Estimation on Risk Factor Loading based on Mixed Vine Copula

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Abstract

In this study, we introduce a non-linear estimating approach for risk factor loading. This new estimate is based on mixed vine copula with the aim of separating upside and downside risk exposure. We provide empirical evidence of Chinese stocks that copula-based method fits better than OLS for single-factor model, then we present that adjusted estimate adapted for time-serial weights performs better when fitting factor loadings. For multi-factor model, copula-based method is also superior in explaining the changes of asset return and interpreting the influence of extreme changes from risk factors. By exploring the usage of copulas in factor model regression, we enhance the accuracy for predicting asset returns as well as extending the application of factor model during extreme events.

Key words: mixed vine copula, factor loading, asymmetric correlation

1. Introduction

Explaining cross-sectional differences of asset loading towards different risk factors is one of most basic topics in finance world. Capital asset pricing model proposed by Sharpe[7] and intertemporal pricing model proposed by Merton[4] both imply linear relationship between asset return and economic risk factors, as well as the most-widely used Fama-French model[5]. Risk factor loading is the indicator for influence direction and influence scale.

Linear regression is one of most commonly-applicable methods in financial analysis. The accuracy of this statistical method is dependent on the correct depiction of relationship between response variable and explanatory variables. The key concept behind traditional multivariate factor model is identifying the dependence between a set of risk factors, which is usually non-linear for actuarial financial data. As a result, the risk factor loading derived from linear regression scope is not able to describe real risk impact on asset. The basic least square method is based on normal distribution assumption. Generalized linear model releases distribution assumption to exponential family including binomial, Poisson, gamma and etc(Joe(2010)[18]). However, financial data is usually skewed and fat-tailed, copula functions provide feasible methods to remove the restrictions of exponential distribution and describing the dependence structure in more flexible way(Killiches(2017) [17]).

Non-linear dependence modelling has become more and more popular within the last decades and copulas is one of most widely applied as they allow various marginal distribution assumptions and dependence structure according to Sklar(1959)[3]. Vine copula is designed to model serial dependence (Bedford and Cooke(2002)[2]) and Smith(2015)[1] studied how vine copulas model the dependence structure of more than one time-series. In this paper, we explore a new estimation method of risk factor loading based on mixed vine copula function.Patton [12] modelled asymmetric correlation between stock returns by copulas. According to the researches by Velu[15] and Czado [16],the impact of risk factor is not asymmetry either and it is of importance to differentiate the upside impact and downside impact. Mixed

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copula can separate the comparable sensitivities to risks of upside tail, downside tail and middle part by including Gumbel copula, Frank copula and Clayton copula in the mixed model in this paper. As vine copula can effectively solve the dimension explosion problem, it is specifically useful for multi-factor estimation without requirement of any assumed distribution.

One approach to testing the specification of copula-based methodology is comparing degree of fitting respectively. The main novelty of the estimation approach presented in this paper is the mixed vine copula based estimates for factor loadings with the aim of thoroughly describing the asymmetric relationship between risk factor and asset return and it is further extended to model multi risk factors. This estimation approach is superior to linear method as it models a wider class of correlation with risk factors.

The paper is organized as follows. Section 2 briefly introduces the format of mixed vine copula, and then we construct the estimates of unitary risk factor loadings based on mixed copula in Section 3. Section 4 contains empirical evidence of new estimation approach and compare it with linear regression from the perspective of value at risk and R square. After that, we develop a modified approach to estimate factor loading with self-adjusted time effect in Section 5 and demonstrate and discuss how mixed vine copula is used to explain factor model with multiple risks. Conclusions and implications are offered in Section 6.

2. Mixed vine copula and risk factor loading

2.1. Mixed vine copula

Since in this paper we use mixed vine copula to model the relationship between asset and risk factor, we first give a brief introduction to construction of this type copula. Vine copula is defined as a flexible and applicable method to construct high-dimensional copulas by approximating pair-wise copula with connected vines. Kjersti and Claudia(2009)[?] used pair-copula decomposition to exhibit complex pattern of dependence in the tails, which is named Canonical Vine Copula. In this paper, we use c-vine copula to model the dependence for n assets as follows:

$$c(x_1, ..., x_n) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,...,j-1}(F(x_j|x_1, ..., x_{j-1}), F(x_{j+i}|x_1, ..., x_{j-1})) \tag{1}$$

