# An empirical study of the risk-free rate and the expected consumption growth

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**Abstract**

This paper studies the relationship between the risk-free rate and the expected consumption growth. Using the monthly time series data from 2002.01-2017.12, we obtain the following empirical evidences: 1) In the whole period, US supports the positive intertemporal substitution effect and rejects the negative precautionary saving effect. Accordingly, China rejects the positive intertemporal substitution effect and supports the negative precautionary saving effect. 2) In the subsample period 2002.01-2008.12, US and China generate the consistent results and both support the CRRA asset pricing model. 3) The estimated time discount factors are 0.9995 and 0.9966 for US and China respectively. 4) US has a relative risk aversion than China both in the whole sample and subsample.

**JEL classification numbers:** G12, E21
**Keywords:** Risk-free rate; Consumption growth; Asset pricing; Intertemporal substitution; Precautionary saving

## 1 Introduction

The risk-free rate is an important factor in macroeconomics and finance. When people worry about the uncertainty of the economy or the future, the precautionary saving demand will rise. This will lead to the increase of the investment growth and the decrease of the consumption growth, and therefore result in the descending of the risk-free rate. The economic intuition is obvious that the risk-free rate comoves with the consumption growth in the same direction. From the perspective of consumption-based asset pricing theory, the relationship between them seems to be positive and linear. The purpose of this paper is to verify the effectiveness of the theory using the empirical time series data of US and China. Scholars have found some affecting factors of the risk-free rate, such as the GDP growth rate, the unemployment rate, the inflation rate, the capital

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marketization, the stock market return and volatility, the monetary policy and so on. We implement a detailed literature survey in section 2 and treat some of these factors as the control variable in our empirical setting of section 4.

In general, we need to consider three aspects of factors which are the economic fundamentals, the monetary policy and the capital market. In China, the interest rate liberalization is an important reform policy of the economy and now it is still under way. In the prophase and metaphase stage, the interest rate is expected to go up. In addition, within the economic transformation period the dependence of the investment, the real estate, and the land will dramatically decline. Then the demand of the capital will rise which will cause a high interest rate. We use the one-year government bond rate as the proxy of risk-free rate in the long run and use the inter-bank offered rate (IBOR) or the benchmark deposit rate as the proxy in the short run. Due to the rigid payment problem in China, using the three-month government bond rate will underestimate the risk-free rate after year 2010. Therefore, after 2010 some studies use the wealth-management products rate as the proxy. In US, there is a high degree of capitalization and the interest rate is market-oriented so there are less fluctuations and misestimations in the US risk-free rate. We use one-year government bond rate as the proxy of risk-free rate in the long run and three-month treasury bill rate in the short run.

We take the expected aggregate consumption growth of the whole economy as the independent variable. To estimate the expected consumption growth of this month, we apply last month's consumption growth as the substitute or the average of the last N months consumption growth as the alternative options (N could be 3, 6 or 12). The detail will be discussed it section 4 which is the empirical analysis section.

The software SAS is used for programming and implementing all the regressions. The basic empirical method or technique is the time series analysis and the sensitivity analysis. The method of instrumental variable is supposed to be used for solving the omitted variable problem or reciprocal causation problem.

The remaining of this article is organized as follows. Section 2 contains the literature review and the innovation of this paper. Section 3 displays the theoretical framework and the assumptions. Section 4 describes the data and presents the empirical results. Section 5 concentrates on the robustness check with subsample analysis. Section 6 concludes.

## 2 Literature review

### 2.1 Related theory and literature

Risk-free rate has been discussed in plenty of top journals such as JF, JFE, RSF, JFQA etc. These studies mainly focus on the risk-free rate puzzle that the CCAPM fails to interpret the low risk-free rate in US and on the term structure of interest rates. As for the study of consumption, the household consumption risk and the risk aversion are the hottest topics. There is a gap between the asset pricing theory and the empirical evidence from different countries. This paper tries to directly link the risk-free rate with the consumption growth under the controlling of other important variables. Some related papers are summarized as follows.

Lin and Jen (1980) constructed the theoretical model which linked the consumption, the investment, the market price and the risk-free rate all together. The model was a new version of CAPM, which showed that the risk-free rate is not an exogenous variable and, therefore, must be determined jointly with other endogenous variables. It is a good attempt to explain the risk-free rate and the consumption growth. Because of the lack of data in that period, the empirical evidence was hard to be provided. Lettau and Ludvigson (2001) used the quarterly data of consumption, wealth and found that the fluctuations in the consumption-wealth ratio are strong predictors of excess stock return over a Treasure bill rate. Since the return of risk asset can be predicted by the information of consumption, the risk-free rate should not be excluded in the prediction process. We follow their research and define the consumption as the nondurable goods and service including food and clothes. And we update the data to 2017. Constantinides and Ghosh (2017) showed that shocks to consumption growth are negatively skewed, persistent, countercyclical, and drive asset prices. There are also some studies focusing on the impact of the consumption on house prices, through some channels such as wealth effect (Carroll et al., 2011), mortgage effect (Compbell and Cocco, 2007), substitution effect (Sheiner, 1995) and so on. Whether these effects are existing in risk-free assets is an important research question for further studies.

The literature studying the main influence factors of the risk-free rate are as follows. Watcher (2006) developed a consumption-based model of the term structure of interest rate and discovered that nominal bonds depends on past consumption growth and on expected inflation. Chien and Lustig (2010) introduced limited liability in a model with a continuum of ex ante identical agents that face aggregate and idiosyncratic income risk and found the negative correlation between risk-free rate and the expected excess stock return and positive correlation with the Pstor-Stambaugh liquidity. Chen (2017) decomposed the risk-free rate in intertemporal substitution effect ($-E_t(m_{t+1})$) and the precautionary saving effect ($-\frac{1}{2}Var_t(m_{t+1})$). They found that the intertemporal substitution effect increased sharply in the bad times while the precautionary saving effect was much too small to smooth the risk-free rate.

The literature above contains mainly the research of the top finance journals which focus on the US stock market. There are also some Chinese studies concentrated on this topic but using the Chinese data to implement the empirical analysis. Wang (2002) analyzed Chinese consumption, risk-free rate and the stock index and showed that there are negative relations between stock index revenue rate and consumption growth rate while under the condition of non-marketization of interest rate in China, the changes of interest rate is not connected to consumption. Jing (2007) analyzed the risk-free rate and the time preference (β) theoretically. In their view, risk-free rate is the compensation for time preference, while risk return is the compensation for people's risk aversion characteristics. Li, Wang and Yang (2009) studied the Chinese risk-free rate and the households' consumption growth using the Chinese monthly data from 1998-2008. However, their sample was too small since they actually used the annual historical mean to estimate the asset pricing model and obtained a negative coefficient of relative risk aversion. Deng (2014) studied the asset pricing model with habit formation and empirically found that the risk-free rate of China from 1995-2011 was highly related to the consumption growth and volatility, the stock market return and volatility. These studies pointed out the important problem of the interest rate marketization reform in China. Empirical researches are still necessary to be carried forward in this field.

