Machine learning and time series models for VNQ market predictions

Yu-Min Lian[a,*], Chia-Hsuan Li[a], Yi-Hsuan Wei[a]

Abstract

This study compares the price predictions of the Vanguard real estate exchange-traded fund (ETF) (VNQ) using the back propagation neural network (BPNN) and autoregressive integrated moving average (ARIMA) models. The input variables for BPNN include the past 3-day closing prices, daily trading volume, MA5, MA20, the S&P 500 index, the United States (US) dollar index, volatility index, 5-year treasury yields, and 10-year treasury yields. In addition, variable reduction is based on multiple linear regression (MLR). Mean square error (MSE), mean absolute error (MAE), and mean absolute percentage error (MAPE) are used to measure the prediction error between the actual closing price and the models' forecasted price. The training set covers the period between January 1, 2015 and March 31, 2020, and the forecasting set covers the period from April 1, 2020 to June 30, 2020. The empirical results reveal that the BPNN model's predictive ability is superior to the ARIMA model's. The predictive accuracy of BPNN with one hidden layer is better than with two hidden layers. Our findings provide crucial market factors as input variables for BPNN that might inspire investors in VNQ markets.

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1. Introduction

Real estate investment trusts (REITs) emerged in 1960 when the United States (US) Congress passed legislation permitting this new approach to real estate investment that broadened the channels through which well-capitalized investors could invest—directly through brokerages or indirectly through real estate stock. Since real estate could be securitized as a REIT that is listed and traded in a centralized market, investors without abundant assets could enjoy the profits enabled by centralized market trading, including property appreciation, higher market liquidity, a steady stream of rental income, portfolio diversification, and inflation protection.

After Box and Jenkins (1976) proposed ARIMA time series analysis, many researchers began a variety of studies. Krzemień et al. (2019) used the autoregressive integrated moving average (ARIMA) model to predict Colombian coal prices, with the results of a mean absolute percentage error (MAPE) assessment of the ARIMA model being less than 0.2%, indicating that the forecasted value was very close to the targeted price. Adebiyi et al. (2014) utilized ARIMA to predict the American and Nigerian stock indices and found that ARIMA was more suitable for short-term prediction. Lian and Liao (2015) applied the ICSS AR-GARCH models to analyze the volatility structure of oil futures market returns. The back propagation neural network (BPNN) is also used for prediction in many fields, and researchers have compared its predictive ability with other models. Hsieh et al. (2011) established that non-linearity is a universal phenomenon in financial markets, so applying BPNN to predict stock prices is more effective. Grudnitski and Osburn (1993) adopted BPNN to predict price changes in an average month of gold futures and the S&P 500; the results indicated that BPNN's accuracy rate for gold futures was 61% and for S&P 500 was up to 75%. Nazlioglu et al. (2020) performed research on the price and volatility interaction between the REITs of 19 countries and the crude oil market, and the results supported the significant influence of volatility. Ngo (2017) explored how variations in the US exchange rate impacted REIT returns. The results demonstrated that when US currency appreciated, REIT equity returns had the opposite variation. Chang (2017) investigated whether the REIT index could hedge against inflation risk; the results indicated that during a period of positive co-movement, the REIT index could partially hedge against inflation.

Since linear or non-linear relationships may exist in financial data, the ARIMA model, a type of time series analysis that is not affected by exogenous variables, was used. In addition, BPNN was applied to analyze non-linear data and predict the VNQ price. Both models have been used to make precise predictions of various financial

products. Therefore, the aim of this study is to investigate which model provides more valuable information to investors interested in REIT products. This study makes three major contributions. First, we model the behavior of VNQ prices by the BPNN and ARIMA models to understand the operation of the VNQ market. Our findings are valuable for the investment of other real estate-linked products for which the VNQ price dynamics are expected to follow the proposed models. Second, we apply the multiple linear regression (MLR) to conduct variable reduction and extract the statistically significant factors. Finally, we investigate the prediction performance of the BPNN and ARIMA models, compared with the actual VNQ data to conclude the best predictive model. The empirical results are significant for investors and for the organization of the VNQ market.

The remainder of this paper is organized as follows. Section 2 presents the principles of BPNN and the ARIMA model's methodology. Section 3 describes the empirical results. Section 4 concludes.

