

International CAPM, Dynamic Betas and Optimization of Portfolios: Are countries-risk more profitable?

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Abstract

The paper proposes to check the relationship between risk and return in international spot markets. This subject will be made using time-varying betas estimation, original from theoretical structure of portfolio selection, which will be analyze the time evolution of non diversifiable risk among countries. The sample is composed by 14 countries, among developed and emerging ones. The sample period is from January 2002 to August 2015. We computed the dollar excess return for each country index as well as for the MSCI world index, which is proxy to market return. The risk free rate was a Treasury 30 years. Starting from the theoretical support of International CAPM (ICAPM), we estimated multivariate GARCH (MGARCH) models described in Tse and Tsui(2002), which are able to estimate conditional variances and covariances. They are extensions of the models of Bollerslev et al.(1988), Bollerslev, Engle, and Wooldridge and the model of Engle and Kroner(1995). All emerging markets, except Chile, had a beta higher than 1 as compared with developed countries. The research found that ICAPM is not valid in the context of international stock market, in other words, in average, the country with high risk is not the country with expected excess return.

Keywords: International CAPM; Dynamic Betas; International portfolio.

JEL Classification: F21;F30.

1 Introduction

Risk diversification theory was initially approached by Markowitz's work entitled Efficient Frontier (1952), in which micro-economical foundations were used to analyze how assets behave in front of uncertainties. Fifty years later, the process of globalization brought new components to this debate. One of them was the international diversification process of portfolios. This is to say that, American investors can easily buy assets anywhere in the world using the Internet to do so. Thus, from the 70s and 80s, it started to increase studies on international diversification of assets. So the question is: Is international diversification capable of minimizing risks? Or, are risks capable of maximizing investors' returns? These questions have been discussed in the literature for the last couple of years and there are some contradictions hovering over them. Studies show that international diversification brings gains to investors and, on the other hand, researches prove that international diversification does not improve portfolios' income due to several factors that are discussed further in this article.

[Table 1]

Another question concerned the international capital market is on risks taken on economies with different levels of development. Conventional literature suggests that, in order to invest in emerging markets, apparently more risky than developed ones, it is necessary to have higher returns according to the investors' demand. So another question comes to scene: have risky markets given more returns recently? This is another question that is answered by empirical tests, making use of international CAPM. International Capital Asset Pricing has been the focus of many studies and researches such as [Adler and Dumas(1983)], [Karolyi(1995)], [Santis and Gerard(1997)], [Ng(2004)], [Eun et al.(2008)Eun, Huang, and Lai]. One of the main aspects of their studies is concerned with developing economies in several countries. Just

to mention few, [Karolyi(1995)] developed his work based on American and Canadian markets, while other collected data by observing developing markets [Adler and Dumas(1983)], and son on many other authors. Besides them, it is noted that in the literature, there is a lack of studies discussing international pricing of assets, both on developed and emerging markets. We verified in the past few moments, precisely in the 1990s, a quite considerable growth on the importance assumed by the emerging markets on the global economy. Table 1 illustrates that.

It is perceived that emerging countries participation increased 18,67% in 1991 to 38,71% in 2013. This reinforces the importance of emerging economies in the global scenario, what has been making capital of this emerging economies flow. Part of this capital goes to stock markets, and has been causing a large growth in the index of emerging market spots when compared to levels from the 1990s. These facts also allow better diversification for actives since they can invest part of the resources on markets such as mentioned above. In this sense, the empirical contribution of this works is directed to the analysis of international pricing of actives on developing and emerging market contexts. To be more specific, it is intended here to test international capital asset pricing model (ICAPM) taking into account developing and emerging markets. The goal is accomplished according to the quantity of dynamic betas in the market that is derived from the multivariate GARCH model. Following that, it is estimated the risk premium of each market and also set optimal portfolios that maximize investors returns and/or minimize the risk.

The current study goes beyond the first introduction section and is divided into five more sections. The section number 2 is concerned on the financial literature taking into account previous works dedicated to the same theme but making use of different methodologies that consequently show different alternative results; the section number 3 describes empirical strategies used to estimate portfolios with minimal variance (risk). The section number 4 is exposed the planning and treatment of the sample used to obtain results that are analyzed in the following section of number 5. In the section 6 is regarded to final remarks and closure of the research.

2 Literature review

International capital asset pricing model (ICAPM) was initially carried out by [Adler and Dumas (1983)] with the goal of determining factors that affect portfolio riskiness. In his model, it is verified the influence of inflation and exchange rates on shareholders? global risk. Furthermore, the first intention of this model is to determine if, in empirical terms, nations can be distinguished according to purchasing power parity (PPP) deviations. If it stays the same, differences between countries? inflations are compensate in terms of currency devaluation. It is important to highlight that ICAPM is applicable when the investors use the same index of price in order to reduce returns, a supposition that is not realistic in international levels. Differently, IAPM allows the use of several price indices; in other words, to each nation, price index is used to calculate investors? real return.

The next step of Alder and Dumas?s work is centered on international asset pricing model deviation. In this step, they have shown that investors, who are averse to risk, fully prefer taking risk of domestic inflation rather than exchange rates or price actions uncertainties. It is not clear if the result would be modified, this is to say that, estates, commodities, and precious metals would be included in the matrix of possible investments.

$$\begin{aligned} \mu_i - \sum_{k=n+1}^{n+L} \gamma_{i,k} \mu_k = r \left(1 - \sum_{k=n+1}^{n+L} \gamma_{i,k} \right) + (1 - 1/\alpha^m) \left(\sigma_{i,\pi}^l - \sum_{k=n+1}^{n+L} \gamma_{i,k} \sigma_{k,\pi}^l \right) \\ + (1/\alpha^m) \left[\sum_{j=1}^N \omega_j^m \left(\sigma_{i,j} - \sum_{k=n+1}^{n+L} \gamma_{i,k'} \sigma_{k',j} \right) \right] \quad i = 1, \dots, n \quad \forall l \end{aligned} \quad (1)$$

Where r_i is nominal interest rate on the currency i. μ_i is the value expected from the instantaneous change of the exchange rate of the currency i, against the reference currency $l + 1$. r_{l+i} is the exchange rate of currency used as value unit, until now simply indicated as r. $\gamma_{i,k}$ is the i exchange rate covariance and dollar currency, the national investor rate of inflation l, and r_s is exchange rate covariance with returns

transformed into actives k , included by $k = n + 1$ to N , the covariance with its own exchange rates.

In accordance to [Adler and Dumas(1983)] under risk-neutral, the value expected from the deflated spot rate must be equal to the future rate that will be also deflated, which the covariance between intended rate and deflator is the source of this premium. The second premium is connected to the exchange rate covariance with real return on global marketing portfolio. This model assumes that agents are neutral to risks; however, the majority of the models assume that agents are averse to the risk. The reason for this, according to [Adler and Dumas(1983)], is that the model relates spot and future rates that cannot be tested.

Another issue pointed out by [Adler and Dumas(1983)] was the question of the well-being associated to cambial risk. Authors have studied how cambial risk affected well-being and how it is allocated among individuals. The conclusion is that the impact of risk cambial on well-being depends on stock market condition. It depends on factors such as: government, monetary policies and integration of capital markets with another capital markets in the world.

[Adler and Dumas(1983)] examined marketing segmentation in the article involving capital control, access to local countries, limit of foreign capital in the local company. These facts mainly occur in governments that reject free market or, in populist governments that clearly affect returns and investment risks. [Adler and Dumas(1983)] affirmed that segmentation on international markets of commodities can produce deviation on PPP that might disturb risk allocation all over the world. In this way, this international asset pricing model can contribute to maximize investors' allocation, what means to say that, it indicates the country they should invest the greatest amount of money.

Another questions studied by [Adler and Dumas(1983)] is related to financial decisions in international companies, specifically companies that face problems of imperfections on capital markets and segmentation. [Adler and Dumas(1983)] affirmed that cambial variations can affect companies in many dimensions: through the impact of monetary actives on long and short terms, and legal passive and active. The main results of the investigation on this issue is that the value of a company

with a contract is the same value of a company without a contract, added the value of the same. This question has relation with [Modigliani and Miller(1958)]'s theory, that affirms that the structure of capital does not affect the value of the company.

