International Journal of Information Technology & Decision Making

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--Manuscript Draft--

Manuscript Number:	IJITDM-D-15-00071
Full Title:	A dynamic multi-period mixed-integer non-linear mathematical programming to establish and support military centers during wars: Evolutionary computation approach
Article Type:	Research Paper
Keywords:	OR in defense; Dynamic multi-period location-allocation; Mixed integer programming; Evolutionary computations; War planning
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Abstract

Determining the starting point of a battle and sequentially logistics of military centers are complicated decisions in dynamic situation of wars. In this paper, two mathematical models are proposed to address such decisions. The first model optimizes the starting point of a battle due to several constraints. The second model, which is a dynamic multi-period mixed-integer non-linear mathematical programming (DMP-MINLP), identifies the best location for establishing the supporting warehouses considering the strategic locations of opposite group and the provision of required ammunition for supporting centers. An evolutionary computation, i.e., a modified Genetic Algorithm (GA), and an exact algorithm, called branch and bound (B-B), are developed to solve the proposed models. The proposed GA has several features to handle this complicated DMP-MINLP. Full battle examples, in which the performances of both solution methods are compared, are presented. The B-B method is not capable to solve the medium size instances while proposed GA presents qualified solutions. The results are promising and the proposed models can provide proper strategies and operational solutions for the real world situations.

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1 Introduction

Location can be defined as determining, establishing, and organizing the position of several facilities or centers to offer special services, while considering other facilities and centers due to several constraints (Boloori Arabani and Zanjirani Farahani, [1]). Since the classic positioning models cannot provide a proper answer for real world problems, several developments have been proposed. Among them, expansions related to long-term programming, network planning, multi-period and dynamic programming are worth to be mentioned (Wesolowsky [2]; Erlenkotter, [3]; Boloori Arabani and Zanjirani Farahani, [1]). These are suggested for situations in which the parameters change throughout time in a predictable manner. The aim of these models is to accommodate the facilities with the created conditions in different periods of time (Shulman, [4]; Romeijn and Morales, [5]).

There are several applications of such models in real world problems. In order to show one of the applications of these models in the real world, consider a situation in which a war is underway in a specific region. Planning the war includes several complicated decision in a multi-period and dynamic situation. First problem is determining the proper area for starting the struggle. Sequentially the logistics of the army including establishing supporting warehouses, dispatching ammunitions, designing plans of attacks are next decisions. Due to several constraints, dynamic, and multi-period nature of wars, making proper decisions associated to the aforementioned issues are not trivial tasks.

In this paper, two mathematical programs are proposed to address such decisions. The first model tries to find the suitable location for start of a battle due to several constraints. The second model, which is a dynamic multi-period mixed-integer non-linear mathematical programming (DMP-MINLP), determines the best location for establishing the supporting warehouses considering the strategic locations of opposite group, making decision of attacks, and the provision of required ammunitions for supporting centers. An evolutionary computation, i.e., Genetic algorithm (GA), and an exact algorithm, called branch and bound (B-B), are developed to solve the proposed models. The proposed GA has several features to handle the complicated DMP-MINLP.

The remaining parts of this paper are organized as follows. In Section 2, a brief literature of location-allocation models and its applications are reviewed. The applications of optimization in wars are also reviewed in Section 2. The solution procedures of these models are also reviewed in the same section. In Section 3, proposed mathematical models are developed. Section 4 is allocated to present the proposed structure, operators, and design of customized GA for the proposed mathematical models. The simulated numerical instances and the result of the proposed GA completed with a B-B method are presented in Section 5. In section 6, the conclusions and future research directions are presented.

2 Literature of past works

The literature of related past works is reviewed in this section. The literature review has been accomplished in three sub-sections. In the first sub-section, literature of location-allocation problems, the extensions of location-allocation problem, and their applications is reviewed. In the second sub-section, literature of application location-allocation problems is reviewed. In last sub-section, the solution procedures for location-allocation problems are reviewed.

2.1 Brief Literature of Classic Location-Allocation Problems and Extensions

Davoodi et al., [6] described a compatible discrete space p-center problem. In order to solve this model, they proposed a heuristic Voronoi diagram algorithm. In order to avoid sticking in local optimums, they applied a number of changes in the body of the algorithm.

Davoodi and Mohades, [7] presented a constrained version of coverage problem. The model's aim is to find a minimum number of agents which covers all demand points. In order to solve this model, they proposed a heuristic approach based on p-center problem's solution and Voronoi diagram.

Garcia-Hernandez et al., [8] addressed several methods for the Unequal Area Facility Layout Problem (UA-FLP). They proposed an interactive genetic algorithm (IGA) considering DM's knowledge at each generation.

Rezaei and Zarandi, [9] presented a continuous facility location model in which some of the parameters are in the case of fuzzy data. The authors also proposed a fuzzy modeling method to estimate the required functions in the initial model. Moreover, they used a simulation method for implementation and evaluation of the proposed model.

Rahmati et al., [10] presented a multi-objective location model in which facilities behave as M/M/m queues. The first objective function was minimizing the sum of the collective travel and waiting times. The second objective function was maximizing the idle time of all facilities. The third objective function was minimizing required budget cost. In order to solve the model, the authors proposed three Pareto-based meta-heuristics, multi-objective harmony search (MOHS), non-dominated ranking genetic algorithm (NRGA) and non-dominated sorting genetic algorithm (NSGA-II).

Wen and Kang, [11] presented optimization models for facility location–allocation problem with random fuzzy demands. They have introduced a hybrid intelligent algorithm by integrating random fuzzy simulations, the simplex algorithm and GA in order to solve these random fuzzy models.

Amiri-Aref et al., [12] presented a mixed-integer nonlinear programming (MINLP) model for a stochastic restricted center location-dependent relocation problem. These authors applied two meta-heuristics, i.e., imperialist competitive algorithm (ICA) and the GA, to solve the proposed model.

In dynamic facility location problems, the conditions of some parameters such as the number of applicants, market trends, distribution expenses, patterns of demand, and environmental factors change continuously. In order to deal with these changes, type and the position of facilities should also be reviewed continuously (Boloori Arabani and Zanjirani Farahani, [1]). Wesolowsky, [2] proposed dynamic facility location for the first time. Wesolowsky, [2] modeled a situation in which in each period of time, there were some candidate target points. The objective function was to minimize the transportation costs. Erlenkotter, [3] compared the performance of seven methods for solving dynamic facility location problems. Shulman, [4] studied operational and transportation expenses of facility location problem to minimize the expenses of facility establishment considering capacity limitations. Shulman, [4] presented an algorithm for transferring Lagrange's non-optimized solutions to applicable solutions for Dynamic Capacitated Plant Location Problem (DCPLP).

Romauch and Hartl, [13] formulated Stochastic Dynamic Facility Location Problem (SDFLP) and proposed an exact solution procedure for small size instances based on random dynamic programming. The authors used Monte Carlo simulation method to handle the large size instances.

Albareda-Sambola et al., [14] proposed a multi-period location model for service facilities. The model proposed by the authors allows deciding about opening or closing new and existent facilities in a limited period of time. The authors also proposed Lagrangian relaxation method solve the problem and to relax customers' demands limitations for allocations to facilities.

Gebennini et al., [15] presented a mixed-integer mathematical programming for dynamic location and allocation problem with safety stock in an integrated production–distribution system. The model proposed by these authors determined the optimal number of facilities, facility locations, and customer demands allocation.

