TESTING FOR ENDOGENEITY OF COVID-19 PATIENT ASSIGNMENTS

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Abstract

A considerable number of individuals infected by COVID-19 died in self-isolation. This paper uses a graphical inference method to examine if patients were endogenously assigned to self-isolation during the early phase of COVID-19 epidemic in Ontario. The endogeneity of patient assignment is evaluated from a dependence measure revealing relationships between patients' characteristics and their location at the time of death. We test for absence of assignment endogeneity in daily samples and study the dynamic of endogeneity. This methodology is applied to patients' characteristics, such as age, gender, location of the diagnosing health unit, presence of symptoms and underlying health conditions.

Keywords: COVID-19, Patient Assignment, Endogenous Selection, Conditional Dependence, Value of Human Life, Value in Health.

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1 Introduction

In all countries infected by COVID-19, a considerable number of people died while self-isolating at home. This paper investigates if patient assignments to self-isolation instead of medical care might have contributed to the deadly outcome of the infection during the early phase of the epidemic in Ontario.

In Ontario, 1434 deaths due to COVID-19 were reported between January 23 and May 05, 2020. Among these, 1116 deaths occurred after at least one day of (self)isolation and 318 in hospital. Among the individuals deceased in self-isolation, 262 were diagnosed after death (post mortem) and 854 were diagnosed before they died. The total number of individuals diagnosed prior to death and deceased either in isolation or in hospital, is 1172. These counts are based on the records of the Public Health Ontario (PHO) database containing individual records of 18722 patients diagnosed with COVID-9 over the period of the first 104 days of the epidemic between January 23 and May 05, 2020. The database IPHIS-REPORT.csv combines records from the iPHIS (integrated Public Health Information System) and CORES (Toronto Public Health Coronavirus Rapid Entry System).

The deceased patients and/or their health care providers, at some point, made a choice between being self-isolated at home or hospitalized. In the case of elderly patients, the families might have been involved in making this decision. In particular, 262 deceased who were not diagnosed prior to death, might have decided by themselves to stay home. Our records do not contain information on whether they sought medical assistance. However, the remaining deceased individuals came into contact with health care providers at least once when tested positive for COVID-19. Therefore, their choice between self-isolation and hospitalization was, if not determined, at least influenced by the health care providers. Due to the presumed role of health care providers, the location of individuals prior to their death will be referred

to as the "patient assignment".

At the beginning of the epidemic, little was known of the effects of COVID-19, thus explaining why many patients remained in self-isolation at home, or were sent back home after seeing a doctor. Some of those patients with apparently mild symptoms experienced sudden deterioration of health or even death over a short period of time. Another potential explanation of why only 22% of the deceased died in hospital is the high cost of medical care and/or a concern about an anticipated shortage of beds in the intensive care units (ICU) during the upcoming peak [see e.g. Robert et al. (2011) for the impact of ICU admission on mortality, Palmer et al. (2014) on the Health System Funding Reform and Appendix 4]. During the early phase of the epidemic and under the threat of shortages of ventilators, medical staff might have felt pressured to only hospitalize patients with severe symptoms.

The aim of our paper is to give ex-post insights on patient assignments to self-isolation during the early phase of the epidemic in Ontario. We examine the endogeneity of assignments by revealing relationships between the characteristics of deceased patients and the states of self-isolation or hospitalization. We describe a dependence measure that allows us to quantify the endogeneity, and study its evolution over time. This methodology is applied to qualitative or quantitative characteristics of patients, such as age, gender, location of the diagnosing health unit, presence of symptoms and underlying health conditions. We test the null hypothesis of absence of endogeneity, which is equivalent to testing the independence between assignment to self-isolation and the deceased patients' characteristics.

The endogeneity analysis applied to daily data on infections in Ontario over the early phase of COVID-19 provides little evidence for endogeneity of daily patient assignments to isolation. This suggests that during the early phase of the epidemic, the daily decisions on remaining in self-isolation were mostly unrelated to patients' characteristics. Nevertheless, we find evidence

of endogeneity of daily assignments to isolation for patients over 80 years old and for asymptomatic patients. Over the entire observational period, the absence of endogeneity is rejected for age over 50, absence of symptoms, and the Ottawa location of diagnosing health unit.

The patient assignment also appears unrelated to the increasing demand for ICU beds. We examine the pressure on ICU beds and show that the point-in-time (PIT) difference of counts of available and occupied beds underestimates the potentially needed number of beds. We adjust this PIT measure by adding patients who remained in self-isolation until their death and account for the exponential increase in the number of infections during the early phase of the epidemic.

The paper is organized as follows. In Section 2, we quantify the endogeneity of patient assignments by introducing a dependence measure and test for the absence of endogeneity. In Section 3, this approach is applied to the data on individuals with various characteristics. In Section 4, we examine the pressure on the ICU system. Section 5 concludes. Appendix 1 describes the measure of dependence and the test of independence based on a contingency table. Appendix 2 gives alternative interpretations of the dependence measure. Additional summary statistics are provided in Appendix 3. Appendix 4 relates our outcomes to the structural model of patient care in health economics.

2   The endogeneity of patient assignment

The assignment Y to self-isolation or medical care is a binary variable, which takes values 1 and 0 for isolation and hospitalization, respectively. The state of death is the conditioning variable, denoted by D, which takes values 1 and 0 for death and life, respectively. The qualitative or quantitative explanatory variables Z are the characteristics of patients, such as age, gender, and

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location of the diagnosing health unit. If the assignment Y to self-isolation is random, for example, due to a diagnosis error caused by the lack of experience with COVID-19, then variables Y and Z are independent given $D = 1$. Otherwise, the assignment exhibits endogeneity, which is evaluated from the dependence measure discussed below.

2.1 The dependence measure

Let us consider the variable Y indicating where the patient was assigned at the time of death:

$$Y = \begin{cases} 1, & \text{if he/she was hospitalized,} \\ 0, & \text{if he/she was isolated.} \end{cases}$$

Let us introduce a binary characteristic Z of a patient. This can be the gender (male or female), an indicator of age (age smaller than 70 years or larger than 70 years), or the location (Toronto area or the rest of Ontario). Then, from the observations on Y_i, Z_i conditional on $D_i = 1$, $i = 1, ..., n$ at time t (day or week), we compute a sample-based 2×2 contingency table:

$$\hat{P}_t = \begin{pmatrix} \hat{p}_{00t} & \hat{p}_{10t} \\ \\ \hat{p}_{01t} & \hat{p}_{11t} \end{pmatrix}, \tag{3.1}$$

where $\hat{p}_{00t}$ is the proportion of old individuals who died while self-isolated, $\hat{p}_{10t}$ is the proportion of young individuals who died while self-isolated, $\hat{p}_{01t}$ is the proportion of old individuals who died hospitalized, and $\hat{p}_{11t}$ is the proportion of young individuals who died hospitalized, in the population of deceased at time t. Thus, $\hat{P}_t$ provides an approximation of the joint distribution of (Y, Z) given death $D = 1$ at time t.

If a patient assignment is random, the variables Y and Z are independent[1] given death $D = 1$. The conditional independence is denoted by

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[1]There is an alternative interpretation of the conditional independence condition. Let

$Y \perp Z|D$. Otherwise, there is a conditional dependence that may reveal that patients have been endogeneously assigned to self-isolation. The dependence can change over time, depending on the pressure on a medical care unit.