Equation 1 is the c-vine copula function and its likelihood function is Equation 2

$$L(c) = \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^{T} \log \left[c_{j,j+i|1,...,j-1,t}(F(x_j|x_1, ..., x_{j-1}), F(x_{j+i}|x_1, ..., x_{j-1})) \right] \tag{2}$$

In Equation 2,$c_{j,j+i|1,...,j-1}$ is the copula function of x_i and x_j, and $F(x_j|\cdot)$ is the conditional marginal function of x_j. Normal distribution is biased when sample data is skewed. In this article, we use Generalized Pareto distribution(GPD) to estimate marginal distribution. $R = (R_1, R_2, ..., R_n)$ is the set of asset returns and G_R^θ is the marginal distribution. In spite of location θ, scale $\sigma > 0$ and shape $k \in \mathbb{R}$ of GPD, dependence function $D(u_1, u_2, ..., u_n)$ is also needed to approximate multi-variant joint distribution of tails.

According to the maximum likelihood method to estimate the joint tail distribution by Ledford(1997)[?], we firstly hypothesize that time-series data of asset returns R_1 and R_2 are time-independent. The dependence function D_R^θ of asset return R beyond threshold θ represents the asymmetry of upside correlation and downside correlation, where comprising Gumbel Copula Frank Copula and Clayton Copula. Gumbel Copula is sensitive to positive co-movements and Clayton Copula is better explaining the downside correlation. Correlation derived from Frank Copula is symmetrical and we include Frank Copula in mixed-copula aiming at calibrating the relative upside-sensitive weight and downside-sensitive weight.

Suppose bivariate asymmetric dependence relationship between asset return as:

$$D_R^\theta = \sum_{i=1}^{3} w_i C_i(F_{R_1}^{\theta_1}(x_1), F_{R_2}^{\theta_2}(x_2)) \tag{3}$$

The likelihood function of asset return series R_1 and R_2 within time window T is:

$$L(\{R_{1,t}, R_{2,t}\}_{t\in[1,T]}, \phi) = \prod_{t=1}^{T} L(R_{1,t}, R_{2,t}, \phi) \tag{4}$$

2.2. Factor model

Given the important role of risk factor model in explaining asset return, correlation analysis is critical as it measures the covariance risk among investment portfolio. The multivariate correlation is getting more complicated as the increasing of involved assets. Factor model offers a way to reduce dimensions by focusing on sensitivities to common factors.

For stock i, R_i refers to its return in time t and F_j represents risk factor j. If market contains N stocks which is subject to M common risk factors, factor model is:

$$R_i = \alpha_i + \beta^*_{i,1}F_1 + \beta^*_{i,2}F_2 + \cdots + \beta^*_{i,M}F_M + \epsilon_i, \quad i = 1, 2, ..., N \tag{5}$$

where α_i is excess return that cannot be explained by M risk factors and ϵ_i is residuals which is irrelevant to risk factors. Sensitivities to risk factor changes $\beta^*_{i,j}$ is risk factor loading. Linear risk factor loading estimation is based on the following conditions: 1 $E(F_j) = 0, \quad j = 1, 2, ..., M$; 2 $E(F_i, F_j) = 0, \quad i, j = 1, 2, ..., M, i \neq j$; 3 $E(\epsilon_i) = 0, \quad i = 1, 2, ..., N$; 4 $E(\epsilon_i, \epsilon_j) = 0, \quad i, j = 1, 2, ..., N, i \neq j$.

Theoretically, if multi-factor model can accurately reflect impact from common factors to asset return, it can describe and predict asset return. For realistic financial data like joint distribution between asset return and risk factor, they usually do not follow elliptical distribution, as a result, the linear scope is biased as it does not allow other complicated dependence relationship. Furthermore, outliers have impact on linear factor loading estimation and copulas are suitable for data fitting with extreme values. Consequently, copulas are used for modelling risk factor loadings in following section.

