L – Q – FUZZY QUOTIENT ζ - GROUP

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Abstract :

In this paper, we define a new algebraic structure of L – Q- fuzzy sub ζ – groups and L – Q – fuzzy quotient ζ – groups and discussed some properties. We also defined ζ – Q - homomorphism over L – Q – fuzzy quotient ζ – groups. Some related results have been derived.

Key Words: L – Fuzzy subset, L – Q – Fuzzy subset, ζ – Q – Homomorphism, L – Q – fuzzy Sub ζ – groups, L – Q – fuzzy quotient ζ – groups.

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1. Introduction

Zadeh [12] introduced the notion of a fuzzy subset of a set X as a function from X into [0, 1]. Goguen in [5] replaced the lattice [0, 1] by a complete lattice L and studied L – fuzzy subsets. Rosenfeld [1] used this concept and developed some results in fuzzy group theory. Solairaju and Nagarajan [2, 3] introduced and defined a new algebraic structure of Q – fuzzy groups. Saibaba [4] introduced the concept of L – fuzzy sub ζ – groups and L – fuzzy ζ – ideal of ζ – groups. Sundrerrajan et al [11] studied the concepts of anti Q – L – fuzzy ζ – group, we invite the reader to consult the cited work [6, 7, 8, 9, 10] a non gathers. Here in this paper we introduce the notion of L – Q – fuzzy quotient ζ – group and there define ζ – Q – homomorphism over L – Q – fuzzy quotient ζ – groups.

2. Preliminaries

2.1 Definition:[5] A post (L, $\leq$) is called a lattice if supremum of a, b and infimum of a, b exist for all a, b $\in$ L.

2.2 Definition: A lattice ordered group (ζ – group) is a system G = (G, +, $\leq$) where

1. (G, +) is a group
2. (G, $\leq$) is a lattice
3. The inclusion is invariant under all translations $x \leq y \Rightarrow a + x + b \leq a + y + b$ for all a ,b $\in$ G.

2.3 Definition:[5] Let X be a non empty set L = (L, $\leq$) be a lattice with least element 0 and greatest element 1. An L – fuzzy subset μ of X is a function $\mu : X \rightarrow L$.

2.4 Definition: Let X be a non empty set L = (L, $\leq$) be a lattice with least element 0 and greatest element 1 and Q be a non empty set . A L – Q – fuzzy subset μ of X is a function $\mu : X \times Q \rightarrow L$.

2.5 Definition: An L – Q – fuzzy subset μ of G is said to be an L – Q – fuzzy sub ζ – group (LQFSζG) of G if for any x, y $\in$ G.

1. $\mu(xy, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
2. $\mu(x^{-1}, q) = \mu(x, q)$
3. $\mu(x \vee y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
4. $\mu(x \wedge y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$.

2.6 Theorem: If μ is an L – Q – fuzzy sub ζ – group of G, then $\mu(x, q) \leq \mu(e, q)$ for $x \in G$ and e is the identity element in G.

2.7 Theorem: Let μ be an L – Q – fuzzy sub ζ – group of G, then H = $\{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$ is either empty or a sub ζ – group of G.

Proof:

If no element satisfies this condition, then H is empty. If x, y satisfy this condition, then $\mu(xy^{-1}, q) \geq \min\{\mu(x, q), \mu(y^{-1}, q)\} = \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ and $\mu(e, q) \geq \mu(xy^{-1}, q)$, since μ is an L – Q – fuzzy sub ζ – group of G hence $\mu(e, q) = \mu(xy^{-1}, q)$ thus $xy^{-1} \in H$, let x, y $\in$ H then $\mu(x, q) = \mu(e, q)$ and $\mu(y, q) = \mu(e, q)$. $\mu(x \vee y, q) \geq \min\{\mu(x, q), \mu(y, q)\} \geq \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ then $\mu(x \vee y, q) = \mu(e, q)$ hence $x \vee y \in H$, also $\mu(x \wedge y, q) \geq \min\{\mu(x, q), \mu(y, q)\} \geq \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ then $\mu(x \vee y, q) = \mu(e, q)$ hence $x \wedge y \in H$, therefore H is sub ζ – group of an ζ – group G.

2.8 Definition: An L – Q – fuzzy sub ζ – group μ of G is called an L – Q – fuzzy normal sub ζ – group (LQFNSζG) of G if for any x, y $\in$ G $\mu(xyx^{-1}, q) \geq \mu(y, q)$.

