**Modeling tail-dependence of crypto assets with extreme value theory – Perspectives of risk management in banks**

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**Abstract**

We examine the risk measurement of cryptocurrencies, specifically Bitcoin, Ethereum, and Litecoin. All three currencies have different properties and therefore show slightly different backtesting results. In addition to the classic methods of market risk measurement, historical simulation, and the variance-covariance approach, we also use the Extreme Value Theory to measure risk. However, due to the lack of stationarity and normal distribution of the 1-day and 10-day changes, neither the historical simulation nor the variance-covariance approach shows acceptable backtesting results. Only the Extreme Value Theory with the peaks-over-threshold method delivers satisfactory backtesting results at a confidence level of 99.9 %. In the context of our analysis, the highly volatile market phase from January 2021 was crucial. In this, extreme deflections that have never been observed before in the time series have significantly influenced backtesting. At the same time, the strength of the Extreme Value Theory comes into play here and generates a preferable risk measurement.

**Keywords:** crypto assets, extreme value theory, backtesting, Basel traffic light approach, historical simulation, variance-covariance approach

**JEL:** G17, G21, G28, G32

**1. Introduction**

The evolution of crypto assets began with a distributed ledger or similar technology. It led to several virtual currencies like Bitcoin, Ethereum, Tether, Binance Coin, Cardano Ripple, Dogecoin, Litecoin, Chainlink, Tron, etc. A continuously rising market capitalization reflects the rapid growth of these crypto assets. New all-time highs of price and trading volume or market capitalization have been reached (Basel Committee, 2021).

Although the relative size of the crypto asset market regarding the global financial system is relatively small for financial markets' stability, its absolute size is meaningful. Focused on the significant absolute size of crypto asset markets, it is necessary to develop an appropriate risk measurement to deal with emerging risks of investing in crypto assets for professional market players like banks and investment firms. The BCBS stated that certain crypto assets have a high degree of volatility and could cause high financial risks for such market participants (Basel Committee, 2021).

Concerning most well-known market risk measurements, crypto assets have several statistical characteristics that may not fit such Value at Risk measures. Significantly, the high degree of volatility, tail-dependence, and missing stationarity (Pardalos, 2020a) challenge common risk measurements like historical simulation, variance-covariance model, or monte carlo simulation (Mehmke *et al.*, 2012). Furthermore, the market conduct of crypto asset investors leads to whether the typically observed correlations represent a new asset class (Allen, 2022; Borri, 2019; Guo *et al.*, 2021).

The following academic aspects should be answered:

1. *Are the common Value at Risk approaches an adequate measurement approach?*
First, we perform a statistical analysis of the behavior of crypto assets. Next to the results, we perform a deviation analysis to justify the possible missing fit of common VaR methods.
2. *Does an extreme-value-theory-based Value at Risk provide a better fit for the statistical characteristics of crypto assets?*We compare the standard VaR methods to show which approach best fits the emerging risk of investing in crypto assets. Hence, the extreme value theory provides a highly tail-dependence fit in extreme market situations.

Starting with a literature review, we describe the relevant risk measurement methods like historical analysis and variance-covariance methodology. We also describe different distributions and the extreme value theory concerning the various possibilities of providing a Crypto-Value at Risk. Next to the currently supervisory capital requirement treatment as an intangible asset, we introduce the BCBS 519 and the future expected supervisory treatments. After this literature review, we describe our used Data and Methodology. Afterward, we describe and discuss our results. At least, we conclude our research and highlight the main aspects.

**2. Literature review - Risk management of cryptocurrencies**

**2.1 Introducing cryptocurrencies**

Virtual currencies/ cryptocurrencies are a digital representation of assets that a central bank or agency does not create. There is, therefore, no connection to legal tender guaranteed by a central bank or authority. Unlike traditional currencies, they are based on the idea of ​​a surrogate currency with a finite supply of money. Cryptocurrencies can be created using a predetermined mathematical process to create new value units. This is called mining (Basel Committee, 2021).

The users face each other peer-to-peer on an equal footing. The respective owner manages the Virtual Currency (VC) with his private and public key pairs for authentic transactions. All users can transfer their VC to each other within the network and must regularly communicate the respective target addresses outside the network. However, it is impossible to identify which person owns the VC in the network based on the positions. Transactions, once made, are fundamentally irreversible. In addition to the transmission of VC within the web, it is also possible to physically transfer digits and keys between people, bypassing them onto data carriers (Arslanian, 2022).

**2.2 Supervisory treatment of crypto assets**

Considering the regulatory handling of crypto assets in financial market regulation, there are apparent difficulties. At least in the context of Pillar 1 of the Basel III framework, there are no special requirements for capital backing. CRR III primarily defines counterparty and market risks. The latter can be subdivided into interest rate risks in the trading book, share price risks, and foreign currency risks in various financial instruments. In this context, cryptocurrency price and volatility risks cannot be assigned to market risks or specific financial instruments. It should also be added that the existing requirements for cash, commodities, or FX positions cannot reflect the volatility of crypto assets. The accounting requirements must therefore be taken into account. In line with IAS 38, cryptocurrencies are classified as intangible assets (ECB, 2019). According to Article 38 of the CRR, intangible assets are included as deductions from Common Equity Tier 1 capital. This equates to an RWA weighting of 1.250 % multiplied by 8 % equity backing, corresponding to an estimated 100% for crypto-assets as intangible assets (European Parliament and of the council, 2013).