### 2.2 The contribution of this paper

This paper has four key contributions to the existing studies. Firstly, we fill the gap between classic consumption-based asset pricing theory and the empirical evidence on the relationship of risk-free rate and the consumption growth. Secondly, we use the historical mean of consumption growth to approximate the expected consumption growth and check the effectiveness of different expectation periods. Thirdly, we test the hypothesis and estimate the relative risk aversion coefficient and the time discount factor. This finding will provide some support on the CRRA utility function theory. Finally, we compare the US results with the Chinese results and find the inconsistence of the two countries in the whole sample and the consistence in the subsample. These findings may bring up some policy suggestions on the reform of China's interest rate liberalization.

## 3 Theoretical framework

According to the consumption-based asset pricing model (Cochrane 2005), an investor always targets at maximizing his total utility of today and the future as follows.

$$\mathrm{U}(c_t, c_{t+1}) = \mathrm{u}(c_t) + \beta E_t[\mathrm{u}(c_{t+1})] \tag{1}$$

Many scholars of asset pricing assume that the utility function takes the CRRA form for convenience.

$$\mathrm{u}(c_t) = \frac{1}{1-\gamma} {c_t}^{1-\gamma} \tag{2}$$

Where $\gamma$ is the coefficient of the relative risk aversion.

If the investor tries to maximize the total utility of the investor at time $t$ given the endowment $e_t$, he faces the following conditions.

$$\max_q \mathrm{U}(c_t, c_{t+1}) = \mathrm{u}(c_t) + \beta E_t[\mathrm{u}(c_{t+1})] \quad s.t.$$

$$\begin{cases} c_t = e_t - p_t q \\ c_{t+1} = e_{t+1} + x_{t+1} q \end{cases} \tag{3}$$

Where $p_t$ is the asset price at time $t$, $x_{t+1}$ is the sum of the asset price $p_{t+1}$ and the dividend $d_{t+1}$. The asset can be stocks, bonds, derivatives and so on. $q$ is the

quantity of the asset, we assume that the endowment is divided either into consumption or into investment.
We solve the first order condition and get the basic asset pricing condition which is

$$p_t = E_t[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}] \tag{4}$$

Where $m_{t+1} = \beta \frac{u^{'}(c_{t+1})}{u^{'}(c_t)} = \beta \frac{{c_t}^{\gamma}}{{c_{t+1}}^{\gamma}}$ is the stochastic discount factor or the pricing kernel.
By definition, the return of the asset is given by

$$R_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t} = \frac{x_{t+1}}{p_t} = r_{t+1} + 1 \tag{5}$$

So, we can rewrite the equation (4) as following form

$$E_t[m_{t+1}R_{t+1}] = 1 \tag{6}$$

Since the asset can be anything, we consider the risk-free asset and assume the risk-free rate is relatively stable as the time goes. We must have

$$E_t\left[m_{t+1}R_{t+1}^f\right] = 1 \tag{7}$$

$$R_{t+1}^f = \frac{1}{E_t[m_{t+1}]} = \frac{1}{E_t\left[\beta \frac{{c_t}^{\gamma}}{{c_{t+1}}^{\gamma}}\right]} \tag{8}$$

We assume that $\beta = \frac{1}{e^{\delta}}$, where $\delta$ is a positive parameter and close to zero, so $\beta$ is close to 1. Then equation (8) can be wrote as

$$R_{t+1}^f = \frac{1}{E_t\left[\beta \frac{{c_t}^{\gamma}}{{c_{t+1}}^{\gamma}}\right]} = \frac{1}{E_t\left[e^{\delta} e^{\ln(\frac{{c_t}^{\gamma}}{{c_{t+1}}^{\gamma}})}\right]} = \{E_t[e^{-\delta-\gamma\Delta lnc_{t+1}}]\}^{-1} \tag{9}$$

Where $\Delta lnc_{t+1} = \ln(\frac{c_{t+1}}{c_t})$ is the consumption growth and we additionally assume that it obeys the normal distribution.

$$\Delta lnc_{t+1} \sim Normal(\mu, \sigma^2) \tag{10}$$

Let $z = -\delta - \gamma \Delta lnc_{t+1}$, then z also obeys the normal distribution. According to the two formulas in econometrics, we know that

$$\begin{cases} \mathrm{E}(e^z) = e^{E(z)+\frac{1}{2}\sigma^2(z)} \\ \mathrm{Var}(e^z) = e^{2E(z)+\sigma^2(z)}\left(e^{\sigma^2(z)}-1\right) \end{cases} \tag{11}$$

Therefore equation (9) changes to

$$R_{t+1}^f = e^{\delta+\gamma E_t(\Delta lnc_{t+1})-\frac{1}{2}\gamma^2\sigma_t^2(\Delta lnc_{t+1})} \tag{12}$$

Since $R_{t+1}^f$ is the gross risk-free rate, then the risk-free rate is

$$r_{t+1}^f = \ln\left(R_{t+1}^f\right) \approx R_{t+1}^f - 1 \tag{13}$$

Combining equation (12) and (13), we obtain the relationship between risk-free rate and expected consumption growth described by equation (14)

$$r_{t+1}^f = \ln\left(R_{t+1}^f\right) = \delta + \gamma E_t(\Delta lnc_{t+1}) - \frac{1}{2}\gamma^2\sigma_t^2(\Delta lnc_{t+1}) \tag{14}$$

By now we have displayed the theoretical foundation of the relationship between risk-free rate and consumption growth. All those are based on five assumptions:
1) The CRRA utility function hypothesis of the representative investor.

2) The normal distribution of the consumption growth.
3) The endowment is divided either into consumption or the investment (no trading cost).
4) The time discount factor β has the form as $\beta = \frac{1}{e^{\delta}}$ and close to 1.
5) The stock market is efficient and investors are rational.
Equation (14) is the crucial theoretical foundation we try to verify. This CRRA model is just a benchmark asset pricing model since there are more complicated models such as the Campbell and Cochrane model, the Epstein and Zin model et al. In next section, we will describe the data and the empirical model setting. Then we will test the assumptions and discuss the empirical results.

# 4 Empirical analysis

### 4.1 Data sources

This article collects the US data from the WIND database and Amit Goyal's website (http://www.hec.unil.ch/agoyal/docs/PredictorData2017.xls) while the Chinese data is all from WIND database. All these data are monthly time series from 2002.01 to 2017.12 (192 months in total). The consumption data is the total sales of the nondurable goods and service for each month. We implement the seasonal adjustments. The US risk-free rate is three-month treasure bill rate while the Chinese risk-free rate is the one-year government bond return according to study on Chinese risk-free rate. Since Chinese data frequency of government bond return is daily, we use average of daily returns of one month as the risk-free rate and transform the annual rate to monthly rate. The S&P500 index and the Shanghai Composite Index are used to calculate the monthly stock return and volatility for US and China respectively.

