Modeling of Orthometric Height of Gongola Basin using GPS/Levelling and EGM 2008 and its Geophysical Implications

E. E. Epuh, Oginni and M. Orji
Department of Surveying and Geoinformatics, University of Lagos,
eeepuh@yahoo.com

Abstract
In Gongola basin, GPS observations on Benchmarks (BM) are very scarce and cover only small parts; in terms of spatial scale of the region. It is well understood that most accurate determination of geoid is obtained by dense and well distributed national and regional gravity points. Since Gongola basin is one of the Nigerian Frontier Inland Sedimentary Basins (NFISB), reliable GPS/Levelling and gravimetric geoid heights adjustments studies are required to produce reliable heights for geophysical investigations.
In this research, some new potential for the common adjustment of the available geometric, orthometric and geoid heights using parametric models were used. Each corrective term in the models were equaled to the residual geoid obtained from the separation of the GPS geoid and that obtained from a global geopotential model: Earth Gravity Model (EGM 2008). An extensive emphasis are made to the questionable behavior of orthometric heights in geophysical investigations. Incorrect orthometric height observations/estimations produces a faulty topographic mass density distribution which causes distortion in the observed Bouguer anomalies. The adjustment of the orthometric heights using the 5 parameter corrector model as applied to EGM 2008 produced a ±13.3cm orthometric height defects. This height defect across the basin produced a difference between the observed Bouguer anomaly and the new computed Bouguer anomaly using the corrected orthometric height the difference has a maximum value of 0.522mGal, a minimum value of 0.431mGal, mean value of 0.465 and standard deviation of 0.018mGal. This shows that the observed Bouguer did not meet the 0.01mGal accuracy limit and should be readdressed adequately using the corrected value for future geophysical investigation.

Keywords: orthometric height, EGM 2008, geoid, GPS, parametric models, Bouguer anomaly

1.0 Introduction
Orthometric height of a point is the distance H along a plumbline from the point to the geoid. It is a function of position $(H = H(\theta,\lambda))$. In order to estimate the maximum contribution of the topographic potential to geoid determination, isostatic compensation or geophysical investigation, we can only consider the worst cast, i.e, $(H = H(\theta,\lambda)) = \text{constant}$ as an approximation (Sun, 2000). Since gravity is not constant over a large areas, orthometric height of a level surface is not constant. Traditionally, orthometric heights are obtained using spirit levelling. It could also be obtained by subtracting the geoid height from GPS observation (which ideally gives ellipsoidal height as shown in Fig 1. The determination of precise orthometric height using this traditional methods needs validation in some sense since its error seriously affects the gravity modeling of the earth interior and seismic acquisition parameters. Practical applications need to use a model rather than measurements to calculate the change in gravitational potential versus depth in the earth; since the geoid is below most of the land surfaces (i.e. Helmert orthometric height).

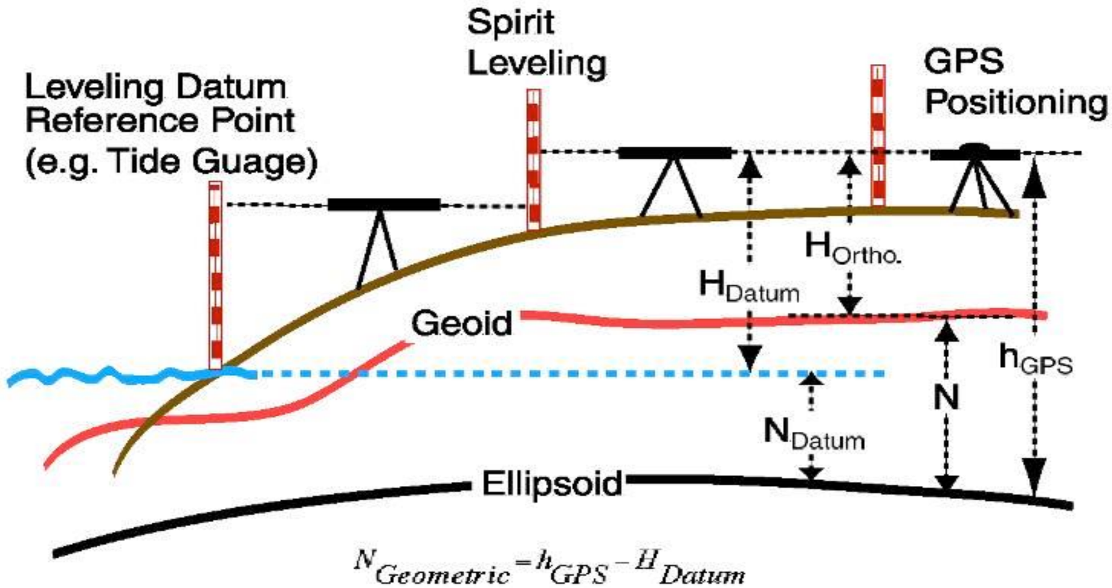


Fig 1. Geometrical relationship between orthometric, ellipsoidal and the geoid heights (Source: Pelvis *et al.*,2008)

According to Tziavos *et al.,* (2011) collocated observation of h,H, and N are used to a) assess the external accuracy of gravimetric geoid models (Featherstone *et al* (2001), b.) construct corrector surfaces in an area of study, so that the transformation between either of the can be made (Sideris *et at., 1992),* and c.) substitute conventional spirit levelling by GPS/Levelling during which there is no need to measure orthometric heights since they are determined by GPS measurements and gravimetric geoid heights (Fotopoulos *et al.*, 2001, Vergos and Sideris, 2002).

In Geophysics, the use of orthometric heights has become tangible after the improvement of knowledge of the Earth's gravity field with satellite missions (e.g Kaula; 1967, Parsons and Daly; 1984, Hager; 1984, Lambeck; 1988). In this context, geoid undulations were found to relate with major geophysical formations. To compute the gravity anomaly (at the geoid), the up/downward continuation requires knowledge of the vertical gradient (along the plumb line) of the Earth's gravity field ($\delta g/\delta H$) interior and sometimes exterior (e.g. for airborne data) to the Earth's gravitating masses (*cf.* Hammer 1970; La Fehr & Chan 1986). In practice, however, this vertical gravity gradient along the plumb line is difficult to estimate accurately, especially inside the topography (*cf.* Vanicek *et al.* 1996; Wang 1997; Sun and Vanicek 1998, Hackney and Featherstone *et al*, 2003). Instead, the vertical gradient of normal gravity ($\delta\gamma /\delta h$), which is recognized as the free-air correction, and the Bouguer gradient are usually used as an approximation. Irrespective of the use of the plate/shell/cap Bouguer models of the topography (which must also be embedded in the terrain correction for consistency), the overriding limitation in this Bouguer correction is the accurate estimation of the topographic mass density (e.g., La Fehr 1991b; Vanicek *et al.* 1999; Huang *et al.* 2001; Hackney and Featherstone, 2003). Incorrect estimation of the topographic mass-density causes distortions in the Bouguer gravity anomalies. Topographic mass-density distribution are usually highly correlated with geological structures, thus causing problems in geophysical interpretations, and also causing aliasing in gravity gridding and prediction in geodesy. The requisite precision in vertical positioning is required to produce a combined elevation correction of $0.2mGalm^{-1}$. To achieve a survey accuracy of ±0.01mGal, the elevation of the gravimeter above the reference ellipsoid must be known to about ±5cm (Lowrie 2007). In the event that geoid undulation are large enough to affect a survey, the station orthometric height must be corrected to true elevation above the ellipsoid.