The study of [Karolyi(1995)] examined the dynamicity of returns in short terms and volatility, with stocks negotiated in New York and Tokyo stock markets. Still in accordance to [Karolyi(1995)], the growth of financial market integration brings to us the need of studies on how stocks returns influence region markets and also, studies on verifying the implications of volatility and pricing affecting those markets. The author studied the dynamic relation that occurs between return of stocks and volatility of the return, on a Standard and Poor (S&P) and TSE 300, which stock indices are from April 1981 to December 1989, comprehending the period that occurred the international crash of 1987. The methodology utilized in this research was the bivariate GARCH model, a family of statistics models originally developed by [Engle(1982)] and [Engle and Bollerslev(1986)]. The author justifies the choice of the model once they consider it capable of translate the innovation of returns that are transmitted from US-Canadian stock markets to another stock markets. In this model, he also points innovations of volatility in one market that has been impacted by the others through an impulsive response function provided by the VAR model.

The data of [Karolyi(1995)] presents stylized facts about financial data: thicker tail, non-linear dependence and clustered volatility. The author states that the series of S&P 500 showed asymmetry and also excess of kurtosis, also indicating that these series do not follow normal distribution. The non-linear dependence identified by [Karolyi(1995)] shows that squared returns have great self-correlation, greater than the returns on level. On the other hand, the econometric model utilized by [Karolyi(1995)] was the GARCH model developed by [Susmel and Engle(1994)]. To the conditional average, the author estimated a VAR process in order to analyze the international movement of transmissions on stock markets. From residual VAR model, it was estimated GARCH model with the main goal of capturing the relation between conditional variances on markets. This model is capable of providing conditional variance, conditional covariance and conditional correlation between the stock markets of USA and Canada. Besides the GARCH model of conditional correla-

tion, [Karolyi(1995)] used BEKK-MGARCH, created by [Engle and Kroner(1995)], to combine results and verify the power of explanation regarding to both models. One of the disadvantages of BEKK model is that it is needed a large number of parameters to estimate.

According to the results of the research developed by [Karolyi(1995)], it is possible to report that the GARCH model had a better adjustment to the process of returns since residual MGARCH are smaller. In fact, the author did not find non-normality for VAR model but great asymmetry and excess of kurtosis. However, unlikely VAR model, the estimated bivariate residual GARCH showed what it is to be considered as normal, with insignificant excess of asymmetry and lightly positive excess of kurtosis.

The author mentioned above simulated the impulsive-response function according to the stock market of Canada and USA, concluding that the Gaussian innovations on those markets are rapidly transmitted through higher responses that, usually in the first day, falls along time. [Karolyi(1995)] also discovered that the magnitude of the responses, are much larger in domestic shocks rather than external ones. This seems to be consistent according to [Karolyi(1995)] results. In his study, [Karolyi(1995)] verified the relation between the returns of Canadian companies listed on the North American market. The main goal of the study perceives if shocks in North American market mostly affects companies presented or those not presented on the stock list. In fact, the author concluded that the magnitude and persistence of S&P 500 innovations have some impacts on the subsequent returns of the stocks listed. It is reinforced that both market were lower than those stocks that were not listed on the Canadian market.

The work of [Karolyi(1995)] analyzed the sample in sub periods in order to verify the degree of integration before and after the capital market regulation of 1980s. The sub periods were 1981-1984, 1984-1987 and 1987-1989. Actually, the author observed changes between both markets in the sense of integration, so, to be more precisely, it has occurred integration on financial markets during 1980s.

According to [Santis and Gerard(1997)], on average, expected gains from international diversification are equal to 2,11% on annual base; they were not significantly affected by the rise of integration level on international markets. Estimations pointed

by the authors were based on North American investors. The method of estimation used to find the results are derived from the GARCH model proposed by [Engle and Kroner(1995)]. So this research attempts to demonstrate optimized portfolios of assets based on international diversification, as well as the contagion between emerging and developing markets. It is know that nowadays it is easy to buy ETFs, what it is considered to be indices of funds based on the income of the country selected to invest. CAPM used by this author is given:

$$E(R_{it}|\vartheta_{t-1}) - R_{ft} = \delta_{t-1}cov(R_{it}, R_{mt}|\vartheta_{t-1}) \quad (2)$$

Where δ_{t-1} is the risk price of the market that comprehends the returns calculated in US-Dollar. [Santis and Gerard (1997)] showed that this approach presumes that investors do not protect themselves from exposing to exchange rate risk, what that is to say that the price of the cambial risk is 0. The same work also mentions that risk price covariance δ_{t-1} should be positively equal to every market in order to indicate if international markets are fully integrated and if the global systemic risk is the only relevant factor in play. Still following the author mentioned above, the ICAPM proposed may include the impact of volatility in country with excess of returns. According to it, the equation shows that

$$R_{it} - R_{ft} = \alpha_i + \delta_{t-1}cov(R_{it}, R_{mt}|\vartheta_{t-1}) + \gamma_i(var(R_{it}|\vartheta_{t-1}) + \varepsilon_{it} \forall i \quad (3)$$

$$R_{it} - R_{ft} = \alpha_i + \lambda' z_{t-1} + \delta_{t-1}cov(R_{it}, R_{mt}|\vartheta_{t-1}) + \varepsilon_{it} \forall i \quad (4)$$

In order to verify if international financial markets are more integrated and if global diversification is a way to minimize risks in Bear market scenario, the authors

used the following specification:

$$E(R_{dt} - R_{USdt}|\vartheta_{t-1}) = \delta_{t-1} [\text{var}(R_{USdt}|\vartheta_{t-1}) - \text{cov}(R_{USdt}, R_{mt}|\vartheta_{t-1})] \quad (5)$$

Where R_{dt} is the return of a diversified portfolio internationally, which includes R_{mt} and R_{ft} in dollar and also have the same volatility of the US portfolio. The authors conclusions are that the bear markets are more contagious than the bull markets, once the bull market correlation is higher. However, according to [Santis and Gerard(1997)], this result is not enough to discard the efficient theory of the internationally diversified portfolio.

[Cappiello and Fearnley(2000)] used ICAPM with regime-switching on GARCH parameters to investigate if investors are unsure about the return on a specific asset risk, and, if it is shown that they are reversed to the risk, they will require market risk premium. The main goal of the author is to estimate the market risk premium to developed capital markets, stochastically. The specification used by this author is given:

$$E(R_i^c) = \gamma_M \text{cov}(R_i^c, R_M^c) + \sum_{j=1}^L \delta_j \text{cov}(R_i^c, R_j^c) \quad (6)$$

Where the γ_M parameter represents the market price risk and δ_j are currency risk prices.

According to [Cappiello and Fearnley(2000)] and important question is the number of assets and the countries to include in the model. The authors state that ICAPM postulates that, in a globalized world, the excess of return in the stock market of a country in dollar i includes the sum of the risk premiums required for all international and foreign investors, containing the country i, besides all the countries integrated to it. The econometric model used by this author is the GARCH with regime-switching proposed by [Hamilton (1990)]. The option for this model lies on the reason why that GARCH (1,1) parameters are instable along time and, thus,

predictions cannot be considered robust. The data utilized by [Cappiello and Fearnley(2000)] are on the last day of negotiation of the week, and they refer to the period of September 7, 1986 to December 31, 1998, to a total amount of samples of 674 observations. The data captured the effect of the financial crisis during the 80s and 90s such as the crash market in 1987, the Asian crisis 1997, the Russian crisis in 1998 and the Latin-American crisis in 1998. The used the excess of return of the market index to USA, Japan Europe and to the world, since these stock markets represent 95The unconditional correlations found by [Cappiello and Fearnley (200)] were: 0,21 between Japanese and US markets, 0,489 between European and North-American stock markets and 0,448 between the Japanese and European markets. This lows correlations suggest that international diversification is beneficial because the stress that affects one market does not affect others at the same intensity.