2. Methodology

2.1. Back propagation neural network (BPNN)
2.1.1. Principle of neuron

An artificial neural network (ANN) is composed of several input layers, one or more hidden layers, and an output layer. Each layer contains several homogeneous neurons, and includes a transfer function that calculates the output results from the input data, with weights representing the relationship importance and the data's correlation between each neuron. The formula for a neural network is as follows:

$$Y_j = (net) = f(\sum_{i=1}^{n} w_{ij} X_i - \theta_j), \tag{1}$$

where Y_j represents the target output, net represents the summation function, and X_i is the input variables. The term w_{ij} is the weight of j hidden layer neurons from i input variables of the input layer. θ_j indicates the bias of the hidden layer neurons, and f is the transfer function.

2.1.2. Framework of BPNN

BPNN is trained with supervised learning consisting of two steps—feed-forward and back propagation. In the feed-forward step, data are transferred from the hidden layer to the output layer. The network's predictive value is calculated using the gradient steepest descent method. When the predicted value differs from the result, the predictive value is subtracted from the target value to obtain a deviation, and then is back propagated to the network. In the back propagation step, the network adjusts the

connected weights and predicts values close to the target value to minimize the loss function. Figure 1 shows the ANN framework.

[Insert **Figure** 1]

2.1.3. Transfer function

The linear output of the former neuron can be transferred to non-linear output through the transfer function, which makes BPNN more capable of highly complex learning. The sigmoid function is widely used for the BPNN transfer function; its output value is between 0 and 1 and has continuous and differentiable characteristics. The sigmoid function is given by the following:

$$(net) = \frac{1}{1+exp^{-net}}, \tag{2}$$

2.1.4. Parameter setting for BPNN

The optimal number of hidden layers and neurons is determined by trial and error. The initial weights and thresholds are randomly produced by uniform distribution. The learning rate corrects the range of the weight adjustment when using the gradient steepest descent method. According to past experience, the learning rate is between 0.1 and 1.0, which creates a favorable convergence. To improve the vibration of the weight vectors or speed of convergence, neural networks usually utilize the momentum term to control the proportion of the weight variation. The value of the momentum term is between 0 and 1.

2.2. ARIMA Model

2.2.1. Definition of ARIMA model

There are three components of the ARIMA model—autoregressive process (AR), integrated process (I), and moving average process (MA). By setting the order of AR(p), the degree of differencing (d), and the order of MA(q), the ARIMA(p,d,q) model is constructed.

I.   Autoregressive process (AR)

The AR model depends linearly on the previous terms, which indicates an autoregressive model of order p. The AR(p) model is defined as follows:

$$y_t = a_0 + \sum_{i=1}^{p} a_i \, y_{t-i} + \varepsilon_t, \tag{3}$$

where a_0 is the intercept of the constant term, p is the order of lagged value, a_i is the coefficient of y_{t-i}, and ε_t is white noise.

II.   Integrated process (I)

Before we conduct time series analysis, the input variables must be stationary. If we perform regression analysis with non-stationary variables, it may produce spurious regression results (Granger and Newbold, 1974). Most financial data are not completely stationary, but we can make them stationary with differencing. This is called an integrated process and denoted by I(d). The I(d) model can be expressed as follows:

$$\Delta X_t = X_t - X_{t-1}, \tag{4}$$

where ΔX_t is first-order difference and $\Delta^d X_t$ indicates d-order differences.

III. Moving average process (MA)

The MA model has a similar structure to the AR model and is comprised linearly of past white noise terms, which is a linear regression model of order q. The MA(q) model can be written as follows:

$$y_t = b_0 + \sum_{i=1}^{q} b_i \, \varepsilon_{t-i} + \varepsilon_t, \tag{5}$$

where b_0 is the intercept of the constant term, q is the order of lagged value, b_i is the coefficient of ε_{t-i}, and ε_t is white noise.

2.2.2. Building the ARIMA model

Time series data must be tested by unit root tests to ensure they are stationary. We applied three unit roots tests—the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS). If there are unit roots in time series y_t, this indicates that y_t has a stochastic trend, indicating that the data are non-stationary and must be processed with differencing. The next step is to decide the degree of differencing and transform the stochastic series into a stationary one. After the integration process, the subsequent step is the white noise test. A Ljung–Box (LB) test is used and the null hypothesis is a random series, testing whether the variables of each term are autocorrelated. If no autocorrelation is indicated, then forecasting with the ARIMA model can proceed. The fourth step is to find the best value of AR(p) and MA(q). In this phase, it is popular to apply the autocorrelation function (ACF) and partial correlation function (PACF) and identify the lag orders of the ARIMA model with ACF and PACF plots. We use ACF and PACF plots to obtain order q of MA and order p of AR. ACF and PACF will converge to zero after a certain q and p number of lags, but when the lag orders of AR(p) and MA(q) are discontinuous or oscillating decay occurs, determining the best value of p and q by plots is not easy. Table 1 presents the criterion of identifying order p of the AR process and order q of the MA process with ACF and PACF plots.