### 4.2 Variables description

(1) Overview of the variables
The explained variable is the risk-free rate $R^f_{t+1}$, the explanatory variables are the expected consumption growth and the variance of the consumption growth, the control variables are the inflation rate, the expected return of stock market and the volatility of the stock return. We use 3 months, 6 months and 12 months historical mean of the consumption growth to estimate the expected consumption growth and apply the same method to the expected stock return. The most convenient way is to use last month's consumption growth as an approximate of the expected consumption growth. We will discuss it in the next subsection. Since the GDP is the endowment of the whole economy and consumption which already contains some information of GDP is directly related to the risk-free rate, so the GDP growth is not included in the control variables for this paper.
The expected consumption growth and the expected return of stock market and the corresponding variances take the following form.

$$E_t(\Delta lnc_{t+1}) = \frac{1}{T}\sum_{i=1}^{T} \Delta lnc_{t+1-i} \tag{15}$$

$$E_t(R_{t+1}) = \frac{1}{T}\sum_{i=1}^{T} R_{t+1-i} \quad (16)$$

$$\sigma_t^2(\Delta lnc_{t+1}) = \frac{1}{T}\sum_{i=1}^{T}(\Delta lnc_{t+1-i} - E_t(\Delta lnc_{t+1}))^2 \quad (17)$$

$$\sigma_t^2(R_{t+1}) = \frac{1}{T}\sum_{i=1}^{T}(\Delta lnR_{t+1-i} - E_t(R_{t+1}))^2 \quad (18)$$

T can be 1, 3, 6, 12 for different length of the expectation of the agent. Table 1 displays the description of the variables.

Table 1: Description of all the variables

| **Variable label** | **Variable expression** | **Explanation** |
|---|---|---|
| *Rfree* | $R_{t+1}^{f}$ | *Risk-free rate* |
| *Cg* | $\Delta lnc_{t+1}$ | *Consumption growth* |
| *E_Cg1* | $E_t(\Delta lnc_{t+1})$ *T=1* | *Expected consumption growth T=1* |
| *E_Cg3* | $E_t(\Delta lnc_{t+1})$ *T=3* | *Expected consumption growth T=3* |
| *E_Cg6* | $E_t(\Delta lnc_{t+1})$ *T=6* | *Expected consumption growth T=6* |
| *E_Cg12* | $E_t(\Delta lnc_{t+1})$ *T=12* | *Expected consumption growth T=12* |
| *V_Cg3* | $\sigma_t^2(\Delta lnc_{t+1})$ *T=3* | *Variance of consumption growth T=3* |
| *V_Cg6* | $\sigma_t^2(\Delta lnc_{t+1})$ *T=6* | *Variance of consumption growth T=6* |
| *V_Cg12* | $\sigma_t^2(\Delta lnc_{t+1})$ *T=12* | *Variance of consumption growth T=12* |
| *Rt* | $R_{t+1}$ | *Stock return* |
| *E_Rt1* | $E_t(R_{t+1})$ *T=1* | *Expected stock return T=1* |
| *E_Rt3* | $E_t(R_{t+1})$ *T=3* | *Expected stock return T=3* |
| *E_Rt6* | $E_t(R_{t+1})$ *T=6* | *Expected stock return T=6* |
| *E_ Rt 12* | $E_t(R_{t+1})$ *T=12* | *Expected stock return T=12* |
| *V_ Rt 3* | $\sigma_t^2(R_{t+1})$ *T=3* | *Variance of stock return T=3* |
| *V_ Rt 6* | $\sigma_t^2(R_{t+1}$ *T=6* | *Variance of stock return T=6* |
| *V_ Rt 12* | $\sigma_t^2(R_{t+1})$ *T=12* | *Variance of stock return T=12* |
| *infl* | *inflation* | *Inflation rate* |

(2) The descriptive statistics

The descriptive statistics of all the variables are shown in Table 2.1 and Table 2.2 below. Table 2.1 displays the descriptive statistics of all the variables for US data, while Table 2.2 displays the descriptive statistics of all the variables for Chines data.

Table 2.1: Descriptive statistics of all the variables for US data

| Variables | Size | Mean | Std | Min | Max | t-statistics |
|---|---|---|---|---|---|---|
| *Rfree* | 192 | 0.0010 | 0.0013 | 0.0000 | 0.0042 | 10.92 |
| *Cg* | 192 | 0.0029 | 0.0092 | -0.0374 | 0.0289 | 4.39 |
| *E_Cg1* | 192 | 0.0028 | 0.0092 | -0.0374 | 0.0289 | 4.22 |
| *E_Cg3* | 192 | 0.0028 | 0.0058 | -0.0326 | 0.0126 | 6.70 |
| *E_Cg6* | 192 | 0.0028 | 0.0043 | -0.0210 | 0.0092 | 8.92 |
| *E_Cg12* | 192 | 0.0028 | 0.0033 | -0.0102 | 0.0074 | 11.56 |
| *V_Cg3* | 192 | 0.0001 | 0.0001 | 0.0000 | 0.0016 | 6.34 |
| *V_Cg6* | 192 | 0.0001 | 0.0001 | 0.0000 | 0.0009 | 8.65 |
| *V_Cg12* | 192 | 0.0001 | 0.0001 | 0.0000 | 0.0005 | 11.62 |
| *Rt* | 192 | 0.0061 | 0.0409 | -0.1832 | 0.1042 | 2.06 |
| *E_Rt1* | 192 | 0.0059 | 0.0409 | -0.1832 | 0.1042 | 2.02 |
| *E_Rt3* | 192 | 0.0062 | 0.0256 | -0.1172 | 0.0767 | 3.34 |
| *E_Rt6* | 192 | 0.0058 | 0.0200 | -0.0903 | 0.0567 | 4.03 |
| *E_ Rt 12* | 192 | 0.0053 | 0.0145 | -0.0473 | 0.0358 | 5.10 |
| *V_ Rt 3* | 192 | 0.0010 | 0.0015 | 0.0000 | 0.0091 | 9.36 |
| *V_ Rt 6* | 192 | 0.0013 | 0.0014 | 0.0000 | 0.0075 | 12.61 |
| *V_ Rt 12* | 192 | 0.0015 | 0.0015 | 0.0001 | 0.0075 | 14.60 |
| *infl* | 192 | 0.0017 | 0.0039 | -0.0192 | 0.0122 | 6.13 |

Note: All the data are monthly time series from 2002.01-2017.12

As shown in Table 2.1, the average risk-free rate of one month is 0.10% with a standard deviation of 0.13%. The average consumption growth rate is 0.29% with a standard deviation of 0.92%. The average expected consumption growth rate is 0.28% with a standard deviation of 0.92%, 0.58%, 0.43%, 0.33% for the expectation periods T=1, 3, 6, 12 respectively. A moving average of longer horizon brings about smaller fluctuations for the expected consumption growth rate. The mean return of the stock market S&P 500 index of one month is 0.61% which means an annual return of 7.57%, and with a standard deviation of 4.09%. The average monthly inflation rate is 0.17% implying an annual inflation rate of 2.06% which is consistent with the circumstances of US economy. And the deviation is 0.39% monthly.

