

FIXED AND DYNAMIC ASSET ALLOCATION IN THE ACCUMULATION PHASE

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ABSTRACT. In this paper, we consider the historical real returns of fixed and dynamic allocation portfolios consisting of equities and short term bonds over thirty year time horizons, where fixed real contributions are made to the portfolios annually. In particular, we consider both the scenario where the investor annually rebalances a portfolio to a fixed ratio as well as the scenario where the investor's annual contribution has a fixed ratio but the portfolio is never subsequently rebalanced. These results provide investors in the accumulation phase historical data that may provide a useful guide to asset allocation decisions. Of particular interest is that, over the 88 thirty-year time intervals considered, dynamic allocation portfolios had a better overall performance than fixed allocation portfolios, and that both fixed and dynamic allocation portfolios strongly benefited from a heavy equity exposure.

One of the most important decisions an investor faces is in regard to asset allocation, and, in particular, the portions of investment allocated to equities and bonds. Of course, the asset allocation chosen by a given investor depends not only on tolerance for risk but also the stage in the life cycle of investing. For investors in the decumulation phase, we are particularly influenced by the “Trinity Study” [4] that provides particularly useful historical insight into the risks associated to various equity/bond allocations over thirty year time horizons. This study leads many to the conclusion that, for the typical investor in retirement, an equity/bond allocation of 60% / 40% enables the annual withdrawal of 4% of the portfolio value at retirement with a very small chance of the portfolio being depleted within thirty years. A subsequent study by Estrada [6] reinforces the conclusions of the Trinity study and moreover provides data supporting the 90/10 allocation advocated by Warren Buffett [3].

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The purpose of this paper is to consider an analogue of the Trinity Study that focuses on the needs of investors in the *accumulation phase*. In particular, we consider the historical real returns associated to a hypothetical investor who contributed on the first business day of every year, for thirty consecutive years, \$1 to a tax-free/tax-deferred account where transaction costs were negligible. We presume that the portfolio consisted entirely of short-term bonds as well as holdings in a broad-based equity index fund modeling the S&P 500. We consider 88 thirty-year periods, beginning with the period from 1900 to 1929 and ending with the period from 1987 to 2016.

We treat two basic strategies the investor may viably consider. The first is that the investor will decide upon a fixed equity/bond percentage allocation and rebalance to that allocation annually. The second is that the investor will decide upon a fixed equity/bond percentage allocation of the annual *contribution* but never subsequently rebalance the portfolio. These options are described by John Bogle in [2] :

Once you have determined a strategic long-term asset allocation, you must decide whether this balance will be relatively fixed or dynamic. There are two principal options. You can (1) keep your strategic ratio fixed, periodically buying and selling stocks and bonds to restore your portfolio to its original allocation, or (2) set an initial allocation and then let your investment profits ride. In the latter case, your initial allocation will gradually evolve to reflect the relative performance of stocks and bonds.

For the stock allocation we assume the investments were in an S&P 500 index fund, and we use the data provided by Robert Shiller [10] in this regard. For the bond allocation we choose to consider short-term United States government securities or their equivalent counterparts. Of course, this poses a challenge as the short term bond market in the United States in the early 1900's functioned very differently than the one of the present day. For the short-term bond series considered in this paper, we use premium commercial paper for the years 1900-1931 where the data is taken from Homer and Sylla [8], followed by three month Treasuries from 1931-1946 where the data is taken from Homer and Sylla, followed by one year Treasuries from 1946-1990 where the data is taken from Homer and Sylla, followed by one year Treasuries from 1990-2016 where the data is provided by Shiller. We remark that an alternate bond series may be obtained by Dimson, Marsh, and Staunton

(see [5] for more information in this regard); the reader should be advised that in [6], Estrada uses the DMS data set. To adjust returns for inflation we use the CPI For All Urban Consumers as provided by the United States Bureau of Labor Statistics.

At the outset of this project, we were particularly curious on two fronts. One, in the Trinity Study, the “bad outcomes” for investors with a high allocation to equities occurred in scenarios where a bear market occurred soon after retirement. We recognized, however, that the bad outcomes for the Trinity Study should correspond to *good* outcomes for an investor in the accumulation phase; in particular that investors with a heavy equity allocation should receive a considerable tailwind if they encounter a bear market early in their accumulation phase. This notion is memorably put by William Bernstein in his *The Four Pillars of Investing* [1]:

A young person saving for retirement should get down on his knees and pray for a market crash, so that he can purchase his nest egg at fire sale prices.

