Interaction between the VaR of cash flow and the interest rate using the ALM

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Abstract

In this paper, we propose an approach to study the impact of the interest rate on the risk of variation in cash flows measured by the value at risk (VaR) using stochastic processes and ALM technics.

This approach provides a decision-making tool for manage asset, liability funds to bankers insurers and all companies operating in the financial sector.

Keywords:

Interest rates, VaR, cash flow, ALM Technics, Stochastic Processes.

1. Introduction

ALM is one of the main tools used to help solve rate variation problems in financial institutions such as banks and insurance companies. He plays a very important role in managing the various activities of the financial institute.

Appropriate liquidity and balance sheet management are a key factor in ensuring the activity of financial institutions and are a tool for managers to make decisions about risk management with variations in the interest rate.

The activities of companies, whether banks, insurance companies or for-profit corporations, generate cash flows affecting the balance sheet as assets or liabilities. This financial flow and its risk are influenced by the variation in the interest rate. In the case of a positive or negative variation, it will have an impact on the assets, liabilities or both at the same time.

The objective of this paper is to study this influence by treating the impact of the change in the interest rate on the risk of this flow

2. Financial flows

The financial flow $\left(F_i / i=1,\ldots,n\right)$ of a company E is the ensemble of n amounts received or paid by it at different times $\left(t_i / i=1,\ldots,n\right)$ in future. In other words:

$$\left\{\left(F_i, t_i\right) / i=1,\ldots,n\right\}, \qquad (1)$$

where:

- F_i is the amount of the ith movement of the financial flow.
- t_i is the maturity of the ith movement of the financial flow.

3. Interest rate

The interest rate is defined as the economic remuneration of time. This is the amount a borrower is willing to pay to his lender in addition to the capital, and it is based on the credit risk of that borrower.

The evolution of the interest rate can be modeled by several stochastic processes whose Vasicek process or model is the most popular. This stochastic process is called the process of return to the mean.

The Vasicek model assumes that the current short-term interest rate is known, while the future values of this rate follow the following equation:

$$dr_t = \eta\left(\bar{r} - r_t\right)dt + \sigma dz_t \qquad (2)$$

Where :

- η : is the rate of return of the interest rate to the average.
- $\bar{r}$: is the average interest rate.
- σ : is the volatility of the interest rate which is assumed to be independent of r_t
- z_t : is a Brownian movement such as $dz_t = \varepsilon_t \sqrt{dt}$ avec $\varepsilon_t \sim N\left(0,1\right)$

4. Elements of Asset and Liability Management (ALM)

Asset-liability management aims at controlling financial risks and the consistency of the various balance sheet items over a time horizon. The objective of the ALM is to contribute to the achievement of the overall objectives of the bank and the insurance company's top management while taking the regulatory constraints into account in order to adopt an adequate financing policy.

5. Value at Risk (VaR)

The VaR is a measure of risk most widely used in financial markets to quantify the maximum loss on a portfolio for a given horizon and confidence level. It depends on three elements:

- The distribution of the portfolio's profits and losses for the holding period.
- The level of confidence.
- The holding period of the asset.

Analytically, the VaR with time horizon t and the threshold probability α the number $VaR(t,\alpha)$ such as:

$$\mathrm{P}\left[\Delta R \leq VaR(t,\alpha)\right] = \alpha \tag{6}$$

where :

- t : Horizon associated with VaR which is: 1 day or more than one day.
- α : The probability level is typically 95%, 98% or 99%.

6. The VaR of financial flow and the interest rate

The financial flow $F_i, i=1,...,n$ from a company E is a set of n amounts received or paid by it at different times $t_i, i=1,...,n$ to the future whose interest rate corresponds to the t_i is r_{t_i}. Interest rates $r_{t_i}, i=1,...,n$ are assumed to be independent.

The present value of this financial flow is given by:

$$V_r = \sum_{i=1}^{n} F_i r_{t_i} = F_1 r_{t_1} + F_2 r_{t_2} + ... + F_n r_{t_n}$$

Consider two future flows: assets A and liabilities P given by :

$$A = \left\{ \left(A_i, r_{t_i} \right), \quad \mathrm{i} = 1,...,n \right\}), \qquad \mathrm{P} = \left\{ \left(P_i, r_{t_i} \right), \quad \mathrm{i} = 1,...,n \right\}$$

The surplus relative to the couple of flow $\{A, P\}$ in relation to the interest rate r_{t_i} , $\mathrm{i}=1,...,n$, noted S_{r_t} , is given by:

$$S_{r_t} = \sum_{i=1}^{n} \left(A_i - P_i \right) \times r_{t_i}$$

or $$S_{r_t} = \sum_{i=1}^{n} AP_i \times r_{t_i} = \sum_{i=1}^{n} F_i \times r_{t_i}$$ where $$F_i = AP_i = \left(A_i - P_i \right)$$