Many other geophysical observations also require precise determination of the orthometric height. Example is the application of erroneous orthometric height in the seismic static corrections in the determination of low velocity layer which introduces significant error. Seismic recording involves a source and receiver separated by an offset distance (Dobrin and Savit, 1988). It is obvious from Fig 2. That the traveltime of a wavelet along a raypath is influenced by the surface elevations of the geophone and the shotpoint. Today, we are acquiring seismic data in more rugged areas and areas that were formerly "no data" areas and pushing existing technologies to the limit of usefulness in this process (Marsden, 1993). The need for higher resolution data demands the determination of near surface problems at the surveying planning stage. This requires the determination of better static corrections. Static corrections involve a constant time shift to the data traces. Corrections made to each seismic trace for elevation (orthometric height) effects (elevation statics) and near-surface low velocity effects (weathering statics) by conceptually moving the shots (shot statics) and receiver (the receiver static) to a common reference surface (datum plane) are necessary for proper data processing. Static corrections are most important in the processing of land data which leads to improved quality in subsequent processing steps which, in turn, impact the integrity, quality and resolution of the imaged sections. With rugged topography, often times, a floating datum is used to carry out elevation and weathering corrections at both the shot and receiver stations to avoid different datum elevations at line intercessions. Such a surface is formed by averaging the orthometric height lying within a circle of inclusion whose diameter is equal to the spread length. However, it is equally likely that a floating datum at least will lie above the orthometric height (topography) or within the weathered layer. Improper floating datum constitutes an error in the required elevation correction. In static correction, which amounts to moving the entire seismic trace up or down in time in order to put the shots and receivers on a flat datum plane and the correction of the near surface velocity anomalies beneath the source or receiver requires a correct orthometric height for effective implementation. The correction procedure involves establishing a datum on which to locate source and receiver. Reflection seismic theory is based on a horizontal datum, but the datum of the field data is the topography (orthometric height). So static correction due to elevation is expressed as a change in travel time. According to (Dubrin and Savit, 1988)

$$\Delta t = \frac{H_s}{v_1} - \frac{H_r}{v_1} \tag{1}$$

where $\Delta t =$ Travel time, $H_s =$ shot point orthometric height

$H_r =$ receiver point orthometric height, v_1 = Velocity

In the near surface velocity anomalies, the presence of anomalous velocities beneath a source or receiver or if the thickness of the weathering layer changes substantially, the amount to be subtracted from the seismic trace time is given by the following formula (Fig 2)

$$\Delta t = \frac{H_a}{v_a} - \frac{H_a}{v_1} = \left(\frac{v_1 - v_a}{v_1 v_a}\right) H_a \tag{2}$$

where $H_a =$ shot point orthometric height, v_a = Velocity

Clearly, there is a need for a systematic study of the orthometric height correction in both geodesy and geophysics, including theoretical and numerical comparisons of the correction algorithms with one another. Based on the above analysis, it is concluded that the correction or validation of orthometric heights for density variation within consolidated crust, introduces large errors and should not be taken as being reliable at the present stage in gravity field modeling. A way to improve these data is the application of a correction model to the residual geoid obtained as a difference between the local geoid (GPS) and that obtained using the Global geopotential model (EGM 2008).

Researchers such as Heiskanen and Moritz (1967), Sideris *et al* (1992), Kearsley *et al.*, (1993), Featherstone *et al*., (1998), Musa (2003), Kiamehr (2006), Forsberg and Madsen (1990), Fotopoulos *et al.*, (1999), Mainville *et al*., (1992), Erol and Celik (2004), have written on detailed geoid modeling techniques. Nwilo *et al.*, (2009) determined geoid in Lagos using geometrical interpolation, while Okiwelu *et al.,* (2011) determined geoid in Nigeria using EGM 2008. Forsberg *et al*., (2004) used a four parameter Helmert transformation model to derive a corrector surface. The corrector surface was consistent across the United Kingdom at the level of between 1 and 2cm. Ozen *et al.*,(2002) utilized a multi quadratic interpolation technique to fit the GPS/levelling data in Tonga, Turkey. Cecilia *et al.*,(2007) derived orthometric heights used corrector surfaces that brought precision to the observed orthometric heights. Isioye *et al.,* (2011) utilized a five to eight parameter model to fit the GPS/Leveling to the EGM 2008 model to improve the determination of orthometric height observed from GPS in Port-Harcourt, Nigeria. Elimann *et al*., (2000), Ahmeed (2009) utilized gravimetric geoid derived from Stokes' integral to determine the orthometric heights. In the work by Forsberg and Kearsley (1989) the discovered the difference of between 12cm and 22cm in orthometric heights derived from GPS determined geoid and gravimetric geoid while Anuar (2003) obtained orthometric height accuracy of 10cm by fitting a polynomial model using GPS/local geoid data and 80cm accuracy in using GPS/Global geopotential model. The Global Geopotential model adopted in this research for orthometric height determination is the Earth Gravity Model (EGM 2008) Pelvis *et al,* (2008). The main goal of this paper is to firstly, investigate the blunders in the observed orthometric heights and its correction using collocation of GPS and EGM 2008 geoid observations. Secondly, this study is carried out to evaluate the effect of the corrected orthometric heights and provide an improved definition of the Bouguer gravity anomalies of the basin used for gravity modeling and other geophysical investigations.

1.2 Description of Study Area

Gongola Basin forms part of the Upper Benue Trough. The study area is located North-West of the Upper Benue Trough in the Gongola Basin. The study area is located North-West of the upper Benue Trough in the Gongola basin. It is bounded by Longitudes 10.3ºE and 10.8ºE, Latitudes 9.9ºN and 11.1ºN. The prospect area is located within the following places; Alkaleri (W), Darazo (NW), Dukku(NE), Larishi (E), and Kwala(S). A substantial part of the area is bounded by Gongola River at the Northwestern part of the prospect. The greater part of the area is low-lying with elevation varying from 254.6m to 543.5m, and characterized by gentle relief with small hills concentrated especially on the eastern riverside of Gongola. Gongola basin falls within the region now known as Nigerian Frontier Inland Sedimentary Basin (NFISB). It is anticipated that the NFISB hold promise of containing reserves of yet undiscovered oil and gas. The Basin is shown in Fig 3.