This also corresponds to the view of the Basel Committee, which uses BCBS 519 to divide crypto assets into two main categories. Table 1 provides an overview of this:

**Table 1**: Overview of the Prudential Treatment of crypto assets

|  |  |  |
| --- | --- | --- |
| **Prudential requirements** | **Group 1 crypto-assets** | **Group 2 crypto-assets** |
| **Group 1a** Tokenised traditional assets | **Group 1b** Cryptoassets with stabilization mechanisms (i.e., stablecoins | Cryptoassets that do not qualify as Group 1 (e.g., bitcoin) |
| **Credit and market risk requirements**  | Capital requirements are at least equivalent to those of traditional assets (with further consideration for capital add-ons)  | New guidance on the application of current rules to capture the risks relating to stabilization mechanisms (with further consideration for capital add-ons)  | New conservative prudential treatment based on a 1.250 % risk weight applied to the maximum of long and short positions |
| **Other minimum requirements (leverage ratio, significant exposures, liquidity ratios)** | Application of the existing Basel Framework requirements with additional guidance where applicable |
| **Supervisory review** | Additional guidance to ensure that risks not captured under minimum (Pillar 1) requirements are assessed, managed and appropriately mitigated (including through capital add-ons |

Source: Basel Committee, 2021.

Group 2 seems particularly relevant for the present article since this includes the typical crypto assets such as Bitcoin, Ethereum, etc. In addition to the regulatory capital deposits of Pillar 1, there are also requirements for Pillar 2. There are various risks to which banks, but also other market players, are exposed:

* market risk
* liquidity risk
* credit risk (especially counterparty risk)
* operational risk (including fraud and cyber threats);
* money laundering / terrorist financing risk; other
* legal and reputation risks.

Summing up the supervisory review, there are high volatility risks, mainly focusing on market price risks.

**2.3 Market risk measurements**

Based on the knowledge of high volatility risks, it now makes sense to place the problem of this expected more heightened level of risk in the theoretical context of the various market price risk models. For this purpose, we focus on the variance-covariance approach, the historical simulation, and the extreme value theory. In addition to the monte carlo simulation, historical simulation and the variance-covariance approach, in particular, are standard methods of measuring market price risk (Huschens, 2017; Wiedemann, 2013).

The VaR can be calculated using absolute, relative, and logarithmic risk factor changes. The present article focuses on absolute differences. Absolute differences can be described as dependent on the level and thus tends to be unsuitable for trend-related changes in value. In contrast, high changes in times of high volatility are suitable in times of lower volatility and could lead to better forecast quality. Finally, transferring historical changes to the future requires stationarity, i.e., stochastic freedom from trends (Miller, 2018; Huschens, 2017).

The historical simulation is based on historical risk factors changes. Based on these historical risk factors changes, the current portfolio is revalued. Furthermore, considering the historical factor changes means that no distribution or correlation assumption is necessary since this information is taken from the historical change and transferred to the current price level. It is characteristic of the historical simulation that the risk value is counted, i.e., based on the amount of relevant data for the x-worst data set matching the confidence interval. An environment detached from the data record can thus already be recorded, which rank is to be used as the risk value (Wiedemann, 2013).

As a parametric approach, the variance-covariance method assumes a normal distribution of the risk factor changes. In the case of several risk factors, the correlation between them must be considered. The variance-covariance method is widespread due to the low data requirements and good feasibility. However, the variance-covariance approach, particularly the normal distribution assumption, has some disadvantages. The financial crisis has already shown that outliers are usually underestimated and cannot be sustained. It should be added that the normal distribution lacks so-called 'fat tails,' which can cover larger characteristics at the edge of the distribution. Concerning the correlation assumption, the variance-covariance method uses the Pearson correlation coefficients, which on the one hand, continues the assumption of linearity of the normal distribution and, on the other hand, also has to be checked in terms of stability (Rüder, 2019; Pesaran, 2016; Gleißner, 2019; Daníelsson, 2006; Romeike, 2020).

Due to a high level of extreme risks, these cannot be adequately represented using the flanks of the normal distribution and, sometimes, the empirical distribution too. Therefore, a so-called heavy-tailed distribution with thicker and broader flanks is required. Due to the rarity of extreme risks and their systemic character, it is possible that these cannot be inferred from history. Therefore, data from the tails of the empirical distribution are used for modeling (Zhao, 2021; Pardalos, 2020b; Ahelegbey *et al.*, 2021). The extreme value theory is based on the two central convergence theorems of Fisher-Tippett and Pickards-Balkema-de Haan. The peaks-over-threshold method (PoT method), as a method of extreme value theory, uses both of these convergence theorems (Embrechts *et al.*, 2008):

* The Fisher and Tippett convergence theorem describes standardized samples' convergence against the generalized extreme value distribution. After proper renormalization, the maxima of a sample of iid random variables converge to an extreme value distribution (Gumbel distribution, Fréchet distribution, or Weibull distribution).
* The Pickards-Balkema-de Haan theorem states that the distribution of excesses above a sufficiently high threshold *u* that fall within the region of attraction of the generalized extreme value distribution can be approximated by the Generalized Pareto Distribution (GPD).

