THE CONVERGENCE OF MANN ITERATION FOR GENERALIZED $\Phi-$HEMI-CONTRACTIVE MAPS

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ABSTRACT. Charles[1] proved the convergence of Picard-type iterative for generalized $\Phi-$accretive non-self maps in a real uniformly smooth Banach space. Based on the theorems of the zeros of strongly $\Phi-$accretive and fixed points of strongly $\Phi-$hemi-contractive we extend the results to Mann-type iterative and Mann iteration process with errors.

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1. INTRODUCTION

In[2], we can see a Mann-type iteration method for a family of hemi-contractive mappings in Hilbert spaces; In[3], we can see a Halpern-Mann type Iteration for fixed point problems of a relatively nonexpansive mapping and a system of equilibrium problems; In[4], we can see that convergence of Mann's type iteration method for generalized asymptotically nonexpansive mappings; In[5], we can see that some Mann-type implicit iteration methods for triple hierarchical variational inequalities, systems variational inequalities and fixed point problems; In[6], we can see a Mann-type iteration method for solving the split common fixed point problem; In[7], we can see that Mann and Ishikawa-type iterative schemes for approximating fixed points of multi-valued non-self mappings.

In 2009, Charles[1] proved the convergence of Picard-type iterative for generalized $\Phi-$accretive non-self maps in a real uniformly smooth Banach space. In this paper, we will consider to extend the result of Charles[1] to Mann-type iterative and Mann iteration process with errors.

In 1995, Liu[8] introduced what he called the Mann iteration process with errors.

In 1998, Xu[9] introduced the following alternative definitions:

Let K be a nonempty convex subset of E and $T : K \to K$ be any map. For any given , the process defined by

$$x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, \quad n \geq 0,$$

where $\{u_n\}$ is bounded sequences in K and the real sequences $\{a_n\}$, $\{b_n\}$, $\{c_n\} \subset [0, 1]$satisfy the conditions

$$a_n + b_n + c_n = 1, \quad \forall n \geq 0 .$$

It called the Mann iteration process with errors.

However, the most general Mann-type iterative scheme now studied is the following: $x_0 \in K$,

$$x_{n+1} = (1 - c_n)\,x_n + c_n T x_n,\ n = 0,\ 1,\ 2,\ ... \tag{1.1}$$

where $\{c_n\}_{n=1}^{\infty} \subset (0,1)$ is a real sequence satisfying appropriate conditions (see, e.g., Chidume[10], Edelstein and O'Brian[11], Ishikawa[12]). Under the following additional assumptions (i) $\lim c_n = 0$; and (ii) $\sum\limits_{n=0}^{\infty} c_n = \infty$, the sequence $\{x_n\}$ generated by (1.1) is generally referred to as the Mann sequence in the light of Mann[13].

2. Preliminaries

Definition 2.1. *[1] Let (E,ρ) be a metric space. A mapping $T : E \to E$ is called a contraction if there exists $k \in [0,1)$ such that $\rho\,(Tx,Ty) \le k\rho\,(x,y)$ for all $x, y \in E$. If $k = 1$, then T is called nonexpansive.*

Definition 2.2. *[1] Given a gauge function φ , the mapping $J_\varphi : E \to 2^{E^*}$ defined by*

$$J_\varphi x := \{u^* \in E^* : \langle x, u^* \rangle = \|x\|\ \|u^*\|\,;\ \|u^*\| = \varphi\,(\|x\|)\}$$

is called the duality map with gauge function φ where X is any normed space.

In the particular case $\varphi\,(t) = t$, the duality map $J = J_\varphi$ is called the normalized duality map.