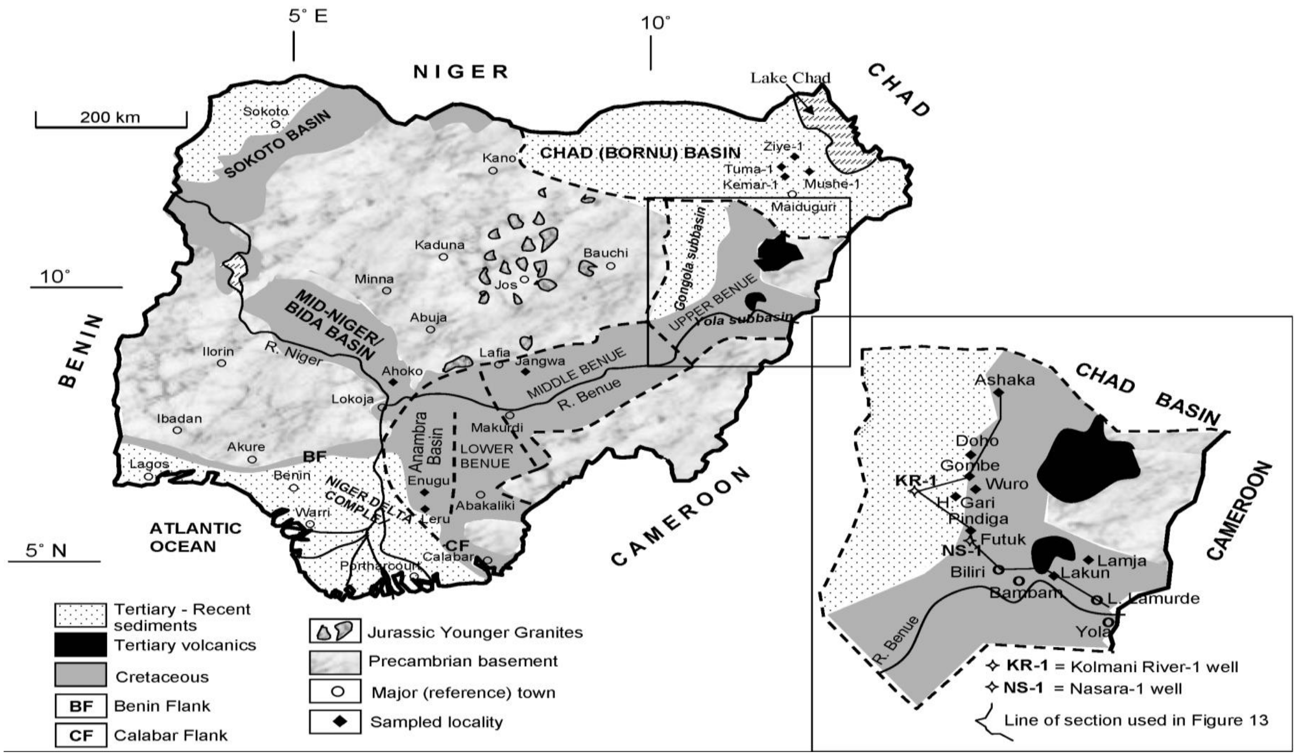


Fig. 3 Sketch geological map of Nigeria showing the inland basins and sample localities. (inset:upper Benue trough magnified (Source: Obaje *et al*., 2006))

3.0 Model Formulation

The equipotential surface of the Earth's gravitational field is called the geoid. It reflects the true distribution of mass inside the earth. The international reference ellipsoid is a close approximation to the equipotential surface of gravity. Fig. 1 shows the relationship between the geoid and the ellipsoid. The fundamental relationship, that binds ellipsoidal height (i.e. GPS heights) and height with respect to a vertical datum established from spirit levelling and gravity survey is given by

$$h_{GPS} - H_{orthometric} - N = \varepsilon \qquad (3)$$

where h is the ellipsoidal height, H is the orthometric height, N is the geoidal undulation and ε is the offset of the vertical datum with respect to the geoid. The application of equation (3) is more complicated due to numerous factors, which cause discrepancies when combining the different height data sets. This is majorly attributed to systematic effects and datum inconsistencies, which can be described by a corrector surface model (Isioye *et al*., 2011). In practice, the various wavelength errors in the gravity solution may be approximated by different kinds of functions in order to fit the geoid to a set of GPS/Levelling points through an integrated Least Squares adjustment. Systematic errors, distortions and datum inconsistencies between orthometric, ellipsoidal and geoid height can be absorbed by fitting GPS/levelling derived geoid height to a gravimetric geoid height using least-squares adjustment and using four and five parameter model. These models according to Tziavos,(2011) produced better results than the polynomial models in the GPS/Levelling geoid collocation adjustments.

$$\Delta N_i = N_i^{GPS} - N_i = h_i - H_i - N_i = a_i^T x + \varepsilon_i \qquad (4)$$

Where N_i is the interpolated geoid height value for the GPS point, considering points from the geoid model that exist in the neighborhood, x is a n×1 vector of unknown parameters (where n the number of the GPS/levelling points), a_i is a n x 1 vector of known coefficients, and ε_i denotes a residual random noise term. The parametric model $a_i^T x$ is supposed to describe the mentioned systematic errors and inconsistencies inherent in the different height data sets. The four and five parameter models according to Fotopoulos (2003) are stated as follows:

The *4-Parameter Model (Model A)* is stated as:

$$a_i = (\cos\phi_i \cos\lambda_i, \cos\phi_i \sin\lambda_i, \sin\phi_{i,} 1)^T$$
$$x = (x_1, x_2, x_3, x_4)^T \qquad (5)$$

$$\Delta N = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i + v_i \qquad (6)$$

Where x_2, x_3 and x_4 x_2, x_3, and x_4 are the shift parameters between two parallel datum and x1 are the changes in semi-major axes of the corresponding ellipsoids, ϕ_1, λ_1 are the latitudes and longitudes, respectively for the Gravity Base stations.

If we add a fifth parameter, the extended 5-parameter model (Model B) can be obtained as:

5-Parameter Model (Model B):

$$a_i = (\cos\phi_i \cos\lambda_i, \cos\phi_i \sin\lambda_i, \sin\phi_{i,} 1, \sin^2\phi_i)^T$$
$$x = (x_1, x_2, x_3, x_4, x_5)^T \qquad (7)$$

$$\Delta N = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i + x_5 \sin^2\phi_i + v_i \qquad (8)$$

$$x_5 = a\Delta f \qquad (9)$$

Where x_1, x_2, x_3 and x_4 are the shift parameters between two parallel datum and x_1, Δf are the changes in semi-major axes and flattening of the corresponding ellipsoids.

Based on limited data acquired, 7-parameter and 8-parameter models were not be formulated the following matrix system of observation equations is now obtained:

$$\begin{array}{ccc} x & A & \Delta N \\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = & \begin{bmatrix} 1\ \cos\phi_1 \cos\lambda_1 \cos\phi_1 \sin\lambda_1 \sin\phi_{1,} 1\sin^2\phi_1 \\ 1\ \cos\phi_2 \cos\lambda_2 \cos\phi_2 \sin\lambda_2 \sin\phi_{2,} 1\sin^2\phi_2 \\ \\ 1\ \cos\phi_4 \cos\lambda_4 \cos\phi_4 \sin\lambda_4 \sin\phi_{4,} 1\sin^2\phi_4 \end{bmatrix} & \begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \\ \Delta N_4 \end{bmatrix} \end{array}$$

$$Ax = \Delta N - \varepsilon \tag{10}$$

Where A is the design matrix composed of one row x_1^T for each observation ΔN_i.

For 4-parameter model, A(6X4) matrix

For 5-parameter model, A(6X5) matrix

The least square adjustment to this equation utilizing the mean squares of the residuals ε_i becomes:

$$X = (A^T PA)^{-1} * (A^T P \Delta N) \tag{11}$$

where $P =$ weight matrix i.e. the inverse of the variance-covariance matrix C of the observation.

This yield:

$$\varepsilon = \Delta N - A\hat{X} = [I - (A^T PA)^{-1} PA^T]\Delta N \tag{12}$$

3.1 Spherical Harmonics Representation of Geoid

A spherical harmonic function is a function that has the same value of θ or ϕ increased by an integral multiple of 2π just like the trigonometric functions in 1 dimension (1 angle). Spherical harmonics are often used to approximate the shape of the geoid. The geoid undulations are evaluated from spherical harmonic coefficients at the surface of the ellipsoid, not taking into account the difference between height anomalies and geoid undulations (Heiskanen and Moritz, 1967, p. 325ff) nor the effect on the geoid of the downward continuation of gravity from the surface (Sjoberg, 1998; Kaban *et al.*, 2004). In spherical coordinates the gravitational potential on the geoid is given as (Featherstone, 1997):

$$V(r,\phi,\lambda)=\frac{GM}{r}\left(1-\sum_{n=2}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda)\overline{P}_{mn}(\sin\phi)\right) \tag{20}$$

where $V(r,\phi,\lambda)=$ gravitational potential to degree N

$GM=$ geocentric gravitational constant, $r=$ geocentric radius, $a=$ equatorial radius

$\phi,\lambda=$ geocentric (spherical) co-latitude and longitude respectively. $\overline{P}_{mm}$ = The fully normaalized associated Legendre polynomials of degree n and order m, and $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the nunerical coefficients of the model based measured data.