Yao et al., [16] proposed a mixed integer non-linear programming for multi-source facility location–allocation and inventory problem. The authors assumed the customer's demand as random variables while the safety stock was considered deterministically in order to satisfy the customers demand. The objective function of model proposed by these authors was to minimize the expected cost.

Albareda-Sambola et al., [17] presented a facility location problem with Bernoulli demands which was formulated by a two-stage random model. The objective function was to minimize the fixed expenses of facilities activities and customers' allocations.

2.2 Applications of Location-Allocation Problem

Vecchio et al., [18] considered the locations of all nodes of a wireless sensor network. They proposed a two-objective evolutionary algorithm based on simulated annealing which takes into certain topological constraints and localization accuracy on the network.

Ghanbari and Mahdavi-Amiri, [19] formulated bus terminal location problem as a p-uncapacitated facility location problem (p-UFLP) with distance constraint. In order to solve this model, they proposed evolutionary algorithms such as memetic algorithms and genetic local search (GLS) algorithms.

Mokryani and Siano, [20] presented a hybrid optimization that combines market-based Optimal Power Flow (OPF) and GA for optimal allocation of wind turbines (WTs). The authors used the GA to select the optimal size and the market-based OPF was used to specify the optimal number of WTs at each alternative bus. They also modeled the stochastic nature of both load demand and wind power generation by hourly time series analysis.

Fazlollahtabar et al., [21] presented a fuzzy mathematical programming model for a supply chain design. The authors assumed demand and cost as fuzzy parameters. In order to solve the model, they applied two ranking functions. The objective of the proposed fuzzy mathematical program was to minimize the total costs by choose the appropriate depots among alternative ones and the allocation of demands to depots and vehicles.

Hajiaghaei-Keshteli, [22] presented a two levels supply chain network which considers distribution centers (DCs) and customers. In order to supply demands of all customers, the proposed model selected some potential places as distribution centers. In order to solve the model, the authors developed two heuristic algorithms called artificial immune algorithm and GA.

Kratica et al., [23] presented a Mixed Integer Linear Programming (MILP) formulation for the capacitated hub location problem (CHLP). In order to solve the model, the authors proposed two evolutionary algorithms (EAs).

Konak et al., [24] proposed a decentralized approach based on agents to maintain connectivity in ad hoc networks (MANET). Locations of agents are dynamically determined using flocking-based behaviors.

Amini and Ghaderi, [25] performed a simulation study to determine best agent flocking behaviors. The authors presented a hybrid algorithm to find optimal locations for dampers within a structural system and proposed an improved version of Harmony Search algorithm and augmented it with the key traits from Ant Colony Optimization (ACO).

Subramanian et al., [26] presented a closed loop supply chains (CLSC) network with uncertain product returns and deterministic demands. The authors also proposed some integer linear programming model for distribution planning and location selection in the CLSC. Subramanian et al., [26] also have developed a heuristic approach based on Vogel's approximation method–total opportunity cost (VAM–TOC), and a priority-based simulated annealing (PBSA) search heuristic to find near optimal solution for the problem.

Location-Allocation in war situations and presenting models for the best decision-making has always been important. Levin and Friedman, [27] reviewed how military supporting units should be organized in order to improve the efficiency. They used a multi-period positioning problem to determine the cache locations and proposed B-B method has been suggested to reduce situational space in a dynamic programming. Betaque et al., [28] reviewed the positioning of naval facilities of different sizes and evaluated the sequential requirements of transportation to meet the demands. Allister, [29] estimated the required time for the landing of different forces using the concept of Tactical Logistics and Distribution System (TLoaDS) proposed by Hamber, [30]. Kang and Gue, [31] also described a simulation model for the disembarkation of naval facilities in the center of naval engineering facility services based on the concept of TLoaDS and the estimation of the time needed for every disembarkation.

Gue, [32] presented a new definition of war for US marines. The emphasis of the model was on the use of very small fighting groups supported by the navy instead of using large fighting groups supported by ground centers. The aim of this logistic planning was to minimize ground facilities. The authors proposed a multi-period mixed integer model for positioning of the facilities to propose a plan for ground distribution system by using the minimum expenses throughout the battle.

Sha and Huang, [33] presented a multi-period location-allocation problem of engineering emergency blood distribution and supply system during high demand times like disasters and earthquakes. The authors solved the problem using a Lagrangian relaxation based algorithm for a real case study in Beijing.

Wilhelm et al., [34] proposed a dynamic strategic supply chain design and planning problem. The proposed model by these authors was presented in the form of a multi-period, multi-product, and multi-level supply chain network design problem. The model was compared with a traditional formulation strategic supply chain.

Ghaderi and Jabalameli, [35] proposed a budget-constrained dynamic un-capacitated facility location–network design problem. The authors developed two efficient heuristics in order to solve the proposed model, which was applied to a case study in the context of health care.

2.3 Solution procedures for location-allocation problems

Doong et al., [36] considered a class of Facility location–allocation (FLA) problem that can assume more realistic conditions in real-life. In order to solve the model, the authors used a hybrid method of GA and sub-gradient technique.

Jawahar and Balaji, [37] proposed a heuristic GA to the multi-period fixed charge distribution problem attached with backorder and inventories. The authors formulated the model as 0–1 mixed integer linear programming and pure integer nonlinear programming. They also compared the proposed GA solutions with approximate solutions and lower bound of LINGO software.

Cadenas et al., [38] proposed a GA for the fuzzy p-median problem. The authors used two populations in the proposed algorithm. In the first one, the solutions with a better crisp transport cost were considered, however in the second population solutions with a better fuzzy satisfaction level were preferred. They also compared the algorithm's results with other heuristic procedures.

Husseinzadeh Kashan et al., [39] presented an optimization algorithm for pure binary optimization field. The authors also have investigated the application of the proposed algorithm on the well-established un-capacitated facility location problem (UFLP). They also found the global optimum of all investigated problems.

Yaghini et al., [40] presented a hybrid meta-heuristic approach by combining a Tabu search and a cutting-plane neighborhood structure for the capacitated p-median problem (CPMP). The authors also used design of experiments approach to tune the parameters of the proposed hybrid algorithm and tested the proposed algorithm on several sets of benchmark instances.

Romeijn and Morales, [4] presented a greedy algorithm to solve a multi-period single-sourcing problem. The proposed algorithm by tried to minimize the expenses of allocations, production, and maintenance.

Gabor and Van Ommeren, [41] presented an approximation algorithm to solve the probabilistic facility location problem. The authors considered total cost of transportation, facility operations, and maintenance as the objective function.

The most important studies related to location-allocation problem and its extension, the applications of location-allocation problem in other fields, and the solutions procedures for location-allocation problems were reviewed. Due to review of literature of past works, none of the studies reviewed above was expressively focused on the case of civil war. In the following sections, two mathematical models are presented for the problems of civil wars.

3 Proposed Dynamic Location-Allocation model for Civil War Conditions

Suppose a battle situation in which there are at least two opposite groups. The planning of battle includes two main issues. First, given group should find the best place for the start of the battle in order to have suitable access to strategic points of the opposite group, such as airports, military centers, ports, refineries, etc. It is clear that the most important factor to achieve victory in a battle or to continue the battle is dependent on having proper equipment and ammunitions for people fighting in the frontline. So the second issue is related to supporting the people who fight in the frontline by establishing proper ammunition dumps. Both issues are modeled in the following sections.