An empirical measure of (conditional) dependence at time t can be defined from a log-linear probability model applied to the 2×2 contingency tables (see Appendix 1). It is given by:

$$\hat{\gamma}_t = \log \left[\hat{p}_{11t} \hat{p}_{00t} / (\hat{p}_{01t} \hat{p}_{10t}) \right], \tag{3.2}$$

as an approximation of its theoretical counterpart :

$$\gamma_t = \log \left[p_{11t} p_{00t} / (p_{01t} p_{10t}) \right]. \tag{3.3}$$

This measure separates the concordant observations (1,1), (0,0) and the discordant observations (0,1), (1,0).

The measure of conditional dependence γ_t (also called the measure of causation [2]) is equal to the regression coefficient of variable Z in a dichotomous logit model for Y (and to the regression coefficient of variable Y in a logit model for Z, by symmetry) (see, Appendix A2.1).

The variables Y, Z are conditionally independent at time t, if and only if $\gamma_t = 0$. They are conditionally positively (or negatively) dependent according to the sign of $\hat{\gamma}_t$ [3].

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us consider a physician who can accurately predict the future severity of the disease denoted by S. Then we have a causal chain : $Z \to S \to Y$, that is, the individual characteristics have an impact on S, and the assignment decision is based on S. Thus the conditional distribution of Y given S and Z is equal to the conditional distribution of Y given S only, which means that variables Y and Z are independent given $S : Y \perp Z|S$. This is the case when $S = D$, that is discussed in our paper.

[2]See e.g. Pearl (2009) for a survey on causal inference and the distinction between measure of association (marginal dependence) and measure of causation (conditional dependence).

[3]For a binary variable Z, the selected indicator of outcome 0,1 has an effect on γ. Indeed, if Z for outcomes 1, 0 is changed to 1-Z, for outcomes 0, 1 indicator, then the sign

We trace out the daily evolution of $\hat{\gamma}_t$ and the associated confidence intervals [see, Appendix 1 for the derivation of an asymptotically valid confidence interval at level 95%] to evaluate the endogeneity of patient assignments over time. This asymptotically valid confidence interval used in this paper, has a simple analytical expression and is valid under the independence hypothesis and its alternative. We test graphically the null hypothesis $\gamma_t = 0$ against $\gamma_t \neq 0$ by verifying if the asymptotically valid confidence interval at level 95% contains 0, which is a test for absence of endogeneity [4]. Thus we do not average the conditional dependence measure with respect to the threshold as proposed in Chatterjee (2020).

2.2 Age Effect

Let us first consider an age indicator Z_a, with thresholds of age equal to $a = 50, 60, 70, 80$. Variable $Z_a = 1$, if the age is greater than a, and 0, otherwise. Figure 1 provides the daily series $\hat{\gamma}_{t,a}$ for patients above each threshold of age along with the evolution of their confidence intervals.

Table 1 shows the counts of assignments among the 1434 deceased by May 04, 1116 of whom died in self-isolation and 318 in hospitals.

Table 1: Joint Counts: Isolation, Age, Given Death.

assignment	> 50	> 60	> 70	> 80
isolation and $>$ age limit	1080	1043	961	781
isolation and $<$ age limit	36	73	155	335
hospital and $>$ age limit	256	203	138	74
hospital and $<$ age limit	62	115	180	244

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of gamma changes to $-\gamma$.

[4]We check if the condition for asymptotic validity of the test given in A.1.2 is satisfied. Alternatively, the quality of the asymptotic approximation can be assessed by applying a permutation test with simulations performed under the null.
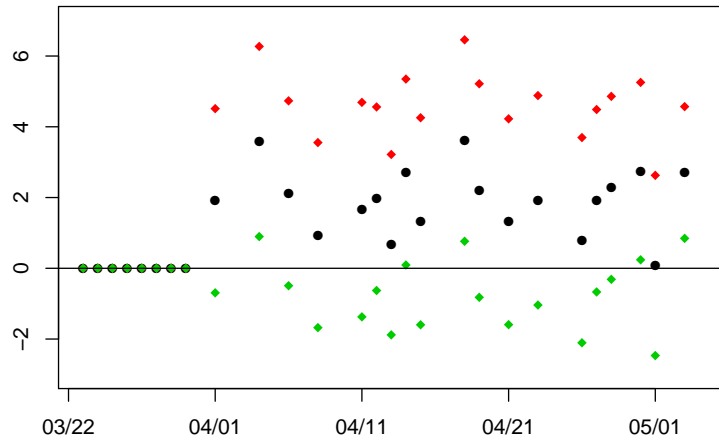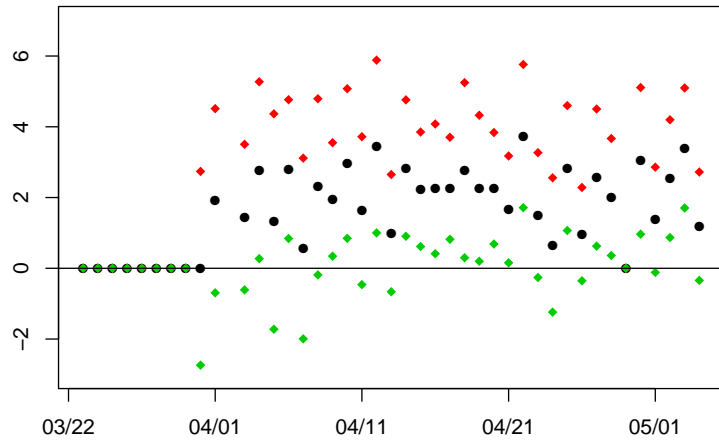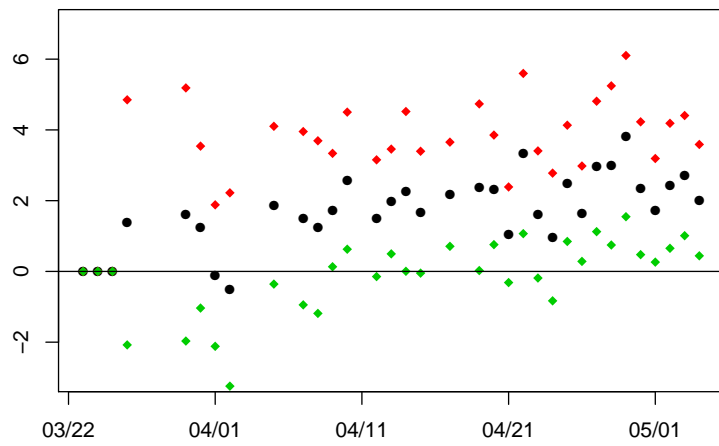
age > 50
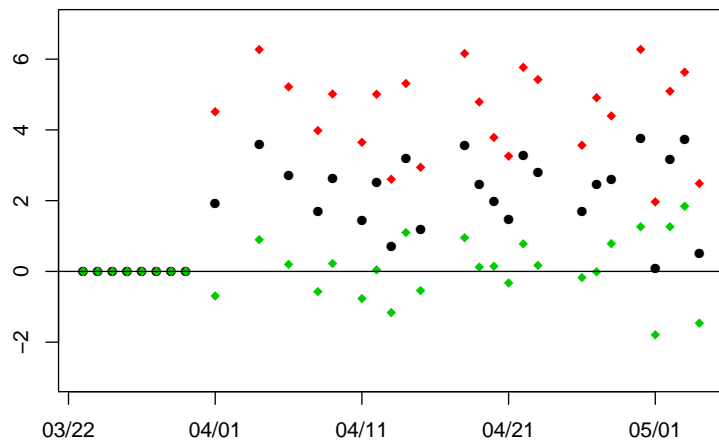
age > 60

Figure 1a: Conditional Dependence Measure, Z=Age

age > 70

age > 80

Figure 1b: Conditional Dependence Measure, Z=Age, Cont.