A second front of considerable interest was in regard to the advantages of annually *rebalancing* a portfolio to achieve a fixed equity / bond percentage allocation throughout the thirty year period. Many financial advisors advocate such a rebalancing as it provides a mechanism for investors to sell equities at inflated prices and to shift bonds into equities after a bear market has taken place. In [9], Burton Malkiel states that

We all wish that we had a little genie who could reliably tell us to “buy low and sell high.” Systematic rebalancing is the closest analogue we have.

John Bogle resonates with this view in [2]:

The advantage of a fixed-ratio strategy is that you automatically lock in your gains and reduce your equity exposure as equity prices increase. Correspondingly, you would increase your equity holdings (with the proceeds of bond sales, or by redirecting new investments) as stocks decline in value, which reduces your equity exposure; this would keep your original balance between risk and reward relatively constant.

Many investors find greater peace of mind with a stable balance of stocks and bonds - a strategy that is counterintuitive but may prove productive - than with taking no action and allowing risk exposure to rise in tandem with the stock market - a strategy that is intuitive but may prove counterproductive.

The outcomes of our analysis are informative on both counts. Indeed, investors with a heavy equity allocation benefitted considerably from encountering a bear market early in their accumulation phase. As a surprise to us, however, we find that rebalancing seems to, over a long time horizon, have had a generally negative effect on a portfolios considered. This was manifested in better overall returns for fixed allocation portfolios with a high equity exposure as well as better overall returns for dynamic portfolios that had a substantial allocation to equities.

The following tables present our findings in detail. Tables 1 and 2 provide the final values of portfolios that were annually rebalanced to satisfy the given equity/bond allocation. (Table 1 covers start dates from 1900 through 1943), and Table 2 covers start dates from 1944 through 1987.) As an example, from Table 1 we find that an investor who contributed \$1 in real terms on January 1 in the years 1933 though 1962, rebalancing the portfolio on January 1 each year so that 80% of the assets would lie in stocks and 20% would lie in bonds, would have a portfolio value of \$111.28 on December 31 of 1962. Tables 3 and 4 provide the final values where the annual contributions satisfied the indicated equity/bond allocation but no rebalancing took place. (We refer to these as *dynamic* portfolios.) Table 5 provides summary statistical information regarding the rebalanced portfolios in Tables 1 and 2, and Table 6 provides summary statistical information regarding the dynamic portfolios featured in Tables 3 and 4. Table 7 provides summary data describing the relative performance of the rebalanced and dynamic portfolios, in particular indicating statistical information regarding the ratio of the terminal value of a rebalanced portfolio and its dynamic counterpart over the 88 historical periods and over varying asset allocations. For each portfolio allocation, each summary table provides the mean, median, standard deviation, maximum, 90th percentile, 75th percentile, 25th percentile, 10th percentile, and minimum associated to the given 88 data points. We took care to provide the data associated to the 50/50 allocation as it is of particular interest to many investors and investment advisors.

TABLE 1. Performance of Annually Rebalanced Portfolios (Start Date 1900–1943)