3.1 First model: selecting the proper start location of the battle

Some preliminary assumptions in modeling procedure are summarized as follows:

- ✓ The strategic points of the opposite group are known in whole area of the war zone.
- ✓ The number of regiments is known during the whole period of planning.
- ✓ Euclidean distance method is used to measure all distances.

3.1.1 Indices and parameters

$i, i' (i, i' = 1,2,..,I)$: fighting regiments in the front line;
$n (n = 1,2,...,N)$: strategic locations of opposite group;

$k (k = 1,2,...,K)$: potential zones (scenarios) for the start of the battle.

The parameters used in the first model are summarized in Table 1.

Insert Table 1 Here

On the basis of the parameters of Table 1, some more indexes are calculated, as shown in eq.(1)-(4).

$$a''^{k}_{1} = \frac{L^k}{2 \times I} \qquad \forall k \in K \qquad (1)$$

$$a''^{k}_{\mathrm{i}} = a''^{k}_{\mathrm{i}-1} + \frac{L^k}{I} \qquad \forall i \in I, i > 1, \forall k \in K \qquad (2)$$

$$\mathrm{D}'^{k}_{\mathrm{ii'}} = \frac{L^k \times |i - i'|}{I} \qquad \forall i, i' \in I, \forall k \in K \qquad (3)$$

$$D^{k}_{in} = \sqrt{\left(a''^{k}_{\mathrm{i}} - a_n\right)^2 + \left(\mathrm{b}''^{k}_{\mathrm{i}} - b_n\right)^2} \qquad \forall i \in I, \forall n \in N, \forall k \in K \qquad (4)$$

Equations (1)-(2) determine the location of every regiment in the front line. The equation (3) is used to calculate the distance between front line regiments, and equation (4) is used to determine the distance between front line regiments and strategic points of opposite group in every potential zone (scenario) for the start of the battle.

3.1.2 Mathematical equations

The decision variable of the proposed model is as follows:

$$Y_{in}^{k}=\begin{cases}1 & \text{if the } i^{th} \text{ regiment attacks the } n^{th} \text{ stratetic point of opposite group in } k^{th} \text{ scenario}\\ 0 & \text{otherwise}\end{cases}$$

$$Min \quad \theta = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{n=1}^{N} C_{T1}\left(1+\alpha_n\right)\left(1+\varepsilon_n{}^k\right) D_{in}{}^{k} Y_{in}{}^{k} \tag{5}$$

s.t.

$$\sum_{k=1}^{K} Y_{in}{}^{k} = 1 \qquad \forall n \in N, \forall i \in I \tag{6}$$

$$\sum_{i=1}^{I} Y_{in}{}^{k} \geq 1 \qquad \forall n \in N, \forall k \in K \tag{7}$$

$$Y_{in}{}^{k} \in \{1,0\} \qquad \forall i \in I, \forall n \in N, \forall k \in K \tag{8}$$

Equations (5)-(8) select the scenario with lowest cost as the best scenario to start of battle. The objective function (5) determines the lowest cost scenario. The set of constraints (6), which is held for each regiment, and for each strategic point of opposite group, guarantees that the assault is done for exactly one scenario. The set of constraints (7) guarantees that at least one regiment attacks the strategic point of opposite group based on a scenario of assault. The strategic point of opposite group usually contains a considerable amount of facilities, equipment, and individuals. The set of constraints (8) determines the type of decision variables.

3.2 Second model: establishing organizing and supporting military centers in wars

In this section a Multi-period Integer Non-linear Programming (MINLP) is proposed to establish, organize, and support the military centers during war.

3.2.1 Assumptions

- ✓ Planning is accomplished for a collection of light ammunitions that are sent in packages;
- ✓ All potential places for establishing supporting centers are known in the war zone;
- ✓ The place of establishing warehouse is determined by position of front-line;
- ✓ The maximum number of warehouses in each period is determined;

- ✓ The location of strategic points is known and fixed;
- ✓ The demand of regiments for light ammunitions follows a uniform distribution pattern;
- ✓ The maximum capacity of each warehouse is known in each period;
- ✓ The number of regiments is known and fixed in all planning periods;
- ✓ The number of fighters in each regiment is known in each planning period;
- ✓ A strategic point is occupied by the regiments one time during planning periods;
- ✓ All distances are calculated using the Euclidean method.

3.2.2 Indices and parameters

$t(t = 1,2,\dots,T)$: planning periods;

$i, i'(i, i' = 1,2,..,I)$: regiment in the front line;

$j(j = 1,2,\dots,J)$: potential location for establishing supporting warehouse;

$n(n = 1,2,\dots,N)$: strategic point of opposite groups.

The parameters used in proposed model are presented in Table 2.

Insert Table 2 Here

On the basis of the parameters of Table 2, some more indexes are calculated, as shown in equations (9)-(11).

$$a''^{t}_{1} = \frac{L}{2\times I} \qquad \forall t \in T \tag{9}$$

$$a''^{t}_{i} = a''^{t}_{i-1} + \frac{L}{I}, \qquad \forall i \in I, \forall t \in T, t > 1 \tag{10}$$

$$d''^{t}_{ii'} = \frac{L\times|i-i'|}{I} \qquad \forall i, i' \in I, \forall t \in T \tag{11}$$

It is notable that the equations (9)-(10) are used to determine the location of each regiment in the front-line in each period of planning. Equation (11) is used to calculate the distance between various regiments in front-line in period *t*.

3.2.3 Decision variables

The following decision variables are considered for the second model:

$$X_j^t = \begin{cases} 1 \text{ if warehouse is established in location j in period t} \\ 0 \text{ Otherwise} \end{cases}$$

$$X_{in}^{\ t}=\begin{cases}1 & \text{if regiment i attacks strategic location n in period t}\\ 0 & \text{Otherwise}\end{cases}$$

$Z_i^{\ t}$: The inventory of ammunition in the regiment i at the end of period t

$Q_{ji}^{\ t}$: The amount of dispatched ammunition from warehouse j to regiment i in period t.

$P_r^{\ t}$: The amount of front-line progress at the end of period t.

$R_F^{\ t}$: The minimum distance between front-line and zero-front of war in period t.

$$Y_j^{\ t}=\begin{cases}1 & \text{if location j is eligible to establish a warehouse in period t}\\ 0 & \text{Otherwise}\end{cases}$$

$$Y_n^{\ t}=\begin{cases}1 & \text{if the strategic point n has been seized in period t}\\ 0 & \text{Otherwise}\end{cases}$$

3.2.4 The proposed multi-period location-allocation model for war planning

Model (12)-(32) is proposed for establishing, organizing, and supporting military centers in war. The proposed model is an extended version of a dynamic multi-period location-allocation mixed integer non-linear mathematical programming.

$$\min \sum_{t=1}^{T}\sum_{j=1}^{J}F_j^{\ t}X_j^{\ t}+\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I}\left(1-p_{ji}^{\ t}\right)\frac{Q_{ji}^{\ t}}{v}C_{T0}d_{ji}^{\ t}+\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I}p_{ji}^{\ t}Q_{ji}^{\ t}C_{T2}+\sum_{t=1}^{T}\sum_{i=1}^{I}h_i^{\ t}Z_i^{\ t}$$
$$+\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{n=1}^{N}C_{T1}\left(1+\alpha_n\right)\left(1+\varepsilon_n\right)d'^{\ t}_{in}X_{in}^{\ t} \quad (12)$$

s.t.