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Figures 1a and 1b show the daily dependence measures along with their confidence intervals over the period of March 22 to May 05, 2020. The coefficients are reported on days with a sufficient number of observations, i.e. a sufficiently high daily count of deaths. The black diamonds indicate the values of $\hat{\gamma}_t$, the green and red indicate the lower and upper bounds of the confidence intervals, respectively. The green diamonds with black contours on the line of zero indicate the missing values of $\hat{\gamma}_t$ due to insufficient numbers of observations and/or zero frequencies. At the beginning of the epidemic, there are many missing values. For this reason, the figures illustrate the period starting from March 22, 2020.

We observe positive values of conditional dependence [5]. From the interpretation of the sign of dependence measure γ in terms of conditional probabilities (see, Appendix A2.2), it follows that the probability to die in isolation is higher for an older individual than for a younger individual. This effect is observed for all age thresholds with almost equal values of $\hat{\gamma}_t$ on average. It is important to analyze the lower bounds of the confidence intervals, which are rather wide due to occasionally limited counts of daily deaths. For $a = 50$, the confidence interval always contains zero, and the assumption of no endogeneity at the daily level is not rejected. However, when the threshold of age increases, for example for $a = 80$, at the end of the observational period (end of April until May 05, 2020), the endogeneity cannot be rejected.

The slight variation of daily dependence measures over time could be related to the pressure on the ICU [see, Section 4]. As we observe no visible trend in $\hat{\gamma}_t$ for all age thresholds a, even over the period of significant pressure between April 4 and April 10, $\hat{\gamma}_t$ appear independent of the pressure on ICU. This suggests that the standards of medical treatment in the ICU have not changed during the epidemic.

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[5]The condition of asymptotic validity of the Gaussian approximation [see, Appendix A1.2] is satisfied except on days before April 11, on April 26, 29 and May 01.

So far, we have considered patients who were diagnosed either before, or after their death. As mentioned in the Introduction, 262 Ontarians died undiagnosed in isolation. The age characteristics of these 262 patients deceased and diagnosed sick of COVID-19 after death are as follows:

quantile	0%	25%	50%	75%	100%
age	24	76	86	92	102

The average age of these patients is 82 years and standard deviation is 14.14. Likely, the elderly deceased were residents of the long term facilities who died of COVID-19 without being diagnosed. They are probably the early victims of the COVID-19 crisis in the retirement homes in Ontario.

Let us now consider only the 1172 patients who were diagnosed before their death. Among these 1172 deceased, 854 individuals died in isolation.

Table 1a: Joint Counts of Isolation, Age, Given Death and Diagnosed
Before Death

assignment	> 50	> 60	> 70	> 80
isolation and > age limit	830	804	741	611
isolation and < age limit	24	50	113	243
hospital and > age limit	256	203	138	74
hospital and < age limit	62	115	180	244

Figure 8 displaying the dependence measures for the diagnosed individuals is given in Appendix 3. We observe that some dependence measures are shifted upwards, but the general conclusions remain unchanged [6].

2.3 Effect of other individual characteristics and conditioning

The endogeneity test is now applied to other patient characteristics Z. We examine below the endogeneity of assignment given the gender, location of

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[6]The condition of asymptotic validity of the Gaussian approximation [see, Appendix A1.2]is satisfied except for days before April 15, on April 17, 18, 23, 26, and between April 29-May 01.

diagnosing health unit, presence of symptoms and underlying health conditions.

For each variable Z, we first provide the joint distribution of Y, Z conditional on $D = 1$ over the whole period. Next, we show the plots of the daily measures of dependence. We also consider the effect of changing the conditioning set, when the sub-population of interest includes only the deceased who were diagnosed before death.

2.3.1 Gender Effect

Table 2 below shows the joint counts of deceased in either isolation or hospitalization for variable Z of gender.

Table 2: Joint Counts: Gender, Isolation, Given Death.

assignment	count
isolation and men	483
isolation and women	633
hospital and men	155
hospital and woman	163

Figure 2a below illustrates the conditional dependence between gender and isolation.

We observe that the dependence measures are close to zero [7]. Therefore, we can conclude about the absence of endogeneity of patient assignments with respect to gender.

Let us now examine the sample without the 262 patients diagnosed after death.

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[7]The condition of asymptotic validity of the Gaussian approximation [see, Appendix A1.2] is satisfied except for days before April 11, on April 26, 29 and May 01.
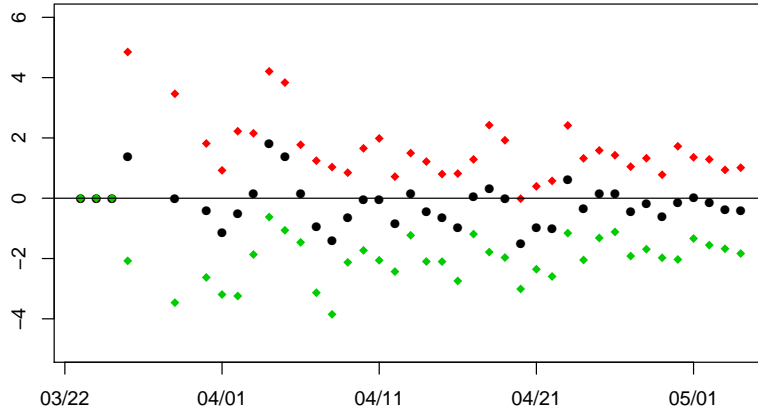
Figure 2a: Conditional Dependence Measure, Z=Gender

Table 2a: Joints Counts of Gender, Isolation, Given Death and Diagnosed Before Death.

assignment	count
isolation and men	366
isolation and women	488
hospital and men	155
hospital and woman	163

As shown in Figure 2.b, the conclusion remains the same if the individuals diagnosed post mortem are removed from the sample [8].

2.3.2 Effect of diagnosis area

Let us now examine the variable Z of Diagnosis Health Unit Area and the endogeneity test applied to the regions of Toronto and Ottawa.

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[8]The condition of asymptotic validity of the Gaussian approximation [see, Appendix A1.2] is satisfied except for days before April 15, on April 17, 18, 23, 26, and between April 29-May 01.
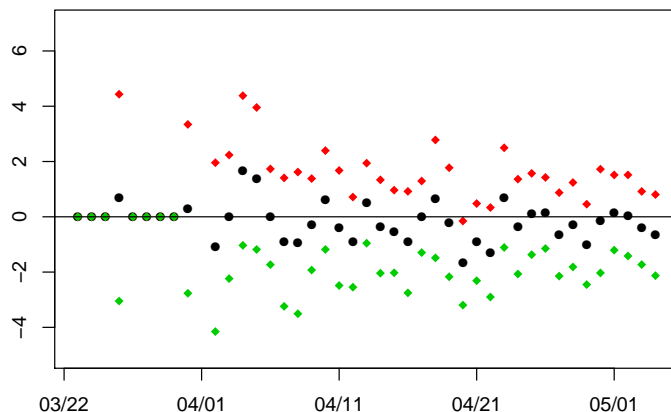
Figure 2.b: Conditional Dependence Measure, Z=Gender, Diagnosed

More precisely, we consider three binary variables :

$Z_1 = 1$, if the individual is diagnosed in Toronto region, 0, otherwise,

$Z_2 = 1$, if the individual is diagnosed in Ottawa region, 0, otherwise,

$Z = Z_1 + Z_2 = 1$, if the individual is diagnosed either in Toronto, or in Ottawa region.