2.9 Theorem: Let G be an ζ – group and μ be an L – Q – fuzzy sub ζ – group of G then the following conditions are equivalent.

1. μ is an L – Q – fuzzy normal sub ζ – group of G
2. $\mu(xyx^{-1}, q) = \mu(y, q)$ for all $x, y \in G$.
3. $\mu(xy, q) = \mu(yx, q)$ for all $x, y \in G$.

2.10 Corollary: Let μ be an L – Q – fuzzy normal sub ζ – group of G, then $H = \{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$ is either empty or a normal sub ζ – group of G.

Proof:

By Theorem 2.7 H is a sub ζ – group of G, then for any $x \in G$ and $y \in H$ $\mu(xyx^{-1}, q) = \mu(y, q) = \mu(e, q)$ since μ is an L – Q – fuzzy normal sub ζ – group of G and $y \in H$ hence $xyx^{-1} \in H$ thus H is a normal sub ζ – group of G, therefore H is either empty or a normal sub ζ – group of G.

2.11 Lemma: Let μ be an L – Q – fuzzy sub ζ – group of G**.** Then $x\mu$ **=** $y\mu$ if and only if $\mu(x^{-1}y, q) = \mu(y^{-1}x, q) = \mu(e, q)$ for all $x, y \in G$ and $q \in Q$.

Proof: Straightforward**.**

2.12 Definition: Let G_1, G_2 be any two ζ – groups. Then the function $\Psi: G_1 \to G_2$ is said to be ζ – Q – homomorphism if for all $x, y \in G_1$

1.. $\Psi(xy, q) = \Psi(x, q)\ \Psi(y, q)$

2. $\Psi(x \vee y, q) = \max\{\Psi(x, q), \Psi(y, q)\}$

3. $\Psi(x \wedge y, q) = \min\{\Psi(x, q), \Psi(y, q)\}$.

2.13 Definition: An L – Q – fuzzy subset μ of X is said to bare sup property if, for any subset A of X, if there exist $a_0 \in A$ such that $\mu(a_0, q) = \vee_{a \in A}\ \mu(a, q)$.

2.14 Definition: Let Φ be a function from a set X into a set Y. An L – Q – fuzzy subset μ of X is called Φ - invariant if $\Phi(x, q) = \Phi(y,q)$ then $\mu(x,q) = \mu(y,q)$ where $x, y \in X$ and $q \in Q$.

2.15 Definition: Let G_1, G_2 be any two ζ – groups. Then the function $\Psi: G_1 \to G_2$ is said to be ζ – Q – isomorphism if for all $x, y \in G_1$

1.. $\Psi(xy, q) = \Psi(x, q)\ \Psi(y, q)$
2. Ψ is bijection.

3. Some Results of L – Q – Fuzzy Quotient ζ – Group

3.1 Theorem: Let μ be an L – Q – fuzzy sub ζ – group of G with identity e. Let $H = \{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$. Consider the map $\mu^* : G / H \to L$ defined by $\mu^*(xh, q) = \vee\ \mu(xh, q)$ for all $h \in H$, $x \in G$ and $q \in Q$. Then

1. H is a normal sub ζ – group of G.
2. The map μ^* is well defined.
3. μ^* is an L – Q – fuzzy sub ζ – group of G/ H.

Proof:

Since μ is an L – Q – fuzzy normal sub ζ – group of G

1. H = {x $\in$ G, q $\in$ Q; μ(x, q) = μ(e, q)}let y $\in$ H , x $\in$ G and q $\in$ Q then μ(y, q) = μ(e, q), nowμ (xyx $^{-1}$, q) = μ (y, q) = μ(e, q), since μ is an L – Q – fuzzy normal sub ζ – group of G, Hence xyx $^{-1}$ $\in$ H.
Let x, y $\in$ H then μ(x, q) = μ(e, q) = μ(y, q)
μ (x $\vee$ y, q) $\geq$ min{ μ(x, q), μ(y, q)}= minμ(e, q), μ(e, q)} = μ(e, q) hence μ (x $\vee$ y, q) $\geq$ μ(e, q), then μ (x $\vee$ y, q) μ(e, q) thus x $\vee$ y $\in$ H.
And μ (x $\wedge$ y, q) $\geq$ min{ μ(x, q), μ(y, q)}= minμ(e, q), μ(e, q)} = μ(e, q) hence μ (x $\wedge$ y, q) $\geq$ μ(e, q), then μ (x $\wedge$ y, q) μ(e, q) thus x $\wedge$ y $\in$ H. Therefore H is a normal sub ζ – group of G.