Also, given that the spherical harmonic expansion of the normal gravitational potential is expressed according to Featherstone (1997) as:

$$U^{*}=\frac{GM^{e}}{r}\left(1-\sum_{n=2}^{N}\left(\frac{a^{e}}{r}\right)^{n}(\overline{J}_{n0})\overline{P}_{mn}(\sin\phi)\right) \tag{21}$$

the anomalous (disturbing) potential T is given as

$T=V-U^{*}$

By introducing the Brun's formula (Heiskanen and Moritz, 1967)

$$N=\frac{T}{\gamma} \tag{22}$$

$N=$ geoid undulation, γ=normal gravity

N can be expressed in spherical harmonics as (Rapp *et al*, 1991)

$$N=\frac{GM}{r\gamma}\left(\sum_{n=2}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda)\overline{P}_{mn}(\sin\phi)\right) \tag{23}$$

The EGM 2008 presents a spherical harmonics expansion of the geopotential to degree and order 2159 (Pelvis *et al.*,, 2008). The availability of such GGMs poses new potentials in order to validate available orthometric heights and subsequently correct blunders in the levelling database (Tziavos *et al*., 2013)

Combination Solution based on **57 months** of GRACE satellite-tosatellite tracking data and a global set of $5'\times5'$ Δg **(~ 9.3M values)**. Expressed in **Ellipsoidal Harmonic Coefficients,** EGM08 is complete to degree and order 2159 **(~ 4.7M coefficients)**. When converted to **Spherical Harmonic Coefficients**, additional terms up to degree 2190 and order 2159 arise. The **Spatial Resolution** (half wavelength) of EGM08 is (nominally) **9.3 km** × **9.3 km** on the equator, which is **6 times higher** than that of EGM96. The resolving power of any model depends highly on the data used for its development, whose properties are geographically dependent. The **Accuracy** of EGM08, as gauged from comparisons with **independent** data, is **3 to 6 times higher** than that of EGM96, depending on the functional (e.g., geoid undulation, deflection of the vertical) and on the geographic area in question .

3.0 Methodology

3.1 Gravity Data Acquisition

The station interval of 500m was used and a total of 1813 gravity stations were observed acquired from SNEPCO (Fig 4a). The acquired data lies within 625000 mE to 700000mE and 1096818mN to 1225000mN ($(\lambda = 10.3^o E - 10.8^o E, \phi = 9.9^o N - 11.8^o N)$. The station data include location details in Northings and Eastings (referenced to Minna Datum) and also WGS84 longitude and latitude, Observed gravity (in mGal), Ellipsoidal Height (i.e. GPS Height) and Orthometric height. Fig 4b shows the gravity base station distribution. Table 1 showing the attributes of the base stations. Fig 4 shows the distribution of the base stations. The orthometric height obtained on the field using spirit levelling is shown in Fig 5. The geoid undulation for Gongola basin computed using the EGM 2008 algorithm is shown in Fig 6.

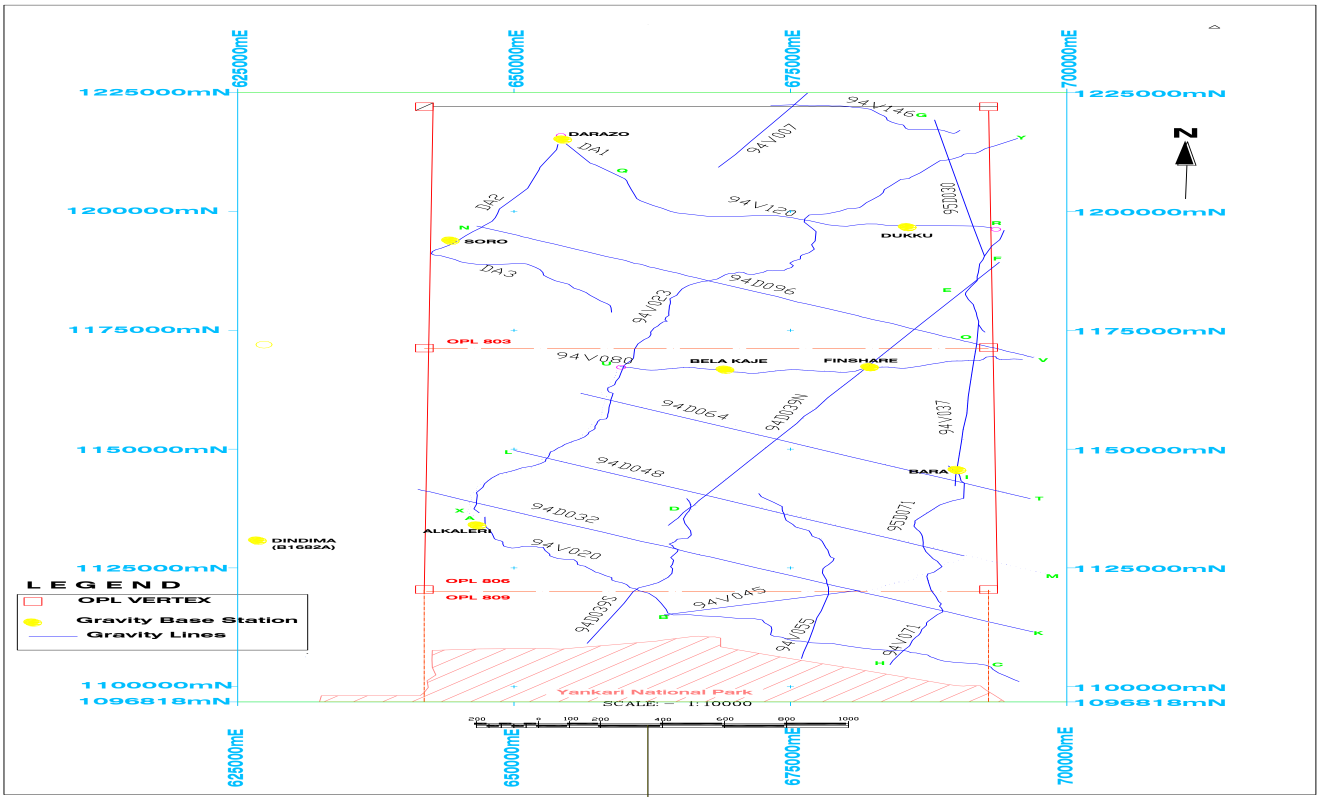


Fig 4a: Gongola Basin Gravity Basemap (Source: SNEPCO, 1995)