3. New estimation approach

Risk factor loading (β)abstracts two aspects of information about relationship between asset return and risk factor: change direction and sensitivity. In order to be consistent with these two aspects, we decompose factor loadings into two parts. The first part is aimed to measure the possible upside co-varying probability d_{up} and downside probability d_{down}; the second part is upside and downside sensitivity of asset to risk factor represented by e_{up} and e_{down}. Hence Risk factor loading to factor j is shaped as equation 6.

$$\beta^*_j = d_{up}e_{up} + d_{down}e_{down}, \quad j = 1, 2, ..., M \tag{6}$$

where the sign of $|d| \leq 1$ refers to the change direction. When d_{up} is positive(negative), asset up-tail return changes in the same direction of risk factor, similarly when d_{up} is positive, asset downside-tail return changes in the same direction of risk factor. When the sign of d is negative, asset return and risk factor return change in opposite direction. The absolute value of d_{up} and d_{down} reflects changing scale. We include Gumbel Copula, Clayton Copula and Frank Copula in the mixed copula function as equation 7.

$$C(x, y) = \sum_{i=1}^{3} w_i C_i(x, y; \alpha_i), \quad \sum_{i=1}^{3} w_i = 1 \tag{7}$$

Parameters $\alpha_i, i = 1, 2, 3$ in equation 7 are estimated using expectation maximization algorithm(EM). Correlation coefficient τ derived from mixed vine copula is to describe the direction of co-varying of asset and risk factor. Although the new estimation approach does not include correlation in the middle part, Frank Copula is still important for distinguishing the relevant scale of upside correlation and downside correlation thorough weight allocation. The weight $w_{up} = w_1$ and parameter α_1 of Gumbel is estimated according to equation 7, which are used to compute upside Kendall correlation coefficient τ_{up}, and weight and parameters from Clayton ($w_{down} = w_2$ and α_2) are used to compute downside Kendall correlation coefficient τ_{down}. Then we obtain the expression of d_{up} and d_{down} in the following equation.

$$\begin{aligned} d_{up} &= w_{up}\tau_{up} \\ d_{down} &= w_{down}\tau_{down} \end{aligned} \tag{8}$$

Indicators d_{up} and d_{down} depict direction and probability of co-varying between asset return and risk factor while $e_{up}(e_{down})$ is defined as delta up(down) of asset return faced with risk factor increasing(decreasing) per unit. We firstly define counts of same-direction moves c as:

$$c(\tau) = \sum_{t=1}^{T} \phi_{t,j}(\tau) \tag{9}$$

where

$$\phi_{t,j}(\tau) = \max\left(0, sign\left(\tau \frac{R_{t+1} - R_t}{F_{t+1,j} - F_{t,j}}\right)\right) \tag{10}$$

When $\tau = \tau_{up}$ and $\tau = \tau_{down}$, the sensitivities to risk factor j is e_{up} and e_{down} respectively, which are expressed as follows:

$$\begin{aligned} e_{up} &= \frac{\sum_{t=1}^{T} \frac{R_{t+1}-R_t}{F_{t+1,j}-F_{t,j}} \phi_{t,s}(\tau_{up})}{c(\tau_{up})} \\ e_{down} &= \frac{\sum_{t=1}^{T} \frac{R_{t+1}-R_t}{F_{t+1,j}-F_{t,j}} \phi_{t,s}(\tau_{down})}{c(\tau_{down})} \end{aligned} \tag{11}$$

The sample window is T and $\phi_{t,j}$ only reveals parameters when asset moves in the same direction of risk factor. To sum up, the final factor loading estimation is presented in equation 12.

$$\beta_j^* = w_{up}\tau_{up} \left| \frac{\sum_{t=1}^{T} \frac{R_{t+1}-R_t}{F_{t+1,j}-F_{t,j}} \phi_{t,s}(\tau_{up})}{c(\tau_{up})} \right| + w_{down}\tau_{down} \left| \frac{\sum_{t=1}^{T} \frac{R_{t+1}-R_t}{F_{t+1,j}-F_{t,j}} \phi_{t,s}(\tau_{down})}{c(\tau_{down})} \right| \tag{12}$$

Equation 12 is constructed on mixed copula including upside-sensitive copula and downside-sensitive copula. By doing so, the new estimate does not only involve relationship between asset return and risk factor, it also takes extreme effect and asymmetry of asset-risk correlation into consideration.