$$Z_i^t=Q_i^t \qquad \forall i\in I,\ t=1 \quad (13)$$

$$Z_i^t=Z_i^{t-1}+\sum_{j=1}^{J}\left(1-p_{ji}^{\ t}\right)Q_{ji}^{\ t}-M_i^{\ t}D_i^t \qquad \forall i\in I, \forall t\in T,\ t>1 \quad (14)$$

$$\sum_{j=1}^{J}\left(1-p_{ji}^{\ t}\right)Q_{ji}^{\ t}\geq M_i^{\ t}D_i^t \qquad \forall i\in I, \forall t\in T \quad (15)$$

$$\sum_{j=1}^{J}X_j^{\ t}\leq W^t \qquad \forall t\in T \quad (16)$$

$$\sum_{j=1}^{J}X_j^{\ t}\geq 1 \qquad \forall t\in T \quad (17)$$

$$Q_{ji}^{\ t}\leq KX_j^{\ t} \qquad \forall i\in I, \forall j\in J, \forall t\in T \quad (18)$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I}X_{in}^{\ t}=1 \qquad \forall n\in N \quad (19)$$

$$\sum_{i=1}^{I} Q_{ji}^{\ t} \le cap_j^{\ t} \qquad \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{t} \in \mathrm{T} \tag{20}$$

$$\begin{cases} X_j^{\ t} \le Y_j^{\ t}, & \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{t} \in \mathrm{T} \\ Y_j^{\ t} \ge \dfrac{R_F^{\ t} - u_j^{\ o}}{\left| R_F^{\ t} - u_j^{\ o} \right|}, & \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{t} \in \mathrm{T} \\ Y_j^{\ t} \ge 0, & \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{t} \in \mathrm{T} \end{cases} \tag{21}$$

$$\begin{cases} X_{in}^{\ t} \le 1 - Y'^{\ t}_{n}, & \forall i \in I, \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \\ Y'^{\ t}_{n} \ge \dfrac{R_F^{\ t} - u'^{\ o}_{n}}{\left| R_F^{\ t} - u'^{\ o}_{n} \right|}, & \forall i \in I, \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \\ Y'^{\ t}_{n} \ge 0, & \forall i \in I, \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \end{cases} \tag{22}$$

$$\begin{cases} S^t \le RM_n^{\ t} \times \left| \left(1 - KY'^{\ t}_{n}\right)\left(1 - K\sum_{i=1}^{I} X_{in}^{\ t}\right) \right|, & \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \\ S^t \ge RM_n^{\ t} \times \left| \left(1 - KY'^{\ t}_{n}\right)\left(1 - K\sum_{i=1}^{I} X_{in}^{\ t}\right) \right|, & \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \\ S^t \le R_x - \left(R_F^{\ t} - r\right), & \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \end{cases} \tag{23}$$

$$\mathrm{P}_{\mathrm{r}}^{t} = S^t - r \qquad \forall \mathrm{t} \in \mathrm{T} \tag{24}$$

$$R_F^{\ t} = R_z - r, \quad \mathrm{t=1} \tag{25}$$

$$R_F^{\ t+1} = R_F^{\ t} + P_r^{\ t}, \qquad \forall \mathrm{t} \in \mathrm{T},\ \mathrm{t>1} \tag{26}$$

$$RM_n^{\ t} = \left| u'^{\ o}_{n} - R_F^{\ t} \right|, \qquad \forall \mathrm{n} \in \mathrm{N}, \forall \mathrm{t} \in \mathrm{T} \tag{27}$$

$$b''^{\ t}_{i} = R_F^{\ t}, \qquad \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{t} \in \mathrm{T} \tag{28}$$

$$d_{ji}^{\ t} = \sqrt{\left(a'_j - a''_i\right)^2 + \left(\mathrm{b}'_j - b''^{\ t}_{i}\right)^2}, \qquad \forall i \in I, \forall \mathrm{j} \in J, \forall t \in T \tag{29}$$

$$d'^{\ t}_{in} = \sqrt{\left(a''_i - a_n\right)^2 + \left(b''^{\ t}_{i} - \mathrm{b}_n\right)^2}, \qquad \forall i \in I, \forall n \in N, \forall t \in T \tag{30}$$

$$X_j^{\ t}, X_{in}^{\ t}, Y_j^{\ t}, Y''^{t}_{n}, \in \{0,1\}, \quad \forall i \in I, \forall \mathrm{j} \in \mathrm{J}, \forall n \in N, \forall t \in T \tag{31}$$

$$Z_i^{\ t}, \mathrm{Q}_{ji}^{\ t} \ge 0, \qquad \forall i \in I, \forall \mathrm{j} \in \mathrm{J}, \forall t \in T \tag{32}$$

Objective function (12) minimizes total cost of the battle. The first part of the objective function (12) tries to minimize the whole fixed cost of establishing warehouses in all periods of planning. The second term tries to minimize the cost of ammunition allocation and transportation from warehouses to regiments in front-line. The third section minimizes the cost of dispatched ammunitions from warehouses to regiments, which are destroyed by the enemy throughout the transportation route. In the fourth part of the objective function (12), the cost of holding inventories in each regiment of front-line are minimized in the planning periods. The fifth section intends to minimize the cost of attack from various regiments in front-line to strategic points of the opposite group in the planning periods. It is clear that in each planning period, the cost of attack is directly related to the distance between front-line and strategic points of opposite group, the importance of these points for the opposite group, and the difficulty of access to these points.

The set of constraints (13) shows the amount of inventory at the start of the battle in each regiment of the front-line. The set of constraints (14) presents the balance of inventory in each planning period in each regiment. The inventory of a regiment in a period is equal to the inventory of the previous period plus the ammunitions received from all supporting warehouse minus the consumed ammunition in the i^{th} regiment. The set of constraint (15), which is held for all regiments and all planning periods, guarantees that the required ammunition of each fighting regiment in the front-line is supplied by supporting warehouses. The set of constraints (16) provides the maximum number of supporting warehouses in each period. The set of constraints (17) indicates that at least one of the supporting centers (warehouses) must be established in each period, in order to address the demands of ammunitions by the fighting regiments. The set of constraints (18) expresses that a potential location for supply of ammunition in the j^{th} location can provide services to the i^{th} fighting regiment if a warehouse has been established in the j^{th} location. In this way, the demands of fighting regiments can be assigned to that location.

The set of constraints (19), which holds for all strategic points, assures that only one attack is accomplished on a given strategic point by all regiment in all periods of planning. The set of constraints (20), which holds for all supporting warehouses and all periods of planning, indicates that, in each planning period, the amount of ammunition that each supporting warehouses offers to all fighting regiments cannot exceed the capacity of the supporting center.

The set of constraints (21) expresses that a warehouse can only be established in a conquered location. In other words, it is not eligible to establish a warehouse beyond the war front-line. In fact, when $Y_j^{\,t}$ is equal to zero, i.e. there is no progress, it is not eligible to establish a warehouse in location j. The definition of $Y_j^{\,t}$ is $Y_j^{\,t} = \max\left\{0, \frac{R_F^{\,t} - u_j^{\,o}}{\left|R_F^{\,t} - u_j^{\,o}\right|}\right\}$ $\forall j \in J, \forall t \in T$.