The results of [Cappiello and Fearnley(2000)] indicate a risk price of a positive market and a small insignificant exchange risk price estimated through a model without regime-switching. In addition to it, the market price risk continues positive but not significant and, on the other hand, the cambial price risk remains low and insignificant. The dynamic analysis of the risk premium conducted by the authors shows some results. Rise occurred on the risk premium of the USA during the summer/fall of 1986, given the turbulence of the market, the stock market crash in October 1987, the Gulf War in 1990/91, 1997/98 and Latin-American, Russian and Asian crisis. Still, according to [Cappiello and Fearnley(2000)], in 1987, there was rise regarded to the risk premium in the Japanese stock market affected by the North-America market. The factors that impacted the increase in volatility in the European market were the crash in 1987, the Gulf War and the Asian and Latin-American crisis. Some results of [Cappiello and Fearnley(2000)] on the GARCH model with regime-switching are quite interesting. The researchers have found that the first regime is less persistent than the second. Once the first regime lasts 9,5 weeks, the second one lasts 39,8 weeks. Still, according to [Cappiello and Fearnley(2000)], the model without regime-switching is slow in face to react to shocks in certain ways, ot is relatively slow when it comes to readjust exposures to risk and optimized prices in the portfolio.

[Ng(2004)] derives a dynamic version of ICAPM when the expected returns vary along time. The author used data from the return of assets and exchange rates to USA, Japan, German and United Kingdom. The main contributions to the research, according to [Ng(2004)] were: in the first place, the development of An ICAPM that is empirically treatable; second, the identification and investigations on the importance of intertemporal coverage of the future risks and the real exchange rate; third. The model aligns to the standard CAPM with ICAPM. And forth, the dynamic CAPM offers theoretical bases to risk factors highly used, such as exchange rates, inflation, dividend rate and future premium in the explanation of international asset returns.

[Ng(2004)] reports that ICAPM appears because of the theory of homogeneous expectations is violated in the contexts of international markets. This occurs because there are deviations on the PPP, showing that changes on the exchange rate are not compensated by the change on the level price of the countries. The outstanding difference of this authors work is that the stable variable might have a price, what does not occur in the model of [Adler and Dumas (1983)]. The model used by [Ng(2004)] differs from the traditional ICAPM because the coverage of the future exchange rate risk was reduced to a factor of coverage, involving the future movement in the exchange rate index. The model proposed by him is given:

$$R_{pj,t+1}^r = R_{pj,t+1}^1 \frac{P_t^1}{P_{t+1}^1} \frac{Q_{jt}}{Q_{jt+1}} \quad (7)$$

Where $R_{pj,t+1}^r$ is the real return to investors of the country j ; $R_{pj,t+1}^1$ is the nominal return to investors of the country j which portfolio is expressed in its own currency, P_t^1 is the level of prices in time t in currency on 1, Q_{jt} is the exchange rate in real time t .

Transforming the equation (6) into log:

$$r_{pj,t+1}^r = r_{pj,t+1}^1 - \pi_{t+1}^1 - \Delta q_{jt+1} \quad (8)$$

Still following [Ng(2004)] the equation 9 shows that real return to foreign investors depends on exchange rate and inflation. If the parity of purchasing power is valid, $\Delta_{qjt+1} = 0$, in this case, the domestic return is equal to the foreign return, discounting inflation. The same uses the consumer preferences of [Epstein and Zin(1989)], where γ_j is the aversion coefficient to the relative risk and σ_j is the elasticity of intertemporal substitution. The objective function is defined by:

$$U_{jt} = \left((1 - \beta_j) C_{jt}^{(1-\gamma_j)/\theta_j} + \beta_j (E_t U_{jt+1}^{1-\gamma_j})^{1/\theta_j} \right)^{\theta_j/(1-\gamma_j)} \quad (9)$$

Where $\theta_j = (1 - \gamma_j) / (1 - 1/\sigma_j)$. The Euler equation related to the maximization of the equation above is:

$$1 = E_t \left(\left(\beta_j \left(\frac{C_{j,t+1}}{C_{j,t}} \right)^{-\frac{1}{\sigma_j}} \right)^{\theta_j} \left(\frac{1}{R_{pj,t+1}^r} \right)^{1-\theta_j} R_{i,t+1}^r \right) \quad (10)$$

Adapting to ICAPM, where returns are in reference currency, we have:

$$1 = E_t \left(\left(\beta_j \left(\frac{C_{j,t+1}}{C_{j,t}} \right)^{-\frac{1}{\sigma_j}} \right)^{\theta_j} \left(\frac{1}{R_{pj,t+1}^1 \frac{P_t^1}{P_{t+1}^1} \frac{Q_{jt}}{Q_{jt+1}}} \right)^{1-\theta_j} R_{i,t+1}^1 \frac{P_t^1}{P_{t+1}^1} \frac{Q_{jt}}{Q_{jt+1}} \right) \quad (11)$$

Supposing that returns, consumption growth rate, inflation rate and exchange rate have log-normal conjoint distributions and homoscedastic, the equation 12 can be decomposed into two equations according to [Ng(2004)]:

$$E_t (\Delta c_{j,t+1}) = \mu_{p,j} + \sigma_j E_t (r_{pj,t+1}^1 - \pi_{t+1}^1 - \Delta q_{jt+1}) \quad (12)$$

$$E_t (r_{i,t+1}^1 - r_{f,t+1}^1) + \frac{V_{ii}}{2} = \frac{\theta_j}{\sigma_j} V_{icj} + (1 - \theta_j) V_{ipj} + \theta_j (V_{i\pi^1} + V_{iqj}) \quad (13)$$

Where $\mu_{p,j}$ is the term of variance that measures the uncertainties of consumption in relation to the returns in the real market. So it is assumed to be constant. V_{ii} is the variance of the returns asset i. V_{icj} is the asset i covariance with relative consumption of the country j. V_{ipj} is the covariance between the return of the asset i and the return of the optimized portfolio p of the country j in the reference currency.

The next step of [Ng(2004)] research defined the equation that can be used to substitute the covariance of the returns on assets with consumption in the second Euler equation (14).

$$c_{jt+1} - E_t c_{jt+1} = (E_{t+1} - E_t) (r_{pj,t+1}^1 - \pi_{t+1}^1 - \Delta q_{j,t+1}) + (1 - \sigma_j) (E_{t+1} - E_t) \left(\sum_{k=1}^{\infty} \rho_j^k (r_{pj,t+k+1}^1 - \pi_{t+k+1}^1 - \Delta q_{j,t+k+1}) \right) \quad (14)$$

[Ng(2004)] affirms that, from the definition of 0_j , the new equation of pricing is:

$$E_t (r_{i,t+1}^1 - r_{f,t+1}^1) + \frac{V_{ii}}{2} = \gamma_j V_{i,pj^1} + (1 - \gamma_j) (V_{i,qj} + V_{i,\pi}) + (\gamma_j - 1) (V_{i,hpj^1} - V_{i,h\pi} - V_{i,hqj}) \quad (15)$$

The implication of the equation 16 show that, according to [Ng(2004)], in nominal terms in currency of reference, the asset risk premium (adjusted to half of its own variance) depends on the covariance of assets between the returns of the market portfolios, with weight γ_j and with real depreciation adjusted by the inflation in the reference currency, with weight $1 - \gamma_j$, and with innovation in future market returns discounted, except the future inflation and real depreciation with weight $\gamma_j - 1$.

According to [Ng(2004)] the equation 16 is not capable of calculating the return

rate required from several stocks because the weights of the portfolio kept by the investor are not observed. Thus, the solution found by the author was to specify the model using the following formula:

$$E_t(r_{i,t+1} - r_{f,t+1}) + \frac{V_{ii}}{2} = \gamma V_{i,m} + (1 - \gamma) V_{i,q} + V_{i,\pi} + (\gamma - 1) (V_{ihm} - V_{i,hq}) \quad (16)$$

Where V_{im} is the covariance of returns on assets with innovations of real market returns; $V_{i,hm}$ is the covariance of news with future real return of market and $V_{i,hq}$ is the covariance with news on future rates and real exchange rate. The equation 15 is known as dynamic ICAPM (DICAPM). [Ng(2004)] also affirms that DICAMPM explains the risk premium of assets by the covariances of the assets, with real market return, inflation in reference currency, changing in the index of real exchange rates (exchange rate risk), and with real future market returns and real future depreciation (intertemporal hedging components).