[Insert **Table** 1]

2.2.3. Model selection

When more than two models are selected during the ARIMA model-building process, we need to compare their goodness-of-fit and forecasting ability to determine the optimum model. Therefore, this study adopts the Akaike information criterion (AIC) and Schwartz Bayesian information criterion (BIC). The model with the minimum criteria value will be selected and viewed as exhibiting a better fit. This enables any misjudgment of the values of p and q using ACF and PACF plots to be avoided, as it is a more objective method.

2.2.4. Performance assessment

To evaluate and compare the performance of different models, forecasting key performance indicators (KPIs)—including MSE, mean absolute error (MAE), and MAPE—are applied. They all calculate the difference between the actual and predicted values. A lower estimator of MSE, MAE, and MAPE reflects better accuracy in model prediction.

3. Empirical Analysis

We apply BPNN and time series models to predict the VNQ price trend and evaluate prediction performance using MSE, MAE, and MAPE. The training set covers January 1, 2015–March 31, 2020, and the forecasting set covers April 1, 2020–June 30, 2020. The data of BPNN and ARIMA models refer to trading information during the sample period provided by Yahoo! Finance.com.

3.1. Back propagation neural network (BPNN)

3.1.1. Input variables

Table 2 presents the 11 input variables selected for BPNN and their descriptions. MLR was used to conduct analysis, eliminating the input variables that exhibited non-significant correlations with the daily VNQ closing price. Table 3 indicates that variables X_1, X_6, X_8 are statistically non-significant. Therefore, we retain variables $X_2, X_3, X_4, X_5, X_7, X_9, X_{10}, X_{11}$ to proceed with the follow-up analysis.

[Insert **Tables** 2–3]

3.1.2. Structure of BPNN

With respect to parameters setting of BPNN, the training times is 2,000, sigmoid function is used as a transfer function, testing five learning rates: 0.1, 0.3, 0.5, 0.7, and 0.9 and three momentum factors: 0.5, 0.7, and 0.9 using the trial and error method to

select the best parameters and predictive values for the BPNN models.

On the basis of an empirical rule, hidden layers are set within 1~2 of BPNN have the best convergence effect. Therefore, we used trial and error on 1 and 2 hidden layers. For the 1-hidden layer model, we set 1~14 hidden layer nodes; for the 2-hidden layer model, 1~14 hidden layer nodes under the first hidden layer and 2~7 hidden layer nodes under the second hidden layer. Table 4 illustrates the parameter settings of hidden layers and hidden layer nodes.

[Insert **Table** 4]

3.1.3. Output variables

The output variables for the BPNN model are the VNQ's closing stock prices.

3.1.4. Performance assessment of BPNN

We implement MSE, MAE, and MAPE as performance indicators to assess BPNN's forecasting ability. The lower the estimators yielded by MSE, MAE, and MAPE, the closer the predictive and actual prices are, which indicates a better goodness-of-fit for the BPNN we trained. Under the MSE, MAE, and MAPE criteria, Tables 5–7 indicate the top three BPNN models with one hidden layer; Tables 8–10 present the top three BPNN models with two hidden layers. BPNN with one hidden layer exhibits a lower predictive error than the 2-hidden layer model, indicating that the forecast ability of the 1-hidden layer BPNN model is better than that with two hidden layers.

Figures 2 and 3 present run charts of 1- and 2-hidden layer BPNN models during the training period. We select the BPNN model with the best predictive power; the network structure with one hidden layer is (08-04-01), with a learning rate of 0.5 and a momentum factor of 0.9. As for BPNN with two hidden layers, the best structure is (08-02-05-01), with a learning rate of 0.5 and a momentum factor of 0.7. Overall, both the 1- and 2-hidden layer BPNN models' goodness-of-fit are excellent during the training period.

[Insert **Tables 5–10 and Figures** 2–3]

3.2. Estimated results from the ARIMA model
3.2.1. Unit root tests

ADF, PP, and KPSS were used to test if the data series was stationary. The initial data series was not stationary and could not be used. After first-order differencing, the data series appeared stationary. Table 11 demonstrates the unit root test results.

3.2.2. LB test

According to Table 12, the p-value is less than the 0.05 confidence level. As a result, the I(1) series meets the autocorrelation condition for time series analysis.

