale value of the property.

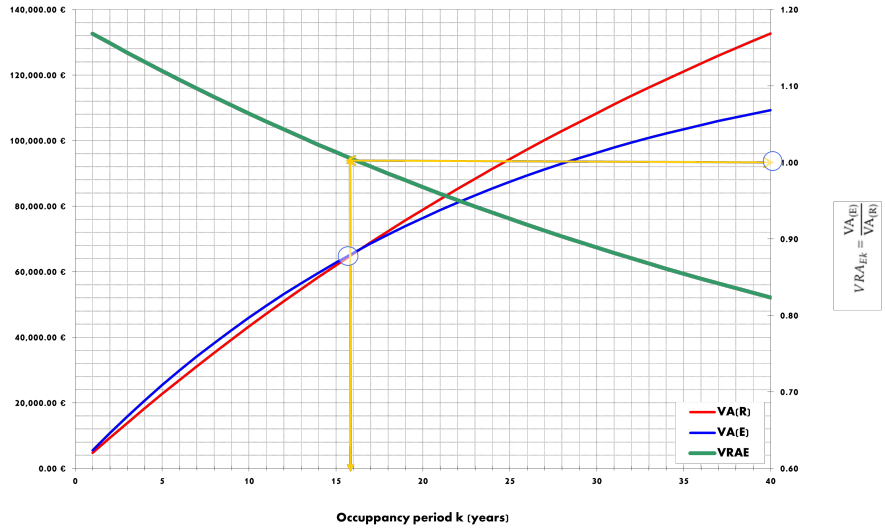

Figure 5: Relative advantage of leasing versus purchase with loan VRA_{Ek}

Equation (26) allows calculating the minimum transaction value based on the gross annual rent:

$$VBR = \frac{t_{ae} \times R_i\,(1 - T_{IR})}{t_{ae} \times (T_{IM} + \tau_c \times T_M) + t_a^2} = \frac{0,05116 \times 48\,000 \times (1 - 0,20)}{0,05116 \times (0,002 + 0,4 \times 0,006) + 0,05^2} = 720\,931€$$

Therefore, the average value assigned to each unit is approximately € 72,100.

4.3.3 Example 3 – Purchase with Loan or Lease

We present a study to analyze the advantage of potentially purchasing an apartment with a monthly rent of € 400.00, which increases at a constant rate of 2% per year.

We consider the option of buying the apartment using a mortgage loan. The bank loan, for a period of 40 years, incurs interest charges, insurance costs, and commissions, resulting in an annual effective interest rate of 5.6% per year. The loan amount corresponds to the value of the apartment: € 125,000.00.

Considering the location of the property, it is estimated to appreciate at a rate of 3%. However, for the studied period of 40 years, an inflation rate of approximately 2% per year is anticipated.

In these terms, the cash flows for each scenario are discounted at an annual rate of $t_a = 5\%$.

The obtained results are represented in Figure 4.

Note that the yield ratio between the annual gross rent and the sale value is 3.84

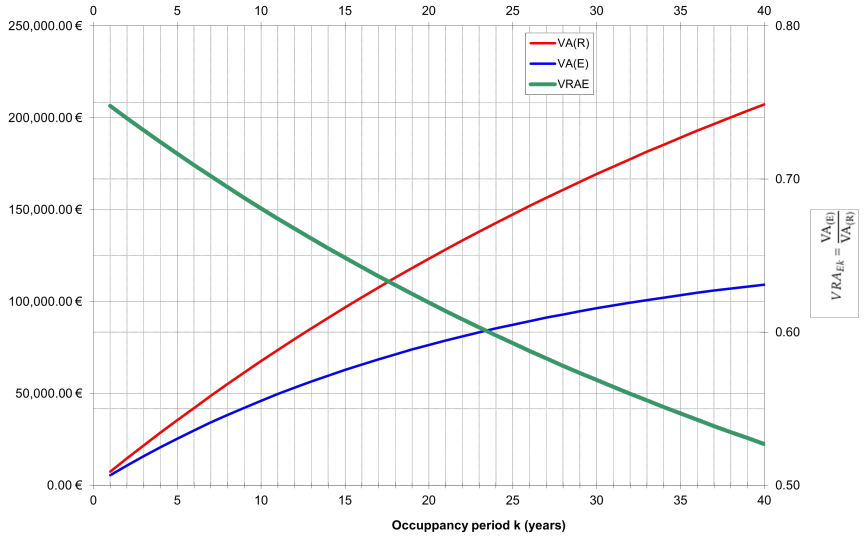

Figure 6: Relative advantage of leasing versus purchase with loan - $VRA_{Ek} < 1$ - yield equal to 6%.