Table 3: Joint Counts of Location, Isolation, Given Death

assignment	count		count		count
isolation out of Toronto and Ottawa	682	in Toronto	295	in Ottawa	139
isolation in Toronto or Ottawa	434	out of Toronto	821	out of Ottawa	977
hospital out of Toronto and Ottawa	160	in Toronto	147	in Ottawa	11
hospital in Toronto or Ottawa	158	out of Toronto	171	out of Ottawa	307

The counts are crude summaries of the propagation of disease in these two areas. We observe that both the magnitude and patterns are significantly different. From the Public Health Ontario (2020 a) report, it follows that the total number of COVID-19 cases before July 4 was 13372 in Toronto as compared to 2109 in Ottawa, with populations of 3, 120, 358 and of 1, 054,656, respectively. The peaks of daily number of deaths, were different

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too: Toronto had the maximum of 43 deaths on April 19 as compared to 12 deaths on May, 1 in Ottawa. [9]

The plots of dependence measures for deceased patients who were diagnosed in the Diagnosis Health Unit Area 3895 of Toronto and the Diagnosis Health Unit Area 2251 of Ottawa are given in Figures 3a and 3b below [10].
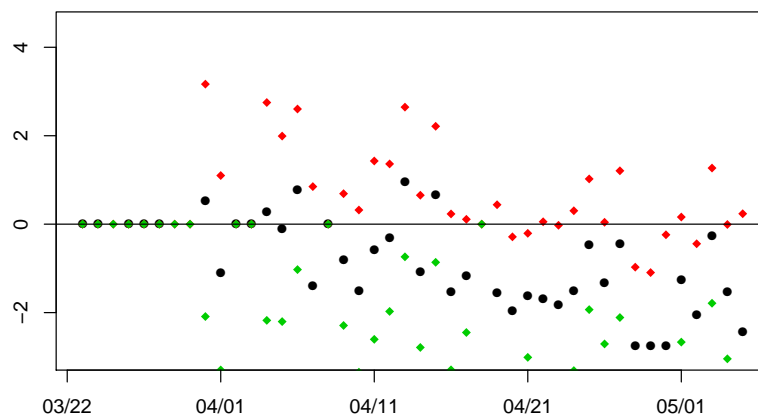

Figure 3a: Conditional Dependence Measure, Z=Diagnosed in Toronto

The daily dependence measures for deceased patients who were diagnosed in Ottawa [resp. in either Diagnosis Health Unit Area 3895 of Toronto, or 2251 of Ottawa] are given in Figure 3c (resp. Figure 9, Appendix 3).

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[9]The other areas with high maximum daily counts of deaths are in general close to Toronto and their peak times are similar. These are: Peel area, maximum count 13 on April 30, for a population of 1, 605, 952, Durham area, maximum count 9 on April 20, for a population of 712, 402, York area, maximum count 7 on April 27 for a population of 1, 225, 797. Waterloo area had also a high maximum daily count of death of 9 on April 23 for a population of 584, 361 [see the interactive map on PHO (2020) a].

[10]The condition of asymptotic validity of the Gaussian approximation is satisfied except on days before April 11, on April 26, 29 and May 01.
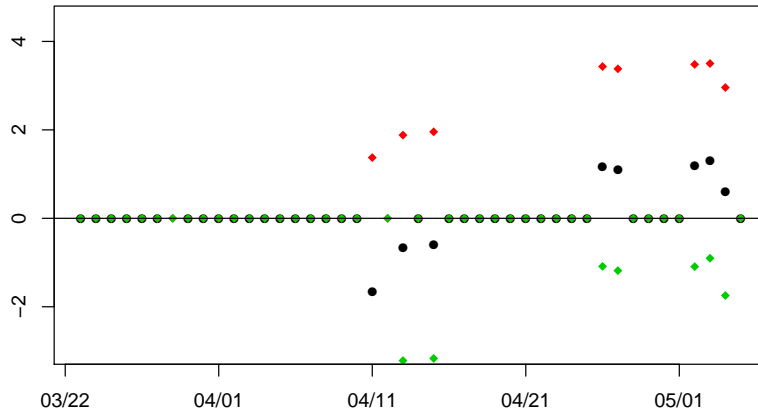
Figure 3b: Conditional Dependence Measure, Z=Diagnosed in Ottawa

Figures 3a and 3b reveal negative dependence for the Toronto location and no significant dependence for the Ottawa location. The observed negative dependence implies that, all things equal otherwise, the severe cases have a larger probability to be hospitalized in Toronto than outside Toronto. The dependence measures are not significant for the Ottawa region. This is largely a consequence of rather low daily death counts. Figure 3c reveals the aggregation issue when both regions are put together, which can be explained from the interpretation of γ in a dichotomous logit model [see, Appendix 2]. The dependence measure in Figures 3a and 3b, γ_{1t}, γ_{2t}, say, correspond to logit models with regression terms $\gamma_1 Z_1$, and $\gamma_2 Z_2$, respectively. In Figure 3c, the γ coefficients correspond to the regression term $\gamma Z = \gamma Z_1 + \gamma Z_2$, assuming implicitly that $\gamma_1 = \gamma_2 = \gamma$.

Let us verify if the conditional dependence outside of Toronto and Ottawa remains significant in the sample without the 262 deceased patients who were diagnosed post mortem. Table 3a below presents the relevant counts:
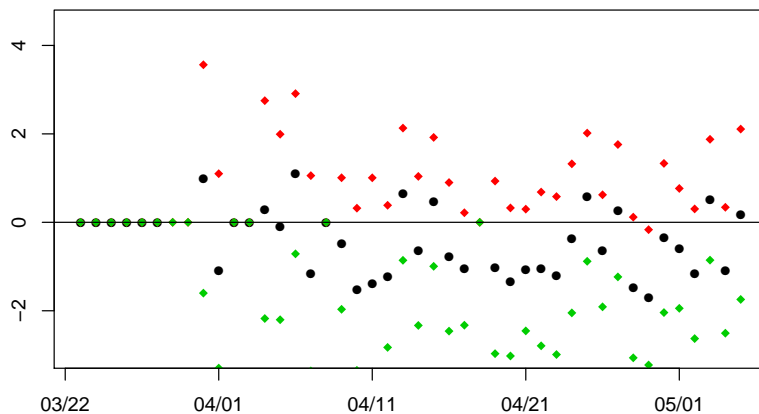
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Figure 3c: Conditional Dependence Measure Z=Diagnosis Health Unit Area Toronto and Ottawa

Table 3a: Joint counts of Location, Isolation, Given Death and Diagnosed Before Death.

assignment	count
isolation, diagnosed out of Toronto and Ottawa	535
isolation diagnosed in Toronto or Ottawa	319
hospital diagnosed out of Toronto and Ottawa	160
hospital diagnosed in Toronto or Ottawa	158

The dependence measures are displayed in Figure 9, Appendix 3. We observe that more $\hat{\gamma}_t$ are located above the line of 0, although the confidence interval still contains 0 on all days.

2.3.3   Effect of symptoms

Let us now examine the conditional dependence given the absence of symptoms such as cough, fatigue, fever, headache, shortness of breath, sore throat or other. After removing the missing values, we get a sample of 817 individuals out of whom 470 died in isolation and 347 in hospital. Table 4 below presents the relevant counts:

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Table 4: Joint Counts: Symptom, Isolation, Given Death.

assignment	count
isolation, no symptoms	53
isolation symptoms	417
hospital no symptoms	13
hospital symptoms	334

The dependence measures are calculated over 10 day intervals starting from March 22 to ensure the validity of results, displayed in Figure 4.
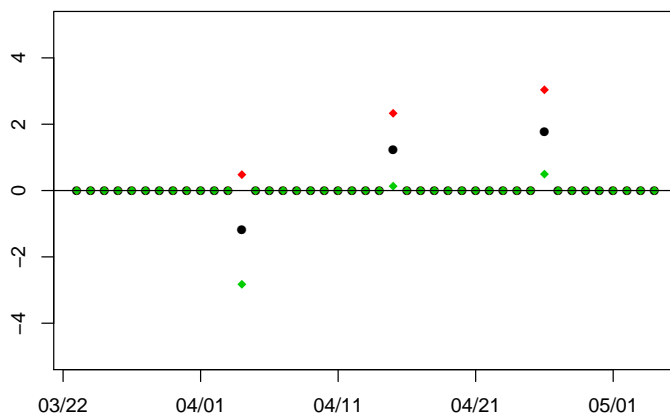

Figure 4: Conditional Dependence Measure, Z=Asymptomatic

We find evidence in favour of endogenous assignment to isolation for the asymptomatic individuals, starting from mid-April.