Table 2.2: Descriptive statistics of all the variables for Chinese data

| **Variables** | **Size** | **Mean** | **Std** | **Min** | **Max** | **t-statistics** |
|---|---|---|---|---|---|---|
| *Rfree* | 192 | 0.0021 | 0.0006 | 0.0008 | 0.0034 | 48.65 |
| *Cg* | 192 | 0.0112 | 0.0575 | -0.1536 | 0.1654 | 2.70 |
| *E_Cg1* | 192 | 0.0120 | 0.0585 | -0.1536 | 0.1654 | 2.84 |
| *E_Cg3* | 192 | 0.0120 | 0.0322 | -0.0861 | 0.0841 | 5.16 |
| *E_Cg6* | 192 | 0.0121 | 0.0195 | -0.0407 | 0.0554 | 8.57 |
| *E_Cg12* | 192 | 0.0119 | 0.0034 | 0.0020 | 0.0233 | 49.13 |
| *V_Cg3* | 192 | 0.0024 | 0.0027 | 0.0000 | 0.0154 | 11.95 |
| *V_Cg6* | 192 | 0.0030 | 0.0021 | 0.0007 | 0.0092 | 20.28 |
| *V_Cg12* | 192 | 0.0034 | 0.0009 | 0.0019 | 0.0076 | 51.98 |
| *Rt* | 192 | 0.0036 | 0.0809 | -0.2828 | 0.2425 | 0.62 |
| *E_Rt1* | 192 | 0.0033 | 0.0810 | -0.2828 | 0.2425 | 0.57 |
| *E_Rt3* | 192 | 0.0034 | 0.0520 | -0.1578 | 0.1410 | 0.91 |
| *E_Rt6* | 192 | 0.0030 | 0.0417 | -0.1265 | 0.1229 | 1.00 |
| *E_ Rt 12* | 192 | 0.0026 | 0.0322 | -0.1031 | 0.0980 | 1.12 |
| *V_ Rt 3* | 192 | 0.0038 | 0.0053 | 0.0000 | 0.0315 | 9.98 |
| *V_ Rt 6* | 192 | 0.0048 | 0.0048 | 0.0002 | 0.0190 | 14.10 |
| *V_ Rt 12* | 192 | 0.0056 | 0.0046 | 0.0005 | 0.0199 | 16.79 |
| *infl* | 192 | 0.0022 | 0.0060 | -0.0130 | 0.0260 | 5.08 |

Note: All the data are monthly time series from 2002.01-2017.12

As shown in Table 2.2, China is much different with US in many variables. The average risk-free rate of one month is 0.21% with a standard deviation of 0.06%. China has higher risk-free rate with lower volatility than US. The average consumption growth rate is 1.12% with a standard deviation of 5.75%. It is not surprising that the consumption growth rate is three times higher than US because of the high GDP growth rate in China (Over 7% each year). The average expected consumption growth rate is 1.20% with a standard deviation of 5.85%, 3.22%, 1.95%, 0.34% for the expectation periods T=1, 3, 6, 12 respectively. A moving average of longer horizon also brings about smaller fluctuations for the expected consumption growth rate and it is more obvious than US. The mean return of the stock market Shanghai composite index of one month is 0.36% which means an annual return of 4.41%, and with a standard deviation of 8.09%. The risk of the Chinese stock market is higher than US. The average monthly inflation rate is 0.22% implying an annual inflation rate of 2.67% which is consistent with the circumstances of Chinese economy. And the deviation is 0.60% monthly.

### 4.3 Model set up

The empirical model is based on the theoretical framework of asset pricing in section 3. Equation (14) is a benchmark relation between the risk-free rate and the expected consumption growth with the CRRA utility function. The risk-free rate is decomposed to two components which are intertemporal substitution effect and precautionary saving effect. For more general utility functions such as Campbell and Cochrane model and the Epstein and Zin model, the expected stock return (risk assets) is also an influence factor of the risk-free rate. Since the purpose of this paper is to provide some empirical evidences for the relationship between risk-free rate and the expected consumption growth rate, we treat the expected stock market return as one of the control variables. In addition, we take the volatility of the stock return into consideration. By the reason of using nominal variables (including risk-free rate and consumption growth rate), we also incorporate the inflation rate (derived by CPI) into control variables. Then the empirical model is set up as follows.

$$r_{t+1}^{f} = \beta_0 + \beta_1 E_t(\Delta lnc_{t+1}) + \beta_2 \sigma_t^2(\Delta lnc_{t+1}) + \gamma' X_{it} + \varepsilon_{t+1} \tag{19}$$

Where $r_{t+1}^{f}$ is the explained variable, $E_t(\Delta lnc_{t+1})$ and $\sigma_t^2(\Delta lnc_{t+1})$ are the explanatory variables, $\beta_1$ and $\beta_2$ are two coefficients respectively. The $X_{it}$ is the vector of control variables and $\gamma'$ is the vector of the coefficients. Table 3 displays the variable categories.

Table 3: Variable categories

| Variables | Variable categories | Variables contained |
|---|---|---|
| $r_{t+1}^{f}$ | explained variable | *Rfree* |
| $E_t(\Delta lnc_{t+1})$ | explanatory variables | *E_Cg1, E_Cg3, E_Cg6, E_Cg12* |
| $\sigma_t^2(\Delta lnc_{t+1})$ | explanatory variables | *V_Cg3, V_Cg6, V_Cg12* |
| $X_{it}$ | control variables | *E_Rt1, E_Rt3, E_Rt6, E_Rt12*<br>*V_Rt3, V_Rt6, V_Rt12, infl* |

### 4.4 Empirical results of US

The software SAS is used for the data cleaning, data arrangement and the regression process. The data is monthly time series from 2002.01 to 2017.12. We implement the regression with different expectation periods T=3, 6, 12. In general, significance of the coefficients increases dramatically when the expectation period

T goes up from 3 to 12. When T=1, the estimated coefficient is 0.0055 with a t-statistics of 0.54 which is not significant. Different options of regressions with control variables are included for comparison. The results are presented in Table 4.1, 4.2, 4.3 respectively.

