Estimation of the static vertical subgrade reaction modulus k_s from CPT

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ABSTRACT: A methodology for the estimation of the static vertical subgrade reaction modulus (k_s) for cohesionless soils from the Cone Penetration Test (CPT) has been introduced in 2013 (Barounis et al.) and 2015 (Barounis and McMahon) and has recently been integrated (Barounis and Philpot, 2017). In this paper, the conclusions from the early two papers are utilized for developing an integrated methodology based on the correlation between q_c and N_{60} (Robertson, 2012). The fundamental concepts and the theory of the proposed methodology are presented with a step-by-step procedure in this paper. The methodology returns one value termed K_F, which is the equivalent spring stiffness for any foundation depth and shape under consideration. The methodology produces values that are as conservative as the traditional SPT approach proposed by Scott (Scott, 1981). The methodology is applied on numerous sandy sites in New Zealand for different foundation typologies.

1 INTRODUCTION

As an initial step for facilitating the earthquake-resistant design of a building and its foundations, the static spring stiffness, or static vertical subgrade reaction modulus, on the surface of an assumed homogeneous half-space is typically evaluated for the given site. Then, by applying numerous dynamic modification factors to the static spring stiffness (ASCE 41-13), the dynamic spring stiffness can be evaluated. Dynamic modification factors are applied to account for the frequency of the excitation force, the embedment of the foundation and the foundation shape. The most fundamental step in this process for the structural engineer, is to determine if the foundation system under analysis is rigid or flexible. Many references are available for facilitating this step (refer to ASCE 41-13, 2014 or ACI 336). Thus, the estimation of the static spring stiffness is an important step in undertaking both static and earthquake resistant design of foundations. This paper is focused only on the static spring stiffness of flexible foundation systems.

The static vertical subgrade reaction modulus k_s is a conceptual relationship, which is defined as the soil pressure exerted σ divided by the deflection δ (Bowles, 1997).

$$k_s = \frac{\sigma}{\delta} \qquad (\mathrm{MN/m^3}) \qquad (1)$$

The static vertical subgrade reaction modulus is not an actual engineering property of the soil, such as Poisson's ratio, as it varies with the width and shape of the foundation (Terzaghi, 1955). The principle underlying the definition of k_s is the resistance a soil layer provides as some deflection is imposed on it due to the applied stress, analogous to a spring shortening at some imposed load. In structural engineering applications, k_s is used to model the soil stiffness in the vertical plane when soil-foundation-structure interaction considerations are included in the structural analysis. Typically, as best practice suggests, structural engineers adopt k_s values recommended by a geotechnical engineer. The structural engineer further tests the sensitivity of the model for k_s values ranging between 0.5 k_s and 2.0 k_s (ASCE 41-13, 2014). The geotechnical engineer needs to assess the k_s range for the particular situation. These values are also to be accompanied by ultimate foundation capacity estimations.

A methodology for the estimation of k_s for sands from the Cone Penetration Test (CPT), as an alternative to the conventional estimation relying on SPT as proposed by Scott (Scott, 1981) for a 300mm plate, was proposed by Barounis et al. (2013). From recent research on the applicability of the proposed CPT methodology (Barounis and Armaos, 2016), it was demonstrated that the produced k_s values are stiffer than the values produced by Scott. The stiffer springs from CPT are conservative for the seismic response of the structure, while the softer springs from SPT are conservative for foundation deformation and their effects to the superstructure.

This paper presents an integrated methodology for the estimation of k_s values for flexible shallow foundations on cohesionless soils based on the correlation between q_c and N_{60} (Robertson, 2012). The methodology produces similar values to the SPT method proposed by Scott for the k_{300} of a 300 mm plate. The final value, corrected for foundation shape, K_F is similar to the value produced by using the conventional SPT-based method by Scott.

For a detailed explanation of the background theory to the methodology, please refer to previous papers from Barounis et al. (2013, 2015, 2016 and 2017) on this topic. The additional theoretical basis for the method is explained in the next paragraph.