2.3.4 Effect of underlying health conditions

A similar analysis can be performed with respect to the presence of at least one underlying health condition, such as anemia or hemoglobinopathy, asthma, cancer, cardiovascular condition, UMC, chronic liver disease, COPD,

diabetes, immune system disorder, neurological disorder, obesity, postpartum, pregnancy, renal conditions, tuberculosis and other illness. There are 538 deceased individuals to be examined after eliminating the missing values, 277 of whom died isolated and 261 hospitalized.

Table 5: Joint Counts: Health Condition, Isolation, Given Death.

assignment	count
isolation, underlying	14
isolation, no underlying	263
hospital, underlying	26
hospital, no underlying	235

The dependence measures calculated over 10 day intervals to ensure the validity of results, are shown in Figure 5.
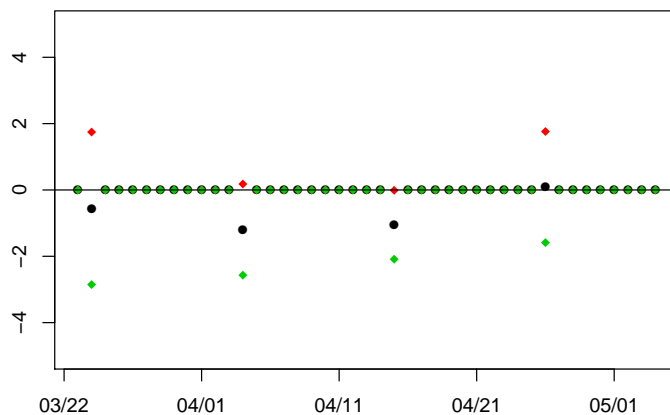

Figure 5: Conditional Dependence Measure, Z=Underlying Health Conditions

We find evidence suggesting that in early April (that is the period of pressure - see Section 4 below) patients with underlying health conditions were endogeneously not assigned to isolation.

2.3.5 Dependence over the entire sampling period

The effects of various characteristics Z are summarized in Table 6 where the dependence measure is computed over the whole period T. These values are all statistically significant at 5%, except for the last one. The statistical significance is related to the larger number of observations from which these dependence measures are computed, as compared to the daily dependence measures.

Table 6: Classification of Variables by $|\hat{\gamma}_T|$ over the Whole Period.

Z	$\hat{\gamma}_T$	std error
Age > 60	2.209	0.186
Age > 70	2.146	0.151
Age > 50	2.125	0.250
Age > 80	2.115	0.152
Ottawa	1.369	0.319
Symptoms	1.183	0.318
Toronto	-0.873	0.131
Health condition	- 0.732	0.343
Ottawa-Toronto	0.439	0.132
Gender	*-0.221*	*0.127*

The negative sign of the dependence measure suggests that patients who were either diagnosed in Toronto or had some underlying health conditions, were endogenously assigned to hospitalization rather than to isolation.

The above ordering by conditional independence is based on the analysis with a single explanatory variable. In the future, it would be insightful to examine the dependence with several characteristics considered jointly and include the cross-effects of gender and age, for example. This would lead to a new class of cross-variable selection algorithm based on our measure of conditional dependence [see Azadkia, Chatterjee (2020), Section 5 for a similar approach with quantitive regressors and the interpretation in terms of logit regressions (see Appendix 2]. At this point, due to the limited number of available data (i.e. the limited number of individual deceased in Ontario) and the missing individual characteristics in the dataset, the validity of a multivariate analysis cannot be ensured [11].

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[11]This also explains why machine learning techniques are inapplicable.

3   Measuring the pressure on ICU beds

Let us now examine how the numbers of occupied ICU beds would have changed if the patients deceased in isolation were hospitalized and treated for COVID-19. A simple approach to measuring the pressure due to COVID-19 on the health system consists in comparing the number of occupied ICU beds (including beds with ventilators and/or intubation) with the total number of available ICU beds. We adjust this simple measure as follows:

i) we add patients deceased in isolation who might not have been correctly diagnosed and hospitalized early enough.

ii) we account for the exponential increase of infections during the epidemic. This effect is amplified by the long duration (about 14-20 days) of treatment for COVID-19 in the ICU.

The measure of pressure that accounts for the two aspects given above is an ex-post measure of pressure rather than a projection of the demand for ICU beds [see e.g. Gibney (2020), Shoukat et al. (2020)].

i) Adjusting for deceased in self-isolation

For each individual deceased from COVID-19 in self-isolation, the time that patient would have stayed in ICU if he/she were hospitalized, is set equal to 14 days. Then, by summing up those individuals, we find the count of ICU beds that would have been potentially occupied in the past. These ex-post daily counts are plotted in Figure 6 over the period from January 23 to May 01, 2020.

The red curve represents the counts of occupied beds and the green curve shows the counts of beds potentially occupied if individuals who died in isolation were hospitalized. The black line represent the sum of both curves.

ii) Adjusting for the dynamics

To adjust for the propagation of the disease, we first consider the daily increase of the count of occupied beds (see Figure 1). This increase is approximately $x\%$ with $x = 0.2$ over the period. We predict the potential total demand (PTD) at horizon of 20 days (3 weeks) by computing the quantity :
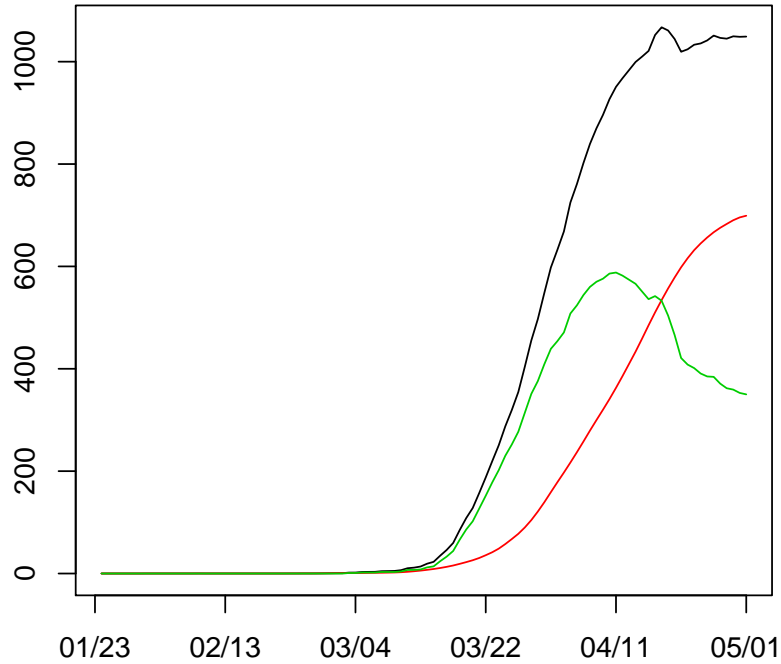
PTD at 20 days

Figure 6: Count of Occupied and Potentially Occupied Beds
red:beds occupied, green:beds potentially occupied, black:sum of curves

=[Count of occupied beds+ Count of potentially occupied beds] x $[1 + 0.2\%]^{20}$.