Proposition 2.3. *[14] If a Banach space E has a uniformly Gateaux differentiable norm, then $J : E \to E^*$ is uniformly continuous on bounded subsets of E from the strong topology of to the weak*topology of E^*.*

Definition 2.4. *[15] A mapping $T : E \to E$ is called strongly pseudo-contractive if for all $x, y \in E$, the following inequality holds:*

$$\|x - y\| \le \|(1 + r)\,(x - y) - rt\,(Tx - Ty)\| \tag{2.1}$$

for all $r > 0$ and some $t > 1$. If $t = 1$ in inequality (2.1), then T is called pseudo-contractive. As we know that T is strongly pseudo-contractive if and only if

$$\langle (I - T)\,x - (I - T)\,y, j\,(x - y) \rangle \ge k\|x - y\|^2 \tag{2.2}$$

holds for all $x, y \in E$ and for some $j\,(x - y) \in J\,(x - y)$, where $k = \frac{1}{t}\,(t - 1) \in (0,1)$.

Definition 2.5. *[1] Recall that an operator $T : D\,(T) \subseteq E \to E$ is strongly accretive if there exists some $k > 0$ such that for each $x, y \in D\,(T)$, there exists $j\,(x - y) \in J\,(x - y)$ such that*

$$\langle Tx - Ty, j\,(x - y) \rangle \ge k\|x - y\|^2\ . \tag{2.3}$$

Proposition 2.6. *[1] A mapping $T : E \to E$ is strongly pseudo-contractive if and only if $(I - T)$ is strongly accretive, and is strongly $\varphi-$pseudo-contractive if and only if $(I - T)$ is strongly $\varphi-$accretive. Then T is generalized $\Phi-$pseudo-contractive if and only if $(I - T)$ is generalized $\Phi-$accretive.*

Definition 2.7. *[1] Let E be an arbitrary real normed linear space. A mapping $T : D(T) \subseteq E \to E$ is called strongly hemi-contractive if $F(T) \neq \emptyset$, and there exists $t > 1$ such that for all $r > 0$,*

$$\|x - x^*\| \leq \|(1 + r)(x - x^*) - rt(Tx - x^*)\| \tag{2.4}$$

holds for all $x \in D(T)$, $x^* \in F(T)$. If $t = 1$, then T is called hemi-contractive. Finally, T is called generalized $\Phi-$hemi-contractive, if for all $x \in D(T)$, $x^* \in F(T)$, there exists $j(x - x^*) \in J(x - x^*)$ such that

$$\langle (I - T)x - (I - T)x^*, j(x - x^*)\rangle \geq \Phi(\|x - x^*\|). \tag{2.5}$$

It follows from inequality (2.5) that is T generalized $\Phi-$hemi-contractive if and only if

$$\langle Tx - x^*, j(x - x^*)\rangle \leq \|x - x^*\|^2 - \Phi(\|x - x^*\|) \ , \ \ \forall n \geq 0. \tag{2.6}$$

Definition 2.8. *[1] Let $N(T) = \{x \in E : Tx = 0\} \neq \emptyset$. The mapping $T : D(T) \subseteq E \to E$ is called generalized $\Phi-$quasi-accretive if, for all $x \in E$, $x^* \in N(T)$, there exists $j(x - x^*) \in J(x - x^*)$ such that*

$$\langle Tx - Tx^*, j(x - x^*)\rangle \geq \Phi(\|x - x^*\|). \tag{2.7}$$

Proposition 2.9. *[1] If $F(T) = \{x \in E : Tx = x\} \neq \emptyset$,the mapping $T : E \to E$ is strongly hemi-contractive if and only if $(I - T)$ is strongly quasi-accretive; it is strongly $\varphi-$hemi-contractive if and only if $(I - T)$ is strongly $\varphi-$quasi-accretive; and T is generalized $\Phi-$hemi-contractive if and only if $(I - T)$ is generalized $\Phi-$quasi-accretive.*

Proposition 2.10. *[1] Let E be a uniformly smooth real Banach space, and let $J : E \to 2^{E^*}$ be a normalized duality mapping. Then*

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, J(x + y)\rangle$$

for all $x, y \in E$.