The set of constraints (22) ensures that, if in a given period of planning (i.e., period *t*) the front-line progresses beyond a strategic center (i.e., center *n*), that center is considered to be seized. Therefore, in the following periods of planning, no fighting regiments are assigned to the seized center. In fact, when Y'^{t}_{n} is equal to zero, in the formula $Y'^{t}_{n} = \max\left\{0, \frac{R_F^{\ t} - u'^{\ o}_{n}}{\left|R_F^{\ t} - u'^{\ o}_{n}\right|}\right\} \ \forall n \in \mathrm{N}, \mathrm{t} \in \mathrm{T}$, the nth strategic center has not been seized until tth period of planning and some of the regiments may be assigned to it.

In order to extend the war toward the strategic centers of the opposite group, which has not yet been seized, we can obtain the distance between the enemy's strategic centers and the war front-line for all strategic points. Then, the nearest center which has been seized is selected and the fight will continue with the aim of seizing that center. This guarantees that our progress is limited only to the war zone and does not allow the period advancement to become larger than the coordinates of last strategic center which we seek to seize. These conditions are determined by S^t in the following equation.

$$S^{t} = \min\left\{RM_{n}^{\ t} \times \left|\left(1 - KY'^{\ t}_{n}\right)\left(1 - K\sum_{i=1}^{I} X_{in}^{\ t}\right)\right|, R_{x} - \left(R_F^{\ t} - r\right)\right\} \ \forall \mathrm{n} \in \mathrm{N}, \mathrm{t} \in \mathrm{T}$$

The set of constraints (23)-(24) directs the war toward the nearest strategic centers of the opposite group, which have not yet been seized. The constraint (25) shows the position of the front-line at the start of the battle. The constraint (26) updates the position of the front-line during the planning periods. The constraint (27) is used to determine the minimum distance between the strategic points of the opposite group and the front-line in each period of planning. The set of constraints (28) updates the locations of the regiments in the front-line during the war progresses in planning periods. The set of constraints (29) computes the distance between the fighting regiments in the front-line and the ammunition supporting warehouses. The set of constraint (30) calculates the distance between the fighting regiments in the front-line and strategic points of the opposite group. Equations (31)-(32) indicate the type of decision variables.

4 Solution method: proposed evolutionary algorithm

In this section, an evolutionary based GA is designed to solve the proposed multi-period location-allocation model for war planning problem. GAs are derived based on Genetics and

Darwin's Evolution Theory. A GA is based on the survival of the fittest and, as such, has several applications for engineering problems (Mitchell, [42]). One of the most common applications of GAs is the optimization of a specific goal, subject to several constraints; this is also the reason why a GA is adopted in this work. In the following, the structure of the GA proposed to solve the multi-period war planning problem is detailed.

4.1 Structure of the chromosome

The suggested chromosome for the problem includes three sections. The first one is related to the potential locations of the supporting centers (i.e., warehouses), to be established. The binary value of each gene depends on whether the supporting center is active in that specific period of time and in that location. Assuming $j=1,\ldots,J$ potential locations for establishing the supporting centers, and $t=1,\ldots,T$ periods of planning, the first part of the chromosome has $J \times T$ *loci* and is presented in Figure 1.

Insert Figure 1 Here

The second part of chromosome, which is a two dimensional matrix, shows the assignment of regiments to various strategic points of the opposite group. The binary value of a gene in the second part of the chromosome is set at 1 if the considered regiment attacks a specific strategic point in a given period of planning. In figure 2, the second part of the proposed chromosome is shown, considering $n=1,\ldots,N$ strategic points for opposite group, $i=1,\ldots,I$ regiments and $t=1,\ldots,T$ periods.

Insert Figure 2 Here

The third part of the proposed chromosome shows the way ammunitions are assigned and dispatched by the established supporting warehouses. The value of a gene in third part of the chromosome is an integer number larger than, or equal to, zero, reflecting the extent to which the fighting regiments' demands have been met by the supporting warehouses,. In Figure 3, the third part of chromosome is shown for a situation where there are $j=1,\ldots,J$ supporting warehouses, $n=1,\ldots,N$ strategic point of opposite group and $t=1,\ldots,T$ periods of planning.

Insert Figure 3 Here

4.2 Initialization of the population

As mentioned, the chromosome designed for to solve the problem in exam has three main parts. The first and second parts include binary values, while third one is filled with integer values. Random binary values are assigned to the first two parts of the chromosome, according to the size of the problem. Integer values are assigned to the third part of the chromosome, according to the size of the problem.

4.3 Computation of the fitness function

The fitness function of a chromosome is calculated based on the values assigned to the genes. Equation (32) is the basis for the computation of the fitness function.

4.4 Constraint handling strategy

The penalty function method is used in order to handle several constraints of the proposed model (12)-(32). First, the fitness function is obtained using equation (32) for every chromosome in the population. Then, based on the values of genes in chromosome and considering the constraints, the violation of the chromosome is calculated and assigned to infeasible chromosomes. Finally the fitness function of violated chromosome is penalized using equation (33).

$$Z_{new} = Z_{old} + \left(\beta^{Ite} \times Violation\right) \tag{33}$$

where Z_{new} is the penalized fitness function of violated chromosome, *violation* is sum of all violations of chromosome considering constraints (13)-(32), Z_{old} is the value of fitness function of the chromosome before considering violation, β is a parameter called the "pressure coefficient", and *Ite* is the iteration number of the GA algorithm. It is notable that, for a given value of violation, the penalty depends on the iteration of the algorithm: as the iterations go on, the penalty value will become greater for infeasible chromosome. This will decrease the probability of presence of infeasible solutions in last iterations of the algorithm.

4.5 Selection Method

In the proposed GA, a combined selection strategy is accomplished to select the chromosomes in the mating pool. A binary tournament selection and a roulette wheel selection method are randomly used based on a predefined parameter of the algorithm. This will increase the quality of exploration phase of the algorithm.

4.6 Recombination: cross-over and mutation operators

In the first two binary parts of each chromosome, the single-point cross-over method is simply used. The cross-over in the third section of the chromosome is done based on (34)-(35).

$$Child_1 = \{\alpha \times Parent_1\} + \{(1-\alpha)\times Parent_2\} \tag{34}$$

$$Child_2 = \{\alpha \times Parent_2\} + \{(1-\alpha)\times Parent_1\} \tag{35}$$

where $Child_1$ and $Child_2$ are the re-produced off-springs made by two specific parents (i.e., $Parent_1$ and $Parent_2$), α is a matrix parameter, its dimension is equal to the dimension of third part of chromosome belongs to $\alpha \in [-\gamma, 1+\gamma]$ being γ a parameter called “search pressure”.

A simple binary mutation is accomplished for the first and second parts of the chromosomes. The mutation of third part of the chromosome, i.e. a normal mutation, is accomplished based on equation (36).

$$Child = Child + \{\sigma \times N(0,1)\} \tag{36}$$

where σ is a parameter which describes the variance of value in which the genes can change. It is defined as $\sigma = \mu \times (U - L)$, where U, and L are upper and lower bounds of feasible values for gene and μ is a parameter of the mutation operator which controls the amount of change.

4.7 Termination criterion

In order to stop the optimization process and obtain the results, a maximum number of iterations is fixed for the proposed GA.