**Fig. 1**: PoT method: $X\_{1}, X\_{2}, …, X\_{n}$ and excesses $Y\_{1}, Y\_{2}, …, Y\_{N\_{u}}$

Source: Zeranski, 2005

The data $X\_{1}, X\_{2}, …, X\_{n}$ are assumed to be iid realizations of the random variable $X$. If a realization of $X$ exceeds the threshold $u$, the realization is called exceedance, and the difference $Y\_{i}=X\_{i}-u$ is called excess (see figure 1). The determination of the threshold value $u$ faces a trade-off problem. The selected threshold should be in the upper-value range of the data. The Pickands-Balkema de Haan theorem also shows that the GPD can approximate the desired distribution of the excesses if *u* is chosen large enough. On the other hand, if a high threshold is determined, there are not enough data to estimate the distribution parameters. These circumstances can lead to a very high variance in the estimate. The Mean Excess Function (MEF) is used to determine *u*. The MEF is defined as the expected value of all excesses:

$e\left(u\right)=E\left(\left.X-u\right|X>u\right), u\geq 0$.

For a sample of $X\_{1}, X\_{2}, …, X\_{n}$, the empirical MEF function is defined as:

$$e\_{n}\left(u\right)=\frac{1}{N\_{u}}\sum\_{i\in ∆\_{n}(u)}^{n}(X\_{i}-u)$$

where *Nu* is the number of excesses. The excess mean value function *e(u)* of the selected functions for a progressive *u* can be displayed graphically in the Mean Excess Plot (Berge, *et al.*, 2006; Embrechts *et al.*, 2008; Saeed Far and Abd. Wahab, 2016). In addition to the threshold, we need to estimate the GPD parameters $ξ $und $β$. The maximum likelihood method (ML method) for determining estimates of the GPD is the most widely used Plot (Berge *et al.*, 2006; Embrechts *et al.*, 2008; Saeed Far and Abd. Wahab, 2016).

To determine the maximum value change under the risk probability $p\in (0,1)$, the results from the confidence level to be maintained $α=1-p$ as the p-quantile $Q\_{p}$. The estimator of the p-quantile is expressed by $\hat{Q}\_{p}$ and found by inverting:

$$\hat{Q}\_{p}=u+\frac{\hat{β}}{\hat{ξ}}\left(\left(\frac{n}{N\_{u}}\*p\right)^{-\hat{ξ}}-1\right).$$

**2.4 Current state of research**

In terms of the current state of research, three research areas can be identified:

* Diversification effects with crypto assets
* Risk/return spillover
* Volatility forecast

Most of these papers focus on specific effects but do not scope the ability of common risk measurement methods to assess possible specific risks of arising crypto assets. Starting with Corbet *et al.*, 2018, the authors analyze the relationship between popular cryptocurrencies and other financial assets. They stated crypto assets could lead to diversification effects and risk-return advantages. Although diversification benefits could be derived, these are only in short investment horizons observable. Borri, 2019 focuses on the conditional tail-risk for cryptocurrencies and finds that these are highly exposed to tail-risk within crypto markets (Sun *et al.*, 2021; Corbet *et al.*, 2018; Borri, 2019).

Relating liquidity effects, cryptocurrencies do have a small impact on optimal portfolios. In addition, there is reasonable evidence to imply the existence of downside risk spillover between Bitcoin and four assets (equities, bonds, currencies, and commodities), which seems to be time-dependent. These main findings have implications for participants in the Bitcoin and traditional financial markets for asset allocation and risk management. For policymakers, the results suggest that Bitcoin should be monitored carefully for financial stability (Zhang *et al.*, 2021). Sun *et al.*, 2021 study focuses on cryptocurrencies in Private Equity (PE) company portfolios and investment factors of PE managers. They find that price volatility does not lower institutional investors' confidence as long as the market can offer timely and accurate price change information to meet investors' price consciousness. Furthermore, especially cryptocurrencies with a high familiarity provide diversification benefits.

Most of such research investigates one or two specific cryptocurrencies. Most research is done on the bitcoin time series. This could be motivated by the familiarity and the length of the time series of bitcoin. Many studies have fully emphasized and analyzed the VaR in the Bitcoin market, providing many valuable tools for risk measurement (Ardia *et al.*, 2019; Stavroyiannis, 2018; Troster *et al.*, 2019; Ahelegbey *et al.*, 2021; Gao *et al.*, 2022; Jiménez *et al.*, 2020).