According to the author mentioned above, DICAPM nests ICAPM, dynamic CAPM and static CAPM as special cases. The equation 15 is the formula for pricing assets used by empirical investigation. The same used data monthly collected from July 1978 to April 1998, from Morgan Capital Index Stanley (MSCI) - the index of world market deflated by the Consumer Price Index (CPI) of the USA. The data were used as a proxy to the real return in the world market. The real exchange rate index was built using nominal cambial rates from G7 countries and local inflations. The reference currency was the American dollar. The author estimates the future real return of the market portfolio of global capital and the depreciation of the real cambial rate through VAR(1) model in order to test the equation 15, since he needed to obtain expected values from these variables. In sequence, [Ng(2004)] utilized the VRA impulsive response function to capture innovations on returns. He found out that shocks on returns of the assets, inflation and future premium have negative effects on the innovation of future expected returns discounted; however, innovations on the income of cambial rates and real dividend have positive effects. The same author estimated the aversion coefficient relative to risk, measuring the degree of

aversion to the agents risk. The parameter γ calculated by the research was 5,99, similar result to the work of [Hodrick et al. (1999) Hodrick, Ng and Sengmueller], they found 5,06 coefficient to returns of the G7. In accordance to price risk, the author compares the restrictive model to the non-restrictive one and concludes that there is a slightly reduction on the price risk associated to the covariance of the return of assets with real return on the world market portfolio on the non-restrictive model.

[Ng(2004)] analyses the resources of risks and errors on prices. He comes to the conclusion that errors on prices are low, since it varied from 0,026% per month on the US capital markets, to -0,015% on Japanese the cambial market. According to the same author, intertemporal hedging components are important to the returns on assets because, in the USA, the intertemporal coverage for returns on assets was considerable, decreasing the returns expected from 0,755 to 0,624%. Finally [Ng(2004)] tested if CAPM is valid for his data. The result is that the same was not rejected by the data.

[Cappiello et al.(2006)Cappiello, Engle, and Sheppard] propose a new model called AG-DCC which adds the asymmetry parameter to the dynamic of the correlation matrix. The goal here is to verify if negative shocks on returns affect more the conditional correlation rather than positive shocks. The sample of the authors goes from 1987 to 2002 as weekly frequency. The authors identify an increase correlation between France, German and Italy, and also correlation between them and the United Kingdom. One of the conclusions is that asymmetry is higher on the returns of the stock indices rather than on the titles.

[Eun et al.(2008)Eun, Huang, and Lai] used data that include monthly prices of assets and returns, the number of assets in circulation to companies listed, exchange rates and MSCI indices of the 10 most important countries for the stock market, this is to say that, those with a large volume of negotiation, during the period of January 1980 to December 1999. For risk free rate, the authors used the title rate of the US-treasury T-bill. Statistics described in the work, the authors verified that only in German and Italy, small-caps stocks have higher volatility than blue-chip stocks. Besides, the USA has low volatility among large-caps stocks. These findings

indicate that maybe the North-American market was considered the safest for the investors. Another important fact highlighted by the authors was that international diversification is probably more efficient with small and large stock combinations rather than only with stocks of large-sized companies. They also studied if small-caps funds can be explained by MSCI indices of the country. Thus, the authors used the following specification:

$$R_i = \alpha_i + \beta_i^{AU} MSCI^{AU} + \dots + \beta_i^{US} MSCI^{US} + \epsilon_i \quad (17)$$

Where R_i is the return on the small-cap fund of the country j-th and $MSCI^{AU}$ denotes the return on Australian MSCI and $MSCI^{US}$ is the return on US MSCI. The null hypothesis tested by [Eun et al.(2008)Eun, Huang, and Lai] is that $\alpha = 0$ and $\sum_i \beta_i = 1$. According to the authors, the null hypothesis was rejected for Canada, France, German, Japan, Holland and United Kingdom at a 1% level of significance, and for Australia and Italy 5%. For the US, the null hypothesis was rejected at 10%. To comprehend the relation between return of small-cap fund and global MSCI index and stock market index of every country, [Eun and Shim(1989)] made use of the following equation:

$$R_{ij} = \alpha_{ij} + \beta_{ij}^W R^W + \beta_{ij}^c R_i^c + \epsilon_{ij} \quad (18)$$

Where R_{ij} is the return on funds of the country j-th; R^w is th return on the World MSCI index and US, and Ric is the parcel of the national index of the country i. The author concluded that the global and local facts influenced the returns on small-caps funds. [Eun et al.(2008)Eun, Huang, and Lai] evaluated the capacity of small-caps funds to reduce the variance of the international portfolio, following the methodology of [Solnik(1974)].

The results indicated that the variance of the portfolio when compared to the portfolio with only large-caps stocks. One of the conclusions of the first authors

was that neighboring countries have higher correlation since the integration of these markets is larger. For example, the correlation calculated between France-German was 0,69, while the correlation US-Japan was 0,27. Furthermore, the work estimated the optimized portfolio following [Markowitz(1952)], with and without short sellings. The countries with more weights were Holland and USA. The sample was composed by Australia, Canada, France, German, Hong Kong, Italy, Japan, Netherlands, United Kingdom and USA.

3 Empirical Strategy

3.1 Multivariate GARCH

The research will use the MGARCH model following the approach of [Tse and Tsui(2002)]. This method allows the estimation of dynamic betas from the conditional covariance and conditional variance obtained. The choice of model is justified because the authors incorporate correlations varying in time, while satisfying the condition that the conditional variance matrix is positive definite. The MGARCH model of [Tse and Tsui(2002)] is an innovation model of Bollerslev (1988) and the model of [Engle and Bollerslev(1986)].

Considering $r_t, t = 1, \dots, T$ the set of multivariate observations of excess returns of stock index measured in reference currency and the MSCI world index, each with K elements, where $r_t = (r_{1t}, \dots, r_{Kt})$. The conditional variance, assuming that r_t time-varying, is defined by:

$$Var(r_t | \Phi_{t-1}) = \Omega_t \tag{19}$$

Where Φ_t represents the set of information at time t . The variance of the elements of Ω_t is represented by σ_{it} , for $i = 1, \dots, K$ and the covariances are represented by σ_{ijt} , where $1 \leq j \leq K$. Defining D_t the diagonal matrix in which the i th diagonal element is σ_{ijt} , you can define $\varepsilon_t = D_t^{-1}r_t$. ε_t represents the standardized residual

and it is assumed that it is IID with zero mean and variance matrix $\Gamma_t = \{\rho_{ijt}\}$. Therefore, the correlation matrix for r_t is denoted by $\Omega_t = D_t\Gamma_tD_t$. The conditional variance follows formulation vech-diagonal developed by Bollerslev (1988). Then, each term in the conditional variance follows an univariate GARCH (p, q) given by equation:

$$\sigma_{it}^2 = \omega_i + \sum_{h=1}^q \alpha_{ih} r_{i,t-h}^2 + \sum_{h=1}^p \beta_{ih} \sigma_{i,t-h}^2 \quad (20)$$

Where $\omega_i, \alpha_{ih}, \beta_{ih}$ are nonnegative and $\sum_{h=1}^q \alpha_{ih} + \sum_{h=1}^p \beta_{ih} < 1$ for all $i = 1, \dots, K$. The conditional correlation matrix time-varying is defined by the equation:

$$\Gamma_t = (1 - \theta_1 - \theta_2) \Gamma + \theta_1 \Gamma_{t-1} + \theta_2 \Psi_{t-1} \quad (21)$$

Where $\Gamma = \{\rho_{ijt}\}$ is a positive definite matrix parameters, of size $K \times K$, time variant with unitary diagonal elements and Ψ_{t-1} is a matrix which the elements are functions of the lagged observations of r_t . The parameters θ_1 and θ_2 are nonnegative and it is assumed that the restriction of which $\theta_1 + \theta_2 \leq 1$.