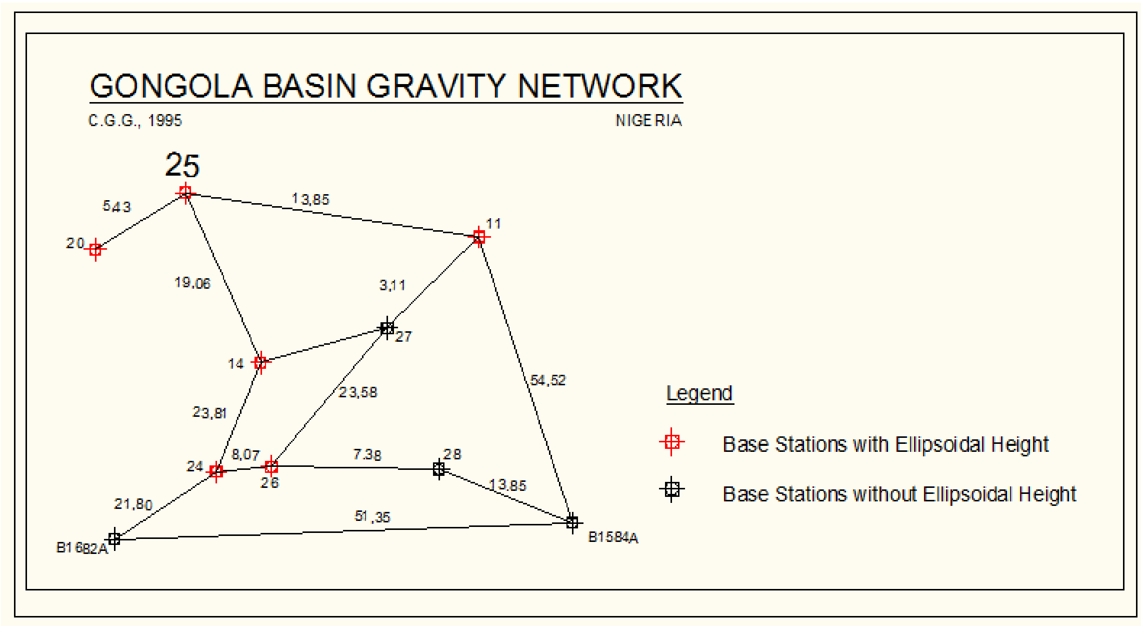


Fig 4b: showing the gravity base station distribution (Source: SNEPCO 1995)

Table 1: The base station Coordinates, orthometric, Ellipsoidal and Geoid heights and Normal Gravity

Gravity Base station	X	Y	Orthometric Height	Lat	Long	Ellipsoidal Height	Normal Gravity
B1584A	732290.3	1130022.09	635.08	10.28893	11.12093	N/A	978032.26
B1682A	625651.39	1132022.93	465.77	10.23958	10.14725	N/A	978083.61
SNEP 11	686908.74	1196864.42	421.8	10.82347	10.70898	439.34	978086.78
SNEP 14	669246.6	1166426.09	494.4	10.54914	10.54607	512.28	978081.58
SNEP 20	644372.63	1193465.77	496.7	10.79465	10.31985	515.04	978095.21
SNEP 24	646990.65	1133847.11	420.7	10.25549	10.34146	439.18	978105.41
SNEP 25	634248.27	1214957.73	510.1	10.98856	10.41107	528.21	978100.63
SNEP 26	647190.48	1135778.98	382.81	10.27295	10.34335	400.87	978113.47
SNEP 27	682054.16	1166943.09	392.39	10.66379	10.55308	N/A	978089.89
SNEP 28	689312.7	1146795.72	323.1	10.37057	10.72909	N/A	978206.1

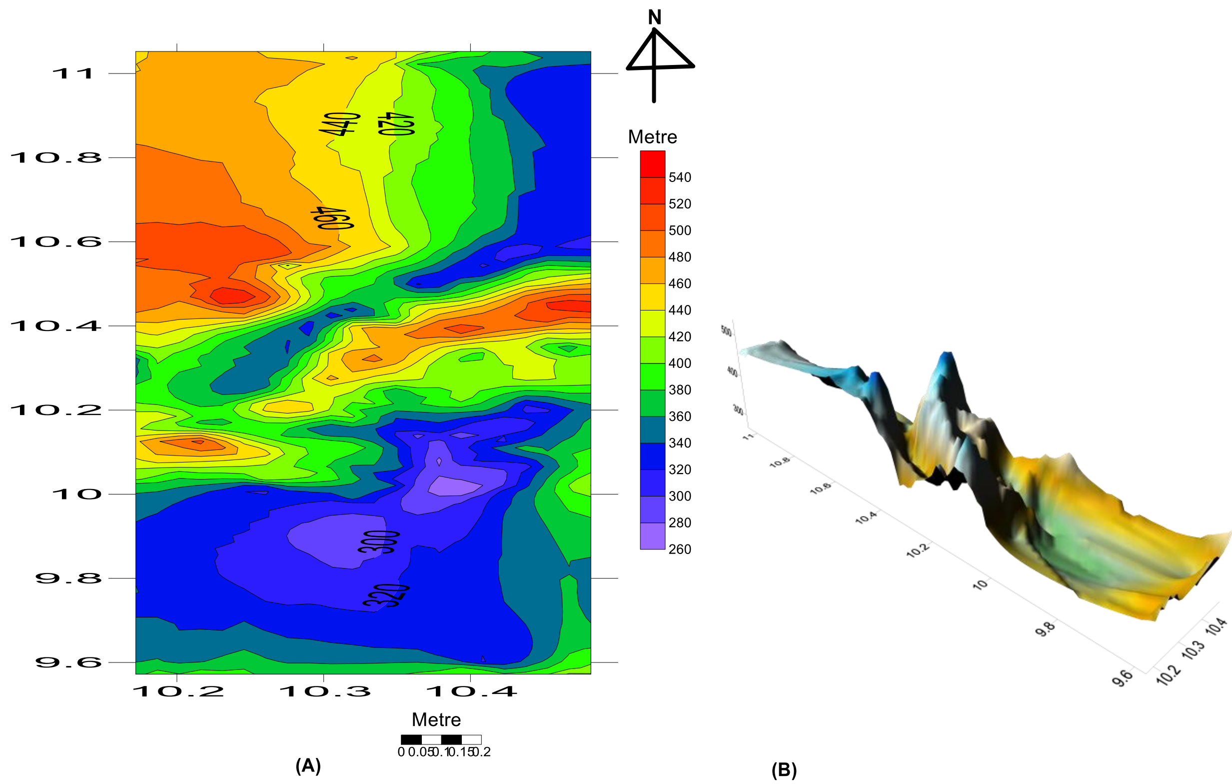


Fig. 5: Contour Map showing Field/Observed Orthometric Height

3.3 EGM 2008 Geoid Undulation

URL1 (2008) is an **EGM 2008 calculator at ICGEM.** Is an http://icgem.gfzpotsdam.de/ICGEM/ICGEM.html. which can be downloaded at URL1. The default ICGEM installation comes with a 10' grid file. The 25' file was downloaded and used with ICGEM to obtain more accurate interpolation. More information on EGM 2008 and downloadable grid files can be found at URL1.