Ahelegbey *et al.* (2021) examine the specific tail risks of cryptocurrencies and use particular measurements. Therefore, they use the extreme downside hedge (EDH) and the extreme downside correlation (EDC) methods and focus on cryptocurrencies' relationship. They show a positive and statistically significant relationship between the tail risk of the crypto assets and the weighted average market index. Based on the EDH, they identified two groups of assets with a characteristic attribute. One group has a speculative behavior like Bitcoin, EOS, and Litecoin and are "givers" of tail risk. The other group could be characterized by a professional outlook like Ethereum, Tron, and Rippe and are mainly receivers of contagion. Based on the EDC, the two groups are split up into four groups:

1. "speculative" and "diversification," e.g., Bitcoin;
2. "professional" and "complementary," e.g., Ethereum;
3. "speculative" and "complementary," e.g., Eos and Litecoin; and
4. "professional" and "diversification," e.g., Ripple and Tron.

Concerning our research question about specific tail risks of crypto assets, Bitcoin is mainly an agent of tail contagion and leads to vulnerable assets, e.g., Ethereum or Litecoin. Focusing on our empirical research, we take this classification into account. Based on the clusters, there is speculation, diversifying, complementary, and professional tail risk and backtesting results (Ahelegbey *et al.*, 2021).

Gao, Ye and Guo (2022) focus their research on value-at-risk- and expected-shortfall-forecasting and modeling Bitcoin risk with a regime-switching conditional autoregressive value-at-risk (CAViaR) model. The paper is based on the empirical finding of dynamic tail risk and prior evidence that bubbles, e.g., a bubble index, contain essential information on systemic risk. Based on daily Bitcoin data between 2013 and 2021, the authors perform in-sample estimates and out-of-sample forecasts of Bitcoin returns and find out that tail risks are observable and lead to under- or overestimation. The authors construct a Markow regime-switching (MS) model with the time-varying transition probability considering asset price bubble information. The backtesting shows that the modeled impact of a bubble index leads to good VaR and ES results. Furthermore, there is some evidence for a form of regime change (Gao *et al.*, 2022).

Jiménez *et al.* (2020) use different semi-nonparametric and parametric distributions, such as volatility models, for modeling Bitcoin risk. They focused on semi-nonparametric risk management, which was never used on cryptocurrency before, and compared the forecast quality with GAS models and GARCH processes. The results show that the SNP technique takes skewness, kurtosis, and extreme events into account and is superior to GAS and GARCH models. Although GAS and GARCH models also provide good results, the authors stated a time-consuming measurement and parametrization process. Considering model complexity, the simple semi-nonparametric approach outperforms the less flexible parametric methods (Jiménez *et al.*, 2020).

Summing up our literature review, most papers research the downside or tail risk of crypto assets. There is a wide range of different risk measurement models observable. While some researchers prefer parametric and complex risk measurement methods, others pointed out that well-parametrized simpler models outperform more time-consuming measurements. At least, none of those mentioned research uses the extreme value theory to measure the tail risk of crypto assets. Mainly, Ahelegbey *et al.* characterize four types of crypto assets that could explain different backtesting results of various cryptocurrencies.

**3. Data & methodology**

Our research focuses on the cryptocurrencies Bitcoin, Ethereum, and Litecoin because of their comparatively long time series within a minimum of 5 years overall cryptos. We use an identical time series length (05.03.2017 – 04.03.2022).

Starting with the daily and 10-day-returns, we perform several normal distribution tests (Kolmogorov-Smirnov Test, Shapiro- Wilk Test, and Anderson Darling Test) to investigate potential tail risk. The normal distribution assumption is sufficient to estimate risk with the variance-covariance approach. Therefore, we use absolute returns.

Considering risk modeling using historical simulation and the variance-covariance approach, we believe in three different confidence intervals of 95.0 %, 99.0 %, and 99.9 %. We distinguish between a cumulative data history and a rolling data history of 365 days with the underlying data series. While the built-up data history includes more observations and can take different market phases into account, the rolling data history reacts more sensitively to short-term market changes. In stress phases, the rolling history can adapt to market changes, such as increased volatility at short notice.

We extend our risk measurement to include extreme value theory based on the above methodology. With this theory, we are pursuing the goal of being able to estimate extremely rare and high-loss events. Therefore, we only consider a holding period of one day to avoid interim loss compensation through longer times and potential autocorrelation effects. It is inherent in the method that a rolling data history does not entail any advantages. The adjustment of the extreme value theory arises from determining the threshold, which also requires a sufficiently long data history. The threshold determination identifies values ​​above the threshold so that values ​​below are ignored, regardless of the length of the data history. We take a new threshold estimate in each observation point if statistically necessary. This forms the basis of our risk assessment.

We use the Basel traffic light approach to evaluate the risk models. The Basel traffic light ranks the number of violations of the predicted values ​​based on the probability of the first type of error (probability that a correct model is wrongly rejected) into green, yellow and red zone. While the yellow zone indicates random outliers, a systematic error can be assumed in the red zone, requiring a model adjustment. Depending on the zone in which a model is assigned, the equity to be covered is determined with an increased multiplier. Due to the holding period of 365 days, we adjust the Basel traffic light approach by a binomial distribution and the confidence level.

**4. Results and discussion**

***a) Are the common Value at Risk approaches an adequate measurement approach?***Starting with the normal distribution test of the three crypto assets Bitcoin, Ethereum, and Litecoin, we use absolute changes for a holding period of 1 and 10 days. All normal distribution tests (Kolmogorov-Smirnov, Shapiro-Wilk, and Anderson Darling) show that the null hypothesis for the absolute 1- and 10-day returns is significant. The assumption of the normal distribution has, therefore, to be rejected.

