Hydromagnetic Flow of a Non-Newtonian Fluid in a Porous Medium Induced by a Moving Plate in Presence of Inclined Magnetic Field

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Abstract

In this study, the hydromagnetic flow of incompressible non-Newtonian fluid in a porous medium induced by a moving plate in the presence of inclined magnetic field was analyzed. The study is aimed to determine the temperature and velocity profiles of the fluid flow. Also, the effect of the permeability parameter, X, Reynolds number, Re, Prandtl number, Pr, Eckert number, Ec, and magnetic field parameter, M, on the flow variables. Moreover, the effect of varying the inclination angle on the flow variables is discussed.

Mathematics Subject Classification : xxxxx

Keywords: Hydromagnetic, Non-Newtonian Fluid, Porous Medium, Magnetic Field.

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1. Introduction

Flow behavior of non-Newtonian fluids are different from those of Newtonian fluids and when seen for the first time they appear abnormal or even paradoxical [4]. The flows of non-Newtonian fluids in a porous medium have gained a lot of importance in recent years. The flow in a porous medium has got more applications in engineering and science such as groundwater hydrology, reservoir engineering, petroleum engineering, agricultural irrigation and drainage,

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chemical reactors, and recovery of crude oil from the pores of reservoir rocks. Studies related to hydromagnetic flow of a non-Newtonian in a porous plate induced by a plate in presence of inclined magnetic field has been conducted by other scientist and mathematicians previously as follows below.

[8] studied Hydromagnetic turbulent flow of a rotating system past a semi-infinite vertical plate with Hall current. In this study it was found that Grashof number enhances both secondary and primary velocities profiles but has a small effect on the temperature profiles. Also it was revealed that the rotational parameter accelerates the secondary velocity profiles while the primary velocity is being diminished. And due to the presence of Hall current magnetic field is strongly affected. Similarly, [5] analyzed Unsteady Hydromagnetic convective flow past an infinite vertical porous flat plate in a porous medium. In this study it was concluded that a growing magnetic parameter or permeability parameter retards the transient velocity of the flow field at all points, the Grashof number for heat transfer enhances the transient velocity of the flow field at all points, the permeability parameter enhances the skin friction and the rate of heat transfer at the wall, the effect of increasing magnetic parameter is to decrease the skin friction and the rate of heat transfer at the wall in the flow field and he effect of growing Prandtl number is to decrease the transient temperature of the flow field at all points in the flow field while a growing permeability parameter reverses the effect. In the same vein, [3] analyzed Hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and soret effect. It was concluded that the rate of heat transfer decreases with increase in the heat generation parameter and the rate of mass transfer is less for the fluid with large viscosity. Subsequently, [11] discussed the solution of MHD flow past a vertical porous plate through a porous medium under oscillatory suction. It was revealed that the magnetic field parameter slows down the velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field. It was also found that the concentration distribution decreases at all points of the flow field with the increase of the Schmidt number. Thereafter, [6] discussed MHD flow in porous media over a stretching surface in a rotating system with heat and mass transfer. It was concluded that the rates of heat transfer and mass transfer on the stretching sheet embedded in a porous medium in a rotating system is influenced by the Magnetic field parameter, Soret number,

mass Grashof number, thermal Grashof number, Injection parameter, Rotational parameter, Permeability parameter, Dufour number, Reynolds number, Radiation parameter, the Eckert number, Schmidt number and time.

[9] discussed some exact solutions of non-Newtonian fluid in porous medium with hall effect having prescribed vorticity distribution function. In this study it was concluded that the exact solutions of the governing equations of incompressible second grade fluid in a porous medium with the Hall currents under the assumption that vorticity distribution is proportional to the stream function perturbed by a quadratic term. In the same vein, [2] studied MHD fluctuating flow of non-Newtonian fluid through a porous medium bounded by an infinite porous plate. It was concluded that the magnetic field parameter slows down the velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field, the Schmidt number has a delaying effect on velocity and concentration distribution of the flow field, the Grashof number for mass transfer accelerates the velocity of the flow field at all points and the effect of permeability parameter of the porous medium is enhanced the velocity of the flow field at all points. Similarly, [7] discussed Hydromagnetic flow and heat transfer over a porous oscillating stretching surface in a viscoelastic fluid with porous medium. It was revealed that an oscillatory sheet embedded in a fluid-saturated porous medium generates oscillatory motion in the fluid. It was observed that the behavior of temperature was monotonic with time rather than oscillatory. It was also revealed that temperature and thermal boundary layer thickness decreases with increasing viscoelastic parameter and mass suction/injection parameter. It was also revealed that the amplitude of velocity increases with increasing the viscoelastic parameter. Subsequently, [1] analyzed MHD flow of water-based brinkman type nanofluid over a vertical plate embedded in a porous medium with variable surface velocity, temperature and concentration. In this study it was observed that an increase of permeability parameter of the porous plate leads to a decrease of the velocity field. Thereafter, [10] discussed heat transfer on MHD rotating non-Newtonian fluid flow through parallel plate porous channel. In this paper it was revealed that the resultant velocity increases with increase of Ekman number or viscoelastic fluid parameter. It was observed that the fluid temperature increases with the increase of magnetic field parameter, darcy parameter, viscoelastic fluid parameter and Prandtl number. Furthermore, it was revealed