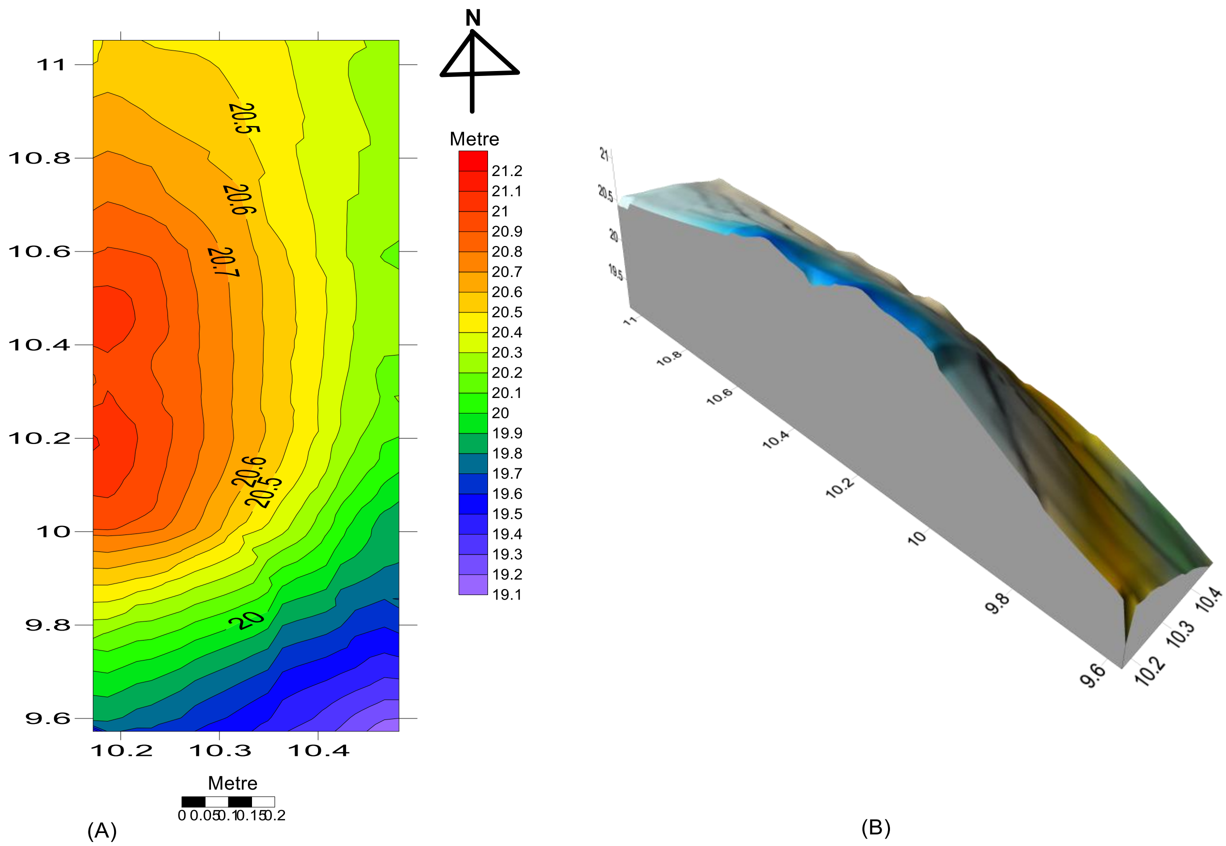


Fig. 6: Gongola Basin Geoid Undulation Map using EGM 2008. (C.I=0.01m)

Table 2: Gravity base stations with Geoidal Undulations from EGM 2008

S/N	Gravity Base Stations	X	Y	Geoidal Undulation
1	SNEP 11	686908.74	1196864.42	19.588
2	SNEP 14	669246.6	1166426.09	19.979
3	SNEP 20	644372.63	1193465.77	20.599
4	SNEP 24	646990.65	1133847.11	20.603
5	SNEP 25	634248.27	1214957.73	20.326
6	SNEP 26	647190.48	1135778.98	20.598

4.0 RESULTS AND DISCUSSION

4.1 Results

This section shows result obtained for adjusted parameters for Model A (4-parameter model) and Model B (5-parameter) datum shift transformation models and their estimates.

$$\Delta N = N^{GPS} - N^{EGM} \quad \textbf{(24)}$$

From the results obtained above for Model A and Model B, 4 parameters and 5 parameters respectively. Model B (i.e. 5 parameter model) gave model with best fit with minimum standard deviation of 0.1338 as to 0.1343 of Model A (i.e. 4 parameter model). The main problem of using these models is that the final residuals ε hold a combined amount of random errors related to GPS observation, levelling operation and Geoid. Therefore, this result does not show the real potential of these models, the final residuals cannot be taken as the exact error of the transformation models. The standard deviation of the residuals is taken as an indication of absolute accuracy of the models.

Table 3: Differences between GPS geoid heights and EGM2008 geoid heights with residuals

Gravity Base station	Lat ϕ	Long λ	N_GPS (m)	N_EGM(m)	ΔN
SNEP 11	10.82347	10.70898	17.54	19.588	-2.048
SNEP 14	10.54914	10.54607	17.88	19.979	-2.099
SNEP 20	10.79465	10.31985	18.34	20.599	-2.259
SNEP 24	10.25549	10.34146	18.48	20.603	-2.123
SNEP 25	10.98856	10.41107	18.11	20.326	-2.216
SNEP 26	10.27295	10.34335	18.06	20.598	-2.538
STD(σ)					0.1767

Residuals are defined as the difference between the *observed* values and the values that are *predicted* by a model. When a model is fit that is appropriate for a particular data, the residuals approximate independent random errors. To calculate fit parameters for a linear model, the sum of the squares of the residuals are minimized to produce a good fit, which is known as least-squares fit.

From Table 3, the difference between the GPS and EGM 2008 geoid height are presented. It shows an average geoid error of 2.2m. The results in Table 4 shows the summary of the input and output after fitting using the parameter models. It is evident from the table that the best fit is obtained using the five parameter model as it presented a minimized residual for all the stations. The reduction by 4cm of the std was achieved while the range reduces by 20cm. Examining the residuals before the fit, the large mean, and std of the height differences is well noticed. Fig 6, shows the graphical representation of the residual of the 4 and 5 parameter model. Table 5 shows the statistical analysis in terms of minimum, maximum, mean and std of the residuals before and after fitting. The 5-parameter model gives a minimum std of 0.122m compared to 0.1226m of the 4-parameter model.

Table 4: Differences between GPS/leveling derived geoid height and EGM2008 before and after 4-Parameter and 5_Parameter fitting

Gravity Base station	**Before Fitting**	**After Fitting**	
	ΔN	4-Parameter (residuals in m)	5-Parameter (residuals in m)
SNEP 11	-2.048	0.0136	-0.0036
SNEP 14	-2.099	0.0247	0.0125
SNEP 20	-2.259	0.0008	0.0140
SNEP 24	-2.123	0.2026	0.2032
SNEP 25	-2.216	-0.0053	-0.0073
SNEP 26	-2.538	-0.2198	-0.2188
STD(σ)	0.1767	0.1343	0.1339

Table 5: Statistical analysis of absolute accuracy of Gongola Basin versus 6 GPS/Levelling data

	Before fitting	**After fitting** (residuals) ε(m)	
	ΔN (m)	4-Parameter	5-Parameter
minimum	-2.5380	-0.2198	-0.2188
maximum	-2.0480	0.2026	0.2032
mean	-2.2138	0.0000	0.0000
standard error σ	0.1613	0.1226	0.1222

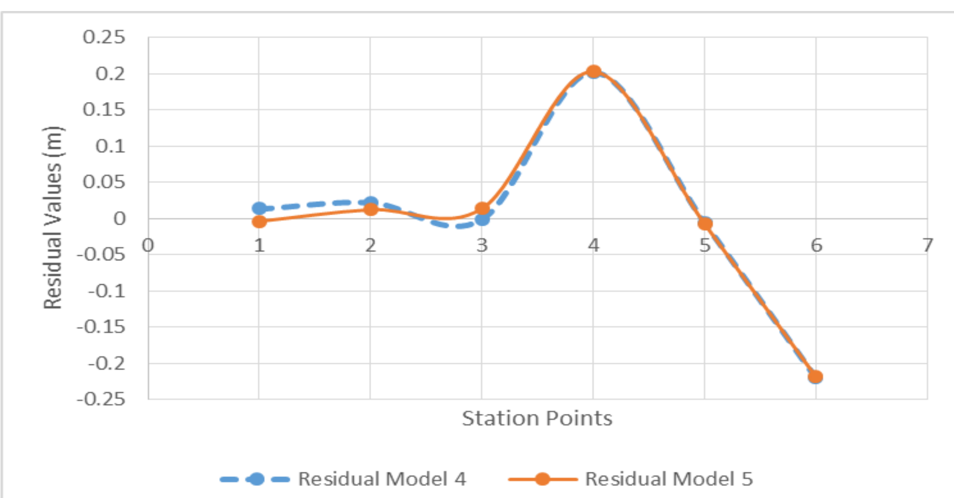


Fig 6: Correlation of the model Residuals

4.2 Validation at Known Points

This aspect of model assessment involves computing orthometric height for gravity base stations and shot points. The 1805 gravity shot points were used for model validation and assessment. This was done by comparing the computed orthometric height values with observed/field values.