Proposition 2.11. *[1] Let $\{\lambda_n\}$ and $\{\gamma_n\}$ be sequences of nonnegative numbers and $\{\alpha_n\}$ be a sequence of positive numbers satisfying the conditions $\sum\limits_{n=1}^{\infty} \alpha_n = \infty$ and $\frac{\gamma_n}{\alpha_n} \to 0$, as $n \to \infty$. Let the recursive inequality*

$$\lambda_{n+1} \leq \lambda_n - \alpha_n \psi(\lambda_n) + \gamma_n, n = 1, 2, ...$$

be given where $\psi : [0, \infty) \to [0, \infty)$ is strictly increasing continuous function such that it is positive on $(0, \infty)$ and $\psi(0) = 0$. Then $\lambda_n \to 0$, as $n \to \infty$.

3. Main Results

In this section, we will consider to extend the result of Charles[1] to Mann-type iterative and Mann iteration process with errors under the following assumptions. First, we extend the result of Charles[1] to Mann-type iterative.

Theorem 3.1. *Suppose K is a closed convex subset of a real uniformly smooth Banach space E. Suppose $T : K \to K$ is a bounded generalized $\Phi-$hemi-contractive map with strictly increasing continuous function $\Phi : [0,\infty) \to [0,\infty)$ such that $\Phi(0) = 0$ and $x^* \in F(T) \neq \emptyset$. For arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ iteratively by*

$$x_{n+1} = (1 - c_n) x_n + c_n T x_n, n = 0, 1, 2, ... \tag{3.1}$$

where $\{c_n\} \subseteq (0, 1)$, $\lim c_n = 0$ and $\sum c_n = \infty$. Then, there exists a constant $d_0 > 0$ such that if $0 < c_n \leq d_0$, $\{x_n\}$ converges strongly to the unique fixed point x^* of T.

Proof. Let r be sufficiently large such that $x_1 \in B_r(x^*)$. Define $G := \overline{B_r(x^*)} \cap K$. Then, since T is bounded we have that $(I - T)(G)$ is bounded.

Let $M = \sup\{\|(I - T) x_n\| : x_n \in G\}$. As j is uniformly continuous on bounded subsets of E, for $\varepsilon := \frac{\Phi\left(\frac{r}{2}\right)}{2M}$, there exists a $\delta > 0$ such that $x, y \in D(T)$, $\|x - y\| < \delta$ implies $\|j(x) - j(y)\| < \varepsilon$. Set $d_0 = \min\left\{1, \frac{r}{2M}, \frac{\delta}{2M}\right\}$.

Claim1: $\{x_n\}$ is bounded.

Suffices to show that x_n is in G for all $n \geq 1$. The proof is by induction. By our assumption, $x_1 \in G$. Suppose $x_n \in G$. We prove that $x_{n+1} \in G$. Assume for contradiction that $x_{n+1} \notin G$. Then, since $x_{n+1} \in K$ $\forall n \geq 1$, we have that $\|x_{n+1} - x^*\| > r$. Thus we have the following estimates:

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq \|x_n - c_n (I - T) x_n - x^*\| \\ &\leq \|x_n - x^*\| + c_n \|(I - T) x_n\| \\ &\leq r + d_0 \cdot M \\ &\leq 2r \ , \end{aligned}$$

$$\begin{aligned} \|x_n - x^*\| &\geq \|x_{n+1} - x^*\| - c_n \|(I - T) x_n\| \\ &> r - c_n \cdot M \\ &> \frac{r}{2} \ , \end{aligned}$$

$$\begin{aligned} \|(x_{n+1} - x^*) - (x_n - x^*)\| &\leq c_n \|(I - T) x_n\| \\ &\leq c_n \cdot M \\ &\leq \frac{\delta}{2} < \delta \ , \end{aligned}$$

