Pythagorean Relation In Triangles and Fermat's Last Theorem

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March, 2023

Abstract

This paper derives n-th Pythagorean relation from the edges of right triangle and the result be applied to any triangles to discover the truely proof of Ferma'ts Last Theorem. When the value of n is equal to 2 we find FLT turns to Pythagorean Theorem, so the proof should be there[1]. If we can make a n-th power relation among the edges of right triangle, then by applying this to any triangle we will find our desire first step. For, nonhypotenuse integers[Appendix 1] general form of binomial equation is sufficient. The following proof should be Pierrie de Fermat's discovered proof based on which he made his most famous quotes in Mathematics on the margin of his favourite book Diophantus' Arithmatica over the year 1637, it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.

1 Introduction

The mathematical presentation of Fermat's Last Theorem is $x^n + y^n = z^n$ has no integer solutions for $n > 2$, where x, y, z are three non-zero positive

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intergers. Since, the variables x, y, z are non zero positive intergers, then they must satisfy one of the relations – summation of two non zero positive integers is greater than, equal or less than the other positive integer. If we can proof FLT for these three relations[Corollary 1] of non zero positive integers then we will find our desire proof.

2 Fermat's Actions

2.1 When $x + y > z$

If the summatation of two non zero positive integers is greater than the other positive integer, they forms a triangle. There are two types of triangle - right angled and non-right angled (acute, obtuse) triangle. First, if we assume x, y, z are the edges of any right triangle, where z is hypotenuse. Then we find,

$$\frac{x}{z} = \sin A; \frac{y}{z} = \cos A$$

Here, A is the acute angle of the right triangle. Now applying n-th power and addition we find $x^n + y^n = z^n(\sin^n A + \cos^n A)$. When $n = 2$, this equation presents

$$x^2 + y^2 = z^2$$

Because when the value of n is equal to 2 then the value of $\sin^n A+\cos^n A$ is equal to 1 but when n is greater than 2 we find the value is not equal to 1. For this, when n is greater than 2, z^n be greater than or less than $x^n + y^n$ but from [Lemma1] we find,

$$z^n = x^n + y^n + k_i$$

Now, if x, y, z are edges of non-right angled triangle then the triangle must be divided into two right angled triangle by drawing a perpendicular. Now applying the above equation on this two right angled triangles seperately and addition, then by calculation we will find for $n > 2$,

$$x^n + y^n \neq z^n$$

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2.2 When $x + y = z$

Appling n-th power on the both sides it turns $z^n = (x + y)^n$, after applying the binomial equation[12] we find

$$z^n = x^n + \sum_{i=1}^{n-1} {}^nC_i x^{n-i} y^i + y^n$$

Since, x, y are non zero positive integers, then z^n must be greater than $x^n + y^n$ that means we find for $n > 2$,

$$x^n + y^n \neq z^n$$

2.3 When $x + y < z$

For this relation we can write $x + y + k_m = z$, where k_m is a constant. Since, it is obvious for from the calculation of sub section 2.2 z^n is greater than $x^n + y^n$, that means for $n > 2$,

$$x^n + y^n \neq z^n$$

3 Elaboration

The above section presents the Fermat's thinking which is the verbal form of the proof and this section is the elaboration. Assusme ABC is a non-right angled triangle, and x, y, z represents the lengths of the edges of the triangle ABC then by drawing perpendicular on suitable[*] edge from opposite top point will divide the non-triangle angled triangle into two right triangles.

Then any edge z will be divided into two parts b and $z - b$, Let a is the length of the perpendicular. Therfore, the length of the edges of one right triangle will be x, a, b and another will be $y, a, (z - b)$ where x, y will be the hypotense of the right triangles repetively.