start	end	100/0	80/20	60/40	50/50	40/60	20/80	0/100
1900	1929	113.97	97.91	82.55	75.27	68.31	55.47	44.23
1901	1930	91.50	83.58	74.76	70.17	65.52	56.28	47.43
1902	1931	55.83	58.94	59.89	59.60	58.84	56.10	52.06
1903	1932	55.05	59.07	61.10	61.37	61.18	59.52	56.42
1904	1933	79.60	79.51	76.22	73.56	70.36	62.68	53.96
1905	1934	68.32	69.45	67.84	66.12	63.88	58.12	51.17
1906	1935	100.43	94.81	85.62	80.09	74.16	61.69	49.30
1907	1936	125.56	112.64	96.55	87.95	79.28	62.44	47.20
1908	1937	81.16	79.85	74.63	70.85	66.49	56.62	46.15
1909	1938	91.39	87.57	79.76	74.77	69.30	57.60	45.90
1910	1939	91.53	87.12	78.92	73.82	68.31	56.67	45.18
1911	1940	79.46	77.10	71.27	67.39	63.04	53.52	43.75
1912	1941	63.10	62.42	58.90	56.29	53.24	46.28	38.81
1913	1942	67.66	64.80	59.25	55.76	51.95	43.83	35.73
1914	1943	77.81	71.89	63.43	58.64	53.68	43.75	34.48
1915	1944	86.60	77.74	66.70	60.83	54.95	43.63	33.56
1916	1945	112.94	95.93	77.77	68.89	60.42	45.20	32.71
1917	1946	81.15	70.96	59.31	53.40	47.62	36.91	27.78
1918	1947	70.33	61.72	51.91	46.93	42.05	32.99	25.23
1919	1948	70.90	61.57	51.38	46.32	41.43	32.46	24.88
1920	1949	78.13	66.29	54.21	48.43	42.96	33.19	25.19
1921	1950	87.36	71.29	56.12	49.20	42.83	31.87	23.30
1922	1951	91.69	73.34	56.73	49.34	42.64	31.32	22.66
1923	1952	96.18	75.87	58.08	50.32	43.34	31.73	22.97
1924	1953	89.76	71.36	55.28	48.24	41.91	31.28	23.18
1925	1954	122.06	91.63	67.05	56.87	48.00	33.79	23.58
1926	1955	145.70	105.25	74.32	61.99	51.49	35.20	23.94
1927	1956	142.01	102.09	72.09	60.24	50.20	34.69	23.97
1928	1957	123.29	90.43	65.41	55.41	46.87	33.49	24.07
1929	1958	164.72	114.12	77.98	64.24	52.85	35.77	24.39
1930	1959	168.12	115.77	78.94	65.07	53.64	36.60	25.32
1931	1960	168.17	115.36	78.71	65.01	53.76	37.07	26.02
1932	1961	181.93	123.32	83.28	68.48	56.40	38.59	26.92
1933	1962	159.16	111.28	77.62	64.88	54.33	38.46	27.74
1934	1963	177.49	121.79	83.48	69.22	57.52	40.12	28.54
1935	1964	188.44	128.43	87.58	72.46	60.09	41.78	29.65
1936	1965	194.43	131.88	89.71	74.20	61.55	42.89	30.58
1937	1966	169.56	118.28	82.92	69.69	58.77	42.40	31.35
1938	1967	177.12	122.74	85.56	71.73	60.37	43.39	31.98
1939	1968	176.25	122.37	85.62	71.95	60.72	43.93	32.61
1940	1969	142.24	103.38	75.75	65.15	56.28	42.65	33.14
1941	1970	134.38	99.14	73.80	64.00	55.74	42.93	33.89
1942	1971	133.11	99.04	74.29	64.64	56.46	43.68	34.56
1943	1972	136.61	101.69	76.24	66.29	57.83	44.58	35.06

TABLE 2. Performance of Annually Rebalanced Portfolios (Start Date 1944–1987)

start	end	100/0	80/20	60/40	50/50	40/60	20/80	0/100
1944	1973	95.84	76.77	61.78	55.58	50.15	41.23	34.45
1945	1974	62.77	54.88	47.99	44.93	42.14	37.31	33.41
1946	1975	76.59	64.63	54.47	50.04	46.02	39.10	33.48
1947	1976	73.70	62.78	53.42	49.29	45.53	38.97	33.58
1948	1977	55.89	50.45	45.38	43.02	40.79	36.71	33.13
1949	1978	52.70	48.21	43.87	41.79	39.79	36.04	32.65
1950	1979	48.81	45.22	41.61	39.83	38.08	34.73	31.59
1951	1980	49.86	46.09	42.25	40.35	38.48	34.86	31.45
1952	1981	39.58	38.87	37.77	37.10	36.37	34.76	32.99
1953	1982	45.40	44.07	42.29	41.27	40.18	37.82	35.32
1954	1983	47.94	46.48	44.51	43.38	42.16	39.51	36.68
1955	1984	47.12	46.34	45.02	44.19	43.26	41.14	38.75
1956	1985	54.49	52.37	49.73	48.25	46.69	43.36	39.87
1957	1986	66.92	62.24	57.16	54.52	51.84	46.47	41.18
1958	1987	59.38	56.67	53.39	51.59	49.69	45.70	41.55
1959	1988	64.40	60.55	56.24	53.95	51.61	46.81	41.99
1960	1989	72.27	66.56	60.57	57.53	54.47	48.43	42.58
1961	1990	65.27	61.35	56.99	54.70	52.35	47.56	42.75
1962	1991	81.20	73.50	65.70	61.83	58.02	50.62	43.65
1963	1992	81.12	73.18	65.25	61.36	57.53	50.16	43.27
1964	1993	85.46	76.05	66.96	62.57	58.32	50.28	42.93
1965	1994	81.37	72.85	64.58	60.58	56.69	49.31	42.52
1966	1995	103.53	88.92	75.55	69.36	63.51	52.85	43.55
1967	1996	122.85	102.04	83.82	75.66	68.12	54.79	43.66
1968	1997	149.61	119.97	95.13	84.37	74.62	57.95	44.59
1969	1998	186.48	143.64	109.35	95.01	82.32	61.34	45.31
1970	1999	200.06	151.49	113.49	97.86	84.19	61.91	45.22
1971	2000	174.32	135.48	104.26	91.15	79.53	60.23	45.38
1972	2001	144.09	116.67	93.53	83.45	74.32	58.67	46.11
1973	2002	108.71	92.89	78.45	71.82	65.61	54.48	45.03
1974	2003	130.33	106.94	86.69	77.74	69.56	55.38	43.87
1975	2004	124.15	101.88	82.66	74.19	66.45	53.06	42.19
1976	2005	123.55	100.57	81.04	72.51	64.77	51.48	40.81
1977	2006	128.67	103.63	82.71	73.69	65.56	51.75	40.81
1978	2007	112.65	92.70	75.65	68.18	61.37	49.63	40.13
1979	2008	67.73	62.73	57.06	54.12	51.18	45.46	40.16
1980	2009	80.25	71.38	62.27	57.81	53.49	45.41	38.23
1981	2010	84.21	73.47	62.93	57.90	53.11	44.33	36.71
1982	2011	76.33	67.19	58.08	53.70	49.49	41.71	34.89
1983	2012	79.77	69.13	58.83	53.97	49.35	40.93	33.68
1984	2013	90.19	75.93	62.71	56.65	51.00	40.96	32.61
1985	2014	93.16	77.72	63.61	57.20	51.25	40.80	32.18
1986	2015	81.14	69.13	57.83	52.58	47.66	38.84	31.39
1987	2016	89.10	74.03	60.41	54.26	48.57	38.62	30.46