that increase of the Grashof number leads to the increase both primary and secondary velocity of the fluid flow. It was revealed that magnitudes of the rate of heat transfer were enhanced by Ekman number, darcy parameter, Grashof and Prandtl number.

To this end, this study considers hydromagnetic flow of a non-Newtonian in a porous plate induced by a plate in presence of inclined magnetic field. The temperature and velocity profiles of the fluid flow are determined. Also, the effect of angle of inclination of the magnetic field and non-dimensional numbers on the flow variables are discussed.

2. Mathematical Formulation

This study considers unsteady two-dimensional hydromagnetic flow of incompressible non-Newtonian fluid in a porous medium induced by a moving plate in the presence of inclined magnetic field. The porous medium is saturated with non-Newtonian fluid and the plate moves along $x-$axis with a constant velocity u_0. The plate induces the flow of the fluid in the porous medium. A uniform magnetic field B_0 is applied in a direction of the flow which is inclined at an angle α to the plate. The regime is illustrated in Figure 1 below.
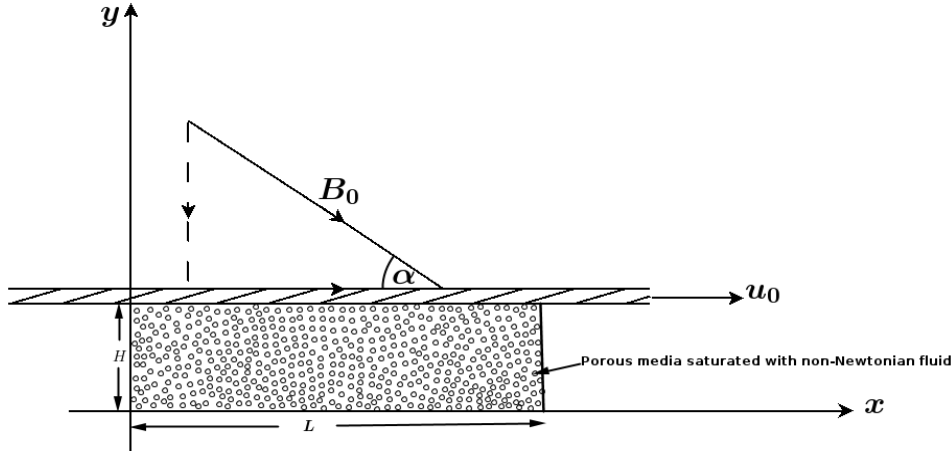

Figure 1: Flow Configuration

The following assumption are considered in order to model the problem.

1. The fluid is incompressible and the flow is unsteady.

2. The plate is moving with a constant velocity u_0 in the presence of inclined magnetic field B_0 at angle α to the direction of the flow.

3. The measurements of the plate and the medium are of comparable lengths.

4. The force due to electric field is negligible compared to the Lorentz force.

5. The moving plate is a non-permeable magnetic material i.e. it allows magnetic lines of force to pass through.

6. The porous medium is isotropic and non-magnetic.

Based on the above assumptions, the governing equations are as follows:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equations:

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \mu\frac{u}{K_p} + \sigma B_0{}^2 \sin\alpha(u\sin\alpha + v\cos\alpha) \tag{2}$$

Equation (2) is the momentum equation in the $x-$axis.

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right) = \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \mu\frac{v}{K_p} + \sigma B_0{}^2 \cos\alpha(u\sin\alpha + v\cos\alpha) \tag{3}$$

Equation (3) is the momentum equation in the $y-$axis.

Energy Equation:

$$\rho C_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left[4\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right] \tag{4}$$

The viscosity μ of the non-Newtonian fluid in this case is considered as a function of temperature i.e.

$$\mu = \mu_0 \left[1 + \gamma^* \left(T - T_{\mathrm{p}} \right) \right] \tag{5}$$

where μ_0 is the ambient fluid's viscosity, γ^* is a constant expressed as $\gamma^* = \frac{1}{\mu}\left(\frac{\partial \mu}{\partial T}\right) T_f$ with T_f being the fluid's film temperature.