4. Empirical result of single risk factor loading estimation

We present the performance comparison between linear estimation and mixed vine copula-based estimation approach proposed in the above section. In order to testify its validity in different market condition. We select ten stocks listed on Chinese stock market topped in valid data. Firstly, we examine one-factor model for Fama-French three factors *market*, *SMB* and *HML*, momentum factor *Mom*, reversal factor *Rev* and volatility factor *Vol*. The sample window is from January 1995 to June 2018. The return data in this study all refers to log return.

Table 1 reports the R-square and factor loading by linear OLS estimation and mixed vine copula-based estimation in full sample window. Panel A represents empirical result of daily return sample while panel B reports the empirical result of monthly return sample. Column (1) lists sample stock code and column(2) to column(7) indicate R-square of each risk factor. From the result, we can conclude that R-square computed by mixed copula estimation is higher than that computed by linear regression for all risk factor. The increase of R-square for *market SMB* and *HML* is the largest with 12.65% 24.14% and

22.49%. Volatility factor *Vol* model has least R-square improvement of 0.14%. Monthly return sample data leads to similar result: R-square in *market*-factor model has experienced maximum enhancement of 9.33%. It is because mixed copula reflects asymmetric influence of risk factor to asset return.

In order to testify the robustness of mixed copula estimation approach for risk factor loading, we further demonstrate R-square for rolling window. The rolling window for daily data is 1 month and for monthly data is 12 months and empirical result is reported in table2. The usage of mixed vine copula leads to better fitting for *smb* with 16.76% increase in R-square for daily return, 42.18% increase for monthly return. As for volatility factor *vol*, there is 16.76% enhancement in R-square for daily return, and only 1.02% increase for monthly return. R-square for market factor *market* increases 13.6% for daily return and 7.47% for monthly return.

5. Improved estimation with serial adjustment

During the period of high volatility, the volatile risk factor has more intense influence on asset return. Factor loading computation is corresponding to sample window selection. Taking 36-month window for example, when extreme event happens in the last month(at time t), copula-based estimate is exposed to average changing condition of 36-months and pays little attention to unexpected change in the last month. Aiming at addressing this problem, we improve the estimates by adding serial adjustment. The first step is to divide the rolling window into N sub-window, the weight for sub-window n is $W_n = n/(n \times (n+1)/2)$. Latest sub-window is allocated with larger weight and distant sub-window is allocated with smaller weight, allowing reflection of impact from nearer extreme event. The improved factor loading estimation $\beta_j^\dagger$ is expressed in equation13.

$$\beta^\dagger = \sum_{n=1}^{N} W_n \beta_n^* \tag{13}$$

where

$$\beta_n^* = w_{up,n} \tau_{up_n} |e_{up,n}| + w_{down,n} \tau_{down} |e_{down,n}|$$
$$W_n = n/(n \times (n+1)/2)$$

Table 3 provides average R-square of listed stocks for different risk factors by adjusted mixed copula based estimates. Row (1) to row (4) reports respective R-square for 3 sub-window, 5 sub-window, 8 sub-window and 10 sub-window. The comparison result shows that when dividing sample periods into 5 sub-windows, the fitting effect is best. This is because too few sub-windows with smaller sample data per window is difficult to describe real dependence structure while too many sub-window is exposed to data noise. In the following examination, we all divide the rolling window into 5 groups.

Adjusted estimation method has a better explanatory effect to risk factor loadings because when risk factor undergoes extreme changes, the adjusted method is more sensitive to latest abnormal changes and mirrors it in factor loading. In order words, adjusted approach better explains the risk condition. It is of significance as abnormal changes in risk factor is usually companied with abnormal changes in asset price. With the purpose of testifying the fitting effect for extreme events, we compute VaR and CVaR with 0.95 confidence level and compare it with historical VaR and CVaR. Given historical return R_i for stock i, its distribution function is F_{R_i}. Assume the fitted asset return is $\hat{R}_i$ with distribution $F_{\hat{R}_i}$. The measurement of VaR and CVaR at confidence level $\alpha \in (0, 1)$ is as follows:

$$\begin{aligned} VaR_\alpha(R) &= max(R_i : F_R(r) \leq \alpha) \\ CVaR_\alpha(R) &= E[r | r < VaR_\alpha(R)] \end{aligned} \tag{14}$$

The empirical result is represented in table 4. Both VaR and CVaR computed from fitted return by copula method and adjusted copula method are closer to historical value than linear regression. Furthermore, copula estimates after serial adjustment has a better fitting effect for risk value since it captures latest extreme risk event by over-weighting.