It is observed that Ψ_{t-1} is analogous to r_{t-1}^2 of GARCH (1,1). However, with Γ_t is, according to Tse and Tsui (2002), a standardized measure, Ψ_{t-1} needs depend on standardized residuals lagged ε_t . Defining $\Psi_t = \{\psi_{ijt}\}$, Ψ_{t-1} follows the specification:

$$\psi_{ij,t-1} = \frac{\sum_{h=1}^M \varepsilon_{i,t-h} \varepsilon_{j,t-h}}{\sqrt{\left(\sum_{h=1}^M \varepsilon_{i,t-h} \sum_{h=1}^M \varepsilon_{j,t-h} \right)}} \quad (22)$$

ψ_{t-1} is the correlation matrix of $\{\varepsilon_{t-1}, \dots, \varepsilon_{t-M}\}$. Defining E_{t-1} a matrix $K \times M$ given by $E_{t-1} = \{\varepsilon_{t-1}, \dots, \varepsilon_{t-M}\}$. If B_{t-1} is a diagonal matrix $K \times K$ where the i th diagonal element is $\left(\sum_{h=1}^M \varepsilon_{i,t-h} \right)^{1/2}$ for $i = 1, \dots, K$, we have:

$$\Psi = B_{t-1}^{-1} E_{t-1} E_{t-1}' B_{t-1}^{-1} \quad (23)$$

The conditional log-likelihood ℓ_t of the observation r_t is given by:

$$\begin{aligned}\ell_t &= -\frac{1}{2} \ln |D_t \Gamma_t D_t| - \frac{1}{2} r_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} r_t \\ \ell_t &= -\frac{1}{2} \ln |\Gamma_t| - \frac{1}{2} \sum_{i=1}^K \ln \sigma_{it}^2 - \frac{1}{2} r_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} r_t\end{aligned}\tag{24}$$

Define $\theta = (\omega_1, a_{11}, \dots, a_{1q}, b_{11}, \dots, b_{1p}, \omega_2, \dots, a_{Kq}, \rho_{12}, \dots, \rho_{K-1,K}, \theta_1, \theta_2)$ as the vector of parameters and maximizing ℓ in relation to θ , we have $\hat{\theta}$, where $\ell = \sum \ell_t$.

From MGARCH, we can estimate the dynamic beta of each market, dividing the conditional covariance between world market index and domestic market index by the conditional variance of the world market index. The dynamic beta of country measures the sensibility of the country index with the MSCI world index.

3.2 Portfolio Optimization

Portfolios are optimized by the model of [Markowitz(1952)].

Be r_{t+I} the return on stock index, the return of the portfolio between t and $t + 1$ is given by: $r_{p,t+I} = \sum_{i=1}^N w_{i,t} R_{i,t+I} = w' r$. Admitting that $R_t \sim N(\mu_t, \Sigma_t)$ and $\mu_t = \{\mu_{1,t}, \dots, \mu_{N,t}\}$ e $\Sigma_t = \{\sigma_{ij,t}\}$ the average and covariance, respectively. The return of the portfolio $R_{p,t} = w_t' r_t$ follows one normal with average $\mu_{p,t} = w_t' \mu_t$ and covariance $\sigma_{p,t}^2 = w_t' \Sigma_t w_t$. The investor, therefore, encounter themselves with following restrictive minimization:

$$\begin{aligned}\min_w & w' \Sigma w - \frac{1}{\gamma} E[r_{p,t+I}] \\ \text{s.a.} & w' \mu = 1 \\ & w_i \geq 0 \quad \forall i = 1, \dots, N,\end{aligned}\tag{25}$$

Where $w \in \mathbb{R}^N$ is the vector of portfolio weights, $E[r_{p,t+I}]$ is the sample mean of portfolio returns, $w' \Sigma w$ is the sample variance of returns; γ represents the relative degree of aversion to risks and also the restriction of short selling. The restriction

$l'w = 1$ indicates the sum of the weights must be equal to 1 ($l \in \mathbb{R}^N$), its a vector N-dimensional ones. For each degree of risk aversion γ there is an optimized portfolio of investments.

3.3 Minimum-Variance

Minimum-variance portfolio consists in a specific case of strategy of Minimum-Variance which the investors degree of risk aversion is infinite ($\gamma = \infty$), so the attention is paid closely to the risk minimization associated to the portfolio:

$$\begin{aligned} \min_w w' \Sigma w \\ \text{s.t. } l'w = 1 \\ w_i \geq 0 \quad \forall i = 1, \dots, N, \end{aligned} \tag{26}$$

3.4 Dynamic Portfolio Beta Weighted

Be $b_i = 1/\beta_{it}$. So the weight of the asset i in the portfolio in time t is:

$$w_{it} = \frac{b_{it}}{\sum b_{it}}$$

In this sense, how higher the beta is, the lower is the weight of the country in the portfolio.

4 Data

It was collected indices of emerging and developing countries, as table shows. The period of analysis starts in January 2002 and extends to August 2015. Given the research on capital markets of several countries, where there are many local holidays influencing days of negotiation, it was used data per month. As market index R_{mt} it is used MSCI World Index. As free-risk rates, it was utilized the return of an American treasury of 30 years, the treasury 30.

[Table 2 here]

Once collected the indices, it was calculated the return in dollar and then the exceeding return. The exchange rate was collected according to FMI site. The data on indices were collected according to Yahoo Finance.

The return composed in dollar is estimated through the equation:

$$r_{it} = \ln(P_{i,t}/E_{i,t}) - \ln(P_{i,t-1}/E_{i,t-1}) \quad (27)$$

5 Results

5.1 Descriptive Statistics

Number 1 table presents descriptive statistics to excess returns of indices studied in worlds MSCI index. It was utilized the treasury 30 as free risk return, American treasure title with 30 year of deadline. It is verified that the indices of countries such as Italy, South Africa, Australia, United Kingdom, USA and Spain had a large negative exceeding return in dollar, indicating that the treasure title was one of the best investment option when compared to these indices. Emerging countries such as Brazil, Mexico, China and Chile had great positive exceeding return in dollar. [Table 3 here]

In relation to the standard deviation we highlight emerging countries such as South Africa, Brazil and China presenting standard deviation up to 8%. It is observed countries such as USA, Australia and Canada, presenting lower standard deviation when compared to the others, with value down to 5%. Thus, it is perceived higher volatility in emerging markets and then European markets. For instance, the Italian market had 6,9% for standard deviation, what it might be repercussion of the crisis that Europe is facing since 2011.

5.2 Estimated parameters MGARCH and beta analysis

Observing that the estimated parameters by GARCH model we are able to analyze what is the country that presents the greatest persistence in volatility. Chile, United Kingdom and Australia have a coefficient b higher than 0.8, indicating that 80% of volatility in $t - 1$ replicated in t ; in other words, when there is volatility shock, it takes a bit longer to fade away during the time. We also note that the Brazilian market presents b coefficient small, 0.11, indicating a small parcel of current volatility that is passed to future.

Another emerging market that presented low persistence was the Mexican one, with coefficient 0.38. The developing market that presented the lowest persistence was the Canadian one, with coefficient of 0.32. [Table 4 here]

Figure 1 in appendix B brings the betas of the countries surveyed, varying in time (time-vary). Beta measures non-diversifiable risk of each country. So, the higher beta, the higher is the exceeding return that the investor would be requiring to invest in that country. We note emerging markets like Brazil and South Africa with betas at higher levels most of the time. It is also observed greater stability in US beta, where it always remains close to 1, not presenting great variation peaks as it is supported in Table 3, where the American beta has a standard deviation of 0.07, the lowest among all countries. Among the largest standard deviation, it is highlighted South Africa betas whose value was 0:59. Regarding to the average, it is verified that Australia had the lowest average beta among the countries. Countries like Italy, France, Spain and Germany had betas higher than 1, what is indicated for investors with more moderate risk profile. Countries with lower beta such as USA, UK, Chile, Canada, Australia, China and Japan, are indicated to more defensive investors with more risk aversion. Investors analyze betas in order to build their investment portfolios. Assuming risk aversion and forming portfolio with dynamic weights, country with the highest beta will receive less weight, minimizing non-diversifiable investment risk.