Table 4.1: The regression on risk-free rate using US data when T=3

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0010*** | 0.0009*** | 0.0009*** | 0.0009*** | 0.0009*** | 0.0011*** |
| | (9.37) | (8.20) | (8.16) | (7.52) | (7.45) | (7.83) |
| *E_Cg3* | 0.0169 | 0.0174 | 0.0265 | 0.0100 | 0.0210 | 0.0070 |
| | (1.06) | (1.09) | (1.36) | (0.61) | (1.08) | (0.35) |
| *V_Cg3* | | 0.7646 | 0.7590 | 0.7089 | 0.6981 | 0.7691 |
| | | (1.09) | (1.08) | (1.02) | (1.00) | (1.13) |
| *E_Rt3* | | | -0.0036 | | -0.0046 | -0.0063 |
| | | | (-0.82) | | (-1.04) | (-1.44) |
| *V_Rt3* | | | | | | -0.1956*** |
| | | | | | | (-2.98) |
| *infl* | | | | 0.0442* | 0.0471* | 0.0387 |
| | | | | (1.83) | (1.94) | (1.62) |
| *N* | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0059 | 0.0121 | 0.0156 | 0.0294 | 0.0350 | 0.0541 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 4.1 indicate that the coefficients of *E_Cg3* and *V_Cg3* are all positive. For example, in column (2) the coefficients are 0.0174 and 0.7696 while the t-statistics are 1.09 and 1.09 respectively. In column (3) and (5), the effect of *E_Rt3* is negative which implies that the risk-free rate and the expected stock return move in the opposite direction. In column (4) and (5), the variable *infl* is significant in 10% level and moves in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt3* is also significant but it reduces the significance of explanatory variables by a large margin. Since it is not included in the equation of CCAPM model, this result is consistent with the model implication. The $R^2$ is improved from 0.0059 to 0.0541 with the control variables added gradually.

Table 4.2: The regression on risk-free rate using US data when T=6

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0009*** | 0.0008*** | 0.0008*** | 0.0007*** | 0.0007*** | 0.0013* |
| | (8.36) | (6.02) | (5.89) | (5.61) | (5.41) | (7.64) |
| *E_Cg6* | 0.0334 | 0.0421* | 0.0687** | 0.0364 | 0.0673** | 0.0307 |
| | (1.56) | (1.92) | (2.29) | (1.65) | (2.26) | (1.06) |
| *V_Cg6* | | 1.1939* | 1.0834 | 1.0067 | 0.9215 | 1.4241** |
| | | (1.68) | (1.52) | (1.50) | (1.29) | (2.10) |
| *E_Rt6* | | | -0.0084 | | -0.0100 | -0.0180*** |
| | | | (-1.30) | | (-1.54) | (-2.85) |
| *V_Rt6* | | | | | | -0.3709*** |
| | | | | | | (-5.03) |
| *infl* | | | | 0.0411* | 0.0456* | 0.0454** |
| | | | | (1.74) | (1.93) | (2.04) |
| $N$ | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0126 | 0.0272 | 0.0359 | 0.0427 | 0.0547 | 0.1679 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 4.2 indicate that the coefficients of *E_Cg6* and *V_Cg6* are still all positive and significant in column (2), (3) and (5). For example, in column (2) the coefficients are 0.0421 and 1.1939 while the t-statistics are 1.96 and 1.68 respectively. In column (3) and (5), the effect of *E_Rt6* is negative which implies that the risk-free rate and the expected stock return move in the opposite direction. In column (4) and (5), the variable *infl* is also significant in 10% level and moves in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt6* is also significant but it reduces the significance of explanatory variables by a large margin. For example, the t-statistics of *E_Cg6* decreases from 2.26 to 1.06 in column (5) and (6). Since it is not included in the equation of CCAPM model, this result is consistent with the model implication. The $R^2$ is improved from 0.0126 to 0.1679 with the control variables added gradually.

Compared with the case T=3, when T goes up from 3 to 6 the coefficients are significant now and the $R^2$ increases from 0.0541 to 0.1679.

Table 4.3: The regression on risk-free rate using US data when T=12

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0008*** | 0.0004*** | 0.0005*** | 0.0005*** | 0.0005*** | 0.0016*** |
| | (6.78) | (3.04) | (3.14) | (2.82) | (2.91) | (7.74) |
| *E_Cg12* | 0.0744*** | 0.1058*** | 0.1405*** | 0.0990*** | 0.1356*** | 0.0189 |
| | (2.70) | (3.59) | (3.59) | (3.34) | (3.47) | (0.50) |
| *V_Cg12* | | 2.3173*** | 1.8675** | 2.2043** | 1.7212* | 3.4137*** |
| | | (2.70) | (2.03) | (2.57) | (1.87) | (4.07) |
| *E_Rt12* | | | -0.0129 | | -0.0137 | -0.0262*** |
| | | | (-1.35) | | (-1.43) | (-3.06) |
| *V_Rt12* | | | | | | -0.5938*** |
| | | | | | | (-7.53) |
| *infl* | | | | 0.0369 | 0.0386* | 0.0440** |
| | | | | (1.61) | (1.68) | (2.19) |
| *N* | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0370 | 0.0727 | 0.0816 | 0.0853 | 0.0953 | 0.3067 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 4.3 indicate that the coefficients of *E_Cg12* and *V_Cg12* are all positive and significant at level of 1% in column (2), (3), (4), (5) and (6). For example, in column (2) the coefficients are 0.1058 and 2.3173 while the t-statistics are 3.59 and 2.70 respectively. In column (3) and (5), the effect of *E_Rt12* is negative which implies that the risk-free rate and the expected stock return move in the opposite direction. In column (4) and (5), the variable *infl* is significant in 10% level and moves in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt12* is also significant. However, it reduces the significance of explanatory variables even sharply. For example, the t-statistics of *E_Cg6* decreases from 3.47 to 0.50 in column (5) and (6). Since it is not included in the equation of CCAPM model, this result is consistent with the model implication. The $R^2$ is improved from 0.0370 to 0.3067 with the control variables added step by step.

Compared with the case T=6, when T goes up from 6 to 12 the coefficients are more significant now. The significant level changes from 5% to 1% while the $R^2$ increases from 0.1679 to 0.3067.

Since the case of T=12 is the best result, we may make some analyses regarding this result. Firstly, the CRRA model implies the intertemporal substitution effect is positive while the precautionary saving effect is negative. However, our empirical evidence supports the positive intertemporal substitution effect but rejects the negative precautionary saving effect. The estimation of the two corresponding coefficients are 0.1058 and 2.3173 both at 1% significant level. The estimation of coefficient of relative risk aversion γ is 0.1058 in column (2) and 0.1405 in column (3). It reveals that the risk aversion of US investors is relatively low. The estimation of the time preference parameter (the constant) is 0.0004, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9996$, which is consistent with the economic theory.

**4.5 Empirical results of China**

The process is the same with last subsection except the data is Chinese data. The data is monthly time series from 2002.01 to 2017.12. Similarly, we implement the regression with different expectation periods T=3, 6, 12. When T=1, the estimated coefficient is 0.0004 with a t-statistics of 0.56 which is also not significant for Chinese data. Different options of regressions with control variables are included for comparison. The results are presented in table 5.1, 5.2, 5.3 respectively. In general, significance of the coefficients increases dramatically when the expectation period T goes up from 3 to 12.