The series of PTD at 20 days is plotted in Figure 7. By comparing it with the count of occupied ICU beds(red line), we get insights on the magnitude of underestimation under the simple approach.

The above measure of PTD still underestimates the true PTD, as it does not account for the infected and undetected individuals [see Gourieroux, Jasiak
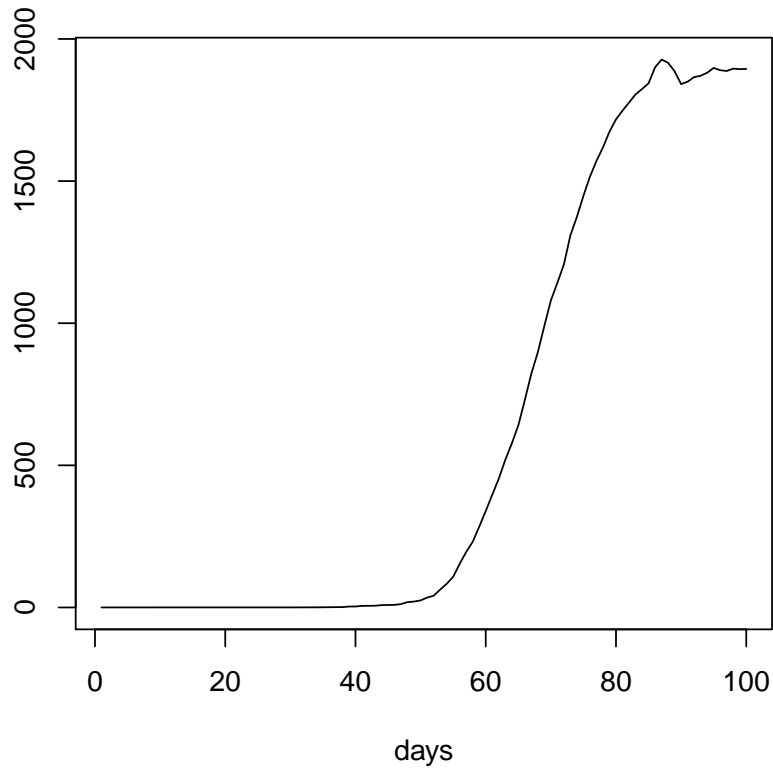
Figure 7: Potential Total Demand at 20 Days

(2020)] and the fact that an increase of detection effort, i.e. the increase of number of tests performed, may also reveal an additional demand for medical care and beds.

The daily PTD series have to be compared with the total number of available ICU beds to see if there was a shortage of beds on some days. Indeed "critical care triage to allow for the rationing of scarce ICU resources might be needed" [Shoukat et al. (2020)]. The numbers of critical care beds equipped with ventilators and available for COVID-19 patients was about 2,811 by April 30 [Office of the Premier (2020)]. However, there were only 415 of them in April 4. Next, their number was increased to "reach the

level of 1300 before the peak" [CBS News (2020)]. Thus, the potential total supply of ICU beds exceeded the potential demand in Ontario, although the low initial supply of beds created pressure between April 4 and April 10. We conclude that there were sufficiently many beds available to treat the patients who died in self-isolation.

4   Concluding Remarks

The measure of conditional dependence allows us to test if patients' characteristics played a role in assigning them to isolation at home or hospitalization at the time of death.

The endogeneity analysis applied to daily individual data on infections in Ontario over the early phase of COVID-19, provides little evidence for endogeneity of daily patient assignments to isolation and suggests that during the early phase of the epidemic, the decisions of remaining in self-isolation were unrelated to patients' characteristics. Nevertheless, after mid-April, we find evidence of endogeneity of assignments for patients over 80 years old and asymptomatic patients. Over the entire observational period, the absence of assignment endogeneity is rejected for age over 50, the location of diagnosing health unit and the absence of symptoms. There is no evidence of assignment endogeneity with respect to gender. Moreover, patients who were diagnosed in Toronto were more likely to be hospitalized than those who were diagnosed in Ottawa.

The approach developed in this paper can be applied to test for endogeneity of other decisions concerning the individual medical histories.

i) The selection of individuals to be tested for COVID-19 needs to be exogenous. Several countries have decided to test mainly symptomatic individuals and/or those in contact with symptomatic persons (tracing), which is endogenous, while Russia, for example, tested daily random samples of population to determine the rate of asymptomatic in the population.

ii) The average time spent in the ICU seems to decrease on average in the world. The dependence measure can be used to test if this is due to enhanced knowledge of COVID-19 and improved treatment (exogenous effect) rather than to endogenous decision of admitting patients with better prospects of recovery.

iii) By changing the conditioning set, the approach may also be used to detect at which stage the assignment decision is taken, for example, by

conditioning on the population of deceased who visited the ER or a walk-in-clinic at the beginning of their infection.

The proposed approach can also be considered as the first-step to deriving patient assignment scores based on the estimation of structural logit models. This second step is beyond the scope of our paper [see, Appendix 4].

The test of endogeneity is important as a doctor may be held liable for causing the patient's loss of a chance to be cured if he/she fails to diagnose the future evolution of the disease [see Government of Canada (2006), subsection "Medical Assistance in Dying-Reporting of Deaths"].

Appendix 1
Measure of Dependence and Independence Test for
2×2 Contingency Tables.

This appendix describes the test of independence between dichotomous variables, which is an alternative to the chi-square test of independence used, for example, in controlled medical experiments [see Mantel, Haenzel (1985) for early reference]. The independence test is used in this paper to test for conditional independence. From its logit model interpretation, it follows that the approach can be extended to include more variables (regressors). Let us consider a (2×2) contingency table for binary variables Y and Z:

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix},$$

which is a 2×2 matrix with positive elements that sum up to one. Its sample, i.e. frequency-based counterpart:

$$\hat{P} = \begin{pmatrix} \hat{p}_{00} & \hat{p}_{01} \\ \hat{p}_{10} & \hat{p}_{11} \end{pmatrix},$$

is computed from n independent observations on binary variables Y and Z. The vector of probabilities is denoted by p and given by:

$p \equiv (p_{00}, p_{10}, p_{01}, p_{11})'$.

Its frequency-based counterpart is denoted by $\hat{p}$.

A1.1 Log-linear probability model of $p_{k,l}$

A log-linear probability model of $p_{k,l}, \ k,l = 0,1$ is obtained from an additive representation of the log-probability in terms of the mean, marginal and cross effects as:

$$\log p_{kl} = \mu + \alpha_k + \beta_l + \gamma_{kl},$$

subject to the constraints :

$$\alpha_1 + \alpha_0 = 0, \beta_1 + \beta_0 = 0, \gamma_{1l} + \gamma_{0l} = 0, l = 0, 1, \gamma_{k1} + \gamma_{k0} = 0, k = 0, 1,$$

and the unit mass restriction [see Nerlove, Press (1973), (1986)].

It follows from simple computation that the expression of $\gamma = \gamma_{11}$ is:

$$\gamma = \log \left[p_{11} \, p_{00} / (p_{01} \, p_{10}) \right]. \tag{a.1}$$

Parameter γ is a measure of dependence between binary variables Y and Z. It can take any value in $(-\infty, +\infty)$. In particular, the measure of dependence is equal to zero, i.e. $\gamma = 0$, if and only if :

$$p_{11} \, p_{00} = p_{01} \, p_{10}$$

$$\Longleftrightarrow \quad \det \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = 0$$

$$\Longleftrightarrow \quad \text{the columns of } P \text{ are linearly dependent}$$

$$\Longleftrightarrow \quad p_{kl} = p_{k.} p_{.l} \;\; \forall k, l, \text{ where } p_{k.} = p_{k1} + p_{k0}, p_{.l} = p_{1l} + p_{0l}.$$

which is the condition of independence for variables Y and Z. Therefore, testing for $\gamma = 0$ is equivalent to testing for independence of Y and Z [see, Section A.3 below].