6. Multiple risk factor estimation

Assuming R is the stock return as dependent variable, risk factors $F_1, F_2, ..., F_m$ as independent variables, then multi-factor regression model have $m + 1$ variables. The variable set is $\{R, F_1, F_2, ..., F_m\}$. According to the definition of conditional copula, the conditional density function on the condition $(F_1, F_2, ..., F_m) = (f_1, f_2, ..., f_m)$ is:

$$h(r|f_1, f_2, ..., f_m|\alpha) = g(R; \alpha_0) \times \prod_{i=1}^{m} c_{r,F_i|F_j}(G_{Y|X_j}(r|f_j), G_{i|F_j}(f_i|f_j); \phi) \tag{15}$$

where α is unknown parameter vector and $G_{Y|X_j}$ are conditional distribution function on the condition $F_j = f_j$; $c_{r,F_i|F_j}$ is the conditional mixed copula density function and ϕ is the parameters to be estimated. The mixed copula based multi-factor regression model for R is $H(f_1, f_2, ..., f_m|\alpha)$ that is integral of r with conditional density function $h(r|f_1, f_2, ..., f_m|\alpha)$. The expression of mixed copula based multi-factor regression model $H(f_1, f_2, ..., f_m|\alpha)$ is equation 16.

$$H(f_1, f_2, ..., f_m|\alpha) = \int rh(r|x_1, x_2, ..., x_m; \alpha)dr \tag{16}$$

Parameter estimation in equation 16 includes parameters related to dependence structure and related to distribution. We use maximum likelihood approach for parameter estimation, the likelihood function is:

$$L(\alpha) = \prod_{t=1}^{T} h(r_t|f_{1,t}, f_{2,t}, ..., f_{m,t}; \alpha) \tag{17}$$

The estimated mixed copula parameters between asset return and each risk factor are reported in table 5. Dependence function among risk factors is also required for multi-factor regression. Table 6 summarizes the parameter estimates between market risk factor and other risk factor. To obtain the final factor loading of multi-factor copula regression, we evaluate dependence structure of every pair-wise risk factor and asset return.

One important indicator measuring regression effect is the distribution of error term. If majority of asset return as dependent variable is interpreted by multiple risk factors, error term of the multi-factor regression is assumed to be normal distributed, otherwise the included risk factors in regression cannot explain asset return effectively. So we analyse MSE under different quantiles as a measure of fitting effect, which is reported in Table 7.

Panel A in table7 reports MSE of ten sample stocks for linear regression and Panel B reports MSE for multi-factor mixed copula regression. For almost all sample stocks, copula regression better explain asset returns than linear regression because mixed vine copula allows simulating correlated relationship by different dependence forms and distribution assumptions. The asymmetric influence of risk factors to asset return and intersect influence between risk factors are depicted by copulas.

7. Conclusion

Factor model is an useful and commonly-applicable method for analysing influence of various risk factors to asset return, which can be calibrated by adding relevant risk factors. The most widely estimation approach is linear regression so the factor loading(regression scope) reflects linear relationship between asset return changes and risk factors changes. Nevertheless, asset return data and other financial data used for risk factor computation is usually not normal distributed so the dependence structure can be in other forms and rather complicated.

Regarding of the problem of asymmetry, we construct a new factor loading estimate using mixed vine copula by separating the upside and downside co-varying direction and sensitivities. Empirical evidence is presented by ten Chinese stocks with longest valid return data. We first testify and compute factor loadings estimates using copula-based method and linear regression, by comparing the R^2 we find out the copula-based method offers a better fitted value for all sample in both constant full sample window and rolling sample window. By examined daily returns and monthly returns, we obtain similar result: risk factor loading is more accurately estimated by copula-based method.

During period of high volatility, the volatile risk factor has more intense influence on asset return. Factor loading computation is corresponding to sample window selection and our constructed estimates only measure averaged effect. With the aim of addressing this problem, we improve the estimates by adding serial adjustment. The improved factor loading estimate better interprets extreme changes of factors so it more accurately depicts assets value condition, supported by comparison result of simulated value of VaR and CVaR by different estimates. Finally, we extend single factor model to multi-factor model by evaluating the pair-wise dependence structure and distribution of factors and between factors and asset return.