TABLE 3. Performance of Dynamic Portfolios
(Start Date 1900–1943)

start	end	100/0	80/20	60/40	50/50	40/60	20/80	0/100
1900	1929	113.97	100.02	86.07	79.10	72.12	58.17	44.23
1901	1930	91.50	82.69	73.87	69.46	65.06	56.24	47.43
1902	1931	55.83	55.08	54.32	53.95	53.57	52.81	52.06
1903	1932	55.05	55.32	55.60	55.74	55.87	56.15	56.42
1904	1933	79.60	74.47	69.35	66.78	64.22	59.09	53.96
1905	1934	68.32	64.89	61.46	59.75	58.03	54.60	51.17
1906	1935	100.43	90.20	79.98	74.86	69.75	59.53	49.30
1907	1936	125.56	109.89	94.22	86.38	78.55	62.87	47.20
1908	1937	81.16	74.15	67.15	63.65	60.15	53.15	46.15
1909	1938	91.39	82.29	73.19	68.64	64.09	54.99	45.90
1910	1939	91.53	82.26	72.99	68.35	63.72	54.45	45.18
1911	1940	79.46	72.32	65.18	61.61	58.04	50.89	43.75
1912	1941	63.10	58.25	53.39	50.96	48.53	43.67	38.81
1913	1942	67.66	61.27	54.89	51.69	48.50	42.12	35.73
1914	1943	77.81	69.15	60.48	56.15	51.82	43.15	34.48
1915	1944	86.60	75.99	65.38	60.08	54.78	44.17	33.56
1916	1945	112.94	96.90	80.85	72.83	64.81	48.76	32.71
1917	1946	81.15	70.48	59.80	54.47	49.13	38.45	27.78
1918	1947	70.33	61.31	52.29	47.78	43.27	34.25	25.23
1919	1948	70.90	61.70	52.49	47.89	43.29	34.09	24.88
1920	1949	78.13	67.54	56.95	51.66	46.37	35.78	25.19
1921	1950	87.36	74.55	61.74	55.33	48.93	36.11	23.30
1922	1951	91.69	77.88	64.08	57.18	50.27	36.47	22.66
1923	1952	96.18	81.54	66.90	59.58	52.25	37.61	22.97
1924	1953	89.76	76.45	63.13	56.47	49.81	36.49	23.18
1925	1954	122.06	102.36	82.66	72.82	62.97	43.27	23.58
1926	1955	145.70	121.35	97.00	84.82	72.64	48.29	23.94
1927	1956	142.01	118.40	94.80	82.99	71.19	47.58	23.97
1928	1957	123.29	103.45	83.60	73.68	63.76	43.91	24.07
1929	1958	164.72	136.66	108.59	94.56	80.52	52.46	24.39
1930	1959	168.12	139.56	111.00	96.72	82.44	53.88	25.32
1931	1960	168.17	139.74	111.31	97.10	82.88	54.45	26.02
1932	1961	181.93	150.92	119.92	104.42	88.92	57.92	26.92
1933	1962	159.16	132.88	106.59	93.45	80.31	54.03	27.74
1934	1963	177.49	147.70	117.91	103.02	88.12	58.33	28.54
1935	1964	188.44	156.68	124.92	109.04	93.17	61.41	29.65
1936	1965	194.43	161.66	128.89	112.51	96.12	63.35	30.58
1937	1966	169.56	141.92	114.28	100.46	86.64	59.00	31.35
1938	1967	177.12	148.09	119.06	104.55	90.03	61.00	31.98
1939	1968	176.25	147.52	118.80	104.43	90.07	61.34	32.61
1940	1969	142.24	120.42	98.60	87.69	76.78	54.96	33.14
1941	1970	134.38	114.28	94.18	84.13	74.08	53.99	33.89
1942	1971	133.11	113.40	93.69	83.83	73.98	54.27	34.56
1943	1972	136.61	116.30	95.99	85.84	75.68	55.37	35.06