The measure of dependence γ is positive (resp. negative) if and only if the underlying binary variables Y, Z are positively (resp. negatively) correlated. Indeed, we have :

$$\text{cov}\,(Y, Z) = E(YZ) - EY\,EZ$$

$$= \quad p_{11} - p_{1.} p_{.1}, \text{ since Y, Z and YZ are Bernoulli variables,}$$

$$= \quad p_{11} - (p_{11} + p_{10})(p_{11} + p_{01})$$

$$= \quad p_{11}(1 - p_{10} - p_{01} - p_{11}) - p_{10} \, p_{01}$$

$$= \quad p_{11} \, p_{00} - p_{10} \, p_{01}.$$

Therefore, $\text{cov}(Y, Z) > 0$, iff $\gamma > 0$.

A1.2 Asymptotic distribution of $\hat{\gamma}$

Let $\hat{\gamma} = \log[\hat{p}_{11} \; \hat{p}_{00}/(\hat{p}_{01} \; \hat{p}_{10})]$ denote the sample counterpart of γ. For large n, the asymptotic distribution of the vector of probabilities $\hat{p}$ is :

$$\sqrt{n}(\hat{p} - p) \sim N(0, \; \text{diag}\,(p) - pp'), \qquad (a.2)$$

where diag p is the diagonal matrix with the components of p as diagonal elements.

The asymptotic distribution of $\hat{\gamma}$ is derived from the above formula by the δ-method. We get :

$$\sqrt{n}(\hat{\gamma} - \gamma) \sim \left[\frac{1}{p_{00}}, -\frac{1}{p_{01}}, -\frac{1}{p_{10}}, \frac{1}{p_{11}}\right] \sqrt{n}(\hat{p} - p).$$

It follows that the asymptotic distribution of $\hat{\gamma}$ is:

$$\sqrt{n}(\hat{\gamma} - \gamma) \sim N\left[0, \frac{1}{p_{00}} + \frac{1}{p_{01}} + \frac{1}{p_{10}} + \frac{1}{p_{11}}\right]. \qquad (a.3)$$

An asymptotically valid confidence interval for γ at level 95 % is:

$$\left(\hat{\gamma} \pm \frac{1.96}{\sqrt{n}} \sqrt{\frac{1}{\hat{p}_{00}} + \frac{1}{\hat{p}_{01}} + \frac{1}{\hat{p}_{10}} + \frac{1}{\hat{p}_{11}}}\right). \qquad (a.4)$$

The above asymptotic confidence interval is valid under the conditions required for the validity of the Gaussian approximation of a multinomial distribution. These are: n large and $n\hat{p}_{kl}$ larger than 10, say, for any k, l. Otherwise, Poisson approximations have to be used.

A1.3 Test of the independence hypothesis

If the validity condition for the asymptotic Gaussian approximation is satisfied, the null hypothesis of independence of binary variables Y and Z can be tested. Testing the null hypothesis of independence of Y and Z is equivalent to testing $H_0 : \gamma = 0$ against $H_A : \gamma \neq 0$. The asymptotically valid test of H_0 at level 5% is as follows. We reject H_0 if

$$\left| \frac{\hat{\gamma}}{(1/\sqrt{n})\left(\sqrt{\frac{1}{\hat{p}_{00}} + \frac{1}{\hat{p}_{01}} + \frac{1}{\hat{p}_{10}} + \frac{1}{\hat{p}_{11}}}\right)} \right| > 1.96$$

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and do not reject, otherwise. Equivalently, H_0 is not rejected if the asymptotically valid confidence interval given above contains zero, and it is rejected, otherwise.

Appendix 2

Conditional Interpretation of Dependence Measure and its Sign

Appendix 1 presented interpretations of γ from the joint distribution of Y and Z. This appendix provides alternative interpretations in terms of the conditional distribution of Y given Z (and Z given Y).

A2.1 The Dichotomous Logit Model

Let us consider a dichotomous logit model for variable Y with binary variable Z as the explanatory variable. This model is defined by:

$$P[Y = 1|Z] \equiv [1 + exp(\alpha + \gamma Z)]^{-1}.$$

Therefore we have:

$$log\frac{P[Y = 1|Z]}{P[Y = 0|Z]} = \alpha + \gamma Z.$$

It follows that:

$$log\frac{P[Y = 1|Z = 1]}{P[Y = 0|Z = 1]} = log\frac{P[Y = 1, Z = 1]}{P[Y = 0, Z = 1]} = \alpha + \gamma,$$

$$log\frac{P[Y = 1|Z = 0]}{P[Y = 0|Z = 0]} = log\frac{P[Y = 1, Z = 0]}{P[Y = 0, Z = 0]} = \alpha.$$

Then, by taking the difference, we get:

$$
\begin{aligned}
\gamma &= log\frac{P[Y = 1, Z = 1]}{P[Y = 0, Z = 1]} - log\frac{P[Y = 1, Z = 0]}{P[Y = 0, Z = 0]} \\
&= log\frac{p_{11}p_{00}}{p_{10}p_{01}}.
\end{aligned}
$$

This provides the interpretation of dependence measure γ as a regression coefficient in a logit model for Y with a binary explanatory variable Z.

A2.2 Conditional Probabilities

The sign of dependence measure γ is easy to interpret in terms of conditional probabilities. Let $logit\ x = log\frac{x}{1-x}$ denote the logit transformation. We get:

$$\gamma = logit P[Y = 1|Z = 1] - logit P[Y = 1|Z = 0].$$

As the logit transformation is an increasing transformation, it follows that:

$$\gamma > 0 \iff P[Y = 1|Z = 1] > P[Y = 1|Z = 0].$$

This equivalence is used for interpreting the empirical results in the text. Also, it is equivalent to:
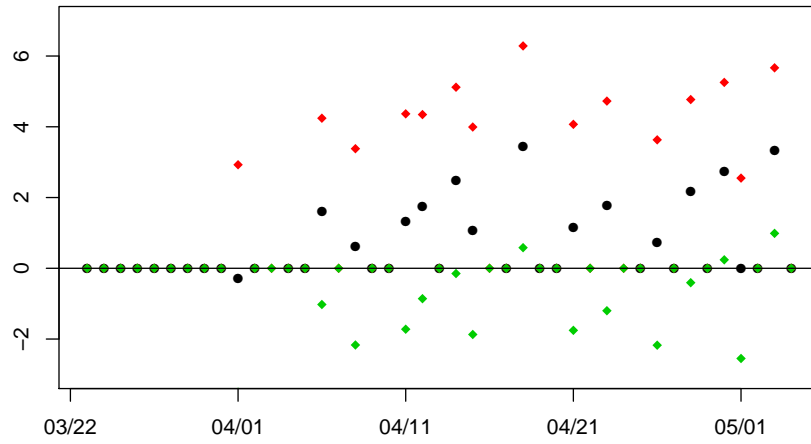
$$P[Y = 0|Z = 1] < P[Y = 0|Z = 0]$$