2. Consider the map μ^* : G / H $\rightarrow$ L defined by μ^* (xh, q) = $\vee$ μ(xh, q) for all h $\in$ H , x $\in$ G and q $\in$ Q then xy $^{-1}$ $\in$ k that is, μ (xy $^{-1}$, q) = μ(e, q) thus μ(xh, q) = μ(yh, q) and hence μ^* (xh, q) = μ^* (yh, q) therefore, the map μ^* is well – defined.

3. (i)μ^* (xh yk , q) = μ^* (xyh, q) = $\vee$ μ(xyh, q) for all h $\in$ H , x, y $\in$ G and q $\in$ Q.

$\geq \vee$ min{$\mu(xh_1, q)$, $\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{$\vee\mu(xh_1, q)$, $\vee\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{μ^*(xh, q) , μ^*(yh, q)}

(ii)μ^* ((xh)$^{-1}$, q) = μ^* (x $^{-1}$h, q) = $\vee$ μ(x $^{-1}$h, q) for all h $\in$ H , x $\in$ G and q $\in$ Q.
= $\vee$ μ(x h, q) = μ^* (x h, q) .

(iii)μ^* (xh $\vee$ yk , q) = μ^* ((x$\vee$y)h, q) = $\vee$ μ((x$\vee$y)h, q) for all h $\in$ H , x, y $\in$ G and q $\in$ Q.

$\geq \vee$ min{$\mu(xh_1, q)$, $\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{$\vee\mu(xh_1, q)$, $\vee\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{μ^*(xh, q) , μ^*(yh, q)}

(iv)μ^* (xh $\wedge$ yk , q) = μ^* ((x$\wedge$y)h, q) = $\vee$ μ((x$\wedge$y)h, q) for all h $\in$ H , x, y $\in$ G and q $\in$ Q.

$\geq \vee$ min{$\mu(xh_1, q)$, $\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{$\vee\mu(xh_1, q)$, $\vee\mu(yh_2, q)$}; $h_1, h_2 \in H$
$\geq$ min{μ^*(xh, q) , μ^*(yh, q)}

Hence μ^* is an L – Q – fuzzy sub ζ – group of G/H.

3.2 Definition: Let μ be an L – Q – fuzzy sub ζ – group of G with identity e. Let H = {x $\in$ G, q $\in$ Q; μ(x, q) = μ(e, q)}. Consider the map μ^* : G / H $\rightarrow$ L defined by

μ^* (xh, q) = $\vee$ μ(xh, q) for all h $\in$ H , x $\in$ G and q $\in$ Q. Then, the L – Q – fuzzy sub ζ – group μ^* of G is called an L – Q – fuzzy quotient ζ – group of μ by H.

3.3 Remark:(1) μ^* is not L – Q – fuzzy normal quotient ζ – group of G/ H.

(2) Consider the map $\mu^* : G / H \rightarrow L$ defined by μ^* (xh, q) = $\vee$ μ(xh, q) for all h $\in$ H , x $\in$ G and q $\in$ Q. Then, μ^* is an L – Q – fuzzy normal quotient ζ – group of G/ H.

3.4 Theorem: If μ^* is an L – Q – fuzzy quotient ζ – group of G/ H, then $\mu^*(xh, q) \leq \mu^*(eh, q)$.

Proof:

Let x $\in$ G, $\mu^*(eh, q) = \mu^*(xx^{-1}h, q)$

$\geq \min\{\mu^*(xh, q) , \mu^*(x^{-1}h, q)\}$

$= \mu^*(xh, q)$

3.5 Theorem: μ^* is an L – Q – fuzzy quotient ζ – group of G/ H iff for all x, y $\in$ G

1. $\mu^*(xhy^{-1}h, q) \geq \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$
2. $\mu^*(xk \vee yk, q) \geq \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$
3. $\mu^*(xk \wedge yk, q) \geq \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$.