2 INTEGRATED METHODOLOGY FOR ESTIMATING THE MODULUS OF SUBGRADE REACTION K_S

2.1 *Fundamental assumptions, advantages and theoretical basis of the methodology*

The fundamental assumptions and theoretical basis for the proposed methodology and its advantages are the following:

- The methodology is applicable to cohesionless soils, only when tested with a CPT that measures penetration resistance at 10 or 20 mm increments with a 35.7 mm diameter cone.
- The theory of springs in series is assumed, modified to consider the configuration of soil layers. This means that the equivalent spring stiffness K_{eq} can be estimated by using the proposed methodology.
- The range of SPT N_{60} values is limited to between 0 and 50 blows. No extrapolation over 50 blows shall be adopted in any case. Thus, an SPT N_{60} of 50 is considered to be effective refusal.
- This upper bound value of SPT produces a maximum $K_{SPT(0.3)}$ (spring for a 300 mm plate based on Scott) value of 90 MN/m^3. Thus, any k_s value produced using this methodology for the actual foundation cannot exceed 90 MN/m^3.
- If values larger than 90 MN/m^3 need to be adopted, either for the 300 mm plate, or for the actual foundation, then actual plate load tests and further site investigations will need to be performed to prove that the subgrade modulus exceeds this value.
- The corresponding q_c (cone tip resistance in MPa) value for any soil with I_c (soil behavior type index) between 1.00 and 2.60 is related to the SPT N_{60} according to the correlation by Robertson, as shown in Figure 1. This figure shows that for increasing I_c values (increasing fines content) at effective SPT refusal, the corresponding q_c value reduces from 35.4 MPa for I_c = 1.00 to 12.5 MPa for I_c = 2.60. As the fines content increases and for a given N_{60} value, the cone resistance and the spring stiffness reduces, along with the soil stiffness E_s (refer to Figure 1).
- The methodology is applicable for circular, square, continuous or rectangular shallow foundations, founded at any shallow depth in the ground including at the ground surface. The equivalent modulus K_{eq} for any foundation shape is estimated based on the simplified formulae provided by Poulos and Davis (Xiao, 2015) that computes the vertical stress distribution beneath the centre of the foundation.
- By using the Poulos and Davis (Xiao, 2015) formulae, essentially a weighting factor is applied to every 10 mm or 20 mm long spring. Hence, the soils nearer the foundation become more critical to the overall response than the soils substantially deeper, or outside the pressure bulb of the foundation.
- The methodology considers an influence depth under any foundation configuration to be the depth at which the vertical stress increase from the foundation becomes equal to 20% of the vertical effective stress (20% rule).
- The methodology is sensitive to stiffness inversions, i.e. dense soils overlying loose soils. The methodology is also sensitive when denser soils are present at some depth from the foundation (within 20% influence depth). In general, the methodology produces good results, even for highly stratified soils or for sandwiched layers of contrasting stiffness.

Figure 1. Relationship between q_c and N_{60} for I_c between 1.00 and 2.60.

The methodology is presented in Figure 2. Detailed explanations for each step of the methodology are given in the subsequent paragraphs.

Step #	Equation	Units	Notes
1	For 10mm increment: $K_{CPT}=100q_c$ For 20mm increment: $K_{CPT}=50q_c$	MN/m³	CPT spring stiffness
2	$K_{CPT\,(0.3)}=0.119K_{CPT}$	MN/m³	Conversion to 300mm plate spring stiffness
3	$K_{CPT\,(SPT0.3)}= K_{CPT(0.3)}$ / CF where for 10mm increment: $CF=0.668(10^{(1.127\text{-}0.282I_c)})$ and for 20mm increment: $CF=0.334(10^{(1.127\text{-}0.282I_c)})$	MN/m³	Conversion to a similar SPT spring stiffness for a 300mm plate as per Scott's correlation, $K_{SPT\,(0.3)}=1.8N_{60}$
4	$K_{eq20\%}=\frac{\sum_{i=1}^{n} Iz_iK_i}{\sum_{i=1}^{n} Iz_i}$	MN/m³	Equivalent spring stiffness for a 300mm plate considering an influence depth as per the 20% rule that corresponds to the foundation geometry under analysis
5	$K_F=K_{eq}$ x $(m+0.5)/(1.5m)$ where m = L/B	MN/m³	Shape correction for the given foundation geometry under analysis

Figure 2. Excerpt from Barounis et al. (2017) paper summarising the five key steps of the methodology.

2.1.1 *1st Step: Estimation of K_{CPT} stiffness*

In this first step, the spring stiffness of the 10 mm long soil element is estimated. As per equation 1, the spring stiffness is the cone resistance q_c divided by the displacement. For a CPT with 35.7 mm diameter and a 10 mm incremental penetration:

$$K_{CPT} = 100q_c \tag{2}$$

This conversion is applied for every q_c measurement until the final CPT depth.