therefore,

$$\|j(x_{n+1} - x^*) - j(x_n - x^*)\| < \varepsilon \ .$$

Then from (3.1), the above estimates and Proposition 2.10 we have that

$$\begin{aligned}
\|x_{n+1}-x^*\|^2 &= \|x_n - c_n(I-T)x_n - x^*\|^2 \\
&\le \|x_n-x^*\|^2 - 2c_n\langle (I-T)x_n, j(x_n-x^*)\rangle \\
&\quad - 2c_n\langle (I-T)x_n, j(x_{n+1}-x^*) - j(x_n-x^*)\rangle \\
&\le \|x_n-x^*\|^2 - 2c_n\Phi(\|x_n-x^*\|) \\
&\quad + 2c_n\|(I-T)x_n\|\,\|j(x_{n+1}-x^*) - j(x_n-x^*)\| \\
&\le \|x_n-x^*\|^2 - 2c_n\Phi\left(\frac{r}{2}\right) + 2c_n\cdot M\cdot\varepsilon \\
&\le r^2 + 2d_0\left[\frac{\Phi\left(\frac{r}{2}\right)}{2} - \Phi\left(\frac{r}{2}\right)\right] \\
&\le r^2
\end{aligned}$$

i.e.,$\|x_{n+1}-x^*\| \le r$, a contradiction. Therefore $x_{n+1}\in G$. Thus by induction $\{x_n\}$ is bounded. Then, $\{Tx_n\}$, $\{(I-T)x_n\}$ are also bounded.

Claim2: $x_n \to x^*$.

Let $A_n = \|j(x_{n+1}-x^*) - j(x_n-x^*)\|$, Note that $x_{n+1}-x_n \to 0$ as $n\to\infty$ and hence by the uniform continuity of j on bounded subsets of E we have that

$$A_n \to 0 \text{ as } n\to\infty.$$

We obtain that

$$\begin{aligned}
\|x_{n+1}-x^*\|^2 &\le \|x_n-x^*\|^2 - 2c_n\Phi(\|x_n-x^*\|) \\
&\quad + 2c_n\|(I-T)x_n\|\,\|j(x_{n+1}-x^*) - j(x_n-x^*)\| \\
&\le \|x_n-x^*\|^2 + 2c_n\left[Z_n - \Phi(\|x_n-x^*\|)\right] \\
&\le \|x_n-x^*\|^2 - 2c_n\Phi(\|x_n-x^*\|) + 2c_nZ_n\ ,
\end{aligned} \tag{3.2}$$

where $Z_n = MA_n \to 0$ as $n\to\infty$.

Let $\lambda_n := \|x_n-x^*\|$ and $\gamma_n = 2c_nZ_n$, then from inequality (3.2) we obtain that $\lambda_{n+1}\le\lambda_n - 2c_n\Phi(\lambda_n)+\gamma_n$, where $\frac{\gamma_n}{c_n}\to 0$ as $n\to\infty$. Therefore, the conclusion of the theorem follows from Proposition 2.11. □

The following corollary follow trivially,since definition 2.5.

Corollary 3.2. *Suppose E is a real uniformly smooth Banach space, and $T:E\to E$ is a bounded generalized $\Phi-$accretive map with strictly increasing continuous function $\Phi:[0,\infty)\to[0,\infty)$ such that $\Phi(0)=0$ and the solution x^* of the equation $Tx=0$ exists. For arbitrary $x_1\in E$, define the sequence $\{x_n\}$ iteratively by*

$$x_{n+1} = (1-c_n)x_n - c_nTx_n, n=0,1,2,...$$

where $\{c_n\}\subseteq(0,1)$, $\lim c_n = 0$ and $\sum c_n=\infty$. Then, there exists a constant $d_0>0$ such that if $0<c_n\le d_0$, $\{x_n\}$ converges strongly to the unique solution of $Tx=0$.

Now, we consider to generalize to a more general case, we extend the result of Charles[1] to Mann iteration process with errors as follows.

Theorem 3.3. *Suppose K is a closed convex subset of a real uniformly smooth Banach space E. Suppose $T : K \to K$ is a bounded generalized $\Phi-$hemi-contractive map with strictly increasing continuous function $\Phi : [0, \infty) \to [0, \infty)$ such that $\Phi(0) = 0$ and $x^* \in F(T) \neq \emptyset$. For arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ iteratively by*

$$x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, n = 0, 1, 2, ... \tag{3.3}$$

where $\{u_n\}$ is bounded sequence in K and $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ are sequences in $[0, 1]$ satisfying the following conditions:

(i) $a_n + b_n + c_n = 1$, $\forall n \geq 0$;

(ii) $\sum_{n=0}^{\infty} b_n = \infty$;

(iii) $\lim b_n = 0$, $c_n = o(b_n)$;

(iv) $\sum_{n=0}^{\infty} c_n < \infty$.