Initial and boundary conditions of this problem are as follows:

$$t = 0 : \begin{cases} u = 0, v = 0, T = T_{\mathrm{w}} & \text{at} \quad y = H, \quad 0 \le x \le L \\ u = 0, v = 0, T = T_{\mathrm{w}} & \text{at} \quad y = 0, \quad 0 \le x \le L \\ u = 0, v = 0, T = T_{\mathrm{w}} & \text{at} \quad x = L, \quad 0 \le y \le H \\ u = 0, v = 0, T = T_{\mathrm{w}} & \text{at} \quad x = 0, \quad 0 \le y \le H \end{cases} \tag{6}$$

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$$t > 0 : \begin{cases} u = u_0, v = 0, T = T_p & \text{at} \quad y = H, \quad 0 \le x \le L \\ u = 0, v = 0, T = T_w & \text{at} \quad y = 0, \quad 0 \le x \le L \\ u = 0, v = 0, T = T_w & \text{at} \quad x = L, \quad 0 \le y \le H \\ u = 0, v = 0, T = T_w & \text{at} \quad x = 0, \quad 0 \le y \le H \end{cases} \quad (7)$$

Temperature of the porous media and that of the walls are assumed to be equal. It is also assumed that temperature of the wall is greater than the that of the plate i.e. $T_w > T_p$.

In this study, the following non-dimensional quantities have been used to transform the governing equations.

$$t^* = \frac{u_0 t}{L}, \quad x^* = \frac{x}{L}, \quad y^* = \frac{y}{H}, \quad u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{u_0}, \quad T^* = \frac{T - T_p}{T_w - T_p}$$

The viscosity variation parameter is expressed as:

$$\gamma = \frac{1}{\mu} \left(\frac{\partial \mu}{\partial T} \right) T_f (T_w - T_p) \quad (8)$$

Hence, (5) becomes:

$$\mu = \mu_0 \left[1 + \gamma \left(\frac{T - T_p}{T_w - T_p} \right) \right] = \mu_0 (1 + \gamma T^*) \quad (9)$$

Using these non-dimensional quantities, equation (2), (3) and (4) in dimensionless form become:

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + \frac{v^*}{A_r} \frac{\partial u^*}{\partial y^*} = \frac{(1 + \gamma T^*)}{\text{Re}} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{1}{A_r^2} \frac{\partial^2 u^*}{\partial y^{*2}} \right) + \frac{\text{Re}}{X} (1 + \gamma T^*) u^*$$
$$+ \text{MRe} \sin \alpha (u^* \sin \alpha + v^* \cos \alpha)$$
$$(10)$$

Equation (10) is the momentum equation in $x-$direction in dimensionless form.

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + \frac{v^*}{A_r} \frac{\partial v^*}{\partial y^*} = \frac{(1 + \gamma T^*)}{\text{Re}} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{1}{A_r^2} \frac{\partial^2 v^*}{\partial y^{*2}} \right) + \frac{\text{Re}}{X} (1 + \gamma T^*) v^*$$
$$+ \text{MRe} \cos \alpha (u^* \sin \alpha + v^* \cos \alpha)$$
$$(11)$$

Equation (11) is the momentum equation in $y-$direction in dimensionless form.

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + \frac{v^*}{A_r} \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{RePr}} \left[\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{1}{A_r^2} \frac{\partial^2 T^*}{\partial y^{*2}} \right] + \frac{(1 + \gamma T^*) \text{Ec}}{\text{Re}} \left[4 \left(\frac{1}{A_r} \frac{\partial v^*}{\partial y^*} \right)^2 \right.$$
$$\left. + \left(\frac{1}{A_r} \frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*} \right)^2 \right]$$
$$(12)$$

Equation (12) is the energy equation in dimensionless form. Equation (6) and (7) in dimensionless form become:

$$
t^* = 0 : \begin{cases}
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad y^* = 1, \quad 0 \le x^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad y^* = 0, \quad 0 \le x^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad x^* = 1, \quad 0 \le y^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad x^* = 0, \quad 0 \le y^* \le 1
\end{cases} \tag{13}
$$

$$
t^* > 0 : \begin{cases}
u^* = 1, v^* = 0, T^* = 0 & \text{at} \quad y^* = 1, \quad 0 \le x^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad y^* = 0, \quad 0 \le x^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad x^* = 1, \quad 0 \le y^* \le 1 \\
u^* = 0, v^* = 0, T^* = 1 & \text{at} \quad x^* = 0, \quad 0 \le y^* \le 1
\end{cases} \tag{14}
$$

Equation (13) and (14) are the initial and boundary conditions respectively in dimensionless form.