2.1.2 *2nd Step: Estimation of the spring $K_{CPT(0.3)}$ for a 300 mm plate by conversion from Step 1*

In this step, a conversion takes place from the CPT spring to the equivalent 300 mm diameter plate spring. This is undertaken according to the formula proposed by ACI (ACI 336, 2002) and Bowles (Bowles, 1997), which relies on principles earlier presented by Terzaghi (1955):

$$K_{CPT(0.3)} = K_{CPT}\frac{D_{CPT}}{300} = K_{CPT}\frac{35.7}{300} = 0.119K_{CPT} \tag{3}$$

Where D_{CPT} is the diameter of the CPT cone in mm. This is consistent with the subgrade reaction modulus theory as a greater loaded area produces lower subgrade reaction values (Terzaghi, 1955 and Bowles, 1997).

2.1.3 *3rd Step: Conversion to a similar SPT spring stiffness value depending on I_c by means of CF*

In this step, a reduction of the spring stiffness is undertaken by using a conversion factor CF. This reduction results in the CPT spring value coinciding with the value estimated from Scott's method.

Robertson (2012) published a correlation between q_c and SPT N_{60} with the soil behavior type index I_c (Robertson, 2015). The relationship is the following:

$$\frac{\left(\frac{q_t}{p_a}\right)}{N_{60}} = 10^{(1.127-0.282I_c)} \tag{4}$$

where $q_t = q_c$ for cohesionless soils and p_a = atmospheric pressure.

By substituting I_c values between 1.00 and 2.60 that correspond to the range of cohesionless soils, p_a of 101 kPa, and SPT N_{60} from 0 to 50 as per the assumptions in section 2.1, the relationship is depicted in Figure 1.

By dividing the $K_{CPT(0.3)}$ with $K_{SPT(0.3)}$, the stiffness ratio of the CPT spring for the 300 mm plate is compared to the SPT spring for the same 300 mm plate. This ratio can be defined as a Conversion Factor (CF):

$$CF = \frac{K_{CPT(0.3)}}{K_{SPT(0.3)}} = \frac{0.119(100q_c)}{1.8N_{60}} = \frac{11.9q_c}{1.8N_{60}} = 6.61\frac{q_c}{N_{60}} \tag{5}$$

And by solving equation (4) for q_c and substituting in (5) we have:

$$CF = 6.61\left(0.101\left(10^{(1.127-0.282I_c)}\right)\right) \tag{6}$$

$$CF = 0.668\left(10^{(1.127-0.282I_c)}\right) \tag{7}$$

Similarly, for a 20 mm incremental penetration:

$$CF = 0.334\left(10^{(1.127-0.282I_c)}\right) \tag{8}$$

This means that CF is dependent on I_c and the incremental testing depth. Applying the CF to the $K_{CPT(0.3)}$ returns a similar spring stiffness for a 300 mm plate diameter to the SPT-based method from Scott (1981). By using the symbol $K_{CPT(SPT0.3)}$ for the SPT spring stiffness produced by the CPT approach, the equation becomes:

$$K_{CPT(SPT0.3)} = \frac{K_{CPT(0.3)}}{CF} \tag{9}$$

The value of CF can be taken from equations 7 or 8 depending on the CPT incremental testing depth.

2.1.4 *4th Step: Equivalent spring stiffness K_{eq} for a 300 mm plate*

The general theory of springs suggests that the equivalent spring stiffness K_{eq} of an infinite chain of springs is given by the formula:

$$K_{eq} = \frac{k_1k_2\ldots k_n}{k_1+k_2+\cdots+k_n} = \frac{\prod_{i=1}^{n}k_i}{\sum_{i=1}^{n}k_i} \tag{10}$$

It must be recognized that equation 10 is insensitive to the configuration of soil layers. In other words, the equivalent spring stiffness would return the same value regardless of the sequence with which the soil layers are configured. The assumptions underlying this formula maybe appropriate to be used in structural mechanics; however, it does not accurately capture the soil and foundation behavior, especially when looser soils are located near the foundation level, or when very dense soils are located at some depth below the foundation, but within the 20% influence bulb. Also, from a numerical perspective, it is impossible for common software to compute the product of 100 individual springs per 1m of CPT. For these reasons, it is proposed to model and capture any layer configuration by using the Boussinesq theory in accordance with the simplified formulae proposed by Poulos and Davis (1974) as presented in Xiao (Xiao, 2015). In essence, the Boussinesq methodology is used to apply a weighting factor to every $K_{CPT(SPT0.3)}$ spring within the influence bulb of the foundation. As depth increases, the associated spring stiffness value becomes less significant for the foundation behavior. Thus, dense deeper soils may not provide substantial stiffness to the foundation, or loose deeper soils may soften the spring substantially. The weighting factor is the well-known influence factor I_z, which takes the value of 1.0 at the foundation depth and diminishes to values that tend to zero with increasing depth.