3.2.3. PACF and ACF

We use PACF and ACF to determine the p and q orders of ARIMA. According to Figure 4, most of the lags exceeding the confidence interval are between 1 and 10, so we construct ARIMA(0,1,1), ARIMA(0,1,2), and ARIMA(10,1,10) models, and so on, to evaluate model fitness.

3.2.4. AIC and BIC

Since there are many ARIMA models selected by ACF and PACF, AIC and BIC were used to test the goodness-of-fit and determine the better prediction models. Based on Table 13, the models within the top four ranks of AIC and BIC were selected and their prediction performance was compared.

3.2.5. Performance

Finally, we compare the prediction deviation with MSE, MAE, and MAPE. Table 14 presents the prediction assessment of the ARIMA model. We find that ARIMA(5,1,2) is the best prediction model for the VNQ price. We test white noise for the standardized residuals of ARIMA(5,1,2). The result shows that the p-value is 0.2082, representing that the standardized residuals of the series are white noise.

[Insert **Tables** 11−14 and **Figure** 4]

3.3. Comparison

Table 15 demonstrates the prediction errors of the best with 1- and 2-hidden layer BPNN models and the ARIMA model. Whether with MSE, MAE, or MAPE, BPNN exhibits a smaller loss function than ARIMA, which reveals that BPNN is more accurate. With either one or two hidden layers, the trend in the predicted VNQ price is similar to the actual value. As depicted in Figure 5, the ARIMA model is a linear data-generating process, so its predictive line exhibits a steady and slowly rising trend. The empirical analysis demonstrates that BPNN demonstrates better forecasting performance.

[Insert **Table** 15 and **Figure** 5]

4. Conclusions

Python was used to program BPNN and ARIMA to construct price forecasting models for VNQ. With BPNN, the transfer function was the sigmoid function and the input variable selection was done with MLR analysis to reinforce the predictive power of the network structure, from which the past 1-day closing price, daily trading volume, and S&P 500 were eliminated. Conversely, the input variables are statically significant at a 5% significance level and, as follows, P_{t-2}, P_{t-3}, *MA5*, *MA20*, *DXY*, *VIX*, *^FVX*, and *^TNX* (refer to Table 2). According to the empirical results, the best BPNN structure is with one hidden layer, four hidden layer nodes, a learning rate of 0.5, and a momentum factor of 0.9. In the ARIMA model, AIC and BIC were utilized to obtain the top four ARIMA models with the best goodness-of-fit. MSE, MAE, and MAPE were used to comprehensively compare errors, and the best ARIMA (5,1,2) model was tested for white noise. Since the null hypothesis could not be rejected, it indicates that ARIMA (5,1,2) had fully collected data from the sequences and exhibited a good fit. Finally, comparing the forecasting ability of BPNN and ARIMA using three performance assessment indicators, BPNN's predictive errors are significantly lower than those of ARIMA. BPNN outperforms the ARIMA time series model in predicting the VNQ market price. Referring to forecasting run charts, the VNQ price trend predicted by BPNN is also closer to the actual price. This study's results provide influential market factors with higher VNQ price correlations and a superior forecasting model, representing practical advice and references for those considering VNQ market investments.

Acknowledgments

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Table 1. Features of ACF and PACF plots.

Model	ACF	PACF
AR(p)	die-down	cut off after order p
MA(q)	cut off after order q	die-down
ARMA(p,q)	die-down	die-down

Table 2. Selected input variables of the BPNN model.

Variables (X_k)		Description
X_1~X_3	$P_{t-1}P_{t-2}P_{t-3}$	Past three days closing price
X_4~X_5	MA_5MA_{20}	5 and 20 days moving average
X_6	Vol	Daily trading volume
X_7	DXY	U.S. dollar index
X_8	$S\&P500$	S&P 500 Index
X_9	VIX	volatility index
X_{10}	$^\wedge FVX$	U.S. treasury yield 5 years
X_{11}	$^\wedge TNX$	U.S. treasury yield 10 years

Table 3. Correlation tests between input variables using multiple linear regression analysis.