3. Numerical Simulations and Discussions

In this study, the Crank-Nicolson method was used to obtain the numerical scheme for solving the governing equations. The knowledge of central difference for solving PDEs was used to determine the corresponding finite difference equation (FDE) of each governing equation. The obtained FDEs were implemented in a computer program which was executed at different values of non-dimensional numbers and the angle of inclination of the magnetic field, α, to determine the velocity and temperature profiles of the fluid flow. The effect of the angle of inclination of the magnetic field, α, and non-dimensional numbers like Reynold's number, Re, Magnetic parameter, M, Permeability parameter, X, Eckert number, Ec, and Prandtl number, Pr on the determined velocity and temperature profiles are also discussed in this section as follows.

From Figure 2 and 3, it is observed that an increase in the angle of inclination of the magnetic field (α) leads to an increase in the magnitude of both primary and secondary velocity profiles. The magnetic field is inclined at an angle, α, to the direction of the flow. Therefore, an increase in magnitude of the angle of inclination of the magnetic field, α, (which is measured in a clockwise direction from the plate) leads to an increase in both primary and secondary velocity profiles of the fluid flow.
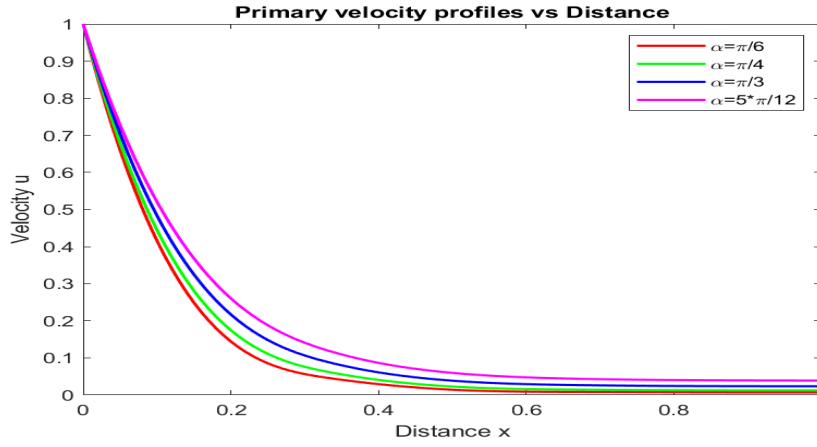
Figure 2: Effect of angle of inclination on primary velocity.
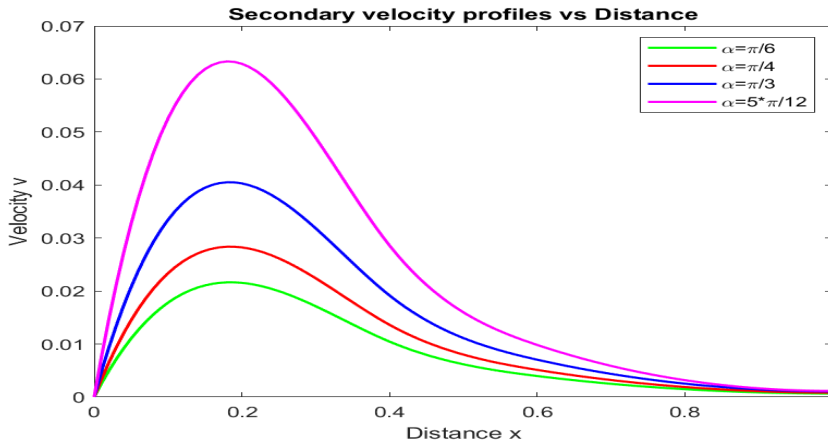

Figure 3: Effect of angle of inclination on secondary velocity.