Then, there exists a constant $d_0 > 0$ such that if $0 < b_n$, $c_n \leq d_0$, $\{x_n\}$ converges strongly to the unique fixed point x^* of T.

Proof. Let r be sufficiently large such that $x_1 \in B_r(x^*)$. Define $G := \overline{B_r(x^*)} \cap K$. Then, since T is bounded we have that $(I - T)(G)$ is bounded.

Let $M = \max\{\sup \|(I - T) x_n\|$, $\sup \|x_n - u_n\| : x_n \in G\}$. As j is uniformly continuous on bounded subsets of E, for $\varepsilon := \frac{\Phi(\frac{r}{2})}{4M}$, there exists a $\delta > 0$ such that $x, y \in D(T)$,$\|x - y\| < \delta$implies $\|j(x) - j(y)\| < \varepsilon$. Set $d_0 = \min\left\{1, \frac{r}{4M}, \frac{\delta}{4M}, \frac{\Phi(\frac{r}{2})}{8Mr}\right\}$.

Claim1:$\{x_n\}$ is bounded.

Suffices to show that x_n is in G for all $n \geq 1$. The proof is by induction. By our assumption, $x_1 \in G$. Suppose $x_n \in G$. We prove that $x_{n+1} \in G$. Assume for contradiction that $x_{n+1} \notin G$. Then, since $x_{n+1} \in K$ $\forall n \geq 1$, we have that $\|x_{n+1} - x^*\| > r$. Equation (3.3) becomes

$$x_{n+1} = x_n - b_n (I - T) x_n - c_n (x_n - u_n). \tag{3.4}$$

Thus we have the following estimates:

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq \|x_n - x^*\| + b_n \|(I - T) x_n\| + c_n \|x_n - u_n\| \\ &\leq r + d_0 (M + M) \\ &\leq 2r \, , \end{aligned}$$

$$\begin{aligned} \|x_n - x^*\| &\geq \|x_{n+1} - x^*\| - b_n \|(I - T) x_n\| - c_n \|x_n - u_n\| \\ &> r - d_0 (M + M) \\ &> \frac{r}{2} \, , \end{aligned}$$

$$\begin{aligned}\|(x_{n+1}-x^*)-(x_n-x^*)\| &\le b_n\|(I-T)x_n\|+c_n\|x_n-u_n\| \\ &\le 2d_0M \\ &\le \frac{\delta}{2}<\delta\ ,\end{aligned}$$

therefore,

$$\|j(x_{n+1}-x^*)-j(x_n-x^*)\|<\varepsilon\ .$$

Then from (3.4), the above estimates and Proposition 2.10 we have that

$$\begin{aligned}\|x_{n+1}-x^*\|^2 &= \|x_n-b_n(I-T)x_n-c_n(x_n-u_n)-x^*\|^2 \\ &\le \|x_n-x^*\|^2-2c_n\langle x_n-u_n, j(x_{n+1}-x^*)\rangle \\ &\quad -2b_n\langle (I-T)x_n, j(x_{n+1}-x^*)\rangle \\ &\le \|x_n-x^*\|^2+2c_n\langle x_n-u_n, j(x_{n+1}-x^*)\rangle \\ &\quad +2b_n\langle (I-T)x_n, j(x_{n+1}-x^*)-j(x_n-x^*)\rangle \\ &\quad -2b_n\langle (I-T)x_n, j(x_n-x^*)\rangle \\ &\le \|x_n-x^*\|^2-2b_n\Phi(\|x_n-x^*\|) \\ &\quad +2b_n\|(I-T)x_n\|\,\|j(x_{n+1}-x^*)-j(x_n-x^*)\| \\ &\quad +2c_n\|x_n-u_n\|\,\|x_{n+1}-x^*\| \\ &\le r^2+2d_0\left[M\cdot\varepsilon-\Phi\left(\frac{r}{2}\right)+2Mr\right] \\ &\le r^2+2d_0\left[\frac{\Phi\left(\frac{r}{2}\right)}{2}-\Phi\left(\frac{r}{2}\right)\right] \\ &\le r^2\end{aligned}$$