In addition to the assumption of normal distribution, we examine whether the returns are stationary over time. This is essential for the ex-ante representativeness of the ex-post data. Otherwise, the historical simulation, in particular, is incorrectly specified since the development of the historical data is subject to trends and ex-ante risk forecasts are likely to be distorted. For this purpose, an augmented Dickey-Fuller test was carried out. The null hypothesis of a non-stationary process was confirmed for absolute 1-day and 10-day returns. These lead to trend-affected processes.

Concerning the basic description of the underlying distribution and the stationarity of the time series, it can first be stated that the rejection of the normal distribution hypothesis means that the variance-covariance approach may have a poorer forecast quality.

Starting with the backtesting, a 1-day holding period and absolute returns for all tested confidence intervals of 95.0 %, 99.0 %, and 99.9 % show an overall poor forecast quality for all crypto assets. The model violations of the red area are sometimes over 20.0 % or 30.0%. Except for the 99.9 % confidence level, the violations are partly higher. In most observations, Ethereum and Litecoin show a better forecast quality in the variance-covariance approach than in the historical simulation. Although the assumption of normal distribution had to be negated, the assumption of normal distribution seems to lead to better forecast results than the historical distribution of the historical simulation. It can be subsumed that the historical observations result in a trend that immediately harms the quality of the forecast and, concerning the impacts, superimposes the strong assumption of normal distribution.

Concerning a generally better forecast quality under the assumption of normal distribution, this cannot be maintained with a 10-day holding period. The best forecast quality is shown with a confidence interval of 95.0 %.

In distinguishing between cumulative and rolling history, we show better backtesting results for the rolling history with a 1-day holding period. Only with Litecoin can this statement not be kept. However, it should be pointed out that Litecoin requires the best forecast quality in backtesting. In contrast, the forecast quality for Bitcoin is worst suited. Interestingly, Litecoin has a significantly better forecast quality for a 1-day than a 10-day holding period.

In summary, it can also be stated that none of the VaR show sufficiently conservative forecast results that are appropriate from a regulatory point of view.

**Table 2**: Forecast quality of historical simulation and variance-covariance approach within 95.0 % confidence level

|  |  |
| --- | --- |
| one-day holding period | ten-day holding period |
| historical simulation | variance-covariance | historical simulation | variance-covariance |
| 95.0 % confidence level | cum. | rolling | cum. | rolling | cum. | rolling | cum. | rolling |
| Bitcoin | green | 65.7% | 68.2% | 66.1% | 71.3% | 68.4% | 69.9% | 70.9% | 70.1% |
|   | yellow | 2.6% | 11.2% | 4.7% | 18.5% | 3.0% | 15.5% | 2.4% | 16.2% |
|   | red | 31.8% | 20.5% | 29.3% | 10.2% | 28.5% | 14.5% | 26.7% | 13.7% |
| Ethereum | green | 65.8% | 61.6% | 68.8% | 66.5% | 73.7% | 61.7% | 74.0% | 80.3% |
|   | yellow | 2.6% | 6.6% | 4.7% | 18.9% | 1.9% | 16.1% | 2.5% | 19.7% |
|   | red | 31.6% | 31.8% | 26.5% | 14.6% | 24.4% | 22.2% | 23.6% | 0.0% |
| Litecoin | green | 73.3% | 72.7% | 100.0% | 79.0% | 73.2% | 73.2% | 78.8% | 75.4% |
|   | yellow | 18.3% | 11.8% | 0.0% | 21.0% | 2.1% | 12.1% | 21.2% | 10.9% |
|   | red | 8.4% | 15.5% | 0.0% | 0.0% | 24.7% | 14.6% | 0.0% | 13.6% |

Source: Own calculations.

**Table 3**: Forecast quality of historical simulation, variance-covariance approach, and extreme value theory within 99.0 % confidence level

|  |  |
| --- | --- |
| one-day holding period | ten-day holding period |
| historical simulation | variance-covariance | Extreme Value Theory | historical simulation | variance-covariance |
| 99.0 % confidence level | cum. | rolling | cum. | cum. |  | cum. | rolling | cum. | rolling |
| Bitcoin | green | 62.7% | 71.8% | 62.2% | 61.9% | 62.8% | 64.4% | 43.6% | 38.5% | 43.3% |
|   | yellow | 4.2% | 11.1% | 2.4% | 2.4% | 6.2% | 3.8% | 26.7% | 27.4% | 18.7% |
|   | red | 33.1% | 17.1% | 35.4% | 35.8% | 31.0% | 31.8% | 29.7% | 34.1% | 38.0% |
| Ethereum | green | 65.7% | 51.9% | 63.7% | 63.7% | 73.0% | 66.0% | 43.8% | 66.0% | 43.4% |
|   | yellow | 4.0% | 21.7% | 2.4% | 2.4% | 2.5% | 5.2% | 39.1% | 2.1% | 30.3% |
|   | red | 30.3% | 26.4% | 33.9% | 33.9% | 24.5% | 28.9% | 17.1% | 31.9% | 26.3% |
| Litecoin | green | 74.7% | 70.6% | 67.6% | 67.6% | 83.6% | 74.9% | 46.3% | 74.8% | 65.4% |
|   | yellow | 4.1% | 15.2% | 7.8% | 7.8% | 16.4% | 1.1% | 36.6% | 1.0% | 5.1% |
|   | red | 21.2% | 14.1% | 24.6% | 24.6% | 0.0% | 24.0% | 17.1% | 24.2% | 29.5% |

Source: Own calculations.