Referring to equation (3), (4) and (6), orthometric height is obtained as expressed below.

$$H_i = h_i - N_i^{EGM\,08} - a_i^T x \tag{25}$$

Where H is the orthometric height, h is the ellipsoidal height $a_i^T x$ is the parametric model.

Table 6 shows comparative analysis of the performance of the two models. Table 7 shows the statistical analysis of the base stations while Table 8 shows that statistical analysis of the 1805 validated shot points. Fig 7 shows the contour maps of the field observed orthometric heights and the model B computed orthometric heights.

Table 6: Validation of Orthometric Height at Base Station

S/N	Station Name	X	Y	Ellipsoidal Height	Geoidal Undulation	Orthometric Heights (m)	Computed Orthometric Heights	
							Model A	Model B
1	SNEP 11	686908.7	1196864	439.34	17.54	421.8	421.740	421.789
2	SNEP 14	669246.6	1166426	512.28	17.88	494.4	494.423	494.410
3	SNEP 20	644372.6	1193466	515.04	18.34	496.7	496.438	496.476

4	SNEP 24	646990.7	1133847	439.18	18.48	420.7	420.966	420.993
5	SNEP 25	634248.3	1214958	528.21	18.11	510.1	509.846	509.977
6	SNEP 26	647190.5	1135779	400.87	18.06	382.81	382.642	382.663

Table 7: Summary of Validation at Base Station

	Observed Orthometric Height (m)	Orthometric Height (m)	
		Model A	Model B
Maximum Value	510.1	509.846	509.977
Minimum Value	382.81	382.642	382.663
Mean Value	454.4183	454.342	454.385
Standard Deviation	52.56492	52.4919	52.5085

Table 8: Summary of Results for Validation Test for 1805 Gravity shot points

	Observed Orthometric Height (m)	Orthometric Height (m)	
		Model A	Model B
Maximum Value	543.500	545.848	545.890
Minimum Value	223.50	226.854	226.818
Mean Value	388.759	391.087	388.759
Standard Deviation	64.921	64.908	64.919

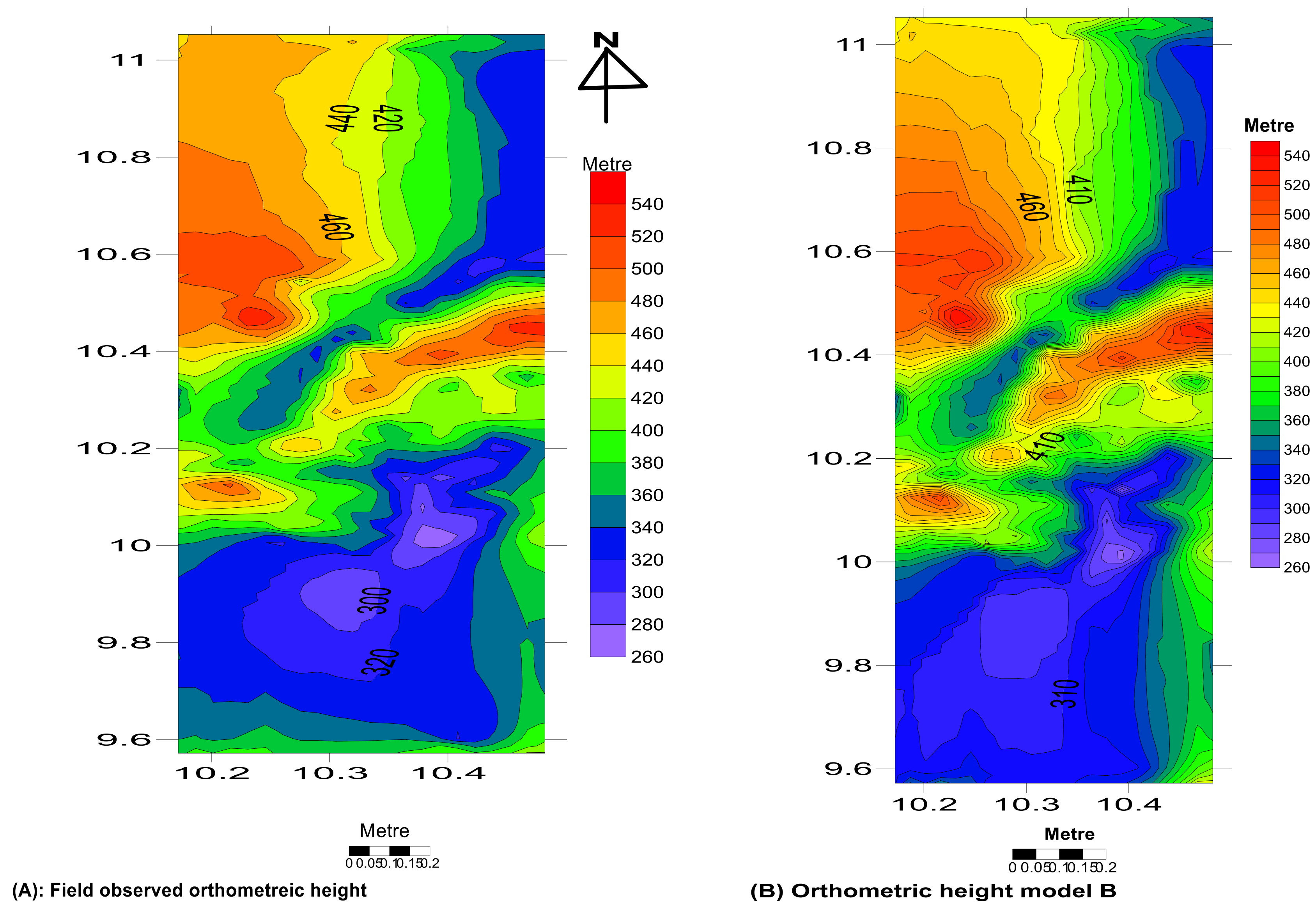
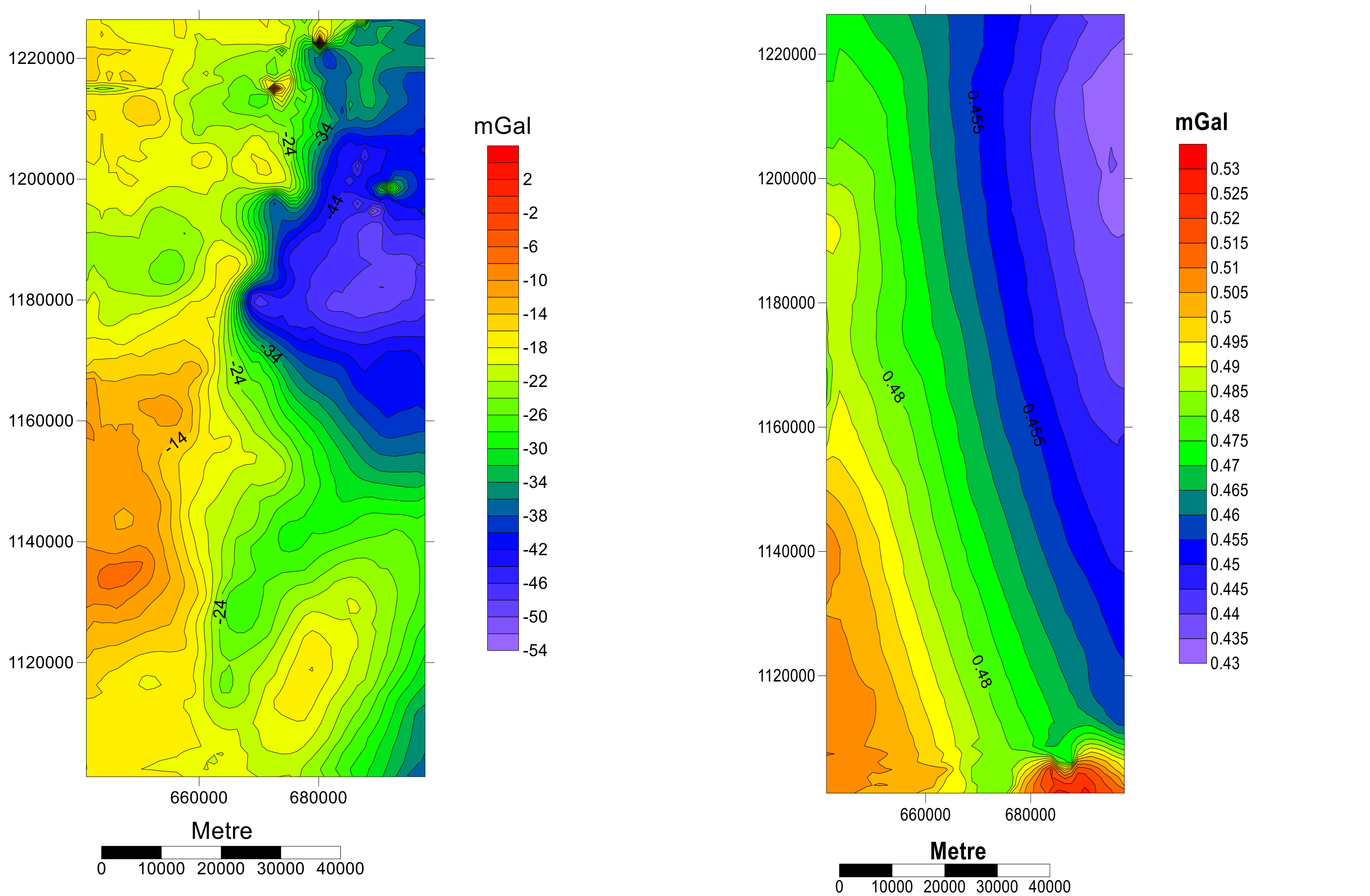