Assuming that the interest rate follows a Vasicek process, i.e :

$$dr_t = \eta\left(\overline{r} - r_t\right)dt + \sigma dz_t$$

Thus, we can develop this model which is used to describe the interest rate as follows:

$$d\left(e^{\eta t} r_t\right) = e^{\eta t} dr_t + r_t \eta e^{\eta t} \Rightarrow e^{\eta t} dr_t = d\left(e^{\eta t} r_t\right) - r_t \eta e^{\eta t}$$

In other,

$$dr_t = \eta\left(\overline{r} - r_t\right)dt + \sigma dz_t \Rightarrow e^{\eta t} dr_t = e^{\eta t}\eta\left(\overline{r} - r_t\right)dt + e^{\eta t}\sigma dz_t$$

Then we get:

$$d\left(e^{\eta t} r_t\right) = \eta \overline{r} e^{\eta t} dt + e^{\eta t}\sigma dz_t \Rightarrow r_t = r_0 e^{-\eta t} + \int_0^t \eta e^{-\eta(t-s)}\overline{r} ds + \sigma\int_0^t e^{-\eta(t-s)} dz_s$$

So the interest rate r_t can be expressed as follows:

$$r_t = \overline{r} + \left(r_0 - \overline{r}\right)e^{-\eta t} + \sigma\int_0^t e^{-\eta(t-s)} dz_s \qquad (3)$$

Knowing that z_t is a process such as $dz_t = \varepsilon_t\sqrt{dt}$ with $\varepsilon_t \to N(0,1)$, so dz_t follows the normal distribution and the variable $\left(\sigma\int_0^t \eta e^{-\eta(t-s)} dz_s\right)$ also follows the normal distribution.

The term $\overline{r} + \left(r_0 - \overline{r}\right)e^{-\eta t}$ is not a random term then $r_t | r_0$ is a random variable that follows the normal distribution.

Using Iso isometry we find that:

$$E\left[\left(\sigma\int_0^t \eta e^{-\eta(t-s)} dz_s\right)\right] = 0 \quad \text{et} \quad E\left[\left(\sigma\int_0^t \eta e^{-\eta(t-s)} dz_s\right)^2\right] = \int_0^t \left(\sigma e^{-\eta(t-s)}\right) ds = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$$

Then the mean and the variance are respectively:

- $E\left(r_t | r_0\right) = \overline{r} + \left(r_0 - \overline{r}\right)e^{-\eta t}$
- $V\left(r_t | r_0\right) = \dfrac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$

So the random variable $r_t | r_0$ follows the normal distribution of mean and variance respectively $\overline{r} + \left(r_0 - \overline{r}\right)e^{-\eta t}$ et $\dfrac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$, i.e :

$$r_t \to N\left(\overline{r} + \left(r_0 - \overline{r}\right)e^{-\eta t}, \sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta t}\right)}\right)$$

Knowing that:

$$V_{r_t} = \sum_{i=1}^{n} F_i r_{t_i} = F_1 r_{t_1} + F_2 r_{t_2} + \ldots + F_n r_{t_n} \quad \text{so} \quad dV_{r_t} = \sum_{i=1}^{n} F_i dr_{t_i}$$

and

$$dr_t = \eta\left(\bar{r} - r_t\right)dt + \sigma dz_t \quad \text{so} \quad dV_{r_t} = \sum_{i=1}^{n} F_i dr_{t_i} = \sum_{i=1}^{n} F_i \left[\eta\left(\bar{r} - r_{t_i}\right)dt_i + \sigma dz_{t_i}\right]$$

$$\Rightarrow \; dV_{r_t} = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)dt_i + F_i \sigma dz_{t_i} \quad \Rightarrow \; \Delta V_{r_t} = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i + F_i \sigma \sqrt{\Delta t_i}\, \varepsilon_{t_i}$$

So $E\left(\Delta V_{r_t}\right) = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i$ and $V\left(\Delta V_{r_t}\right) = \sum_{i=1}^{n} \left(\sigma F_i\right)^2 \Delta t_i$

Let $\alpha_t = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i$ and $\beta_t = \sum_{i=1}^{n} \left(\sigma F_i\right)^2 \Delta t_i$ then $\Delta V_{r_t} \to N\left(\alpha_t, \sqrt{\beta_t}\right)$

The calculation of the VaR at α is given by the following equation:

$$P\left(\Delta V_{r_t} \leq VaR_\alpha\right) = \alpha$$

Knowing that $\Delta V_{r_t} \to N\left(\alpha_t, \sqrt{\beta_t}\right)$ therefore $P\left(\frac{\Delta V_{r_t} - \alpha_t}{\sqrt{\beta_t}} \leq \frac{VaR_\alpha - \alpha_t}{\sqrt{\beta_t}}\right) = \alpha$

$$\Rightarrow \quad \frac{VaR_\alpha - \alpha_t}{\sqrt{\beta_t}} = \tau_\alpha .$$

Thus

$$VaR_\alpha = \sum_{i=1}^{n} \eta F_i \left(\bar{r} - r_{t_i}\right)\Delta t_i + \tau_\alpha \sigma^2 \sum_{i=1}^{n} F_i^2 \Delta t_i \qquad (4)$$

7. Numerical application

In this section we will present a numerical application of VaR calculation expressed by the equation above using the financial flows of the company WAFA ASSURANCE below from the period from 2006 to 2016.