Table 5.1: The regression on risk-free rate using Chinese data when T=3

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0021*** | 0.0022*** | 0.0022*** | 0.0022*** | 0.0022*** | 0.0022*** |
| | (45.25) | (35.44) | (35.74) | (34.46) | (34.76) | (32.35) |
| *E_Cg3* | 0.0010 | 0.0008 | 0.0006 | 0.0003 | 0.0000 | -0.0003 |
| | (0.77) | (0.55) | (0.45) | (0.18) | (0.00) | (-0.18) |
| *V_Cg3* | | -0.0257 | -0.0233 | -0.0249 | -0.0222 | -0.0211 |
| | | (-1.60) | (-1.46) | (-1.55) | (-1.39) | (-1.33) |
| *E_Rt3* | | | -0.0017** | | -0.0018*** | -0.0023*** |
| | | | (-1.98) | | (2.10) | (-2.63) |
| *V_Rt3* | | | | | | -0.0185** |
| | | | | | | (-2.21) |
| *infl* | | | | 0.0077 | 0.0094 | 0.0115 |
| | | | | (0.99) | (1.22) | (1.50) |
| *N* | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0031 | 0.0165 | 0.0365 | 0.0216 | 0.0441 | 0.0685 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 5.1 indicate that the coefficients of *E_Cg3* and *V_Cg3* are positive and negative respectively but not significant. For example, in column (2) the coefficients are 0.0008 and -0.0257 while the t-statistics are 0.55 and -1.60 respectively. In column (3) and (5), the effect of *E_Rt3* is negative and significant at 1% level which implies that the risk-free rate and the expected stock return move in the opposite direction. In column (4) and (5), the variable *infl* is not significant but still move in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt3* is significant but it reduces the significance of explanatory variables by a large margin. Since it is not included in the equation of CCAPM model, this result is consistent with the model implication. The $R^2$ is improved from 0.0031 to 0.0685 with the control variables added gradually.

Table 5.2: The regression on risk-free rate using Chinese data when T=6

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0021*** | 0.0023*** | 0.0023*** | 0.0023*** | 0.0023*** | 0.0025*** |
| | (41.40) | (26.48) | (26.46) | (24.55) | (24.47) | (23.98) |
| *E_Cg6* | -0.0007 | -0.0023 | -0.0025 | -0.0026 | -0.0029 | -0.0028 |
| | (-0.31) | (-1.02) | (-1.08) | (-1.12) | (-1.23) | (-1.24) |
| *V_Cg6* | | -0.0603*** | -0.0593*** | -0.0560** | -0.0536** | -0.0583*** |
| | | (-2.80) | (-2.74) | (-2.46) | (-2.33) | (-2.61) |
| *E_Rt6* | | | -0.0008 | | -0.0010 | -0.0017 |
| | | | (-0.80) | | (-0.92) | (-1.58) |
| *V_Rt6* | | | | | | -0.0316*** |
| | | | | | | (-3.51) |
| *infl* | | | | 0.0046 | 0.0060 | 0.0068 |
| | | | | (0.58) | (0.75) | (0.87) |
| *N* | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0005 | 0.0402 | 0.0435 | 0.0420 | 0.0463 | 0.1056 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 5.2 indicate that the coefficients of *E_Cg6* and *V_Cg6* are both negative in column (2) to (6) but only *V_Cg6* is significant. For example, in column (2) the coefficients are -0.0023 and -0.0603 while the t-statistics are -1.02 and -2.08 respectively. In column (3) and (5), the effect of *E_Rt6* is negative which implies that the risk-free rate and the expected stock return move in the opposite direction. In column (4) and (5), the variable *infl* moves in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt6* is significant and it makes no difference on the significance of explanatory variables The $R^2$ is improved from 0.0005 to 0.1056 with the control variables added gradually.

Compared with the case T=3, when T goes up from 3 to 6 one of the coefficients is significant now and the $R^2$ increases from 0.0685 to 0.1056.

Table 5.3: The regression on risk-free rate using Chinese data when T=12

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| constant | 0.0023*** | 0.0034*** | 0.0034*** | 0.0035*** | 0.0034*** | 0.0036*** |
| | (14.25) | (11.61) | (11.57) | (11.68) | (11.61) | (12.58) |
| *E_Cg12* | -0.0144 | -0.0439*** | -0.0439*** | -0.0463*** | -0.0457*** | -0.0327** |
| | (-1.11) | (-3.13) | (-3.16) | (-3.29) | (-3.26) | (-2.39) |
| *V_Cg12* | | -0.2341*** | -0.2264*** | -0.2349*** | -0.2280*** | -0.2473*** |
| | | (-4.52) | (-4.40) | (-4.55) | (-4.43) | (-5.01) |
| *E_Rt12* | | | 0.0026** | | 0.0022* | 0.0015 |
| | | | (1.99) | | (1.67) | (1.16) |
| *V_Rt12* | | | | | | -0.0396*** |
| | | | | | | (-4.40) |
| *infl* | | | | 0.0107 | 0.0078 | 0.0060 |
| | | | | (1.54) | (1.09) | (0.88) |
| $N$ | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0064 | 0.1034 | 0.1220 | 0.1145 | 0.1276 | 0.2099 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

The results Table 5.3 indicate that the coefficients of *E_Cg12* and *V_Cg12* are both negative and significant at level of 1% in column (2), (3), (4), (5) and (6). For example, in column (2) the coefficients are -0.0439 and -0.2341 while the t-statistics are -3.13 and -4.52 respectively. In column (3) and (5), the effect of *E_Rt12* is now positive which implies that the risk-free rate and the expected stock return move in the different direction. In column (4) and (5), the variable *infl* moves in the same direction with risk-free rate which is consistent with the economic intuition. In column (6), *V_Rt12* is also significant. However, it reduces the significance of explanatory variables a little bit. The $R^2$ is improved from 0.0064 to 0.2099 with the control variables added step by step.

Compared with the case T=6, when T goes up from 6 to 12 both the coefficients are significant now. The significant level changes from no significance and 5% to 1% while the $R^2$ increases from 0.1056 to 0.2099.

Since the case of T=12 is the best result, we may make some analyses regarding this result. Firstly, the CRRA model implies the intertemporal substitution effect is positive while the precautionary saving effect is negative. However, our empirical evidence rejects the positive intertemporal substitution effect but supports the negative precautionary saving effect. The estimation of the two corresponding coefficients are -0.0439 and -0.2341 both at 1% significant level. The estimation of coefficient of relative risk aversion $\gamma$ is -0.0439 in column (2) and (3). It reveals that the risk aversion of Chinese investors is negative which means the risk preference of the Chinese investors. The estimation of the time preference parameter (the constant) is 0.0034, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9966$, which is consistent with the economic theory.

**4.6 The comparison of US and China**

In this sector, we put the results of US and China together in case of T=12 for comparison. Since the volatility of stock market return is not suitable to be the control variable both in theory and our empirical finding, we drop it and consider only the expected stock return and the inflation rate in the regressions. The comparison of US and China is showed at table 6.