Our result have implications for risk factor analysis theoretically and empirically. Factor model is useful for understanding the formation of asset return and the main contribution of this paper is better explaining the asymmetric influence of different risk factors on asset return and further better predict future asset return.

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Table 1: Full sample-window R^2(%) Comparison for OLS and Copula Estimates

Sample Stock	Estimation Method	*market*	*SMB*	*HML*	*Rev*	*Mom*	*Vol*
Panel A. Estimation on daily return sample							
SHANXI FEN WINE	OLS	1.91	0.13	0.14	0.93	0.93	0.30
	Copula	2.80	30.77	0.61	1.28	1.21	0.44
CSD WATER SERVICE	OLS	4.53	0.58	0.12	0.18	0.18	0.19
	Copula	10.89	32.68	91.22	0.25	0.26	0.20
NHPRECL	OLS	6.47	5.89	1.80	0.54	0.54	0.23
	Copula	8.44	19.13	14.01	0.69	0.79	0.28
J.L.C	OLS	9.74	3.22	1.53	0.12	0.12	0.23
	Copula	74.32	72.09	41.07	0.58	0.17	0.47
SCAC	OLS	11.90	5.66	2.50	0.13	0.13	0.17
	Copula	17.00	10.61	16.85	25.01	0.60	0.34
FUDAN FORWARD	OLS	12.66	13.24	8.83	0.25	0.25	0.26
	Copula	39.88	19.22	16.76	0.37	0.37	0.32
SXBN	OLS	1.12	1.66	0.90	0.47	0.47	0.17
	Copula	4.94	13.00	7.46	0.97	0.55	0.25
BEZ	OLS	2.21	0.75	0.13	0.25	0.25	0.45
	Copula	4.56	14.82	1.75	0.36	0.32	0.87
LANSHENG	OLS	8.01	2.03	0.97	0.43	0.43	0.48
	Copula	22.19	42.77	0.98	0.71	0.84	0.66
HUAJIN	OLS	1.05	0.20	0.56	0.94	0.94	0.41
	Copula	1.11	19.65	2.74	1.12	1.27	0.49
Panel B. Estimation on monthly return sample							
SHANXI FEN WINE	OLS	8.23	0.82	0.75	0.75	0.75	0.43
	Copula	32.14	0.97	3.21	0.83	21.07	0.57
CSD WATER SERVICE	OLS	10.23	0.59	0.87	0.87	0.87	0.56
	Copula	11.01	0.72	2.68	1.03	45.9	0.67
NHPRECL	OLS	9.33	0.4	0.19	0.19	0.19	0.13
	Copula	13.36	0.58	4.92	0.39	38.22	0.36
J.L.C	OLS	4.18	1.08	0.33	0.33	0.33	0.28
	Copula	6.24	1.25	2.08	0.42	24.67	0.36
SCAC	OLS	6.31	1.48	0.1	0.1	0.1	0.27
	Copula	35.21	1.55	4.43	0.43	37.72	0.97
FUDAN FORWARD	OLS	7.54	1.21	0.2	0.2	0.2	0.3
	Copula	20.87	1.33	4.49	0.22	30.67	0.4
SXBN	OLS	5.29	0.21	0.16	0.16	0.16	0.13
	Copula	6.36	0.79	2.12	0.29	14.78	0.78
BEZ	OLS	10.17	0.16	0.16	0.16	0.16	0.78
	Copula	23.97	0.75	5.37	0.49	49.4	1.15
LANSHENG	OLS	7.74	0.26	0.62	0.62	0.62	0.11
	Copula	8.42	0.77	2.45	0.7	84.15	0.76
HUAJIN	OLS	10.16	1.26	0.78	0.78	0.78	0.9
	Copula	14.92	1.74	1.62	1.13	25.03	0.95