TABLE 4. Performance of Dynamic Portfolios
(Start Date 1944–1987)

start	end	100/0	80/20	60/40	50/50	40/60	20/80	0/100
1944	1973	95.84	83.56	71.29	65.15	59.01	46.73	34.45
1945	1974	62.77	56.89	51.02	48.09	45.15	39.28	33.41
1946	1975	76.59	67.97	59.35	55.04	50.72	42.10	33.48
1947	1976	73.70	65.68	57.65	53.64	49.63	41.60	33.58
1948	1977	55.89	51.34	46.79	44.51	42.24	37.69	33.13
1949	1978	52.70	48.69	44.68	42.67	40.67	36.66	32.65
1950	1979	48.81	45.36	41.92	40.20	38.48	35.04	31.59
1951	1980	49.86	46.18	42.50	40.66	38.81	35.13	31.45
1952	1981	39.58	38.26	36.95	36.29	35.63	34.31	32.99
1953	1982	45.40	43.38	41.37	40.36	39.35	37.33	35.32
1954	1983	47.94	45.69	43.44	42.31	41.18	38.93	36.68
1955	1984	47.12	45.45	43.78	42.94	42.10	40.43	38.75
1956	1985	54.49	51.57	48.64	47.18	45.72	42.79	39.87
1957	1986	66.92	61.77	56.62	54.05	51.47	46.33	41.18
1958	1987	59.38	55.82	52.25	50.47	48.68	45.12	41.55
1959	1988	64.40	59.92	55.44	53.20	50.96	46.48	41.99
1960	1989	72.27	66.33	60.39	57.42	54.45	48.52	42.58
1961	1990	65.27	60.77	56.26	54.01	51.76	47.25	42.75
1962	1991	81.20	73.69	66.18	62.43	58.67	51.16	43.65
1963	1992	81.12	73.55	65.98	62.19	58.41	50.84	43.27
1964	1993	85.46	76.95	68.44	64.19	59.94	51.43	42.93
1965	1994	81.37	73.60	65.83	61.94	58.06	50.29	42.52
1966	1995	103.53	91.53	79.54	73.54	67.54	55.55	43.55
1967	1996	122.85	107.01	91.17	83.25	75.33	59.50	43.66
1968	1997	149.61	128.60	107.60	97.10	86.60	65.59	44.59
1969	1998	186.48	158.24	130.01	115.89	101.77	73.54	45.31
1970	1999	200.06	169.09	138.13	122.64	107.16	76.19	45.22
1971	2000	174.32	148.53	122.75	109.85	96.96	71.17	45.38
1972	2001	144.09	124.49	104.90	95.10	85.30	65.71	46.11
1973	2002	108.71	95.98	83.24	76.87	70.50	57.77	45.03
1974	2003	130.33	113.04	95.75	87.10	78.45	61.16	43.87
1975	2004	124.15	107.76	91.37	83.17	74.97	58.58	42.19
1976	2005	123.55	107.01	90.46	82.18	73.91	57.36	40.81
1977	2006	128.67	111.10	93.53	84.74	75.95	58.38	40.81
1978	2007	112.65	98.15	83.65	76.39	69.14	54.64	40.13
1979	2008	67.73	62.22	56.70	53.94	51.19	45.67	40.16
1980	2009	80.25	71.84	63.44	59.24	55.04	46.63	38.23
1981	2010	84.21	74.71	65.21	60.46	55.71	46.21	36.71
1982	2011	76.33	68.04	59.75	55.61	51.47	43.18	34.89
1983	2012	79.77	70.55	61.33	56.72	52.11	42.90	33.68
1984	2013	90.19	78.67	67.16	61.40	55.64	44.12	32.61
1985	2014	93.16	80.97	68.77	62.67	56.57	44.38	32.18
1986	2015	81.14	71.19	61.24	56.27	51.29	41.34	31.39
1987	2016	89.10	77.37	65.64	59.78	53.92	42.19	30.46