It can be observed that only 16 years after the purchase does it become more advantageous to choose purchase with financing. If the occupancy period is shorter than that duration, it is more advantageous to opt for leasing.

This duration is marked and corresponds, on the x-axis, to the intersection of the discounted values (left y-axis) of the cash flows for purchase with loan and the rents, which corresponds to the unit value of relative advantage of leasing versus purchase and loan, VRA_{Ek}.

Now, let's assume that for the same apartment, the rent corresponds to a yield of 6.00%. The corresponding monthly gross rent would be:

$$R_i = \frac{125,000.0 \times 6\%}{12} = 625.00 \text{ €}$$

The result obtained is reflected in Figure 6. Comparing with the previous case (Fig. 5), regardless of the occupancy period, it is always preferable to purchase since $VRA_{Ek} < 1$.

Applying expression (**??**) for $k = 15$ years to the described situations for the apartment, the following values are found:

- When the monthly rent is € 400.00 $\rightarrow VRA_{15} = 1.01$
- If the rent is € 625.00 $\rightarrow VRA_{15} = 0.65$.

Let's assume again the rent value of € 400.00 and that the buyer takes out a loan of € 100,000.00 (80% of PVT). We want to determine the maximum

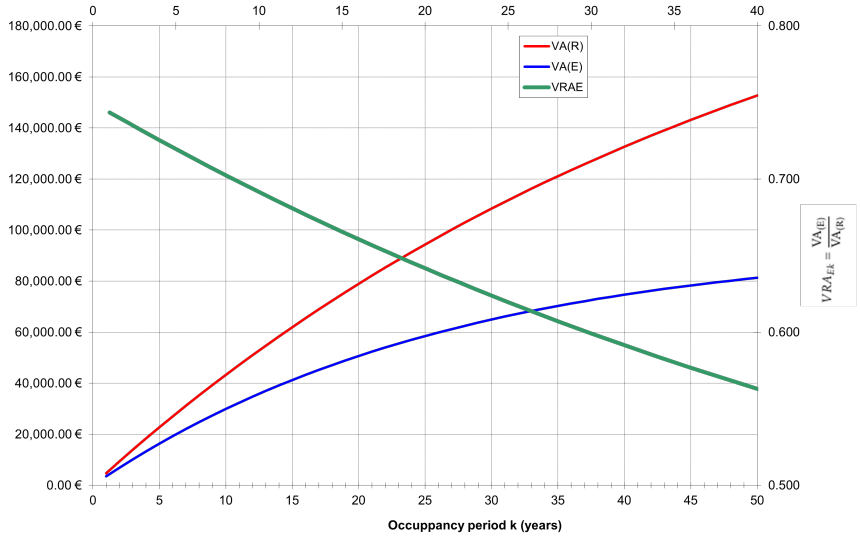

Figure 7: Relative advantage of leasing versus purchase with 80% financing of the purchase value under the same conditions as Figure 5

occupancy period for the advantage of purchase with loan to be superior to the leasing option.

$$\text{VRAEk} = \frac{\text{VA(E)}}{\text{VA}_{(\text{R})}} = \frac{P\dfrac{1-(1+t_a)^{-k}}{t_a} + (\text{PVT1} - C_E) - \dfrac{\text{PVT}k - C_k}{(1+t_a)^k}}{\sum j = 1^k R_1 \times (1+tra)^j \times (1+t_a)^{-j}} = 1$$

Substituting the values and using an iterative approximation method, we find that the value of k satisfying the condition is **14 years**, at which point the outstanding balance is € 85,412.00, and:

$$\text{VRAE}_{14} = \frac{58;279.39}{58;348.94} \approx 1.0$$

Figure 7 presents the different values of $\text{VRAE}_k = \dfrac{\text{VA}_{(E)}}{\text{VA}_{(R)}}$ for $1 < k \leq 40$.

Ultimately, the choice between renting or buying a property will depend on your individual circumstances and personal preferences. It is advisable to conduct careful analysis, considering all the factors mentioned above, and if necessary, seek professional advice before making such an important decision.

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