***b) Does an extreme-value-theory-based Value at Risk better fit the statistical characteristics of crypto assets?***

Starting with the risk measurement of the extreme value theory, the comparison must first be adjusted. Due to the aim of using the extreme value theory to forecast high and very rare claims, the 95.0% confidence level is unnecessary. For better comparability, the 10-day holding period is also omitted. In this way, no interim loss compensation can be considered to reduce risk. Furthermore, the extreme-Value-VaR is only calculated with a growing history since a data history is required to determine the threshold value. The "omission" of data points cannot lead to a higher dynamic than the other VaR methods.

Starting with a confidence level of 99.0 %, Bitcoin shows the best risk measurement with the historical simulation and a rolling 1-year history. With 17.1% in the red zone, the rolling historical simulation has the best risk measurement. This finding also continues with Ethereum, although the rolling historical simulation only reveals slight advantages in comparing the alternative measurements. In the case of Ethereum, the extreme value theory shows a comparable forecast result. With the addition of the yellow and the red regions, the extreme value theory offers the best results. Hence, this approach shows a significantly better forecast quality for Litecoin than other methods.

**Table 4**: Forecast quality of historical simulation, variance-covariance approach, and EVE within 99.9 % confidence level

|  |  |
| --- | --- |
| one-day holding period | ten-day holding period |
| historical simulation | variance-covariance | Extreme Value Theory | historical simulation | variance-covariance |
| 99.9 % confidence level | cum. | rolling | cum. | rolling |  | cum. | rolling | cum. | rolling |
| Bitcoin | green | 37.5% | 11.8% | 10.4% | 6.7% | 75.6% | 64.4% | 43.6% | 38.5% | 43.3% |
|   | yellow | 33.2% | 66.9% | 51.4% | 31.1% | 24.4% | 3.8% | 26.7% | 27.4% | 18.7% |
|   | red | 29.3% | 21.4% | 38.2% | 62.2% | 0.0% | 31.8% | 29.7% | 34.1% | 38.0% |
| Ethereum | green | 61.9% | 24.1% | 61.9% | 18.5% | 75.1% | 66.0% | 43.8% | 66.0% | 43.4% |
|   | yellow | 13.4% | 54.4% | 0.9% | 15.1% | 24.9% | 5.2% | 39.1% | 2.1% | 30.3% |
|   | red | 24.7% | 21.5% | 37.2% | 66.3% | 0.0% | 28.9% | 17.1% | 31.9% | 26.3% |
| Litecoin | green | 72.8% | 39.6% | 61.9% | 8.3% | 75.6% | 74.9% | 46.3% | 74.8% | 65.4% |
|   | yellow | 5.7% | 38.3% | 8.9% | 33.7% | 24.4% | 1.1% | 36.6% | 1.0% | 5.1% |
|   | red | 21.5% | 22.1% | 29.3% | 58.0% | 0.0% | 24.0% | 17.1% | 24.2% | 29.5% |

Source: Own calculations.

Nevertheless, there are no adequate backtesting results for any crypto assets considered. Regarding the temporal distribution of the backtesting outliers, it is obvious that the forecasts become poor from January to March 2021. In this way, 90% and more of the outliers in the observation period from January 2021 can be located across all risk measurement methods. This also goes hand in hand with the general observation that crypto assets have a highly volatile market phase from this time onwards.

**Fig. 2**: Bitcoin: Comparison of risk measurement methods (99.0 % confidence level, one-day holding period)

Source: Own calculations.

**Fig. 3**: Ethereum: Comparison of risk measurement methods (99.0 % confidence level, one-day holding period)

Source: Own calculations.

**Fig. 4**: Litecoin: Comparison of risk measurement methods (99.0 % confidence level, one-day holding period)

Source: Own calculations.

Using a confidence level of 99.9 % shows an improvement in the forecast quality compared to a confidence level of 99.0 %. It is also noticeable that there are no longer any outliers in the red area, which means that an overall satisfactory risk measurement can be attested. The extreme value theory thus also shows clear advantages in backtesting compared to historical simulation and the variance-covariance approach.

Due to the significant improvement in the forecast quality, it can be summarized that the risk measurement of crypto assets, especially in high confidence intervals, appears to be sufficiently conservative and represents a suitable measurement approach. Nevertheless, the historical simulation and the variance-covariance method do not appear to be entirely suitable for adequately measuring cryptocurrency risks.