[Picture 1 here]

[Table 5 here]

After that, static betas were estimated according to minimum ordinary squares,

as it follows in Table 4. It is found that only Brazil, Chile and Australia had very different values in accordance to static and dynamic average beta. For example, the dynamic average beta in Brazil was 1.63, whereas the static beta was 1.48. In Chile the dynamic beta was 0.91 , while the static beta was 0.79 . In the case of Australia we had a static beta of 0.34 and a dynamic average beta of 0.50 . For other countries the values of static and dynamic average betas are very close, as in the US , where the static beta is 0.90 and the dynamic beta is 0.89 .

[Table 6 here]

It is noted that the CAPM hypothesis in which a should be statically zero and only the significant beta is valid for all countries. Besides the static beta through MQO, it was also estimated static betas through quantic regression as it follows the table 7. One of the advantages of quantic regression is its robustness to outliers. It was used quantic regression on the median for the estimation of static betas.

[Table 7]

The estimation of static betas in Japan through quantic regression was a bit different when compared to quantic regression estimated through MQO and medium dynamic beta. The value estimated with the quantilic was 0.91 while MQO estimation was 0.75 and the medium dynamic beta was 0.78. However, in the following section, it is verified the risk return relation in countries with several methods of estimation in order to verify if these methods offers a better adjustment to the model. Figure 2 shows the estimated variance for countries with multivariate GARCH model. It is noted greater variance for emerging countries such as South Africa and Brazil, where values come close to 10% in periods of turbulence in the market like the sub-prime crisis in 2008. We can also highlight Chile as one of the emerging countries which variance works similar to developed countries, with levels not reaching 2% per month, even in moments of financial crisis.

[Picture 2]

5.3 International Diversified Portfolio

From estimating betas and efficient borders [Markowitz (1952)], it is estimated international diversified portfolios. This helps investors to identify which countries are most profitable to invest. In countries like Brazil, during 2014 and 2015, where has been undergoing a serious internal crisis caused by errors in conducting economic policies, it is extremely important to keep active not only in Brazil, but also in other countries. For example, Bovespa stock offers fund ETF IVVB11, which pays the return of S&P US dollar that is accessible even to small investors.

Thus, portfolio 4 is constructed. Portfolio 1 is NAIVE and presents weights $1/N$. Portfolio 2 is the static portfolio of minimum variance according to [Markowitz (1952)] model. Portfolio 3 presents static weights with a better relation risk/ return, also based on [Markowitz (1952)]. Therefore, number 3 is the portfolio with higher risk, but also with higher return when compared to portfolio 2. Portfolio 4 is the portfolio with dynamic weights based on estimating betas in which the action with greater beta receives less weight, so we have a relation of inversion between beta and country's weight in the portfolio composition.

[Table 8 here]

In table 8 is portrayed the weights of each portfolio. We note that the [Markowitz (1952)] model does not recommend investments in European countries, only 10 in Chile among emerging markets. The highest weights of minimum portfolio are given to Australia, what was expected since the country has the lowest standard deviation, as it is shown in Table 1. For portfolio 3, with more risk on searching higher returns, Mexico is now included, and a greater weight is given to investments in Chile. Consequently, it is assigned a lower weight to Australia.

[Picture 3 here]

We note that most of the time, Australia has the greatest weight, since it has the lowest beta. The smallest weights are among countries like South Africa and Brazil, since they have higher beta. In some particular moments, it is noted the China gets weights close to Australia, what it was expected, since China had a defensive beta probably due to factors such as the strong growth despite of being slowing down

from 2015.

[Table 9 here]

Table 9 shows that portfolio 3 provides the best average/variance relationship, with a value of 1.09. Portfolio 2 despite having the lowest standard deviation, has excess of negative return, this is to say that, it had lower return than the American 30-years Treasury title. Portfolio 4 was the second best on ratio M/V, with a value of 0.39. It has been estimated VaR 1% of portfolios in September of 2015 through the GARCH (1,1) model . Portfolio 4 VaR is the highest, with a value of 16:29%; In other words, the investors of portfolio 4, in the worst scenario, have the probability of losing 16.29% in September. The portfolio of minimum variance is predicted to be less risky in September/2015.

5.4 Risk return relationship

The equation for the cross section relationship between average excess return and medium dynamic beta is:

$$E(R_i) = 0.001720 - 0.001460\beta_i$$

(0.60) (-0.57)

In parenthesis, we have t statistic value. So we have negative and not significant relationship, indicating that, in the countries surveyed, CAPM theory is not valid, since it is not found significant relationship between risk and return. We have the example of countries such as South Africa, which had the highest beta with excess of negative return. We also have the opposite example in China, which has the second lowest beta, but it had excess of positive return. Therefore, high betas do not necessarily indicate higher return for the investor. Figure 4 shows the relationship between excess returns and dynamic betas in some countries.

[Picture 4 here]

5.5 Correlation between countries

Facts like those that occurred between late 2015 and early 2016 such as the slow growth in China and the consequent fall in the stock market, have affected capital markets throughout the world. Thus there is a transmission among non-diversifiable risk between countries. In this sense, it seems important to study the correlation within international diversification. So diversification works better when the correlation between two countries is low. Table 10 shows the non-condition correlation between countries [Table 10 here]

Brazil, for example, has low correlation with Japan, China and Australia, but, it has higher correlation with Mexico and the US. In this case, Brazil tends to be influenced more geographically by close countries. However, Brazil also had higher correlation with European markets, with similar values close to 0.6, what disproves the hypothesis of high correlation in close markets. Another important fact is the high correlation between European markets, with values above 0.8. For example, the correlation between Germany and France is 0.88, 0.85 with Italy and 0.81 with Spain. When compared to other countries like Australia and China, German correlation value decreases. With Australia the value was 0.28 and with China was 0.32.

Chinese stock market showed an average correlation of 0.3 with most of the countries surveyed, except with Australia which correlation was 0.09. The result of positive and not very high correlation indicates that, when it occurs a crash in the Chinese market, other markets will be affected, but not so highly intense. It is important to mention that China's stock market had a 0.37 correlation with Brazil, the highest value found. This can be explained by the trade relationship between countries regarding to commodities. The Brazilian stock index Ibovespa, has large weight on Petrobras and Vale, companies that produce commodities. So negative news in China affects commodity prices and consequently the price of the Brazilian companies cited, causing a fall in the Brazilian stock index.

Figure 5 shows the correlation varying in time with the US. There is a decrease in the correlation between Brazil with the latter, from an average 0.7 to 0.6, indicating a lower integration of Brazil with international markets, what it is considered to be

as a result of internal problems facing the country between the years 2014 and 2015, resulting the break in the macroeconomic tripod from 2011. The implementation of the "new macroeconomic matrix" makes Brazil lose big investments in 2015, increasing the volatility and uncertainty in its stock market. In this sense, while the S&P500 index reaches historical highs, the Ibovespa index falls strongly during the years of 2014 and 2015.

[Picture 5 here]

Figure 6 shows the conditional correlation between the countries with Brazilian stock market. Except China, Japan, Australia and South Africa, Brazil has high correlation with both European countries and emerging countries like Mexico and Chile, as well as with the US and Canada. There is a small increase in the conditional correlation between Brazil and Mexico, from 0.65 to 0.75, indicating greater integration between the markets. Although Brazil is impacted by economic news from China, there is a low correlation between the Brazilian and Chinese stock markets. Besides, although the correlation changed from 0.2 in 2005 to around 0.40 in 2015, what it is considered a low number still.

6 Final Remarks

This research proposed to test CAPM on international level. To make that happen, it was estimating dynamic betas of countries that were here studied. The research concludes that, for period and sample surveyed, CAPM is not valid. Answering the question that reads the title, not always the country with the highest beta is the country that offers the highest excess return for the investor. But, despite of this, the research may indicate the power of international diversification to the investor. We simply compare the standard deviation of portfolio 3, 4.03% per month, which is lower than Australias risk, 4.10%, which was the country of lower risks. However Australia had a negative return, around -0.30, while portfolio 3 had a positive return of 0.17% per month. For future researches it is suggested to use a larger number of countries in the sample, as well as divisions into sub-samples in order to verify the validity of CAPM on contexts of international capital markets.