Table 2: Rolling sample-window R^2(%) Comparison for OLS and Copula Estimates

Sample Stock	Estimation Method	*market*	*SMB*	*HML*	*Rev*	*Mom*	*Vol*
Panel A. Estimation on daily return sample with 1 month rolling window							
SHANXI FEN WINE	OLS	38.17	11.60	15.51	5.48	5.48	3.83
	Copula	51.71	31.78	21.42	8.69	8.99	6.56
CSD WATER SERVICE	OLS	39.67	11.59	10.02	4.68	4.68	4.02
	Copula	53.12	30.16	17.17	8.61	7.81	6.11
NHPRECL	OLS	42.88	13.54	9.87	5.14	5.14	4.34
	Copula	56.57	37.06	19.19	8.89	8.57	6.98
J.L.C	OLS	37.88	19.04	11.67	4.57	4.57	4.05
	Copula	50.95	44.54	19.44	8.52	9.1	6.83
SCAC	OLS	39.27	18.28	12.62	4.69	4.69	3.38
	Copula	52.41	37.60	19.07	7.72	7.83	6.46
FUDAN FORWARD	OLS	44.76	23.34	15.41	5.78	5.78	4.66
	Copula	59.78	43.52	24.01	9.26	9.08	7.53
SXBN	OLS	34.45	21.68	15.56	6.44	6.44	3.9
	Copula	46.22	37.57	22.14	10.06	9.91	6.03
BEZ	OLS	36.66	15.23	11.56	5.46	5.46	3.91
	Copula	49.86	21.62	20.86	9.05	8.55	67.35
LANSHENG	OLS	42.63	16.90	13.18	4.83	4.83	4.04
	Copula	57.69	24.46	20.04	9.04	8.58	6.63
HUAJIN	OLS	44.65	14.14	11.82	6.68	6.68	4.09
	Copula	58.73	24.62	19.07	10.78	10.29	6.34
Panel B. Estimation on monthly return sample with 12 month rolling window							
SHANXI FEN WINE	OLS	2.13	1.79	1.42	1.75	1.61	4.3
	Copula	14.31	4.96	4.94	9.88	10.94	5.82
CSD WATER SERVICE	OLS	2.15	3.31	2.00	1.78	2.06	1.85
	Copula	13.04	3.82	2.7	12.82	10.8	2.29
NHPRECL	OLS	2.1	3.93	1.75	1.65	1.81	3.56
	Copula	7.77	6.07	5.8	10.54	7.8	4.83
J.L.C	OLS	2.22	2.36	1.09	1.00	1.02	1.93
	Copula	9.05	3.69	2.47	8.57	7.89	2.81
SCAC	OLS	1.92	7.36	1.31	1.11	1.23	2.4
	Copula	8.49	8.24	2.8	11.96	7.11	3.03
FUDAN FORWARD	OLS	1.26	2.07	1.19	1.15	1.29	2.45
	Copula	5.01	26.3	2.5	7.5	27.55	3.27
SXBN	OLS	3.29	1.7	2.23	1.92	2.33	2.6
	Copula	9.06	3.65	4.08	12.67	6.75	4.72
BEZ	OLS	2.17	1.83	1.24	1.54	1.59	3.12
	Copula	10.96	2.45	5.74	5.86	8.51	4.07
LANSHENG	OLS	3.35	4.34	2.98	2.24	2.55	2.55
	Copula	8.63	10.0	5.26	14.09	7.22	3.41
HUAJIN	OLS	1.9	2.58	1.06	0.65	0.88	2.85
	Copula	10.82	7.88	3.76	7.24	1.05	3.59

Table 3: Averaged R^2 for different sub-windows and risk factor

Sub-window	*market*	*SMB*	*HML*	*Rev*	*Mom*	*Vol*	*MC*	*MCS*
3	0.244	0.009	0.030	0.010	0.206	0.009	0.009	0.009
5	0.272	0.045	0.054	0.045	0.212	0.044	0.032	0.042
8	0.252	0.008	0.037	0.007	0.233	0.008	0.004	0.007
10	0.221	0.079	0.093	0.073	0.182	0.079	0.064	0.077