**Table 5**: Backtesting violations within a 99.0 % confidence level and a one-day holding period

|  |  |  |  |
| --- | --- | --- | --- |
|  | historical simulation | variance-covariance | Extreme Value Theory |
| 99.0 % confidence level | cumulative | rolling | cumulative | rolling |
| Bitcoin | total  | 41 | 22 | 69 | 73 | 30 |
|   | since 01/2021 | 38 | 13 | 62 | 65 | 27 |
| Ethereum | total  | 39 | 29 | 67 | 70 | 22 |
|   | since 01/2021 | 39 | 19 | 63 | 66 | 22 |
| Litecoin | total  | 15 | 20 | 22 | 22 | 7 |
|   | since 01/2021 | 15 | 12 | 22 | 22 | 7 |

Source: Own calculations.

**Table 6**: Backtesting violations within a 99.9 % confidence level and a one-day holding period

|  |  |  |  |
| --- | --- | --- | --- |
|  | historical simulation | variance-covariance | Extreme Value Theory |
| 99.9 % confidence level | cumulative | rolling | cumulative | rolling |
| Bitcoin | total  | 7 | 7 | 45 | 21 | 1 |
|   | since 01/2021 | 6 | 4 | 42 | 14 | 1 |
| Ethereum | total  | 7 | 6 | 46 | 29 | 2 |
|   | since 01/2021 | 7 | 3 | 46 | 19 | 2 |
| Litecoin | total  | 3 | 5 | 13 | 19 | 1 |
|   | since 01/2021 | 3 | 4 | 13 | 13 | 1 |

Source: Own calculations.

Bitcoin tends to show the worst backtesting results, while Litecoin shows the best backtesting results in the two classic methods. Referring to Ahelegbey *et al.*, 2021, Bitcoin can be classified as "speculative" and "diversification". It can be concluded from this that it generates good efficiency advantages, especially in the diversification function. This also goes hand in hand with further research that highlights the diversification benefits of Bitcoin. This is not opposed to the fact that Bitcoin has the worst singular risk measurement in backtesting. It is all the more astonishing that Litecoin as "speculative" and "complementary" shows better backtesting than Ethereum as "professional" and "complementary." A possible explanation is that the "speculative" Litecoin went through a more pronounced volatile market phase at the beginning of 2018 and thus caused more conservative input parameters. In contrast, volatility was less noticeable for Ethereum in early 2018.

**5. Conclusion**

In summary, it can be said that the historical simulation and the variance-covariance method do not appear to be suitable for measuring the risk of crypto assets. On the one hand, this is due to the lack of stationarity in the time series. On the other hand, the normal distribution as the central assumption of the variance-covariance method is not fulfilled either. The unsuitability of the two measurement approaches is particularly evident in a highly volatile market phase from January 2021 since almost all outliers are recorded during this time.

The research question, a) 'Are the common Value at Risk approaches an adequate measurement approach?' is to be answered negatively.

Concerning the research question b) 'Does an extreme-Value Theory-based Value at Risk provide a better fit to the statistical characteristics of crypto assets?' the question has to be answered differentiated. Thus, the research question cannot be confirmed for a confidence interval of 99.0 %. For a confidence interval of 99.9 %, the extreme value theory shows significantly better backtesting so that the hypothesis can be confirmed here.

It is also noticeable here that the three tested cryptocurrencies do not draw completely uniform backtesting results. On the one hand, this may lie in their specific characteristics. On the other hand, the work of Ahelegbey *et al.* can also be taken up, according to which the various assets have different specifications. It can be subsumed that "professional" or less "speculative" crypto assets could tend to include less extreme loss events in the time series. Poorer results in this respect accompany this.

So, it is necessary to adjust the risk measurement depending on the focused currency. In particular, the lack of stationarity and the lack of loss events in the available history should be taken into account. In particular, the extreme value theory can adequately depict the latter property.

**Literature**

[1] Basel Committee (2021), “Prudential treatment of cryptoasset exposures. BCBS 519”, available at: https://www.bis.org/bcbs/publ/d519.pdf (accessed 6 June 2022).

[2] Pardalos, P. (2020a), Mathematical Research for Blockchain Economy: 2nd International Conference MARBLE 2020, Vilamoura, Portugal, Springer proceedings in business and economics, Springer International Publishing AG, Cham.

[3] Mehmke, F., Cremers, H. and Packham, N. (2012), Validierung von Konzepten zur Messung des Marktrisikos: Insbesondere des Value at Risk und des Expected Shortfall, Working paper series / Frankfurt School of Finance & Management, Vol. 192, Frankfurt School of Finance & Management, Frankfurt, M.

[4] Allen, D.E. (2022), “Cryptocurrencies, diversification and the COVID-19 pandemic”, Journal of risk and financial management.

[5] Borri, N. (2019), “Conditional tail-risk in cryptocurrency markets”, Journal of empirical finance.

[6] Guo, L., Härdle, W. and Tao, Y. (2021), A time-varying network for cryptocurrencies, IRTG 1792 discussion paper, 2021, 016, International Research Training Group 1792, Berlin.

[7] Arslanian, H. (2022), The Book of Crypto: The Complete Guide to Understanding Bitcoin, Cryptocurrencies and Digital Assets, Springer eBook Collection, Springer International Publishing; Imprint Palgrave Macmillan, Cham.