The proposed formula has the following form:

$$K_{eq20\%} = \frac{\sum_{i=1}^{n} Iz_i K_i}{\sum_{i=1}^{n} Iz_i} \tag{11}$$

Where I_{zi} is the influence factor that corresponds to a spring stiffness K_i at depth z_i. Equations 12-15 show influence factors for circular, square, continuous and rectangular foundations, respectively (Xiao, 2015):

$$\Delta\sigma z = (q - \sigma_0') \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z}\right)^2}\right)^{1.50}\right] \tag{12}$$

$$\Delta\sigma z = (q - \sigma_0') \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z}\right)^2}\right)^{1.76}\right] \tag{13}$$

$$\Delta\sigma z = (q - \sigma_0') \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z}\right)^2}\right)^{2.60}\right] \tag{14}$$

$$\Delta\sigma z = (q - \sigma_0') \left[1 - \left(\frac{1}{1+\left(\frac{B}{2z}\right)^{\left(1.38+\frac{0.62B}{L}\right)}}\right)^{\left(2.60-\frac{0.84B}{L}\right)}\right] \tag{15}$$

Where B, L and z are respectively the breadth, length and depth below the foundation in metres, $(q-\sigma'_0)$ is the net applied foundation pressure in kPa, q is the gross exerted foundation pressure in kPa and σ'_0 is the effective stress at the foundation depth in kPa. Equation 11 is applied to the depth z at which the stress increase from the net applied pressure equals 20% of the vertical effective stress.

2.1.5 *5th Step: Correction for shape of foundation*

In this step, the final foundation spring stiffness value K_F is estimated. From the four previous steps, a similar spring stiffness value to the SPT approach has been established. The application of a correction factor for the shape of the foundation with length L and breadth B will also result in a similar corrected spring as for the SPT approach. The shape correction factors are presented in Table 1. K_F is determined from the equation:

$$K_F = S_F K_{eq20\%} \tag{16}$$

Table 1: Shape correction factors for various foundation shapes

Foundation Shape	Shape Correction
Circular	1.0
Continuous	(*m*+0.5)/(1.5*m*), *m*=L/B, tends to 0.67 when L/B≥5
Square	1.0
Rectangular	(*m*+0.5)/(1.5*m*), *m*=L/B

3 EXAMPLE CALCULATIONS: FOUR SANDY SITES IN CHRISTCHURCH

The proposed methodology has been applied on four different sandy sites in Christchurch, New Zealand. The foundation shape, dimensions, applied pressures and other key information are presented in Figure 3.

Site	Foundation Shape	Length (m)	Breadth (m)	Depth (m)	Applied Pressure (kPa)	GWT Depth (m)	Influence Depth {20% Rule} (m)
1	Rectangular	10	5	0	50	0.6	7.56
2	Continuous	10	2	0	50	1.0	3.81
3	Square	5	5	0	50	1.2	5.96
4	Circular	N/A	5	0	50	1.0	5.76

Figure 3. Excerpt from Barounis et al. (2017) paper summarising foundation typologies for the four sites.

The summary of results from the proposed methodology are presented in Figures 4-6. Note that the $K_{CPT(SPT0.3)}$ values do not exceed 90 MN/m^3.