From Figure 4 and 5, it is observed that increase in Reynold's number (Re) leads to the increase in the magnitude of both primary and secondary velocity profiles of the fluid flow. Reynold's number is the ratio of inertia force to the viscous force. Increasing Reynold's number leads to the decrease in the viscous force which is the force that opposes the motion of the fluid flow. Hence, since he force that opposes the motion of the fluid flow decreases as the Reynold's number increases, then the velocity profiles of both primary and secondary velocity increase in magnitude as the Reynold's number increases.
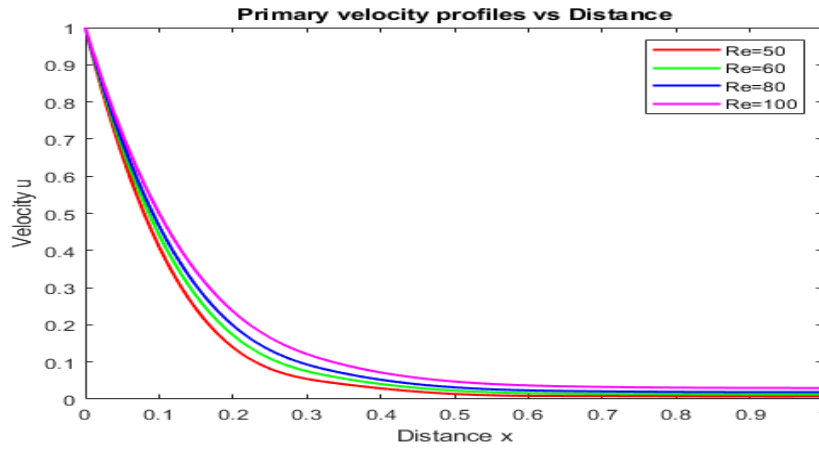
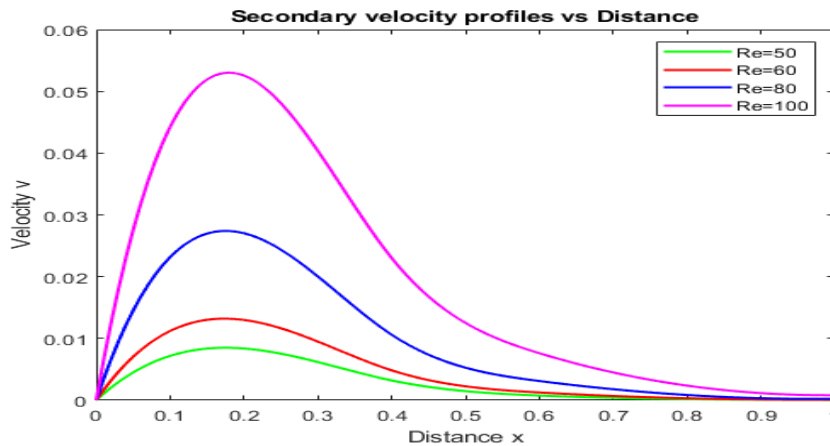Figure 4: Effect of Reynold's number on primary velocity.

Figure 5: Effect of Reynold's number on secondary velocity.

From Figure 6 and 7, it is observed that an increase in Magnetic field parameter (M) leads to an increase in both primary and secondary velocity profiles. If magnetic fields are introduced to an electricity conducting flowing fluid, then the Lorentz force will be introduced to the flow. In this study, magnetic fields are applied in the same direction of the fluid flow and inclined at angle, α. This means that the applied magnetic fields introduce the Lorentz force to the flow. Thus, increasing Magnetic field parameter leads to an increase of the force due to electromagnet which is the Lorentz force in this study. Hence, since the Lorentz force is applied in the same direction with the fluid flow, then an increase in Magnetic field parameter leads to an increase in the magnitude of both primary and secondary velocity profiles.
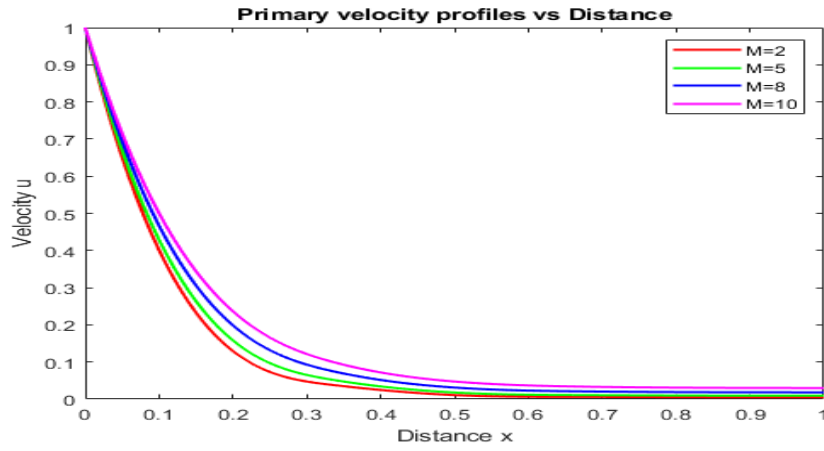
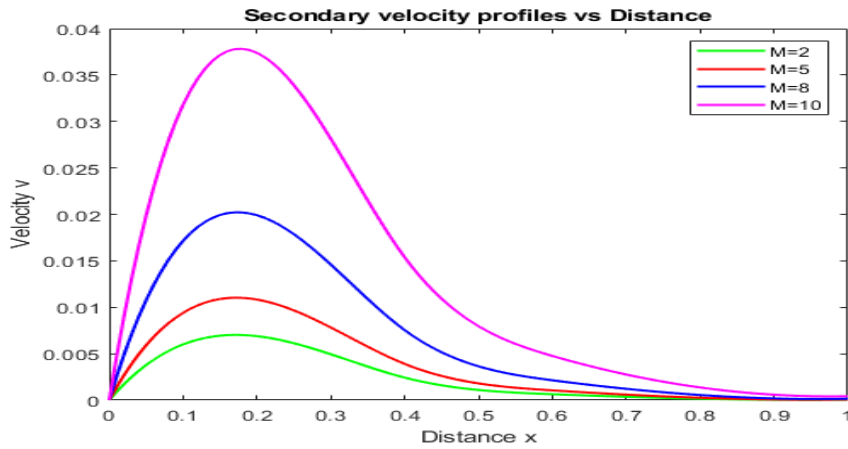Figure 6: Effect of Magnetic parameter on primary velocity.