Variables (X_k)	Coefficient	t-value	p-value	Variables (X_k)	Coefficient	t-value	p-value
Constant	4.141*	4.542	0.000	X_6	−3.8E-9	−0.378	0.705
X_1	−0.031	−1.042	0.298	X_7	0.022*	2.756	0.006
X_2	−0.497*	15.926	0.000	X_8	0.000	1.464	0.143
X_3	−0.953*	31.609	0.000	X_9	−0.046*	−9.241	0.000
X_4	2.502*	34.618	0.000	X_{10}	0.629*	2.442	0.015
X_5	−0.078*	−6.971	0.000	X_{11}	−1.043*	−3.574	0.000

Notes: The model of multiple linear regression is denoted as $Y_t = \alpha + \sum_{i=1}^{11} \beta_i X_{i,t} + \varepsilon_t$, where Y_t is VNQ series at time t, $X_{i,t}$ is the ith input variable at time t, α is intercept, and $\varepsilon_t \sim iid(0, \sigma^2)$. * represents under significance level of 0.05 is statistically significant.

Table 4. Parameter settings of hidden layers and hidden layer nodes.

	1 hidden layer	2 hidden layers
Number of input nodes	8	8
Number of 1st hidden layer nodes	1~14	1~14
Number of 2nd hidden layer nodes	-	2~7
Number of output nodes	1	1

Table 5. The top three 1-hidden layer BPNN models selected by MSE.

MSE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MSE
1	1	08-04-01	0.5	0.9	4.4069
2	1	08-06-01	0.7	0.7	4.7297
3	1	08-04-01	0.7	0.7	4.7358

Table 6. The top three 1-hidden layer BPNN models selected by MAE.

MAE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MAE
1	1	08-04-01	0.5	0.9	1.8067
2	1	08-04-01	0.7	0.7	1.8727
3	1	08-06-01	0.7	0.7	1.8779

Table 7. The top three 1-hidden layer BPNN models selected by MAPE.

MAPE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MAPE
1	1	08-04-01	0.5	0.9	149.4425%
2	1	08-04-01	0.7	0.7	154.1757%
3	1	08-06-01	0.7	0.7	154.8737%

Table 8. The top three 2-hidden layer BPNN models selected by MSE.

MSE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MSE
1	2	08-02-05-01	0.5	0.7	5.6697
2	2	08-02-05-01	0.1	0.5	6.7916
3	2	08-07-03-01	0.7	0.9	6.8971

Table 9. The top three 2-hidden layer BPNN models selected by MAE.

MAE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MAE
1	2	08-02-05-01	0.5	0.7	1.8928
2	2	08-02-05-01	0.1	0.5	2.1985
3	2	08-02-04-01	0.1	0.7	2.2294

Table 10. The top three 2-hidden layer BPNN models selected by MAPE.

MAPE ranking	Number of hidden layer	Network structure	Learning rate	Momentum term	MAPE
1	2	08-02-05-01	0.5	0.7	158.0688%
2	2	08-02-05-01	0.1	0.5	181.2002%
3	2	08-02-04-01	0.1	0.7	183.6018%

Table 11. ADF, PP, KPSS tests on the VNQ price (2015/01/01-2020/03/31).

	ADF	PP	KPSS
Level	$-3.714(10)$	$-3.817(10)$	$9.822(0)*$
1st difference	$-12.046(9)*$	$-40.865(0)*$	$0.027(1)$

Notes: * represents statistical significance at 0.05 level.

Table 12. White noise test on the VNQ price of first-order difference (2015/01/01-2020/03/31).

	p-value
1st difference of series	7.7763e-22

Notes: The null hypothesis H_0 indicates series are purely random processes.

Table 13. AIC and BIC tests on the VNQ price (2015/01/01-2020/03/31).

Ranking	ARIMA	AIC	Ranking	ARIMA	BIC
1	ARIMA(10,1,7)	4042.83	1	ARIMA(4,1,1)	4104.04
2	ARIMA(10,1,9)	4043.68	2	ARIMA(3,1,2)	4105.87
3	ARIMA(10,1,4)	4048.82	3	ARIMA(2,1,3)	4107.22
4	ARIMA(9,1,4)	4049.30	4	ARMIA(5,1,2)	4108.06

Table 14. Prediction assessment on the ARIMA model (2020/04/01-2020/06/30).

Ranking	Model	MSE	MAE	MAPE
1	ARIMA(5,1,2)	13.4917	2.9729	245.3333%
2	ARIMA(10,1,7)	13.8485	3.0188	248.5046%
3	ARIMA(3,1,2)	60.5569	6.5750	520.9969%
4	ARIMA(4,1,1)	60.8947	6.5964	522.6878%
5	ARIMA(9,1,4)	65.7327	6.8753	544.9113%
6	ARIMA(10,1,4)	70.4106	7.1698	568.6037%

Table 15. Comprehensive performance assessment.