5 Results

In order to illustrate the applicability and efficacy of the proposed model and the solution procedure, an example is introduced and analyzed in this section. The results of the proposed GA and a B-B method are compared in this example. The mathematical models, i.e. model (5)-(8) and model (12)-(32), were coded in commercial LINGO software and solved using a B-B algorithm. The proposed GA was coded in MATLAB software.

5.1 *Results of the first model: determining the location of battle start*

Suppose that the opposite group has three strategic points in the battle area. The battle area is shown in Figure 4. Suppose there are two regiments which have the ability to attack from north, south, east and west (i.e., various scenarios of the battle start). There are four potential locations to establish supporting warehouses in. The cost of attack is directly proportionate to the distance between the location of the battle start and the strategic points of the opposite group, the importance of each strategic point and the difficulty to access the strategic points.

Insert Figure 4 Here

In Figure 4, the strategic points and potential locations for warehouse establishment are fixed in all periods of planning during war. The parameters of different scenarios of attack are presented in Table 3.

Insert Table 3 Here

Model (5)-(8) was coded in LINGO and implemented for all scenarios of the starting of the war. The results of implementing model (5)-(8) are represented in Table 4.

Insert Table 4 Here

From the contents of Table 4, it can be concluded that the value of the objective function for the first scenario of attack (i.e., the start of the battle from the south area) has the lowest value among the other scenarios. So, the battle is started from south area.

5.2 *Results of the multi-period location-allocation model for war planning*

In this section, the results of planning for battle and logistical issues of the example in Figure 4 are presented. The example has been analyzed using the B-B algorithm and the proposed algorithm. The example includes three strategic zones, two fighting regiments, four potential locations for the establishment of supporting centers and three periods of planning. The parameters of the example are shown in Table 5.

Insert Table 5 Here

5.2.1 Result of the B-B method

The model (12)-(32) has been coded using LINGO software and analyzed through B-B method. The results are presented in Tables 6 to 11.

As shown in Table 6, the outcome obtained applying the B-B method is a local optimum result. There is a tiny infeasibility in the answer. This little infeasibility value is not important theoretically but, it is not a practical answer in real world situation. As the problem is hard, the LINGO software is not capable to find even a feasible solution for small size instances of the problem using the B-B method.

Insert Table 6 Here

Table 7 presents the way that fighting regiments are allocated to strategic points of opposite group during periods of war planning. Obviously all of attacks are accomplished in the third period of planning. The first regiment is allocated to attack the first and second strategic points of the opposite group, while the second regiment is allocated to attack the third strategic point of opposite group.

Insert Table 7 Here

Table 8 shows the way warehouses are sited in potential locations in order to support the fighting regiments during the periods of planning. For instance, in the second period of planning, warehouses are established in the first and second potential locations.

Insert Table 8 Here

Table 9 shows the inventory of regiments during the periods of planning.

Insert Table 9 Here

Table 10 presents the amount of supplied ammunitions from each warehouse to each regiment during the planning periods. For instance, in the second period of planning, the first regiment receives 15,000 units of ammunitions by the first warehouse.

Insert Table 10 Here

Table 11 presents the amount of advancement toward the strategic points of the opposite group during the periods of planning. The front-line position during the planning periods is also presented.

Insert Table 11 Here

5.2.2 Results of the proposed GA

Model (12)-(32) is a complex mixed integer non-linear mathematical program. Hence, exact solution methods, such as B-B, have no ability to reach the global optimum or even feasible answers for small size instances of it. At the same time, the medium and large size instances of the problem require incredible resources (i.e., software, hardware and CPU time) and cannot be handled in a reasonable manner using exact methods. It should also be noted that the results proposed by the B-B method and Lingo software for small size instances are infeasible. Hence, these solutions cannot be executed operationally. All these considerations justify the design and application of proposed GA for model (12)-(32). The GA for model (12)-(32) has been coded and implemented in MATLAB software. The results of the GA are presented in Tables 12-17.

It can be concluded from Table 12 that the objective function value has been improved, compared to the results of the B-B algorithm. Moreover, the solution is feasible and no violation is reported for the constraints. The CPU time is reasonable for such complicated mixed-integer non-linear mathematical programming.

Insert Table 12 Here

Figure 5 presents the trend of the objective function, plotted by MATLAB software, during the first 1,000 iterations of the GA. As it is evident from that figure, the GA presents a good convergence toward the best known solution presented by LINGO software. Moreover it outperforms the B-B algorithm after 689 iterations.

Insert Figure 5 Here

Table 13 describes the way fighting regiments are allocated to the strategic points of the opposite group by the GA, during the planning periods. Obviously, the result of GA is different from those obtained from the B-B method.

Two attacks are accomplished in second period of planning while one attack is done in third period of planning. The first regiment is allocated to attack the third strategic point of the opposite group in the second period, while the second regiment is allocated to attack the second strategic point of the opposite group in the same period. The first regiment is again allocated to attack the first strategic point of opposite group in the third period.

Insert Table 13 Here

Table 14 reports the way the GA defines the establishment of warehouses in potential locations, in order to support the fighting regiments during the planning periods. The result of GA is different from that obtained by the B-B method. For instance, in the third period of planning, the GA proposes to establish warehouses in the first and second potential locations.

Insert Table 14 Here

Table 15 shows the inventory of regiments proposed by GA during the planning periods. The result of GA is completely similar to those by B-B.

Insert Table 15 Here

Table 16 presents the amount of ammunitions, which is proposed by the GA to transfer from each warehouse to each regiment during planning periods. Again, the result of GA is completely different from that obtained with the B-B. For instance, in the second period of planning, the first regiment receives 16,667 units of ammunition by the first warehouse.

Insert Table 16 Here

Table 17 presents the amount of advancement toward the strategic points of the opposite group, as proposed by the GA, during the planning periods. The position of the front-line, as proposed by GA, is also presented.

Insert Table 17 Here

As clear, the results obtained by the GA are relatively more qualified than the answer gained by the exact method, i.e. the B-B. The value of the objective function achieved by the GA is obviously less than that obtained by the B-B. Also, the result proposed by the GA does not violate constraints of the problem and is completely feasible and practical.

6 Concluding remarks and further research direction

The location-allocation problem and its extensions have been investigated in many real-world applications. The literature of location-allocation problem, its extension, the application of location-allocation problem in other fields and the solution procedures for location-allocation problem were surveyed. A particular extension of the dynamic multi-period location-allocation problem is used in battle, wars and struggle planning. Location-allocation problems play an essential role in dynamic war planning problem, where the optimum region for starting the war, establishing supporting warehouses in potential locations, and transferring ammunitions to fighting regiments are complicated decision during multiple-periods. In this paper, two mathematical programming were proposed to support the aforementioned decisions. The first model tried to determine the best region for starting the war considering the minimum operational cost and several constraints. The second model, which was a complex dynamic multi-period mixed-integer non-linear programming was proposed to determine where, when and how to establish supporting warehouses in potential locations, how to transfer ammunitions to fighting regiments, how, when and from where to attack to strategic points of opposite group, and how to update the progress of regiments and strategic points during multiple-periods of war.