Table 6: The comparison of US and China when T=12

| | US | | | China | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| constant | 0.0004*** | 0.0005*** | 0.0005*** | 0.0034*** | 0.0034*** | 0.0034*** |
| | (3.04) | (3.14) | (2.91) | (11.61) | (11.57) | (11.61) |
| *E_Cg12* | 0.1058*** | 0.1405*** | 0.1356*** | -0.0439*** | -0.0439*** | -0.0457*** |
| | (3.59) | (3.59) | (3.47) | (-3.13) | (-3.16) | (-3.26) |
| *V_Cg12* | 2.3173*** | 1.8675** | 1.7212* | -0.2341*** | -0.2264*** | -0.2280*** |
| | (2.70) | (2.03) | (1.87) | (-4.52) | (-4.40) | (-4.43) |
| *E_Rt12* | | -0.0129 | -0.0137 | | 0.0026** | 0.0022* |
| | | (-1.35) | (-1.43) | | (1.99) | (1.67) |
| *V_Rt12* | | | | | | |
| *infl* | | | 0.0386* | | | 0.0078 |
| | | | (1.68) | | | (1.09) |
| *N* | 192 | 192 | 192 | 192 | 192 | 192 |
| $R^2$ | 0.0727 | 0.0816 | 0.0953 | 0.1034 | 0.1220 | 0.1276 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2017.12.

From Table 6, we obtain some important results. Firstly, the two coefficients of expected consumption growth and the variance of consumption growth represent the intertemporal substitution effect and precautionary saving effect. In the theory of CRRA model, the two coefficients are supposed to one positive and one negative. But the empirical finding reveals that the two coefficients are both positive in US and both negative in China. And the coefficients are both significant at 1% level in US and China. US supports the positive intertemporal substitution effect and rejects the negative precautionary saving effect. China rejects the positive intertemporal

substitution effect and supports the negative precautionary saving effect.
Secondly, the estimation of the time preference parameter (the constant) is consistent with the theory both in US and China. The average estimation of the constant in US is 0.0005, implying the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9995$. The average estimation of the constant in China is 0.0034, implying the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9966$.
Thirdly, the average estimation of the coefficient of expected consumption growth is 0.1273 or -0.0445 for US and China respectively. It means that relative risk aversion $\gamma$ is 0.1273 or -0.0445 in US and China. US investor is risk averse but with a low degree while Chinese investor is a little bit risk preferred.
Fourthly, the risk-free rate comoves with the expected stock return in the opposite direction in US but the same direction in China. In addition, the risk-free rate comoves with the inflation rate in the same direction both in US and in China.
Finally, the $R^2$ increases from 0.0727 to 0.0953 in US and increases from 0.1034 to 0.1276 in China. It is nearly the same for the level of $R^2$ in US and China which is reasonable in the time series regressions.

## 5 Robustness check

In section 5, we perform the following robustness check with subsample analysis. We regress the basic regression (19) on the two subsamples 2002.01-2008.12 and 2009.01-2017.12 with the case of T=12.
The reason of why we perform the subsample regression is that we find the rapid decrease of the expected consumption growth rate in 2008 because of the 2008 financial crisis. This phenomenon is more obvious in US. Figure 1.1 displays the expected consumption growth in US and Figure 1.2 displays the expected consumption growth in China.

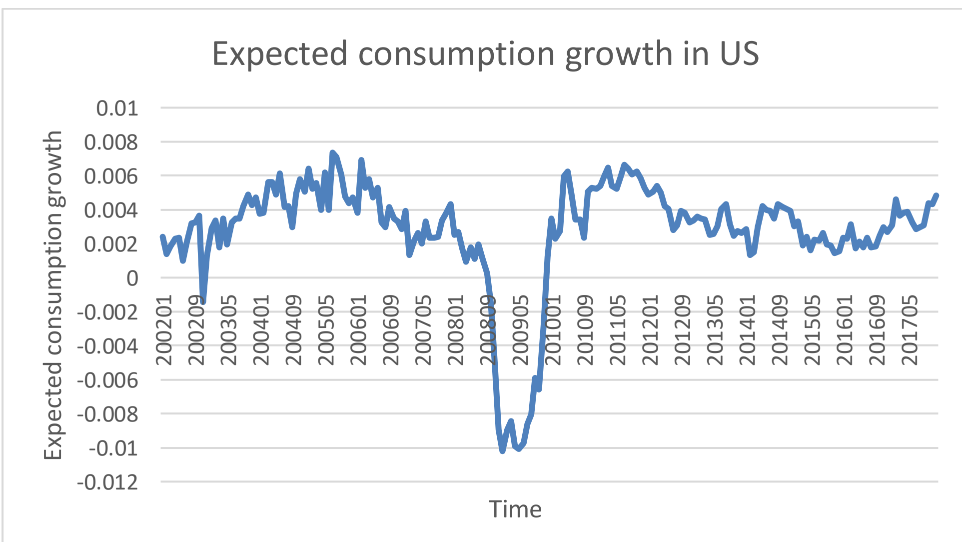


Figure 1.1: The expected consumption growth in US

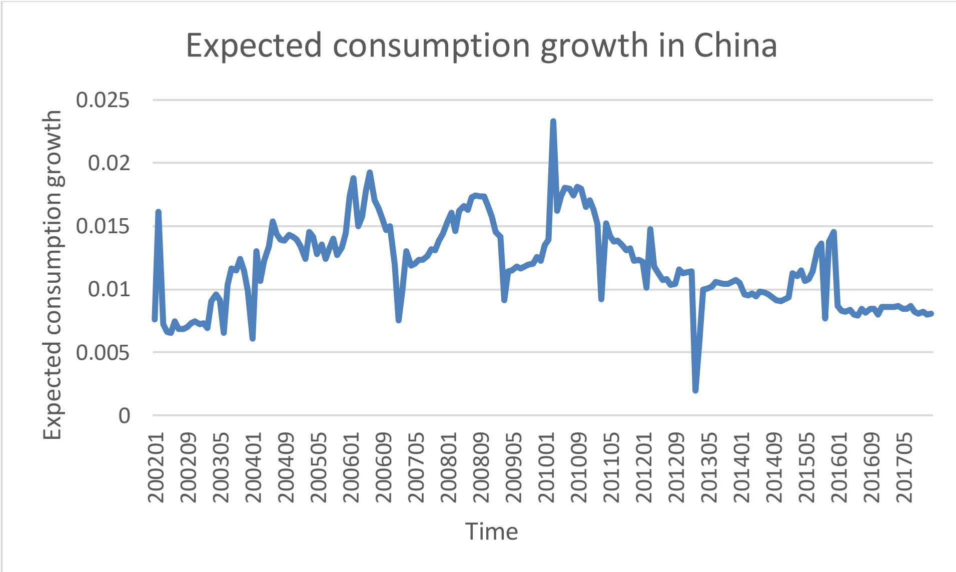


Figure 1.2: The expected consumption growth in China

From figure 1 and 2, we can see that the expected consumption growth of China is higher than US due to the rapid economic growth. What's more, the expected consumption growth decreases sharply in the year 2008, and reaches the bottom at 2008.12. To analyze this, we divide the sample into two samples which are 2002.01 to 2008.12 and 2009.01 to 2017.12. Table 7.1 shows the comparison of US and China when T=12 before 2008.12. Table 7.2 shows the comparison of US and China when T=12 after 2009.01