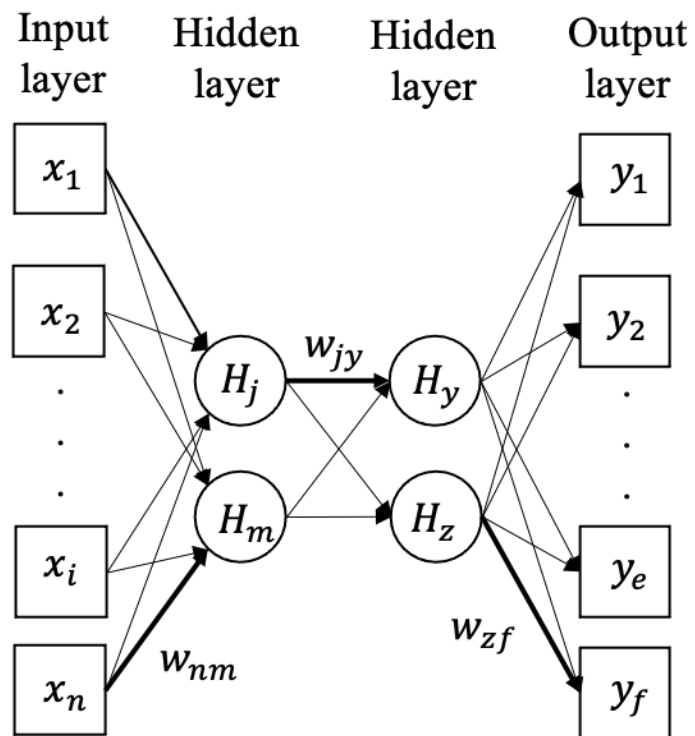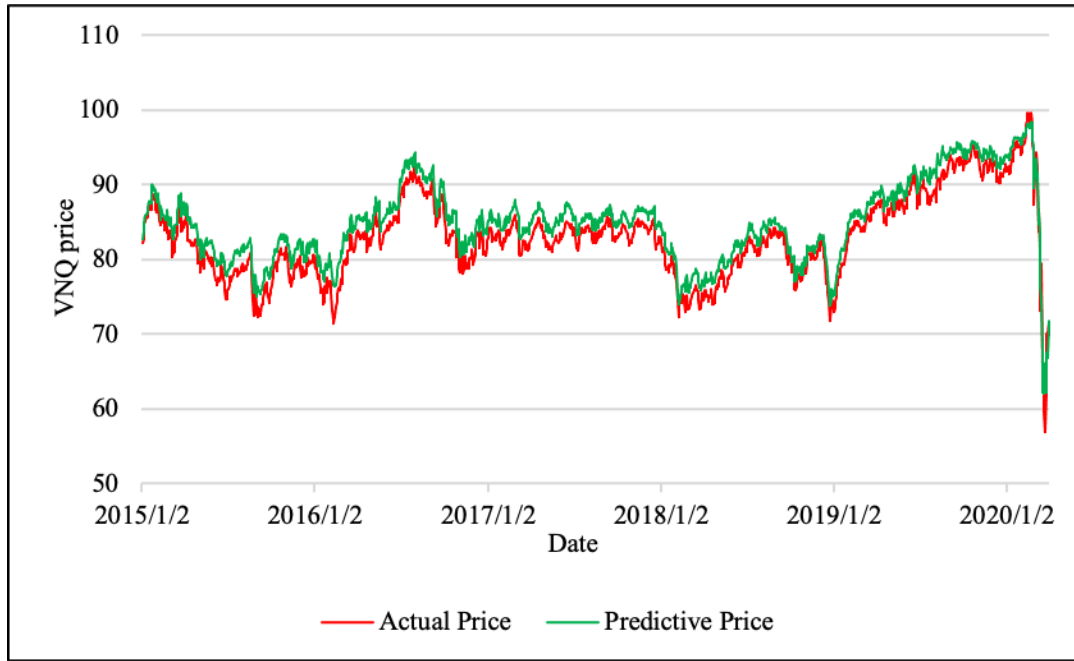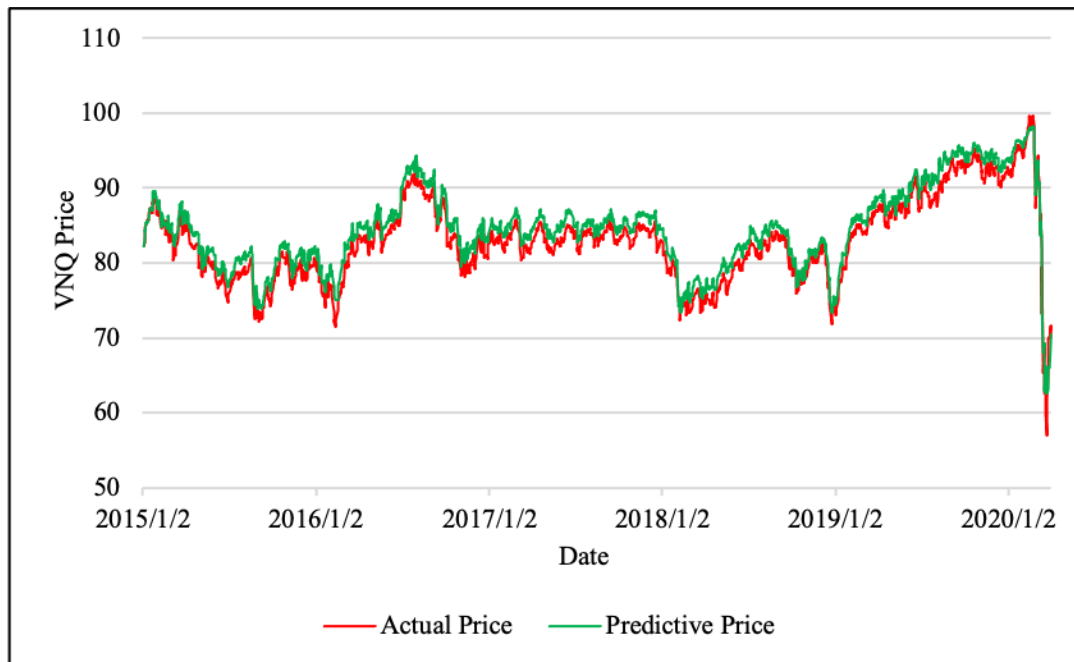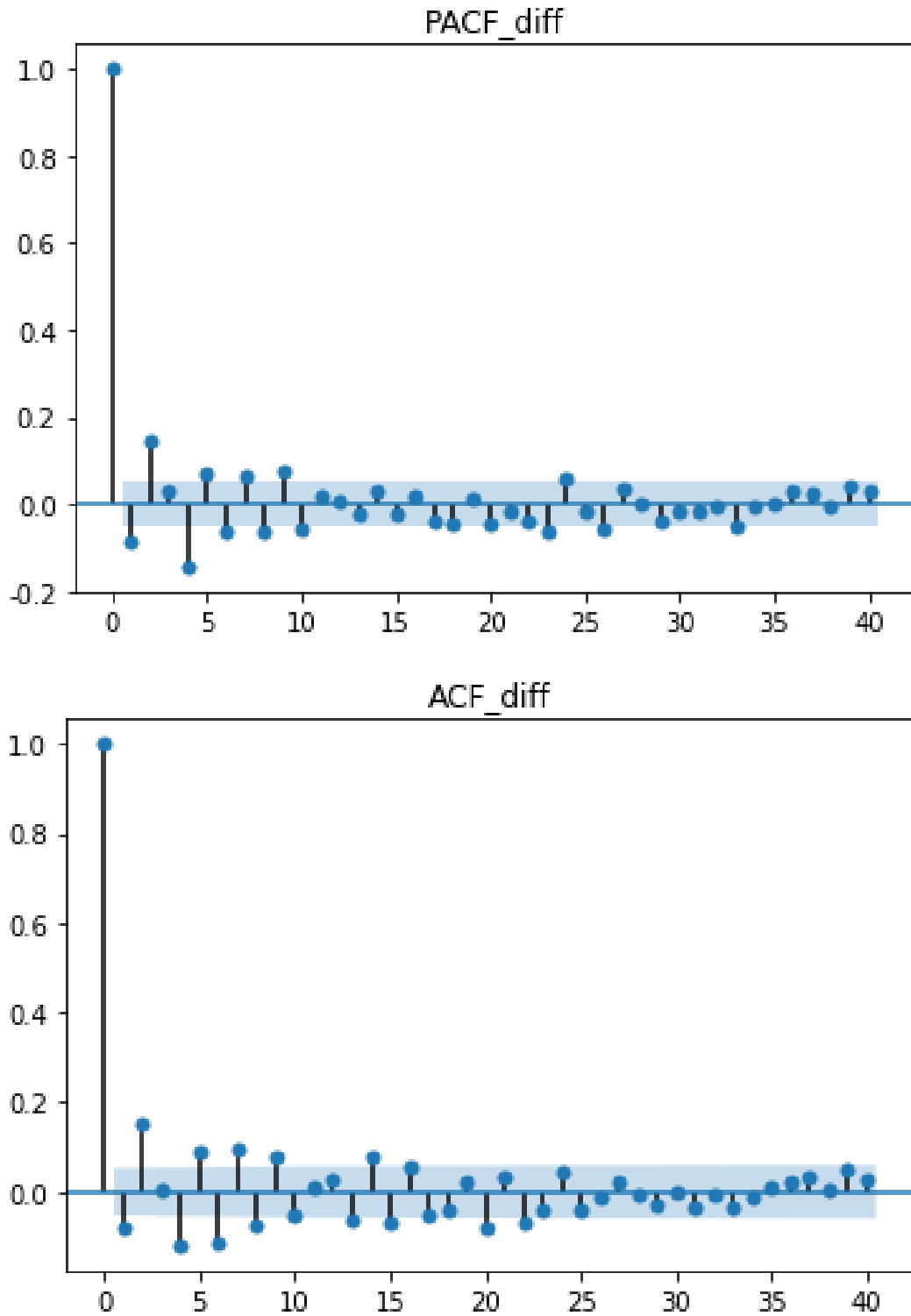| MSE ranking	1	2	3
Model	BPNN(8-4-1)	BPNN(8-2-5-1)	ARIMA(5,1,2)
MSE	4.4069	5.6697	13.4917
MAE ranking	**1**	**2**	**3**
Model	BPNN(8-4-1)	BPNN(8-2-5-1)	ARIMA(5,1,2)
MAE	1.8067	1.8928	2.9729
MAPE ranking	**1**	**2**	**3**
Model	BPNN(8-4-1)	BPNN(8-2-5-1)	ARIMA(5,1,2)
MAPE	149.4425%	158.0688%	245.3333%