Figure 7: Effect of Magnetic parameter on secondary velocity.

From Figure 8 and 9, it is observed that an increase in permeability parameter (X) leads to the decrease in both primary and secondary velocity profiles of the fluid flow. Increase in permeability parameter leads to an increase in the porosity of the porous medium and hence reduces the acceleration of the fluid flow in the porous medium. Decreasing the acceleration of the fluid flow reduces the particles movement as the result it leads to the decrease in the magnitude of both primary and secondary velocity profiles of the fluid flow.
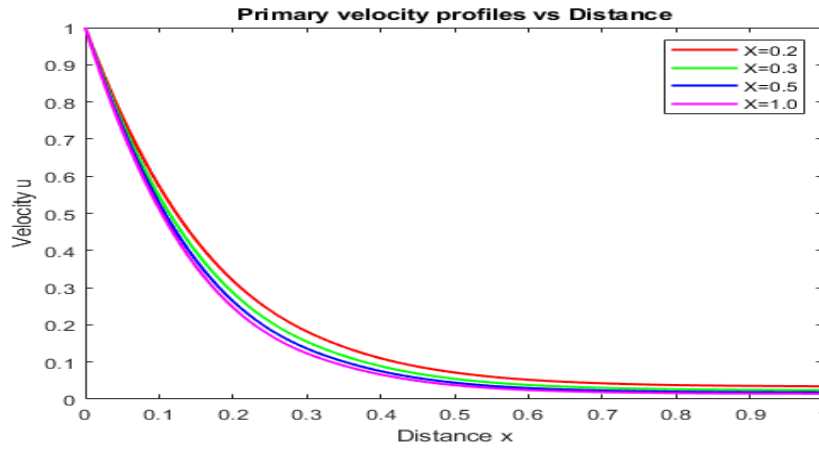
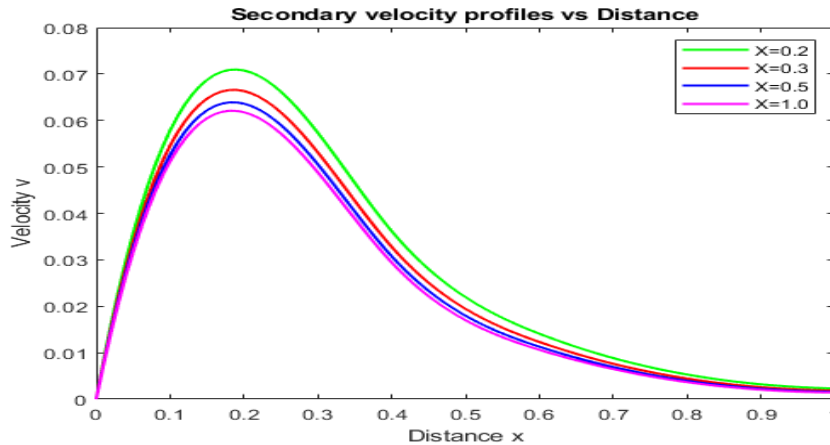Figure 8: Effect of Permeability parameter on primary velocity.

Figure 9: Effect of Permeability parameter on secondary velocity.