Table 7.1: The comparison of US and China when T=12 before 2008.12

| | US | | | China | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| constant | 0.0018*** | 0.0020*** | 0.0018*** | 0.0026*** | 0.0024*** | 0.0026*** |
| | (6.82) | (7.50) | (6.81) | (6.11) | (5.34) | (6.14) |
| *E_Cg12* | 0.1508*** | 0.0175 | 0.1374** | 0.0101 | 0.0169 | 0.0080 |
| | (2.72) | (0.24) | (2.23) | (0.52) | (0.85) | (0.41) |
| *V_Cg12* | -1.6642* | -0.3055 | -1.6824* | -0.1925*** | -0.1620*** | -0.1937*** |
| | (-1.70) | (-0.28) | (-1.71) | (-3.36) | (-2.67) | (-3.39) |
| *E_Rt12* | | 0.0368** | | | 0.0019 | |
| | | (2.61) | | | (1.44) | |
| *V_Rt12* | | | | | | |
| *infl* | | | 0.0152 | | | 0.0080 |
| | | | (0.51) | | | (1.15) |
| *N* | 84 | 84 | 84 | 84 | 84 | 84 |
| $R^2$ | 0.1254 | 0.1940 | 0.1281 | 0.2367 | 0.2561 | 0.2491 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2002.01 to 2008.12.

Table 7.1 shows that the coefficient of expected consumption growth is positive while the coefficient of the variance of consumption growth is negative which is consistent with the benchmark CRRA model. The results of China and US are consistent for this subsample. The difference between them is that in US the coefficient of the expected consumption growth is significant while in China the coefficient of the variance of consumption growth is significant. For US result, the average estimation of coefficient of relative risk aversion γ is 0.1018. The average estimation of the time preference parameter (the constant) is 0.0019, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9981$, which is consistent with the economic theory. The average $R^2$ is 0.1492. For Chinese result, the average estimation of coefficient of relative risk aversion γ is 0.0117. The average estimation of the time preference parameter (the constant) is 0.0025, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9975$, which is consistent with the economic theory. The average $R^2$ is 0.2473.

Table 7.2: The comparison of US and China when T=12 after 2009.01

| | US | | | China | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| constant | 0.0002*** | 0.0003*** | 0.0002*** | 0.0030*** | 0.0034*** | 0.0030*** |
| | (4.25) | (4.26) | (4.06) | (5.51) | (5.79) | (5.53) |
| *E_Cg12* | -0.0115 | -0.0188 | -0.0115 | -0.0669*** | -0.0748*** | -0.0701*** |
| | (-1.13) | (-1.19) | (-1.02) | (-3.17) | (-3.49) | (-3.30) |
| *V_Cg12* | -0.7471 | -0.8302* | -0.7459 | -0.0095 | -0.0925 | -0.0087 |
| | (-1.59) | (-1.69) | (-1.57) | (-0.08) | (-0.70) | (-0.07) |
| *E_Rt12* | | 0.0018 | | | 0.0044* | |
| | | (0.60) | | | (1.67) | |
| *V_Rt12* | | | | | | |
| *infl* | | | 0.0003 | | | 0.0151 |
| | | | (0.04) | | | (1.22) |
| *N* | 108 | 108 | 108 | 108 | 108 | 108 |
| $R^2$ | 0.0240 | 0.0274 | 0.0240 | 0.1025 | 0.1259 | 0.1151 |

Note: In parentheses is the t value, ***, **, and * represent significant levels at 1%, 5%, and 10% respectively. Data is monthly time series from 2009.01 to 2017.12.

Table 7.2 shows that the coefficients of expected consumption growth and the variance of consumption growth are both negative. The results of China and US are still consistent for this subsample. The difference between them is that the coefficient of the expected consumption growth is significant only in China. For US result, the average estimation of coefficient of relative risk aversion $\gamma$ is -0.0101. The average estimation of the time preference parameter (the constant) is 0.0002, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9998$, which is consistent with the economic theory. The average $R^2$ is 0.0754. For Chinese result, the average estimation of coefficient of relative risk aversion $\gamma$ is -0.0706. The average estimation of the time preference parameter (the constant) is 0.0031, which implies the discount factor $\beta = \frac{1}{e^{\delta}} = 0.9969$, which is consistent with the economic theory. The average $R^2$ is 0.1145.

These results of subsamples reveal that the financial crisis event may have a great impact on the estimation of the model coefficients. The model assumptions may not apply to all the periods. In these two subsamples, we find the consistent results between US and China. The empirical evidence supports the CRRA model mainly in the period 2002.01 to 2008.12.

# 6 Conclusion

We analyze the consumption-based asset pricing model with the CRRA utility function and decompose the risk-free rate to three parts which are the time discount factor, the intertemporal substitution, the precautional saving. We use the US and Chinese monthly data from 2002.01 to 2017.12, and obtain some empirical results for these two countries. The CRRA model is idealistic with five assumptions. But we still get some empirical evidence to support it from the macroscopic aspect.

In general, we find that when the expectation period T=12 the results are best since the seasonal volatility can be smoothed by 12-month average. The significance increases gradually when T goes up from 3 to 6 and then to 12. For the whole period sample, we have following results. In US, the coefficients are both positive which supports the positive intertemporal substitution effect and rejects the negative precautionary saving effect. In China, the coefficients are both negative which rejects the positive intertemporal substitution effect and supports the negative precautionary saving effect. The average risk aversion of US is 0.1273 while the average risk aversion of China is -0.0445. In addition, the average discount factor $\beta$ is 0.9995 or 0.9966 for US and China respectively. This result is reasonable and consistent with the theory.

As we focus on the two subsamples divided by the 2008 financial crisis, we observe some more interesting evidences. The results of US and China are consistent in both subsample periods. For the period 2002.01 to 2008.12, the estimation of the two coefficients are one positive and one negative both in US and China which is highly consistent with the CRRA model. The relative risk aversion $\gamma$ is 0.1018 or 0.0117 for US and China respectively. What's more, the average discount factor $\beta$ is 0.9981 or 0.9975 for US and China. For the period 2009.1 to 2017.12, the estimation of the two coefficients are both negative in US and China which rejects the CRRA model on the intertemporal substitution effect. The relative risk aversion $\gamma$ is -0.0101 or -0.0706 for US and China respectively. What's more, the average discount factor β is 0.9998 or 0.9969 for US and China. The results imply that the US investor has higher relative risk aversion than Chinese investor both in whole sample or subsample.

The empirical study on the risk-free rate and the expected consumption growth can bring about some evidence to support the classic consumption-based CAPM model and provide some references for the study of the famous risk-free rate puzzle. With the rapid progress of China marketization of interest rate, the relationship between risk-free rate and the expected consumption growth will be more consistent between US and China.

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