i.e.,$\|x_{n+1}-x^*\|\le r$, a contradiction. Therefore $x_{n+1}\in G$. Thus by induction $\{x_n\}$ is bounded. Then, $\{Tx_n\}$,$\{(I-T)x_n\}$ are also bounded.

Claim2:$x_n\to x^*$.

Let $A_n=\|j(x_{n+1}-x^*)-j(x_n-x^*)\|$, Note that $x_{n+1}-x_n\to 0$ as $n\to\infty$ and hence by the uniform continuity of j on bounded subsets of E we have that

$$A_n\to 0 \text{ as } n\to\infty.$$

We obtain that

$$\begin{aligned}\|x_{n+1}-x^*\|^2 \le& \|x_n-x^*\|^2+2b_n\|(I-T)x_n\|\|j(x_{n+1}-x^*)-j(x_n-x^*)\| \\ &-2b_n\Phi(\|x_n-x^*\|)+2c_n\|x_n-u_n\|\|x_{n+1}-x^*\| \\ \le& \|x_n-x^*\|^2+2b_nM\cdot A_n-2b_n\Phi(\|x_n-x^*\|)+2c_nM\cdot 2r \\ \le& \|x_n-x^*\|^2+2b_n\left[M\cdot A_n+2\frac{c_n}{b_n}Mr-\Phi(\|x_n-x^*\|)\right] \\ \le& \|x_n-x^*\|^2+2b_n\left[Z_n-\Phi(\|x_n-x^*\|)\right] \\ \le& \|x_n-x^*\|^2-2b_n\Phi(\|x_n-x^*\|)+2b_nZ_n\ ,\end{aligned} \tag{3.5}$$

where $Z_n=MA_n+2\frac{c_n}{b_n}Mr\to 0$ as $n\to\infty$.

Let $\lambda_n:=\|x_n-x^*\|$ and $\gamma_n=2b_nZ_n$, then from inequality (3.5) we obtain that $\lambda_{n+1}\le\lambda_n-2b_n\Phi(\lambda_n)+\gamma_n$, where $\frac{\gamma_n}{b_n}\to 0$ as $n\to\infty$. Therefore, the conclusion of the theorem follows from Proposition 2.11. □

The following corollary follow trivially,since definition 2.5.

Corollary 3.4. *Suppose E is a real uniformly smooth Banach space, and $T:E\to E$ is a bounded generalized $\Phi-$accretive map with strictly increasing continuous function $\Phi:[0,\infty)\to[0,\infty)$ such that $\Phi(0)=0$ and the solution x^* of the equation $Tx=0$ exists. For arbitrary $x_1\in E$, define the sequence $\{x_n\}$ iteratively by*

$$x_{n+1}=a_nx_n-b_nTx_n+c_nu_n, n\ge 0,$$

where $\{u_n\}$ is bounded sequence in K and $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ are sequences in $[0,1]$ satisfying

(*i*) $a_n+b_n+c_n=1\ ,\forall n\ge 0\,;$

(*ii*) $\sum\limits_{n=0}^{\infty}b_n=\infty\,;$

(*iii*) $\sum\limits_{n=0}^{\infty}b_n^2<\infty\,,\ c_n=o(b_n)\ ;$

(*iv*) $\sum\limits_{n=0}^{\infty}c_n<\infty.$

Then, there exists a constant $d_0>0$ such that if $0<b_n\ ,\ c_n\ \le d_0,\ \{x_n\}$ converges strongly to the unique solution of $Tx=0$.

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