Table 4: Comparion of VaR and CVaR by Different Estimates

VaR Comparison with 0.95 Confidence Level				
Sample Stock Code	Historical VaR	Linear VaR	Copula VaR	Adjusted Copula VaR
SHANXI FEN WINE	0.0455	0.0378	0.0448	0.0524
CSD WATER SERVICE	0.0456	0.0405	0.0434	0.0485
NHPRECL	0.0487	0.0355	0.0437	0.0569
J.L.C	0.0553	0.0364	0.0468	0.0657
SCAC	0.0547	0.0357	0.0529	0.0719
FUDAN FORWARD	0.0531	0.0371	0.0482	0.0642
SXBN	0.0533	0.0452	0.0501	0.0582
BEZ	0.0568	0.0481	0.0538	0.0624
LANSHENG	0.0503	0.0385	0.0456	0.0574
HUAJIN	0.0524	0.0522	0.0523	0.0525
CVaR Comparison with 0.95 Confidence Level				
Sample Stock Code	Historical CVaR	Linear CVaR	Copula CVaR	Adjusted Copula CVaR
SHANXI FEN WINE	0.1821	0.0895	0.0979	0.1905
CSD WATER SERVICE	0.1542	0.0650	0.1461	0.2352
NHPRECL	0.1368	0.0616	0.0675	0.1427
J.L.C	0.1290	0.0637	0.0947	0.1599
SCAC	0.1175	0.0591	0.0722	0.1306
FUDAN FORWARD	0.1148	0.0582	0.0623	0.1190
SXBN	0.2539	0.0977	0.1109	0.2672
BEZ	0.2091	0.0925	0.1187	0.2354
LANSHENG	0.1385	0.0670	0.0819	0.1534
HUAJIN	0.2708	0.1189	0.1302	0.2821

Table 5: Mixed Copula Parameter Estimates between Asset Return and Risk Factor

Copula	*market*	*SMB*	*HML*	*Rev*	*Mom*	*Vol*	*MC*	*MCS*
Kendall τ								
clayton	0.390	0.153	0.366	1.72E-07	0.383	7.25E-07	7.25E-07	2.66E-07
frank	0.405	0.073	0.414	1.26E-07	0.386	0.093	1.36E-06	7.63E-12
gumbel	0.603	-0.199	0.416	-0.535	0.532	0.029	-0.106	-0.090
α								
clayton	1.277	0.362	1.154	3.44E-07	1.241	1.45E-06	1.45E-06	5.31E-07
frank	1.680	1.079	1.706	1.000	1.628	1.103	1.000	1.000
gumbel	8.009	-1.851	4.386	-6.418	6.350	0.265	-0.959	-0.819

Table 6: Mixed Copula Parameter Estimates between Market Risk and Other Risk Factor

Copula	SMB	HML	Rev	Mom	Vol	MC	MCS
Kendall τ							
clayton	0.268	0.409	1.65E-07	0.620	7.25E-07	7.25E-07	7.25E-07
frank	0.174	0.577	1.40E--07	0.601	1.36E-06	1.36E-06	1.36E-06
gumbel	-0.473	0.663	-0.708	0.744	-0.255	-0.119	-0.131
α							
clayton	0.732	1.383	3.31E-07	3.261	1.45E-06	1.45E-06	1.45E-06
frank	1.211	2.364	1.000	2.506	1.000	1.000	1.000
gumbel	-5.259	9.910	-11.787	13.733	-2.427	-1.083	-1.200

Table 7: MSE of Regression Error Term by Different Quantile

Panel A.Multi-factor Linear Regression				
Sample Stock Code	0.5	0.1	0.05	0.01
SHANXI FEN WINE	7.850	5.459	5.372	5.146
CSD WATER SERVICE	3.896	2.719	2.672	2.550
NHPRECL	3.920	2.728	2.682	2.550
J.L.C	7.984	5.710	5.642	5.393
SCAC	7.984	5.710	5.642	5.393
FUDAN FORWARD	7.564	5.329	5.266	5.105
SXBN	7.552	5.308	5.249	5.095
BEZ	7.782	5.499	5.434	5.229
LANSHENG	6.251	5.661	5.648	4.763
HUAJIN	4.317	4.193	3.266	3.784
Panel B.Multi-factor Copula Regression				
SHANXI FEN WINE	11.329	6.344	6.278	5.738
CSD WATER SERVICE	4.728	5.591	5.527	4.868
NHPRECL	3.323	0.860	0.832	0.534
J.L.C	3.232	1.141	1.087	0.830
SCAC	3.616	1.232	1.178	1.020
FUDAN FORWARD	4.846	2.448	2.404	2.254
SXBN	4.434	3.949	3.908	3.718
BEZ	3.536	1.300	1.246	1.096
LANSHENG	3.574	1.566	3.745	2.751
HUAJIN	2.514	2.590	1.225	1.351