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A Tables

Table 1: PIB Mundial, economia desenvolvidas e mercados emergentes

Ano	Mundo PIB	Avanadas PIB	Emergentes e em desenvolvimento PIB	participao dos emergentes e em desenvolvimento
1991	23,274.13	18,929.68	4,344.46	18.67%
1992	24,355.72	20,360.04	3,995.69	16.41%
1993	24,999.08	20,544.55	4,454.53	17.82%
1994	26,825.49	22,058.42	4,767.07	17.77%
1995	29,824.97	24,366.97	5,458.01	18.30%
1996	30,546.65	24,497.40	6,049.25	19.80%
1997	30,416.67	24,045.68	6,370.99	20.95%
1998	30,201.59	24,079.23	6,122.35	20.27%
1999	31,376.55	25,371.21	6,005.34	19.14%
2000	32,331.33	25,772.46	6,558.87	20.29%
2001	32,129.92	25,494.20	6,635.72	20.65%
2002	33,403.25	26,626.40	6,776.85	20.29%
2003	37,527.89	29,870.42	7,657.47	20.40%
2004	42,228.60	33,093.58	9,135.02	21.63%
2005	45,678.64	34,763.00	10,915.64	23.90%
2006	49,451.67	36,539.71	12,911.96	26.11%
2007	55,827.29	39,944.85	15,882.43	28.45%
2008	61,363.58	42,135.40	19,228.18	31.33%
2009	57,983.31	39,736.08	18,247.24	31.47%
2010	63,467.76	41,523.42	21,944.34	34.58%
2011	70,220.55	44,539.93	25,680.62	36.57%
2012	71,707.30	44,417.08	27,290.22	38.06%
2013	73,982.13	45,338.11	28,644.02	38.71%

Fonte:FMI(2014)

Table 2: Pases e ndices

Country	Index	Ticker
Australia	S&P/ASX 300	AS52
Brazil	Ibovespa Brasil Sao Paulo Stock Exchange Index	IBOV
Canada	S&P/TSX Composite Index	SPTSX
Chile	Index performance for Santiago Stock Exchange	IPSA
China	Shanghai Stock Exchange Composite Index	SHCOMP
France	CAC 40 Index	CAC
Germany	Deutsche Borse AG German Stock Index DAX	DAX
Italy	FTSE MIB Index	FTSEMIB
Japan	Nikkei 225	NKY
Mexico	IPC	IPC
Spain	IBEX	IBX
South Africa	FTSE/JSE Africa All Share Index	JALSH
UK	FTSE 100 Index	UKX
USA	S&P 500 Index	SPX

*Source:*Bloomberg and G20

Table 3: Estatística descritiva para o excesso de retorno

	Ativo	Mdia	Desvio Padro	Min	Max
	MSCI	-0.025%	4.58%	-21.49%	10.01%
frica do Sul		-0.532%	11.06%	-64.77%	23.66%
	Brasil	0.209%	9.22%	-46.44%	20.32%
	Chile	0.368%	6.23%	-25.83%	17.53%
	China	0.288%	8.39%	-28.66%	24.39%
	Canad	0.137%	4.99%	-30.20%	16.35%
	Austrlia	-0.309%	4.10%	-12.87%	11.61%
	Japo	0.087%	5.26%	-21.38%	11.41%
	Alemanha	0.222%	6.90%	-29.41%	19.35%
	Frana	0.027%	6.41%	-22.46%	27.78%
Reino Unido		-0.243%	4.83%	-19.38%	10.96%
	Itlia	-0.441%	6.92%	-25.73%	19.63%
	EUA	-0.007%	4.29%	-18.93%	9.96%
	Mxico	0.465%	6.39%	-37.15%	19.60%
	Espanha	-0.057%	6.79%	-26.61%	16.78%

Table 4: Parmetros Estimados MGARCH(1)

ndice	$\omega \times 10^3$	α	β
msci	0.10	0.21	0.76
jsesaf	1.45	0.24	0.66
ibovbra	4.40	0.40	0.11
ipsachi	0.33	0.03	0.88
ssechina	0.40	0.21	0.75
tsxcan	0.68	0.41	0.32
asx200aus	0.23	0.07	0.80
nikkeijp	0.30	0.14	0.75
daxger	0.22	0.12	0.83
cacfr	1.34	0.25	0.45
ftseuk	0.15	0.14	0.80
ftsemibit	0.21	0.17	0.79
sp500	0.09	0.23	0.73
ipcmex	1.14	0.37	0.38
ibexspa	0.25	0.20	0.76
θ_1	0.0018		
θ_2	0.9486		

Table 5: Estatísticas Descritivas dos Betas dinâmicos

Pas	Mdia	Desvio Padro	Min	Max
frica do Sul	1.62	0.59	0.54	4.37
Brasil	1.63	0.50	0.56	3.31
Chile	0.91	0.24	0.37	1.35
China	0.61	0.34	-0.11	1.56
Canad	0.89	0.25	0.48	1.67
Austrlia	0.50	0.14	0.19	0.72
Japo	0.78	0.19	0.31	1.18
Alemanha	1.34	0.27	0.78	2.02
Frana	1.25	0.35	0.65	4.07
Reino Unido	0.97	0.14	0.71	1.46
Itlia	1.27	0.20	0.83	2.01
EUA	0.89	0.07	0.72	1.13
Mxico	1.13	0.33	0.57	2.23
Espanha	1.24	0.20	0.76	1.76

Table 6: Betas Estticos

Pas	alpha	beta	t alpha	t beta
frica do Sul	-0.0049	1.6358	-0.7723	9.1785
Brasil	0.0025	1.4856	0.5091	9.8791
Chile	0.0039	0.7976	0.9869	7.3114
China	0.0030	0.6391	0.4964	4.0743
Canad	0.0016	0.8848	0.7032	9.9588
Austrlia	-0.0030	0.3475	-1.0213	3.6955
Japo	0.0011	0.7588	0.3452	10.2611
Alemanha	0.0026	1.3156	0.9735	16.1970
Frana	0.0006	1.1744	0.2080	24.4589
Reino Unido	-0.0022	0.9424	-1.2910	20.0106
Itlia	-0.0041	1.2662	-1.3812	20.9874
EUA	0.0002	0.9079	0.1985	46.8701
Mxico	0.0049	1.1110	1.6352	10.1795
Espanha	-0.0003	1.1973	-0.0850	18.4785

Table 7: Betas Esttticos estimados com regresso quantlica

Pas	alpha	beta
frica do Sul	-0.0009	1.6127
Brasil	0.0050	1.5581
Chile	0.0032	0.8888
China	0.0023	0.5851
Canad	0.0039	0.8128
Austrlia	-0.0043	0.4055
Japo	0.0017	0.9125
Alemanha	0.0067	1.2776
Frana	-0.0004	1.1984
Reino Unido	-0.0005	0.9380
Itlia	-0.0050	1.2824
EUA	0.0002	0.8958
Mxico	0.0101	1.0133
Espanha	0.0018	1.2461

Table 8: Pesos da carteira

Country	1	2	3	4(mean)
South Africa	7.14%	0.00%	0.00%	4.19%
Brazil	7.14%	0.00%	0.00%	4.11%
Chile	7.14%	7.47%	25.32%	7.26%
China	7.14%	3.92%	10.61%	15.05%
Canada	7.14%	19.93%	0.00%	7.39%
Australia	7.14%	54.17%	23.44%	13.43%
Japan	7.14%	12.41%	16.60%	8.33%
Germany	7.14%	0.00%	0.00%	4.90%
France	7.14%	0.00%	0.00%	5.25%
UK	7.14%	0.00%	0.00%	6.55%
Italy	7.14%	0.00%	0.00%	5.11%
USA	7.14%	2.10%	0.00%	7.30%
Mexico	7.14%	0.00%	24.03%	5.92%
Spain	7.14%	0.00%	0.00%	5.21%

Table 9: Estatísticas Descritivas das carteiras

Carteira	Mdia	DP	Min	Max	M/V	VaR(Normal)	VaR(t)
1	0.015%	5.040%	-26.808%	12.078%	0.06	-13.80%	-16.74%
2	-0.091%	3.272%	-10.441%	6.631%	-0.85	-10.50%	-12.73%
3	0.177%	4.036%	-21.154%	8.199%	1.09	-12.11%	-14.68%
4	0.101%	5.080%	-21.985%	25.683%	0.39	-16.29%	-19.76%