TABLE 5. Rebalanced Portfolios - Summary Statistics

	100/0	80/20	60/40	50/50	40/60	20/80	0/100
Mean	103.08	84.19	68.54	61.77	55.63	45.05	36.45
Median	89.98	76.94	66.20	61.09	54.05	43.66	34.98
SD	42.02	25.65	15.81	12.81	10.81	8.87	8.21
Max	200.06	151.49	113.49	97.86	84.19	62.68	56.42
P90	170.99	120.52	85.94	75.39	69.38	58.00	45.96
P75	131.03	102.05	78.92	70.34	61.92	52.02	43.34
P25	71.92	64.76	57.13	53.62	47.91	38.30	31.38
P10	55.60	54.13	49.21	45.91	42.11	34.42	24.74
Min	39.58	38.87	37.77	37.10	36.37	31.28	22.66

TABLE 6. Dynamic Portfolios - Summary Statistics

	100/0	80/20	60/40	50/50	40/60	20/80	0/100
Mean	103.08	89.75	76.43	69.76	63.10	49.77	36.45
Median	89.98	77.63	67.15	62.55	58.54	49.52	34.98
SD	42.02	33.35	24.82	20.65	16.62	9.66	8.21
Max	200.06	169.09	138.13	122.64	107.16	76.19	56.42
P90	170.99	143.6	115.37	101.23	87.08	61.21	45.96
P75	131.03	113.13	94.19	84.29	75.06	56.52	43.34
P25	71.92	65.48	57.48	54.36	50.9	42.64	31.38
P10	55.60	54.02	50.31	47.60	43.29	36.61	24.74
Min	39.58	38.26	36.95	36.29	35.63	34.09	22.66

TABLE 7. Ratio of Rebalanced to Dynamic Portfolio Values

	80/20	60/40	50/50	40/60	20/80
Mean	0.96	0.93	0.91	0.91	0.91
Median	0.98	0.95	0.94	0.93	0.95
SD	0.07	0.12	0.13	0.13	0.12
Max	1.08	1.11	1.11	1.11	1.07
P90	1.06	1.08	1.08	1.07	1.04
P75	1.01	1.01	1.01	1.01	1.00
P25	0.92	0.86	0.84	0.83	0.85
P10	0.83	0.72	0.69	0.68	0.71
Min	0.82	0.69	0.66	0.63	0.67

Considerable information can be gleaned from Tables 1 - 7. From them we can draw several lessons that we hope may prove beneficial to an investor in the accumulation phase.

1) With very few exceptions, investors who rebalanced annually reaped no benefits from rebalancing into a mixed equity / bond portfolio. In particular, the *only* 30-year periods that the 80/20 or 60/40 allocation outperformed the 100/0 allocation were the periods from 1903 to 1932 and from 1905 to 1934.

2) Investors choosing to have dynamic portfolios with no annual rebalancing were almost always better served with a 100/0 equity position, the only exception occurring in the 1903-1932 period.

3) Dynamic portfolios had a tendency to outperform rebalanced portfolios. Table 7 reveals that, over all of the mixed equity/bond contribution allocations, in the periods where the rebalanced portfolios outperformed the dynamic portfolios they did so only marginally. However, in periods where rebalanced portfolios underperformed dynamic portfolios they did so substantially.

These results give clear guidance for investors with lengthy investment horizons. First, there appears to be limited benefit and possibly considerable harm associated to rebalancing annually. This is not the viewpoint typically espoused in the literature and media, but the historical record is unambiguous on this point. (We do acknowledge that rebalancing can play a positive role in the context of shorter time horizons.) Second, investors with lengthy investment horizons are better served with a heavy equity allocation, with in fact 100% equity outperforming any other allocation in 86 of the last 88 30-year time periods, and even in those two periods not underperforming other allocations significantly.

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