and to

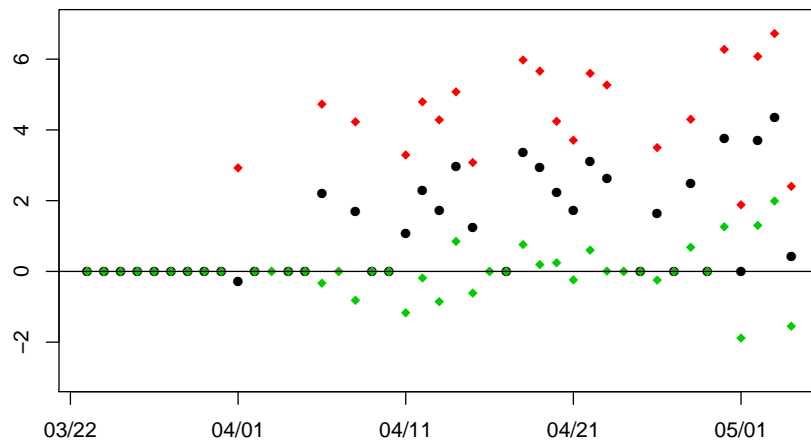$$P[Y^* = 1|Z = 1] < P[Y^* = 1|Z = 0],$$

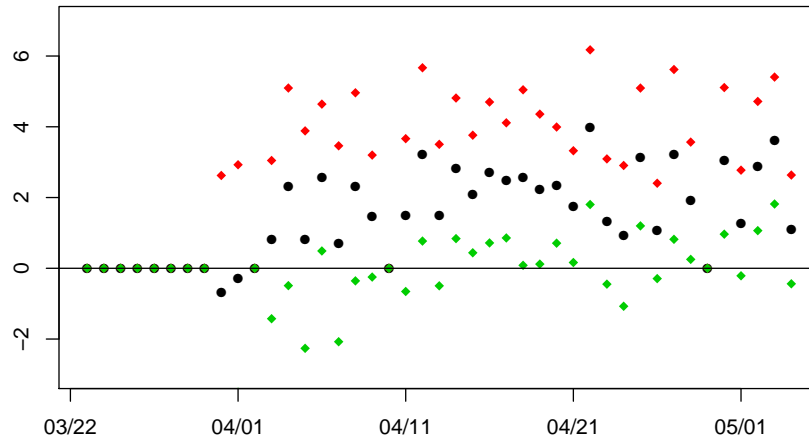where $Y^* = 1 - Y$,
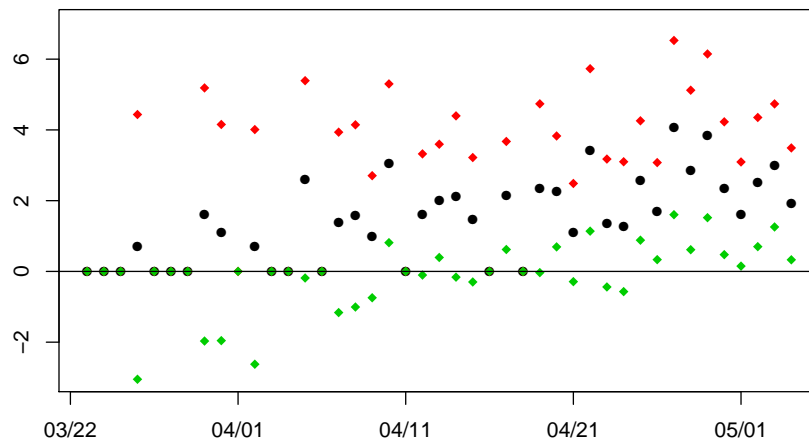
Appendix 3
Additional Figures

age > 50

age > 60

Figure 8a Conditional Dependence Measure, Z=Age, Diagnosed

age > 70

age > 80

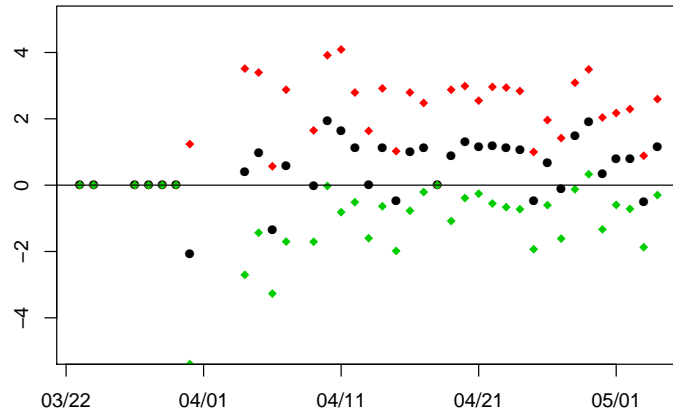Figure 8b Conditional Dependence Measure, Z=Age, Diagnosed Cont.

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Figure 9: Conditional Dependence Measure, Z=outside of Toronto and Ottawa, Diagnosed

Appendix 4

Medical and Economic Scores

In this paper, we developed a descriptive tool for scoring the patient assignment decisions, which is a reduced form of a structural model existing in the economic literature. That structural model considers the utility of a surviving patient based on medical and economic arguments.

Under the shortage and rationing of ICU beds, the patient can be selected on either pure medical, or on medical and economic arguments. This approach is used to manage and control health spending based on a cost/effecti veness (i.e. cost benefit) analysis [see e.g. Garber, Phelps (1997), Abelson (2003), Bergstrom (2006), Hall (2007), Zenios (2009)]. We do not discuss the trade-off between the medical and economic aspects, or the ethical rules for validating such techniques [see e.g. Bayles (1970)].

Let us consider patient i with characteristics Z_i. The Health Economics and Ethics literature determine the expected utility for the society of this individual if he/she is admitted in ICU, and if he/she is not. [12] In a simplified framework, these expected utilities can be decomposed as :

$$U_{1i} = p_1(Z_i)ERL_1(Z_i) \, Val(Z_i), \text{if in } ICU,$$

$$U_{0i} = p_0(Z_i)ERL_0(Z_i) \, Val(Z_i), \text{otherwise},$$

where $p_k(Z_i)$ is the probability to stay alive in, or out ICU, $ERL_k(Z_i)$ is the expected residual lifetime when alive, and $Val(Z_i)$ is the economic value per year of the alive patient.

Then the patient can be selected on the basis of its incremental expected utility for the society, that is :

$$\Delta U_i = U_{1i} - U_{0i} = [p_1(Z_i)ERL_1(Z_i) - p_0(Z_i)ERL_0(Z_i)]Val(Z_i).$$

In this expression the term between brackets is purely medical, based on the predictions about the results of the assignment and treatment. The

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[12]This utilitarian (or social Darwinian) approach is largely used in public economics for road safety [see e.g. Blaeij et al. (2003), Department of Transportation (2014)], for designing life insurance contracts, or for valuation of damages in the loss-of-chance doctrine law.

second term $Val(Z_i)$ is economic, known as the price of human life. In this model, it is implicitly assumed for expository purpose and in order to get a simple decomposition of the expected utility that this value per year does not depend on age, whereas the health status is changing with age [see Pliskin et al. (1980)]. This simplifying assumption is usually made in practice. This value is usually estimated indirectly from market data and willingness-to-pay to stay alive in good health [see e.g. Viscusi, Aldi (2003)].

Then individual 1 will be selected among the two individuals 1,2, if :

$$\Delta U_1 > \Delta U_2.$$

The first component (the medical one) can be estimated by observing the effects of treatment in the past. For COVID 19, it is known that the age is a significant factor [13] for the survival probability p_1, of course also for the ERL, that diminishes with age. The second component is the basis of philosophical and ethical discussions, that are out of the scope of this appendix [see e.g. Bayles (1978), or Baker et al. (2008) for discussions].

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[13]Note that the knowledge of the conditional measure of association brings no information on the measure of association between Y and D.

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