Table 10: Unconditional Correlation between Countries

	Pas	AFS	BRA	Chl	Chi	Can	Aus	Jap	Ale	Fra	GB	Ita	EUA	Mex	Esp
frica do Sul	1.00	0.68	0.68	0.53	0.33	0.72	0.24	0.52	0.52	0.54	0.61	0.53	0.59	0.67	0.51
Brasil	0.68	1.00	0.69	0.69	0.37	0.81	0.14	0.49	0.64	0.60	0.68	0.61	0.66	0.72	0.62
Chile	0.53	0.69	1.00	1.00	0.34	0.62	0.11	0.37	0.57	0.62	0.62	0.52	0.52	0.58	0.50
China	0.33	0.37	0.34	1.00	1.00	0.38	0.09	0.27	0.32	0.29	0.33	0.31	0.31	0.29	0.30
Canad	0.72	0.81	0.62	0.38	1.00	1.00	0.13	0.58	0.70	0.65	0.79	0.69	0.74	0.77	0.66
Austria	0.24	0.14	0.11	0.09	0.13	1.00	1.00	0.22	0.28	0.34	0.32	0.23	0.43	0.22	0.20
Japo	0.52	0.49	0.37	0.27	0.58	0.22	0.22	1.00	0.58	0.57	0.59	0.56	0.58	0.56	0.56
Alemanha	0.52	0.64	0.57	0.32	0.70	0.28	0.28	0.58	1.00	0.88	0.83	0.85	0.85	0.69	0.81
Frana	0.54	0.60	0.62	0.29	0.65	0.34	0.34	0.57	0.88	1.00	0.85	0.87	0.79	0.61	0.81
Reino Unido	0.61	0.68	0.62	0.33	0.79	0.32	0.32	0.59	0.83	0.85	1.00	0.83	0.84	0.70	0.79
Italia	0.53	0.61	0.52	0.31	0.69	0.23	0.23	0.56	0.85	0.87	0.83	1.00	0.78	0.64	0.90
EUA	0.59	0.66	0.52	0.31	0.74	0.43	0.43	0.58	0.85	0.79	0.84	0.78	1.00	0.76	0.74
Mxico	0.67	0.72	0.58	0.29	0.77	0.22	0.22	0.56	0.69	0.61	0.70	0.64	0.76	1.00	0.64
Espanha	0.51	0.62	0.50	0.30	0.66	0.20	0.20	0.56	0.81	0.81	0.79	0.90	0.74	0.64	1.00

B Figures

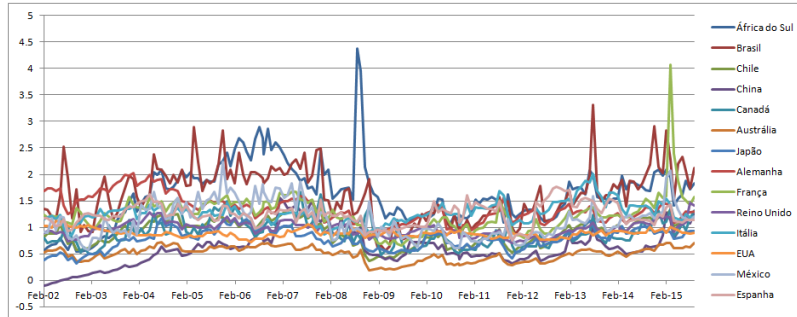


Figure 1: Betas Dinmicos dos Pases

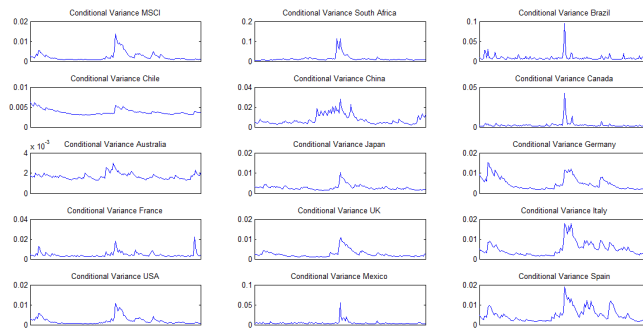


Figure 2: Varincia Condicional dos Pases e do MSCI

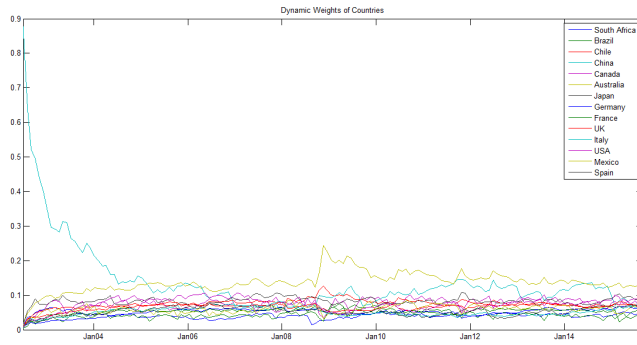


Figure 3: Pesos Portflio 4

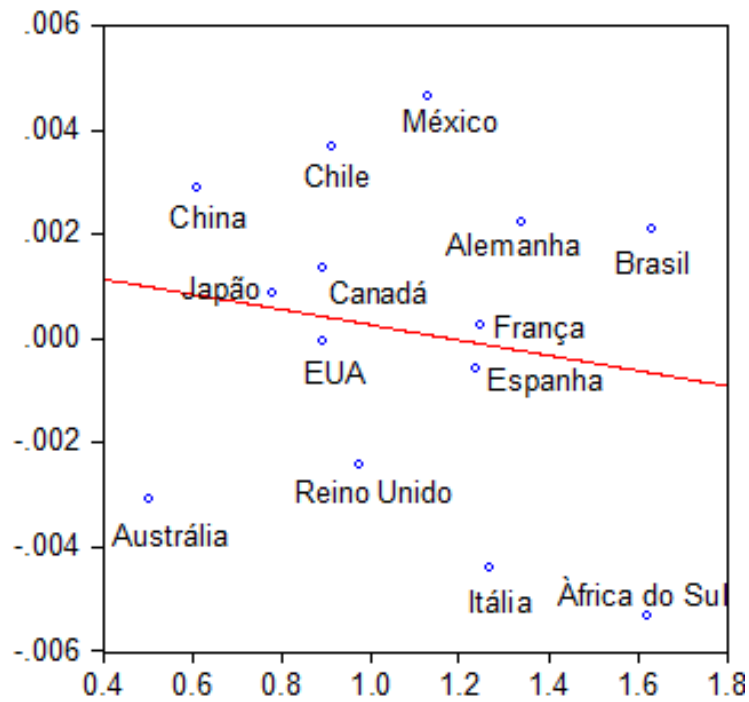


Figure 4: Relao Risco Retorno Betas Dinmicos mdio

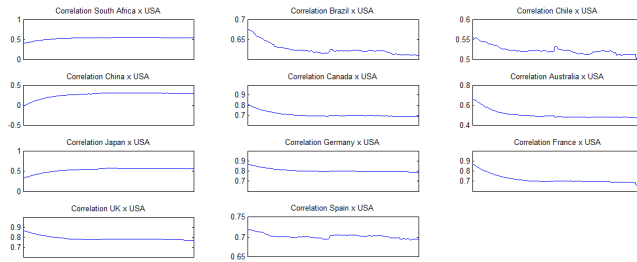


Figure 5: Correlao dos Pases com os Estados Unidos

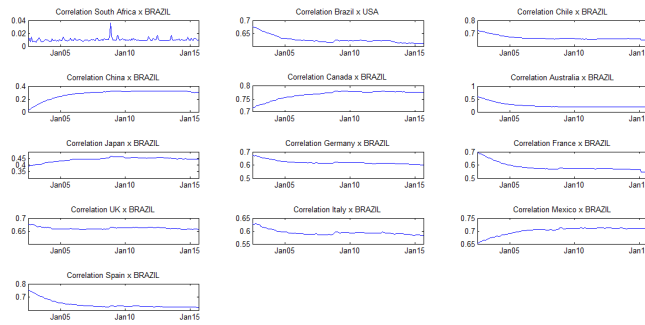


Figure 6: Correlao dos Pases com o Brasil