[8] ECB (2019), “Crypto-Assets: Implications for financial stability, monetary policy, and payments and market Infrastructures”, available at: https://www.ecb.europa.eu/pub/pdf/scpops/ecb.op223~3ce14e986c.en.pdf (accessed 6 June 2022).

[9] European Parliament and of the Council (2013), prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012: CRR III.

[10] Huschens, S. (2017), Risikomaße, Dresdner Beiträge zu Quantitativen Verfahren, 68/17, Saechsische Landesbibliothek- Staats- und Universitaetsbibliothek Dresden; Technische Universität Dresden, Dresden.

[11] Wiedemann, A. (2013), Risikotriade, Competence Center Finanz- und Bankmanagement, Vol. 4, Frankfurt School Verl., Frankfurt am Main.

[12] Miller, M.B. (2018), Quantitative Financial Risk Management, Wiley Finance Ser, John Wiley & Sons Incorporated, Newark.

[13] Rüder, A. (2019), Zinsänderungs- und Bilanzstrukturrisiken: Neue Konzepte Zur Abbildung Von Volumen- und Zinseffekten, Business, Economics, and Law Ser, Gabler, Wiesbaden.

[14] Pesaran, M.H. (2016), Time series and panel data econometrics, First edition, Oxford University Press, Oxford.

[15] Gleißner, W. (2019), Risikoaggregation und Monte-Carlo-Simulation: Schlüsseltechnologie Für Risikomanagement und Controlling, Essentials Ser, Springer, Wiesbaden.

[16] Daníelsson, J. (2006), “Forecasting extreme financial risk”, in Risk management: A modern perspective, Academic Press/Elsevier, Burlington, MA.

[17] Romeike, F. (2020), Erfolgsfaktor Risiko-Management 4. 0: Methoden, Beispiele, Checklisten Praxishandbuch Für Industrie und Handel, 4th ed., Springer Fachmedien Wiesbaden GmbH, Wiesbaden.

[18] Zhao, Z. (2021), “Dynamic bivariate peak over threshold model for joint tail risk dynamics of financial markets”, Journal of business & economic statistics.

[19] Pardalos, P. (2020b), Mathematical Research for Blockchain Economy: 2nd International Conference MARBLE 2020, Vilamoura, Portugal, Springer proceedings in business and economics, Springer International Publishing AG, Cham.

[20] Ahelegbey, D.F., Giudici, P. and Mojtahedi, F. (2021), “Tail risk measurement in crypto-asset markets”, International review of financial analysis.

[21] Embrechts, P., Klüppelberg, C. and Mikosch, T. (2008), Modelling extremal events: For insurance and finance, Stochastic modelling and probability, Vol. 33, 4. corr. printing and 8. printing, Springer, Berlin.

[22] Zeranski, S. (2005), Liquidity at risk zur Steuerung des liquiditätsmäßig-finanziellen Bereiches von Kreditinstituten, Dissertationsreihe / GUC, Gesellschaft für Unternehmensrechnung und Controlling, Vol. 12, GUC, Ges. für Unternehmensrechnung und Controlling, Chemnitz.

[23] Berge, K. / Fröhlich, S. / Locarek-Junge, H. (Ed.) (2006), Risikomanagement aus Bankenperspektive, BWV Berliner Wissenschafts-Verlag, 2006, Berlin.

[24] Saeed Far, S. and Abd. Wahab, A.K. (2016), “Evaluation of Peaks-Over-Threshold Method," Ocean science discussions, pp. 1–25.

[25] Corbet, S., Meegan, A., Larkin, C., Lucey, B.M. and Yarovaya, L. (2018), "Exploring the dynamic relationships between cryptocurrencies and other financial assets" Economics letters.

[26] Sun, W., Dedahanov, A.T., Shin, H.Y. and Li, W.P. (2021), "Factors affecting institutional investors to add crypto-currency to asset portfolios”, The North American journal of economics and finance.

[27] Zhang, Y.-j., Bouri, E., Gupta, R. and Ma, S.-J. (2021), “Risk spillover between Bitcoin and conventional financial markets. An expectile-based approach”, The North American journal of economics and finance.

[28] Ardia, D., Bluteau, K. and Rüede, M. (2019), "Regime changes in Bitcoin GARCH volatility dynamics", Finance research letters.

[29] Troster, V., Tiwari, A.K., Shahbaz, M. and Macedo, D.N. (2019), "Bitcoin returns and risk. A general GARCH and GAS analysis", Finance research letters.

[30] Jiménez, I., Mora-Valencia, A. and Perote, J. (2020), "Risk quantification and validation for Bitcoin", Operations research letters.

[31] Gao, L., Ye, W. and Guo, R. (2022), "Jointly forecasting the value-at-risk and expected shortfall of Bitcoin with a regime-switching CAViaR model", Finance research letters, Vol. 48, p. 102826.

[32] Stavroyiannis, S. (2018), "Value-at-risk and related measures for the Bitcoin", The Journal of Risk Finance, Vol. 19 No. 2, pp. 127–136.