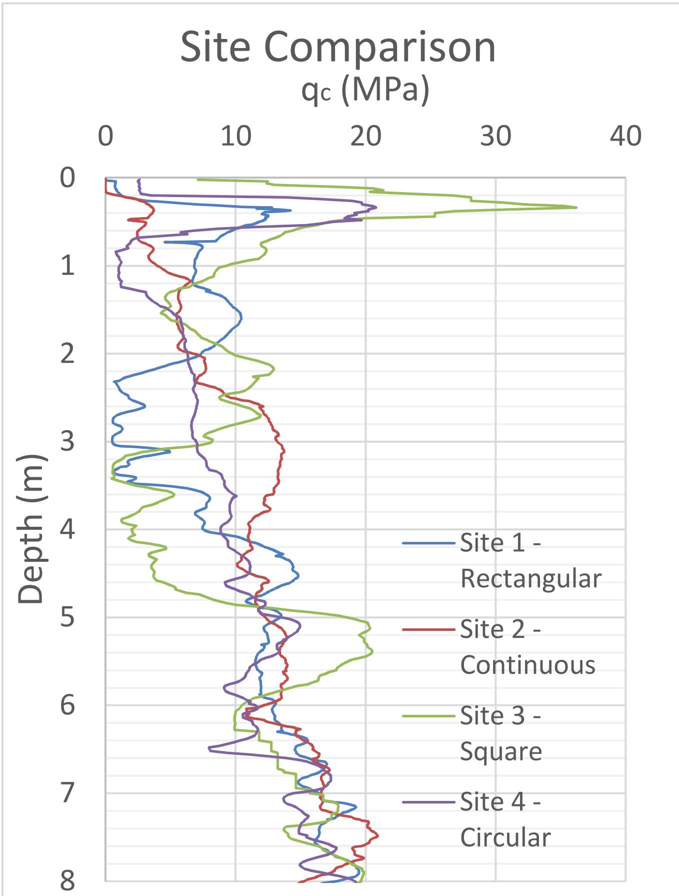

Figure 4. Comparison of q_c across all sites

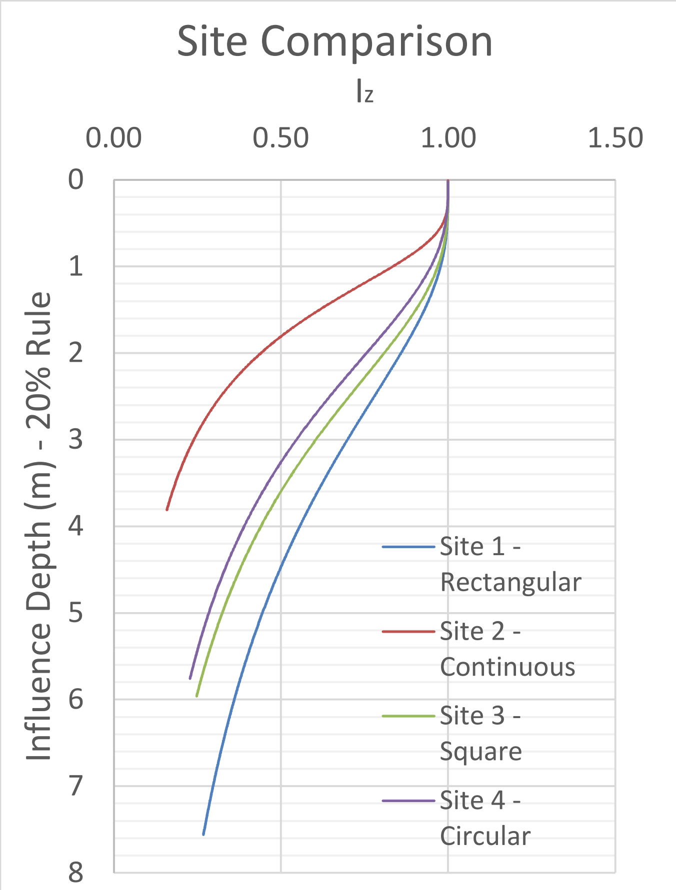

Figure 5. Comparison of influence depths (20% rule) across all sites.