Figure 1. ANN Framework.

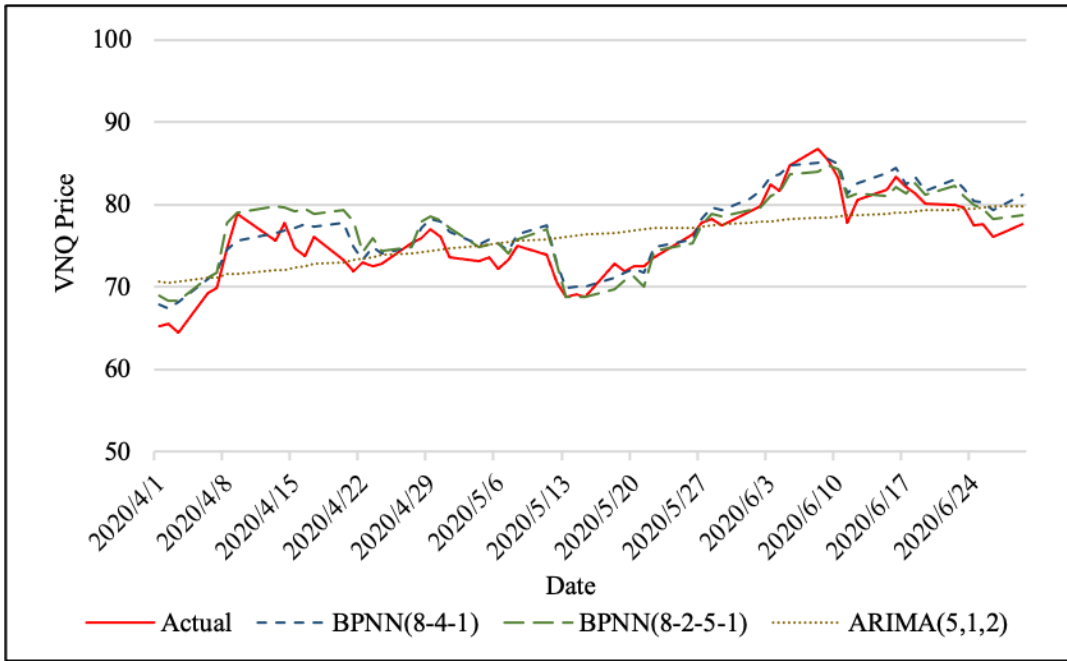**Figure 2.** Training period of the best 1-layer BPNN model.

Figure 3. Training period of the best 2-layers BPNN model.

Figure 4. PACF and ACF of the 1st differenced VNQ price.

Figure 5. Forecast price and actual price trends during the testing period.