3.1

For the triangle having edges x, a, b and A as acute angle,

$$\frac{a}{x} = \sin A; \frac{b}{x} = \cos A$$

Applying n-th power on above both equations and addition, then after childhood calculation we find the following equation,

$$a^n + b^n = x^n(\sin^n A + \cos^n A)$$

When $n = 2$, this equation presents,

$$a^2 + b^2 = x^2$$

Because when the value of n is equal to 2 then the value of $\sin^n A + \cos^n A$ is equal to 1 but when n is greater than 2 we find the value is not equal to 1[Lemma1]. For this, when n is greater than 2, x^n will be greater[Lemma 1] than $a^n + b^n$. Hence we can write,

$$x^n = a^n + b^n + k_1$$

Here, k_1 is a constant and also for the another triangle we will get,

$$y^n = a^n + (z - b)^n + k_2$$

Now applying binomial theorem we will find,

$$(z - b)^n = z^n + \sum_{i=1}^{n-1} {}^nC_i z^{n-i}(-b)^i + (-b)^n$$

Hence, after we setting this value of $(z - b)^n$ into y^n and adding it to x^n, we will find the following skeleton[*] of calculated equation,

$$x^n + y^n = z^n + ...$$

Since, a, b, k_1, k_2 are non-zero positive integers then for $n > 2$,

$$x^n + y^n \neq z^n$$

3.2

Appling n-th power on the both sides of $z = x + y$ it turns $z^n = (x + y)^n$, after applying the binomial equation we find

$$z^n = x^n + \sum_{i=1}^{n-1} {}^nC_i x^{n-i} y^i + y^n$$

This equation presents for non zero positive integers x, y it is obvious z^n is greater than $x^n + y^n$, that means

$$x^n + y^n \neq z^n$$

3.3

For the relation, when $x+y$ is less than z we can write $x+y+k_m = z$, where k_m is a constant. Since, it is obvious from the calculation of the above sub section 3.2, z^n is greater than $x^n + y^n$, that means for $n > 2$,

$$x^n + y^n \neq z^n$$

3.4 Conclusion

In abstract it is described that when the value of n is 2 then it turns to Pythagorean theorem and in the subsection 3.1 it is proved that for only right triangle for the value of n is 2, only

$$x^2 + y^2 = z^2$$

but for the value of n greater than 2, for any other triangle even right triangle and also any other relations for x, y, z find

$$x^n + y^n \neq z^n$$

4 Lemma 1

If u, v, w be three edges of a right triangle where w is the hypotenuse, A is the acute angle of the right trianle then we find from the basic principles of trigonometry

$$\sin A = \frac{u}{w}; \cos A = \frac{v}{w}$$

Applying n-th power,

$$\sin^n A + \cos^n A = \frac{v^n + u^n}{w^n}$$

Since, w is the hypotenuse and u, v be the other edges of a right triangle, then we find $w^2 = u^2 + v^2$. Hence, applying n-th power we find $w^n = (u^2 + v^2)^{\frac{n}{2}}$ and we know for the fraction power[12] the binomial equation the value of w^n be

$$u^n + v^n + \frac{\frac{n}{2}(\frac{n}{2} - 1)}{2!} \frac{v^4}{u^4} + ... + \infty$$

Since u, v are non-zero positive integers then we find

$$w^n > u^n + v^n$$

Now, we see when the vlue of n is equal to 2 then $\sin^2 A + \cos^2 A = 1$ but when n is greater than 2, then we find

$$\sin^2 A + \cos^2 A < 1$$

5 Corollary 1

Now, we find that if x, y, z are three non zero positive integers there may be other relations among them except described in introduction,

$$xy = z; \frac{x}{y^2} = z; x^3 y = z$$

and so on, but they must be satisfied one of the three relations described in introduction. For example, 2, 3, 6 be three non zero integers and multification of 2 and 3 is equal to 6 but they satisfy following,

$$2 + 3 < 6$$

6 Corollary 2

From the derived equation of subsection 2.1 when n is equal to 2, then we find the following the relation,

$$x^2 + y^2 = [a^2 + b^2] + [a^2 + (z - b)^2]$$

From the above section from the calculation we find for x, y, z positive integers for no other realtioins exept right triangle,

$$x^2 + y^2 = z^2$$

7 Appendix

1. Nonhypotenuse Integers- The integers which are not equal to the length of edges of a triangle because to be edges of any triangle it must be satisfy the condition - The summation of two edges of any triangle is greater than the other. For example, 3,4,5 are right as well hypotenuse integers but 2,3,15 are nonhypotenuse integers.

8 Acknowledgement

I like to special thank to Hasan Mahmud for the liability of press meeting and his cooperation on this work.

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