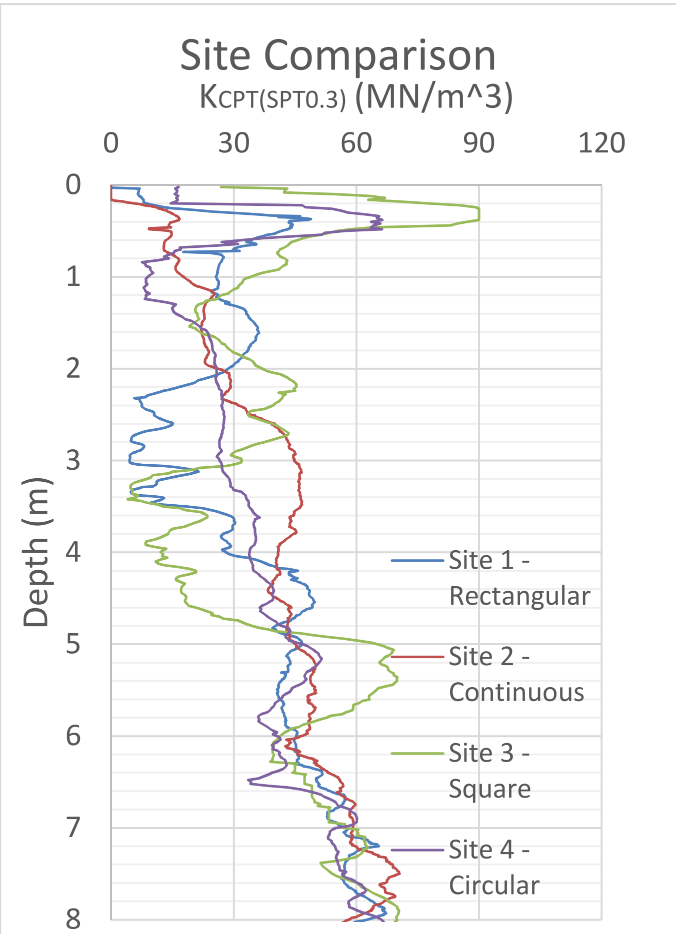

Figure 6. Comparison of $K_{CPT(SPT0.3)}$ values across all sites.

The results of the proposed methodology for the four sites and their agreement with Scott's approach are presented in Tables 2 and 3. The bar above the N_{60} and q_c values in Table 3 indicates that these are depth weighted values by using the Poulos and Davis (1974) influence factors. These values are equivalent N_{60} and q_c values for the influenced soil layers as a whole. These are plotted in Figure 7 indicating the values are consistent with the predominant soil conditions encountered at all four sites.

Table 2: Foundation shape, equivalent spring stiffness and foundation spring stiffness

Site #	Foundation Shape	$K_{eq20\%}$ (MN/m^3)	K_F (MN/m^3)
1	Rectangular	30.07	25.06
2	Continuous	21.14	15.51
3	Square	37.67	37.67
4	Circular	28.86	28.86

Table 3: Depth weighted N_{60} and q_c values and spring stiffness produced by Scott (1981) methodology

Site #	$\overline{N_{60}}$	$\overline{q_c}$	$\frac{\overline{q_c}}{\overline{N_{60}}}$	$K_{eq}20\%$ (Scott)	K_F (Scott)
1	17	8.13	0.49	30.07	25.06
2	12	5.37	0.46	21.14	15.51
3	21	10.56	0.50	37.67	37.67
4	16	7.45	0.47	28.86	28.86

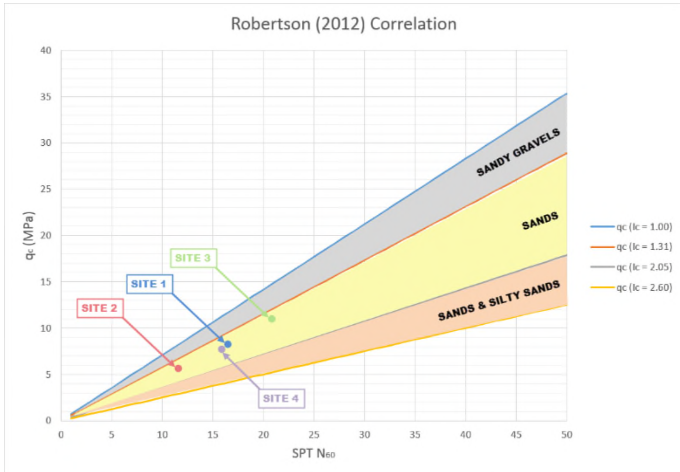

Figure 7. Relationship between q_c and N_{60} for I_c between 1.00 and 2.60 with depth weighted values plotted for all four sites.

4 CONCLUSION

An integrated methodology for the estimation of static spring stiffness from CPT has been presented for flexible shallow foundations on cohesionless soils based on the relationship between q_c and N_{60} (Robertson, 2012). The methodology can be applied to estimate the equivalent spring stiffness $K_{eq20\%}$ for a flexible shallow foundation and then subsequently estimate the K_F value for the actual foundation size and shape. The methodology can only be applied for SPT values ranging from 0 to 50, corresponding to a

q_c that depends on I_c. For facilitating a Winkler foundation type of analysis applicable to flexible foundation systems, ultimate foundation capacity estimations also need to be undertaken by a geotechnical engineer as per well-established available methodologies. The methodology is expected to return similar values, in alignment with the SPT approach as per Scott (1981). However, more research, that includes sensitivity analysis for the proposed methodology, needs to be undertaken to verify the methodology returns reliable spring values for all possible soil configurations. For assessing K_F, one should not rely on one methodology. A number of methods should be applied, of which this could be one.

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