	F1	F2	F3	F4	F5	F6
2006	1 000 457 232,86	43 345 882,71	751 712 237,37	358 385 435,43	535 546 854,47	39 977 696,99
2007	1 129 148 572,00	35 608 429,29	758 245 958,15	401 714 775,10	682 949 812,32	52 330 487,69
2008	1 188 991 849,55	37 615 635,64	768 423 035,45	423 851 648,39	587 056 882,60	27 961 330,07
2009	1 396 390 661,85	39 765 498,38	901 720 102,38	474 836 763,03	1 028 061 612,10	154 267 498,27
2010	1 850 336 661,33	101 863 920,31	1 249 534 438,35	586 405 141,15	688 553 169,53	37 307 656,23
2011	2 114 643 387,73	82 173 954,20	1 408 920 653,90	615 208 768,23	924 609 641,72	208 008 777,91
2012	2 271 516 541,81	56 430 276,03	1 486 121 586,17	717 982 870,52	1 017 221 090,00	404 260 566,07
2013	2 456 622 991,48	19 749 435,01	1 562 056 658,19	764 460 236,87	714 551 385,43	68 786 404,06
2014	2 590 396 273,70	18 016 234,42	1 678 304 389,51	804 423 121,95	615 025 258,10	101 480 107,05
2015	2 440 076 544,89	62 456 800,33	1 543 050 391,84	801 223 719,04	703 964 022,72	202 035 529,43
2016	2 783 651 061,17	56 620 240,37	1 833 957 307,21	932 080 831,33	919 882 144,88	33 280 484,51

The table above is composed of inputs such as premiums and technical products of operations. In addition to, the outputs such as operating expenses and investment expenses. This inputs and outputs are:

- ✓ F1 = premiums
- ✓ F2 = Technical operating products
- ✓ F3 = Benefits and expenses
- ✓ F4 = Technical operating expenses
- ✓ F5 = Investment income allocated to insurance operations.
- ✓ F6 = Charges of investments affected to insurance transactions

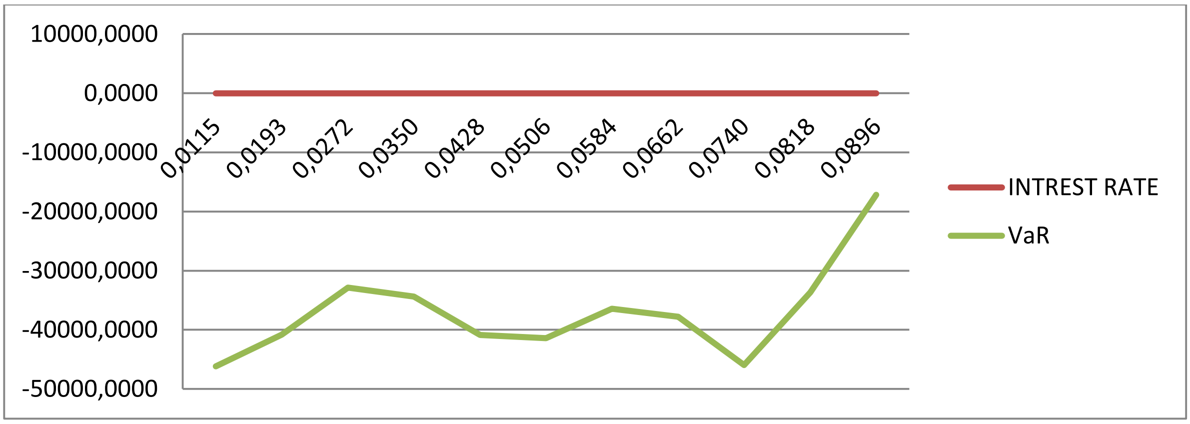

Figure 1: VaR depending on interest rates of company of LIFE INSURANCE

The graphical representation of the VaR given by equation 4 illustrate by the figure below. This figure shows that there is an interaction between the VaR of financial flow and the interest rate. Indeed, we note that the VaR is impacted directly by the interest rate and it is positively correlated to the same rate.

8. Conclusion

In this paper, we have developed an approach based on a mathematical formula to study the impact of the interest rate on the risk of the financial flow using value at risk as a risk measure and ALM to express the variability of a company's financial flows.

This approach allows us to evaluate the risk of financial flows in relation to the variation in the interest rate which gives rise to a decision-making tool for the management of funds, whether at the level of the assets or the liabilities of the company.

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