From Figure 10, it is observed that an increase in Reynold's number leads to the increase in temperature profiles. Reynold's number is the ratio of inertia force to the viscous force. Increasing Reynold's number leads to the decrease in the viscous force which is the force that opposes the motion of the fluid flow. Since the viscous force is decreased then the velocity of the fluid increases which leads to an increase in the collision of the fluid particles. This leads to the dissipation of heat in the boundary layers in the porous medium and hence an increase in the temperature profiles of the fluid flow.
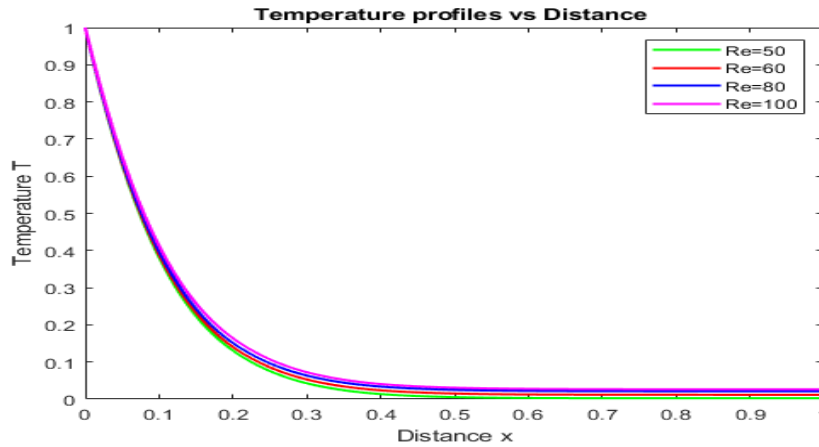
Figure 10: Effect of Reynold's number on temperature.

From Figure 11, it is observed that an increase in Eckert number (Ec) leads to an increase in temperature profiles. Eckert number is the ratio of the kinetic energy to the fluid enthalpy, thus, increasing Eckert number implies that there is high kinetic energy. The kinetic energy is always high when the velocity of the fluid flow is high. When the velocity of the fluid is high, the collision of the fluid particles increases which leads to an increase of the self-heating effects due to dissipation of heat in the boundary layers in the porous medium. Since the self-heating effects due to dissipation is high, it implies that there is an increase in the magnitude of temperature. Hence, Increasing Eckert number leads to an increase of temperature profiles of the fluid flow.
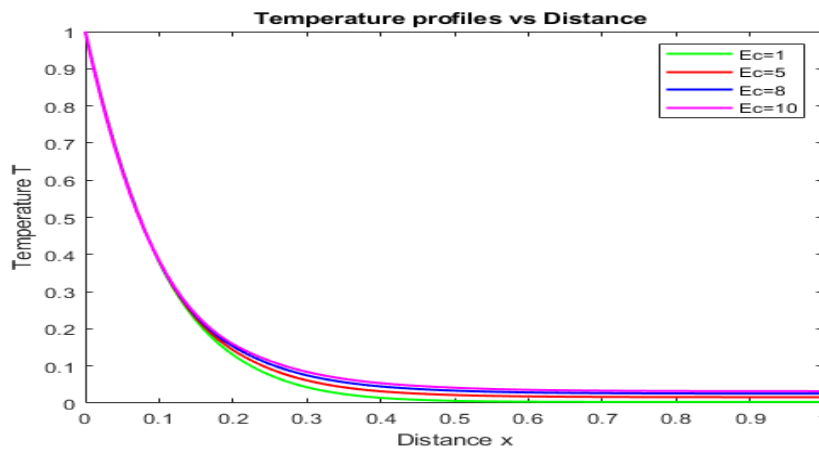

Figure 11: Effect of Eckert number on temperature.

From Figure 12, it is observed that an increase in Prandtl number (Pr) leads to

an increase in temperature profiles. Prandtl number is the ratio of the viscous diffusion rate to the thermal diffusion rate expressed as the ratio of kinematic viscosity to the thermal diffusivity. Increasing Prandtl number implies that, there is low thermal diffusivity of the fluid as the result the fluid expands and the molecules moves apart hence increases the temperature of the fluid. Thus, an increase of the Prandtl number leads to an increase of the magnitude of the temperature profiles of the fluid increase.
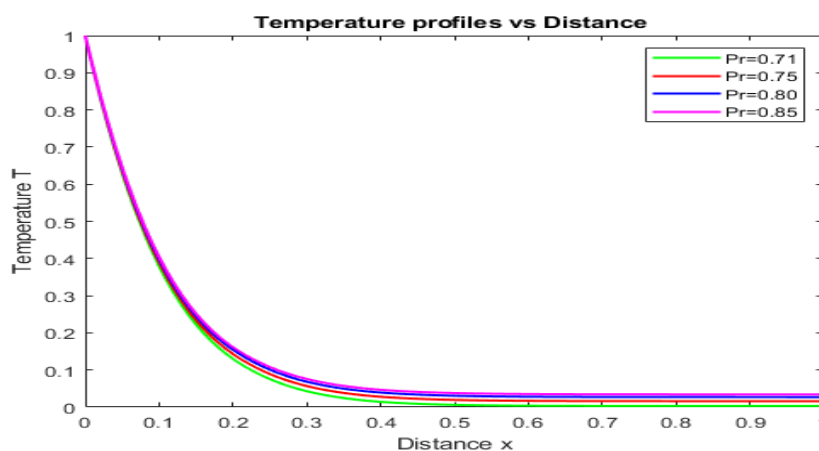

Figure 12: Effect of Prandtl number on temperature.