A simulated illustrative example was presented to demonstrate the applicability and efficacy of proposed models. The first model was coded in LINGO software and solved using an exact algorithm, called the B-B method. The B-B method was not able to find even a feasible solution for small and medium size instances of the second model. So, an evolutionary computation algorithm, i.e. a GA, was designed to solve the instance of the second model using reasonable resources (software, hardware and CPU time). The GA served several features in order to handle the complicated constraints of the second model. The proposed GA was coded in MATLAB software. The results of the GA were compared with those of the B-B method for the second model. As mentioned, the B-B method was not able to provide feasible answers for the presented model. Moreover, the CPU time was not reasonable for B-B method. The proposed GA provided qualified solutions in a reasonable CPU time.

In this paper, most of the parameters of proposed models were assumed as deterministic values. One of the future research directions should be developing the proposed model in uncertain environment which can be probabilistic or fuzzy situations. The extension of proposed models of this study in multi-objective cases can be another interesting issue. Moreover, developing other meta-heuristic methods, especially hybrid meta-heuristic methods, for solving the problem and comparing the results is another interesting future research direction.

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Figure

Figures

Period	*t=1*			…			*t=T*		
Warehouse	1	…	J	1	…	J	1	…	J
Chromosome	1	...	1	0	...	0	0	...	1

Figure 1: Structure of first section of chromosome

	Period	*t=1*			…			*t=T*		
	Strategic Points	*n=1*	…	*n=N*	*n=1*	…	*n=N*	*n=1*	…	*n=N*
regiments	*i=1*	1	...	1	0	...	0	0	...	1
	⋮	⋮	…	⋮	⋮	…	⋮	⋮	…	⋮
	i	1	...	1	0	...	0	0	...	1
	⋮	⋮	…	⋮	⋮	…	⋮	⋮	…	⋮
	i= I	1	...	1	0	...	0	0	...	1

Figure 2: Structure of second section of chromosome

	Period	t=1			…			t=T		
	Strategic Point	*n=1*	…	*n=N*	*n=1*	…	*n=N*	*n=1*	…	*n=N*
Supporting Warehouse	*j=1*	3	…	5	7	…	9	8	…	4
	⋮	⋮	…	⋮	⋮	…	⋮	⋮	…	⋮
	i= J	4	…	6	1	…	7	0	…	9

Figure 3: Structure of third section of chromosome

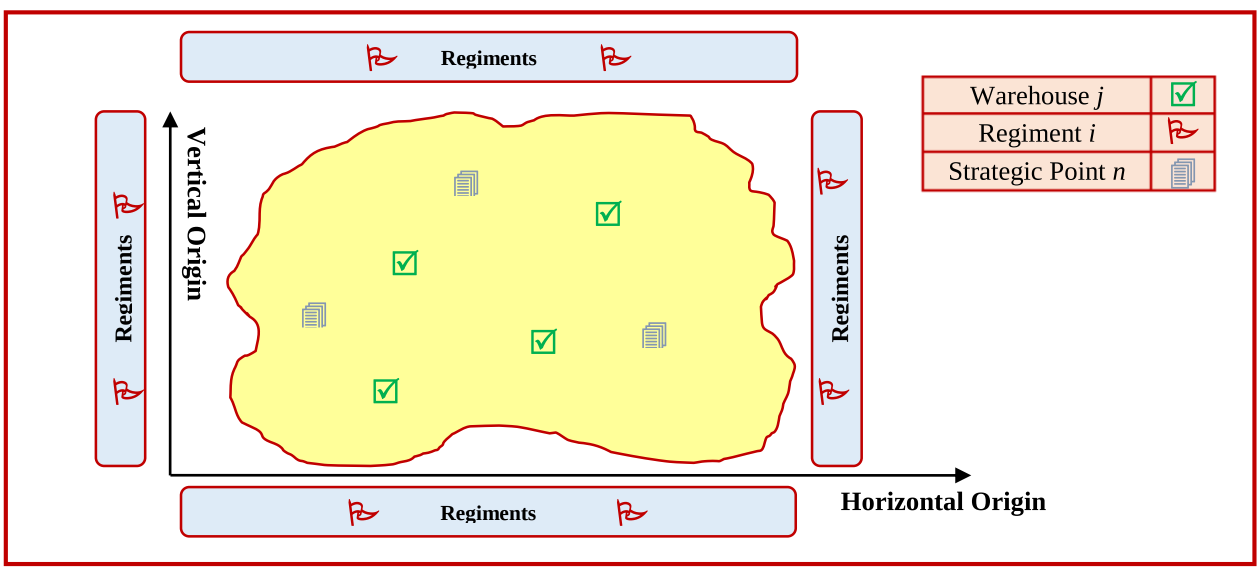

Figure 4: Schematic map of the war zone

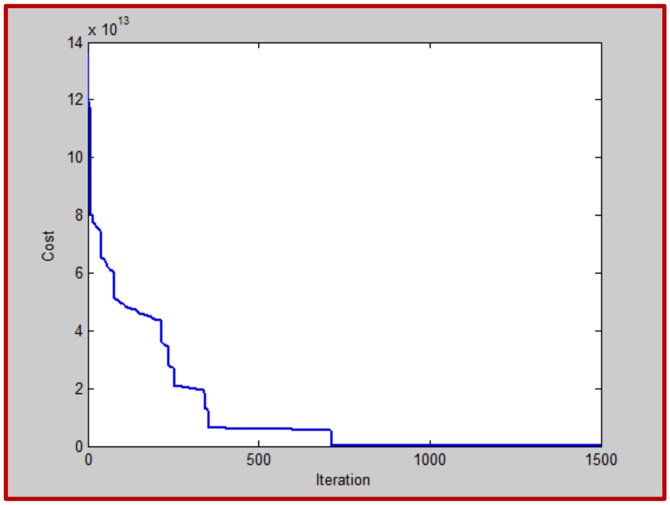

Figure 5: Trend of the objective function in GA iterations

Tables

Table1: Parameters and Indices for the location of battle start

Parameter	Description	Index
L^k	The length of front line for attack in k[th] zone	$\forall k$
$(a''_i{}^k, b''_i{}^k)$	The coordinates of i[th] regiment location at the start of attack in k[th] zone	$\forall i, k$
(a_n, b_n)	The coordinates of n[th] strategic location of the opposite group	$\forall n$
C_{T_1}	The cost of attack from regiment location to a strategic location	
$D_{in}{}^k$	The distance between i[th] regiment and n[th] strategic location of opposite group in k[th] zone	$\forall i, n, k$
$D'_{ii'}{}^k$	The distance between i[th] regiment and i[th] regiment(i$\neq i'$) in k[th] zone	$\forall i, i', k$
α_n	The importance of n[th] strategic location of opposite group	$\forall n$
$\varepsilon_n{}^k$	The degree of difficulty of access to n[th] strategic location of opposite group in k[th] zone (a value between zero and one)	$\forall n, k$