Proof:

($\Rightarrow$) $\mu^*(xhy^{-1}h, q) \geq \min\{ \mu^*(xh, q), \mu^*(y^{-1}h, q)\}$

$\geq \min\{ \mu^*(xh, q), \mu^*(y h, q)\}$

As, μ^* is an L – Q – fuzzy quotient ζ – group of G/ H, then (2), (3) are hold

($\Leftarrow$) If (1) hold then $\mu^*(x^{-1}h, q) = \mu^*(e x^{-1}h, q) \geq \min\{ \mu^*(eh, q), \mu^*(x^{-1}h, q)\}$ $= \min\{ \mu^*(eh, q), \mu^*(x h, q)\}= \mu^*(x h, q)$, therefore $\mu^*(x^{-1}h, q) \geq \mu^*(xh, q)$ for all x $\in$ G. Hence $\mu^*((x^{-1})^{-1} h, q) \geq \mu^*(x^{-1}h, q)$ and $\mu^*(x^{-1}h, q) \leq \mu^*(xh, q)$ thus$\mu^*(x^{-1}h, q) = \mu^*(xh, q)$ for all x $\in$ G.

Now, by (1) replace y by y^{-1} then $\mu^*(xy\ h, q) = \mu^*(x(y^{-1})^{-1}\ h, q)$

$\geq \min\{ \mu^*(xh, q), \mu^*(y^{-1}h, q)\}$

$= \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$ for all x, y $\in$ G.

Also $\mu^*(xk \vee yk, q) \geq \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$ and $\mu^*(xk \wedge yk, q) \geq \min\{ \mu^*(xh, q), \mu^*(yh, q)\}$. Therefore μ^* is an L – Q – fuzzy quotient ζ – group of G/ H.

3.6 Theorem: If μ^*, λ^* are two L – Q – fuzzy quotient ζ – group of G/ H then their intersection is an L – Q – fuzzy quotient ζ – group of G/ H**.**

3.7 Corollary: The intersection of any collection of L – Q – fuzzy quotient ζ – group of G/ H is an L – Q – fuzzy quotient ζ – group of G/ H**.**

3.8 Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \rightarrow G_2$ be an ζ – Q - epimorphism and $\mu^* : G_1 / H \rightarrow L$ be an L – Q – fuzzy quotient ζ – group of G_1/ H. Then $\Psi(\mu^*)$ is an L – Q – fuzzy quotient ζ – group of G_2/ H, if μ^* has a sup property and μ^* is Ψ - invariant and $\Psi(\mu^*) = (\Psi(\mu))^*$.

Proof:

1.$\Psi(\mu^*)(\Psi(x)\Psi(y)h, q) =\Psi(\mu^*)(\Psi(xy)h, q)$

$= \mu^*(xyh, q)$

$\geq \min\{ \mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{ \Psi(\mu^*) (\Psi(x)h, q), \Psi(\mu^*) (\Psi(y) h, q)\}$

2.$\Psi(\mu^*)((\Psi(x))^{-1} h, q) = \Psi(\mu^*)(\Psi(x^{-1})h, q)$

$= \mu^*(x^{-1}h, q)$

$= \mu^*(xh, q)$

$= \Psi(\mu^*)(\Psi(x) h, q)$

3.$\Psi(\mu^*)(\Psi(x) \vee \Psi(y)h, q) = \Psi(\mu^*)(\Psi(x \vee y)h, q)$

$= \mu^*((x \vee y)h, q)$

$\geq \min\{ \mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{ \Psi(\mu^*) (\Psi(x)h, q), \Psi(\mu^*) (\Psi(y) h, q)\}$

4.$\Psi(\mu^*)(\Psi(x) \wedge \Psi(y)h, q) = \Psi(\mu^*)(\Psi(x \wedge y)h, q)$

$= \mu^*((x \wedge y)h, q)$

$\geq \min\{ \mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{ \Psi(\mu^*) (\Psi(x)h, q), \Psi(\mu^*) (\Psi(y) h, q)\}$

Hence $\Psi(\mu^*)$ is an L – Q – fuzzy quotient ζ – group of G_2/ H.

$(\Psi(\mu))^* (yh, q) = \vee \Psi(\mu)(yh, q)\ \forall\ h \in H,\ y \in G_2$ and $q \in Q$.

$= \vee \Psi(\mu)(\Psi(x)h, q)\ \forall\ h \in H,\ x \in G_1$ and $q \in Q$.

$= \vee \mu(xh, q)$

$= \mu^*(xh, q)$

$= \Psi(\mu^*) (\Psi(x)h, q)$

$= \Psi(\mu^*) (yh, q)$

3.9 Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \rightarrow G_2$ be an ζ – Q homomorphism and $\lambda^* : G_2 / H \rightarrow L$ be an L – Q – fuzzy quotient ζ – group of G_2/ H. Then $\Psi^{-1}(\lambda^*)$ is an L – Q – fuzzy quotient ζ – group of G_1/ H and $\Psi^{-1}(\lambda^*) = (\Psi^{-1}(\lambda))^*$.