Fig 7: Orthometric heights



(A) Bouguer Anomaly computed using observed orthometric height

(B) Difference between Observed Bouguer and computed Bouguer using Model B

Fig 8: Showing Bouguer anomaly using field orthometric height

4.3 Geodetic and Geophysical Implications

4.3.1 Geodetic Implications

In this study, the effect of combining GPS ellipsoidal height, EGM2008 geoid undulation and observed orthometric height obtained from 6 gravity base description acquired from SNEPCO has been analyzed statistically and spatially. Using the Transformation equations with 4 parameters and 5 parameters, we have been able to assess which of the model is suitable for obtaining orthometric height to the nearest 15cm. For minimization of residuals, four and five parameter transformation models was used, which absorbed the datum inconsistencies between the orthometric height data, GPS/leveling heights and long wavelength errors due to EGM2008 calculator. The overall suitable residual of ±13.39cm was chosen, between the combined gravity field model EGM2008 and GPS/leveling heights, was achieved with five-parameter transformation model (Model B).

4.3.2 Geophysical Implications

The Bouguer anomaly of the basin is shown in Fig 8a. The map revealed a large-scale negative Bouguer anomaly trending W to E with high amplitudes over the NE zone of the study area. Closer geological and structural observation of this anomaly's axes suggested that its general trend followed an inferred granite intrusion area. The quite different nature of the Bouguer gravity map on the northern side was marked by gravity lows, bounded by relatively steep gradients occurring over or near higher metamorphic formations and other granitic plutons, suggesting the existence of a suture zone between two of the crust's blocks. Positive Bouguer gravity in the southern area tending SW-NW marked the intrusion of dense rocks in this area. The reason for the drastic change in orthometric height is as a result of the 2m average difference between the GPS geoid and that obtained using the Global geopotential model (EGM 2008). The topographic gravitational potential model (Martinec, 1998) which is a function of the topographic mass bounded by the geoid (N) and the earth surface H (orthometric height) increases or decreases with a change in $H(\Delta H)$ with constant density**.** The lateral change in H produces lateral variable density function (Martinec, 1998; Kuhn, 2002) which accounts for the major part of the gravitational effect and thus produces changes in the observed Bouguer anomaly. The improvement in the computation of orthometric height using the corrector model brought a change in the Bouguer gravity anomaly obtained using Models A and B orthometric heights as shown in Table 9. This is because there is a high correlation between the topographic masses and the observed gravity effects. Change in orthometric height causes change in Bouguer anomaly. It also shows that the real mass density distribution from the corrected orthometric heights differ from the observed Bouguer. Martinec, 1998; Tziavos *et al,* 1996 and Kuhn, 200a,b observed that 10% change in density distribution is not acceptable. However, in the Bouguer difference map (Fig 8b), the Bouguer anomaly gradually decreases from 0.54mGal in the Western zone to

0.48mGal the Eastern zone of the project area where the sedimentary thickness is to the depth of 7km (Epuh *et al.*,2011*)*. This is because sedimentary layers often have density values below 2.4g/m^3 while the plutonic rocks such as gneiss have density value of more than 3.0g/m^3. This variation in density in addition to the change in orthometric height produced the change in the Bouguer gravity anomaly value. In the use of model B orthometric height, the difference has a maximum value of 0.522mGal, a minimum value of 0.431mGal, mean value of 0.465 and standard deviation of 0.018mGal. The statistical analysis shows that the mean variation of 0.431mGal in the Bouguer anomaly is greater than the required precision of 0.1mGal precision (Lowrie, 2007). This value is substantial to alter the result in gravity modeling and interpretation and as such, false rock layers could be discovered in the affected area. The orthometric height difference shows a large horizontal gradient exist in the transition zone between areas of negative and positive anomaly values as shown in Fig 8a,b. Another problem is that in sedimentary basin, even for the same rock type, the density can vary laterally (Pagiatakis and Armenakas, 1998, Perkins, 1998). An incorrectly observed orthometric height will adversely affect the gravity reduction (Bouguer anomaly).The determination of orthometric heights using geoid obtained from GGM ensures that the complete gravitational effect of the global topographic masses is taken into account in the corresponding gravity reduction. Thus, the gravity anomaly obtained becomes a true representation of the density distribution in the study area without distortion. From equation 1 and 2, it is obvious that the change in the orthometric heights of the shot and receiver points will affect the value of the change in time. Its effect need to be investigated as further research.

Table 9: Summary of Results for the Difference in Bouguer Anomaly between the observed and the computed using Models A and B

	Model A (mGal)	**Model B (mGal)**
Maximum	**0.537**	**0.522**
Minimum	**0.433**	**0.431**
Mean	**0.466**	**0.465**
STD	**0.019**	**0.018**

5.0 Conclusion

The results show implicitly that in geophysical investigations, accurate orthometric height is required for the determination of the topographic mass distribution in order to achieve a reliable field of gravity anomalies. Precise Bouguer anomaly is a precursor for geophysical investigation using gravity data. The five (5) parameter corrector model used in this research could be adopted for future orthometric height modeling for geophysical investigation. The correction of the orthometric height will however affect the seismic static correction since its computation is also based on the orthometric height of the shot and receiver seismic observation points. Further investigation in this area is hereby encouraged.

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