4. Conclusion

In this section, a summary of the effects of non-dimensional numbers and the angle of inclination of the magnetic field on velocity and temperature profiles of the fluid flow is presented.

Increasing the inclination angle, α, leads to an increase in the magnitude of both primary and secondary velocity profiles of the fluid flow. This implies that the velocity varies directly proportional with the inclination angle, α.

Increasing Reynold's number, Re, Magnetic parameter, M, leads to an increase in both primary and secondary velocity profiles of the fluid flow. This shows that the velocity varies directly proportional with Reynold's number, Re, Magnetic parameter, M. Increase in Permeability parameter, X, leads to the decrease in both primary and secondary velocity profiles of the fluid flow. This implies that the velocity varies inversely proportional with the Permeability parameter, X.

Increasing Reynold's number, Re, Eckert number, Ec, and Prandtl number, Pr leads to am increase in magnitude of the temperature profiles of the fluid flow.

This means that, temperature of the fluid flow varies directly proportional with the Reynold's number, Re, Eckert number, Ec, and Prandtl number, Pr.

Acknowledgements

References

[1] F. Ali, M. Gohar, and I. Khan. Mhd flow of water-based brinkman type nanofluid over a vertical plate embedded in a porous medium with variable surface velocity, temperature and concentration. *Journal of Molecular Liquids*, 223:412–419, 2016.

[2] D. Bose and U. Basu. Mhd fluctuating flow of non-newtonian fluid through a porous medium bounded by an infinite porous plate. *Applied Mathematics*, 6(12):1988, 2015.

[3] K. Chand, R. Kumar, and S. Sharma. Hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and soret effect. *Advances in Applied Science Research*, 3(4): 2169–2178, 2012.

[4] P. P. Dabrowski. *Boundary-layer flows in non-Newtonian fluids.* PhD thesis, 2009.

[5] S. Das, M. Maity, and J. Das. Unsteady hydromagnetic convective flow past an infinite vertical porous flat plate in a porous medium. *Int J Energy Environ*, 3:109–118, 2012.

[6] K. Giterere. *Magnetohydrodynamic Flow In Porous Media Over A Stretching Surface In A Rotating System With Heat And Mass Transfer.* PhD thesis, 2013.

[7] S. U. Khan, N. Ali, and Z. Abbas. Hydromagnetic flow and heat transfer over a porous oscillating stretching surface in a viscoelastic fluid with porous medium. *PloS one*, 10(12):e0144299, 2015.

[8] M. Kinyanjui, E. M. Marigi, and J. K. Kwanza. Hydromagnetic turbulent flow of a rotating system past a semi-infinite vertical plate with hall

current. *International Journal of pure and Applied mathematics*, 2(8), 2012.

[9] M. Kumar. Some exact solutions of non-newtonian fluid in porous medium with hall effect having prescribed vorticity distribution function. *Global Journal of Science Frontier Research: A Physics and Space Science*, 14 (5), 2014.

[10] B. Lakshmanna and S. Venkateswarlu. Heat transfer on mhd rotating non-newtonian fluid flow through parallel plate porous channel. *International Journal of Applied Engineering Research*, 13(10):8200–8204, 2018.

[11] A. A. Moniem and W. Hassanin. Solution of mhd flow past a vertical porous plate through a porous medium under oscillatory suction. *Applied Mathematics*, 4(04):694, 2013.

Nomenclature

Symbol	Meaning
B_0	Magnetic field strength, [wbm^{-2}]
u, v	Components of velocity vector [ms^{-1}]
H	Height of the porous media, [m]
L	Length of the porous media [m]
ϕ	Viscous dissipation function, [s^{-1}]
σ	Electrical conductivity, [Ω^{-1}m^{-1}]
ρ	Fluid density, [Kgm^{-3}]
k	Thermal conductivity of the fluid, [WK^{-1}m^{-1}]
M	Magnetic parameter
K_p	Darcy permeability, [m^2]
Pr	Prandtl number.
Re	Reynolds number.
Ec	Eckert number
Ar	Aspect Ratio
X	Permeability parameter
T	Absolute free temperature of the fluid, [K]
T$_w$	Temperature of the wall, [K]
T$_p$	Temperature of the plate, [K]
C_p	Specific heat at a constant temperature, [JKg^{-1}K^{-1}]