Proof:

1.$\Psi^{-1}(\lambda^*)(xyh, q) = \lambda^* (\Psi(xy)h, q)$

$= \lambda^*(\Psi(x) \Psi(y) h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*) (xh, q), \Psi^{-1}(\lambda^*) (y h, q)\}$

2.$\Psi^{-1}(\lambda^*)(x^{-1}h, q) = \lambda^* (\Psi(x^{-1})h, q)$

$= \lambda^*((\Psi(x))^{-1} h, q)$

$= \lambda^*(\Psi(x) h, q)$

$= \Psi^{-1}(\lambda^*) (xh, q)$.

3.$\Psi^{-1}(\lambda^*)((x \vee y)h,q) = \lambda^*(\Psi(x \vee y)h,q)$

$= \lambda^*((\Psi(x) \vee \Psi(y)) h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*) (xh, q), \Psi^{-1}(\lambda^*) (y h, q)\}$

4. $\Psi^{-1}(\lambda^*)((x\wedge y)h,q) = \lambda^*(\Psi(x\wedge y)h,q)$

$= \lambda^*((\Psi(x) \wedge\Psi(y))\, h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*)(xh, q), \Psi^{-1}(\lambda^*)(y h, q)\}$

Hence $\Psi^{-1}(\lambda^*)$ is an L – Q – fuzzy quotient ζ – group of G_1/H.

$(\Psi^{-1}(\lambda))^*(xh, q) = \vee\, \Psi^{-1}(\lambda)(xh, q)\ \forall\, h \in H,\ x \in G_1$ and $q \in Q$.

$= \vee\, \lambda(\Psi(x)h, q)\ \forall\, h \in H,\ x \in G_1$ and $q \in Q$.

$= \lambda^*(\Psi(x)h, q)$

$= \Psi^{-1}(\lambda^*)(xh, q)$

3.10Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \rightarrow G_2$ be an ζ – Q - homomorphism and λ be an L – Q fuzzy normal sub ζ – group of G_2 such that $\mu = \Psi^{-1}(\lambda)$, then $\Phi: G_1 / \mu \rightarrow G_2 / \lambda$ such that $\Phi(x\mu, q) = \Psi(x, q)\lambda$ for every $x \in G_1$ and $q \in Q$ is an ζ – Q- isomorphism.

Proof:

Clearly Φ is onto as Ψ is onto

Let $x\mu, y\mu \in G_1 / \mu$, $\Phi(x\mu, q) = \Phi(y\mu, q)$ then $\Psi(x, q)\lambda = \Psi(y, q)\lambda$ and $\lambda(\Phi^{-1}(x)\Phi(y), q) = \lambda(\Phi^{-1}(y)\Phi(x), q) = \lambda(\Phi(e), q)$ hence $\lambda(\Phi(x^{-1}y), q) = \lambda(\Phi(y^{-1}x), q) = \lambda(\Phi(e), q)$ then $x\mu = y\mu$ by 2.11 Lemma, therefore Φ is one – one. $\Phi((x\mu)(y\mu), q) = \Psi((xy)\mu, q) = \Psi(xy, q)\lambda = (\Psi(x, q). \Psi(x, q))\lambda = (\Psi(x, q)\lambda). (\Psi(x, q)\lambda) = \Phi(x\mu, q)\, \Phi(y\mu, q)$. Now $\Phi((x\mu\vee y\mu), q) = \Psi((x\vee y)\mu, q) = \Psi(x\vee y, q)\lambda = (\Psi(x, q)\vee \Psi(x, q))\lambda = (\Psi(x, q)\lambda)\vee (\Psi(x, q)\lambda) = \Phi(x\mu, q) \vee\Phi(y\mu, q)$. And $\Phi((x\mu\wedge y\mu), q) = \Psi((x\wedge y)\mu, q) = \Psi(x\wedge y, q)\lambda = (\Psi(x, q)\wedge \Psi(x, q))\lambda = (\Psi(x, q)\lambda)\wedge (\Psi(x, q)\lambda) = \Phi(x\mu, q) \wedge\Phi(y\mu, q)$ clearly Φ is an ζ – Q- homomorphism and hence Φ is an ζ – Q- isomorphism.

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