Table 2: Parameters and indices for supporting centers settlement

Parameter	Description	Index
W^t	The number of allowed warehouses which can be established in t[th] period	$\forall t$
$F_j{}^t$	The fixed cost of warehouse establishment in j[th] location in t[th] period	$\forall j, t$
L	The length of front-line	
(a_n, b_n)	The coordinates of n[th] strategic location of opposite group	$\forall n$
(a'_j, b'_j)	The coordinates of j[th] supporting warehouse	$\forall j$
$(a''_i{}^t, b''_i{}^t)$	The coordinates of i[th] regiment in t[th] period	$\forall i, t$
$p_{ji}{}^t$	The percentage of ammunition which is dispatched from j[th] warehouse to i[th] regiment in t[th] period (a value between zero and one)	$\forall i, j, t$
$M_i{}^t$	The number of fighters of i[th] regiment in t[th] period	$\forall i, t$
$D_i{}^t$	The probabilistic amount of ammunition required by every fighter of i[th] regiment in t[th] period (uniform distribution)	$\forall i, t$
$d_{ji}{}^t$	The distance between supporting warehouse of j[th] warehouse and i[th] regiment in t[th] period	$\forall i, j, t$
$d'_{in}{}^t$	The distance between i[th] regiment and n[th] strategic location in t[th] period	$\forall i, n, t$
$d''_{ii'}{}^t$	The distance between i[th] regiment and i'[th] regiment in t[th] period	$\forall i, i'$
$Cap_j{}^t$	The capacity of j[th] warehouse in t[th] period	$\forall j, t$
$h_i{}^t$	The holding cost of unit of inventory in i[th] regiment for t[th] period	$\forall i, t$
v	The capacity of vehicle which is used for transporting ammunitions from warehouses to regiments	
C_{T_0}	The cost of vehicle transportation per unit of distance	
C_{T_1}	The cost of transportation per unit of distance from regiment to a strategic location	
C_{T_2}	The cost of explosion of ammunitions dispatched from warehouses to front-line regiments	
α_n	The importance of n[th] strategic location of opposite group	$\forall n$
ε_n	The degree of difficulty of access to n[th] strategic location of opposite group (a value between zero and one)	$\forall n$
K	A large positive number	
$u_j{}^0$	The minimum distance between j[th] warehouse and zero-line of war	$\forall j$

$u'_n{}^O$	The minimum distance between n[th] strategic location and zero-line of war	$\forall n$
$RM_n{}^t$	The minimum distance between n[th] strategic location and the front line in t[th] period	$\forall n, t$
S^t	The distance between the nearest un-seized strategic point and the front-line in t[th] period	$\forall t$
r	The minimum of allowed radius of front-line related to a seized strategic location	
R_x	The distance between the furthest strategic center and front-line	
R_z	The distance between the nearest strategic center and front-line	
$Q_i{}^1$	The amount of inventory of i[th] regiment at the start of the battle	$\forall i$

Table 3: Parameters of war zone

Scenario	Parameter																			
	b''_2	b''_1	a''_2	a''_1	b_3	b_2	b_1	a_3	a_2	a_1	ε_3	ε_2	ε_1	α_3	α_2	α_1	I	N	L	C_{T_1}
K=1	0	0	60	40	60	45	25	45	60	35	0.8	0.7	0.5	1	0.9	0.7	2	3	40	5000
K=2	60	20	110	110	60	45	25	45	60	35	0.5	0.3	0.8	1	0.9	0.7	2	3	80	5000
K=3	60	20	0	0	60	45	25	45	60	35	0.5	0.9	0.3	1	0.9	0.7	2	3	80	5000
K=4	100	100	60	40	60	45	25	45	60	35	0.35	0.55	0.8	1	0.9	0.7	2	3	40	5000

Table 4: Result of scenarios of the starting of the war

Scenario	K=1 South attack	K=2 East attack	K=3 West attack	K=4 North attack
Total Cost	2,135,536	2,769,736	2,182,007	2,504,125

Table 5: Parameters of the example for multi-period location-allocation model

Period	Parameter													
	Cap_4	Cap_3	Cap_2	Cap_1	h_2	h_1	D_2	D_1	M_2	M_1	F_4	F_3	F_2	F_1	W
t=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
t=2	20000	23000	25000	18000	2500	2000	280	300	40	50	15000	20000	15000	10000	3
t=3	25000	20000	25000	15000	2000	2200	250	350	45	60	20000	15000	12000	8000	4

Table 5 (Continued): Parameters of the example for multi-period location-allocation model

Parameter																				
$u'_3{}^O$	$u'_2{}^O$	$u'_1{}^O$	b'_4	b'_3	b'_2	b'_1	a'_4	a'_3	a'_2	a'_1	R_z	R_x	C_{T_2}	C_{T_1}	C_{T_0}	r	J	T	v	K
60	45	25	50	30	15	10	60	45	60	40	25	60	2000	750	450	5	4	3	10	10^9

Table 5 (Continued): Parameters of the example for multi-period location-allocation model

			j=1	j=2	j=3	j=4
$p_{ji}{}^{t}$	t=1	i=1	0	0	0	0
		i=2	0	0	0	0
	t=2	i=1	0.1	0.3	0.4	0.5
		i=2	0.2	0.4	0.3	0.5
	t=3	i=1	0.15	0.2	0.4	0.3
		i=2	0.25	0.25	0.4	0.45
		$u_j{}^{O}$	10	15	30	50

Table 6: Result of Branch and Bound for Multi-period location-allocation model

Objective function	Type of answer	Constraint number	Variable numbers	Degree of infeasibility	CPU Time (seconds)
55,783,390	Local optimum	238	20	0.00007826901	10

Table 7: Results of Branch and Bound method for the plan of attack

Period	Regiment	Strategic Points		
		1	2	3
1	1	0	0	0
	2	0	0	0
2	1	0	0	0
	2	0	0	0
3	1	1	1	0
	2	0	0	1

Table 8: Results of Branch and Bound method for establishing supporting warehouses

		Locations			
		1	2	3	4
Perio d	1	0	0	0	0
	2	1	1	0	0
	3	1	1	1	0

Table 9: Results of Branch and Bound method for Inventory of Regiments

		Periods		
		1	2	3
Regiment	1	500	500	500
	2	400	400	400

Table 10: Result of Branch and Bound method for supplied ammunition

Periods of Planning			Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4
1	Regiment	1	0	0	0	0
		2	0	0	0	0
2	Regiment	1	15000	3000	0	0
		2	0	11714.29	0	0
3	Regiment	1	15000	0	0	0
		2	6666.667	0	15000	0

Table 11: Results of Branch and Bound method for the advancement and front-line position

	Periods		
	1	2	3
Advancement Value	20	5	35
Front-line Position	0	20	25

Table 12: Results of Proposed GA for establishment of supporting centers

Objective function	CPU time (second)	Degree of infeasibility	Crossover rate	Mutation rate	Population size	Iterations
54,592,501.6547	44.192976	0	0.8	0.1	30	689

Table 13: Results of Proposed GA for the plan of attack

Period	Regiment	Strategic Points 1	Strategic Points 2	Strategic Points 3
1	1	0	0	0
	2	0	0	0
2	1	0	0	1
	2	0	1	0
3	1	1	0	0
	2	0	0	0

Table 14: Results of the proposed GA for establishing supporting warehouses

		Locations			
		1	2	3	4
Period	1	0	0	0	0
	2	1	1	0	0
	3	1	1	0	0

Table 15: Results of the proposed GA for inventory of regiments

		Periods		
		1	2	3
Regiment	1	500	500	500
	2	400	400	400

Table 16: Results of the proposed GA for supplied ammunitions

				Warehouse			
				1	2	3	4
Periods of Planning	1	Regiment	1	0	0	0	0
			2	0	0	0	0
	2	Regiment	1	16,667	0	0	0
			2	1,332	21,558	0	0
	3	Regiment	1	14,974	5,090	0	0
			2	0	15,000	0	0

Table 17: Results of the proposed GA for the advancement and front-line position

	Periods		
	1	2	3
Advancement Value	20	0	20
Front-line Position	0	20	20